On the state agreement problem for multiple unicycles

Dimos V. Dimarogonas and Kostas J. Kyriakopoulos

***Abstract*— In this contribution, a feedback control strategy that drives a system of multiple nonholonomic kinematic unicycles to agreement is introduced. Each agent is assigned with a specific subset of the rest of the team, called the agent's communication set, that includes the agents with which it can communicate in order to achieve the desired objective. The proposed nonholonomic control law is discontinuous and time-invariant and tools from nonsmooth stability theory and graph theory are used to check the stability of the overall system. Similarly to the linear case, the convergence of the multi-agent system relies on the connectivity of the communication graph that represents the inter-agent communication topology.**

I. INTRODUCTION

Multi-agent Navigation is a field that has recently gained increasing attention both in the robotics and the control communities, due to the need for autonomous control of more than one mobile robotic agents in the same workspace. While most efforts in the past had focused on centralized planning, specific real-world applications have lead researchers throughout the globe to turn their attention to decentralized concepts. The motivation for this work comes from many application domains one of the most important of which is the field of micro robotics ([20],[11]), where a team of a potentially large number of autonomous micro robots must cooperate in the sub micron level.

Among the various specifications that the control design aims to impose on the multi-agent team, convergence of the multi-agent system to a desired formation is a design objective that has been pursued extensively in the last few years. The main feature of formation control is the cooperative nature of the equilibria of the system. Agents must converge to a desired configuration encoded by the inter-agent relative positions. Many feedback control schemes that achieve formation stabilization to a desire formation in a distributed manner have been proposed in literature, see for example [28],[17],[15],[9],[7] for some recent results. Of particular interest is also the so-called agreement or rendezvous problem, in which agents must converge to the same point in the state space ([23],[12], [24],[5],[13], [21],[16]).

There have been many approaches to the state agreement problem under both the vehicle motion modelling and the control design perspective. In most cases, single integrator (holonomic) models of motion are taken into account, while the information exchange topology has been considered both static and dynamic, as well as bidirectional or unidirectional. A recent review of the various approaches of the state agreement problem for linear models of motion is [25]. The agreement problem for general nonlinear models has been considered in [18].

In this contribution, a feedback control strategy that drives a system of multiple nonholonomic unicycles to agreement is introduced. The problem treated in this work is similar to the problem solved in [17]. In that reference, the authors use a time varying periodic smooth controller, inspired by the work in [29], to solve the agreement problem. Inspired by our previous work on decentralized navigation of multiple nonholonomic agents [19],[6],[27] we propose in this paper a distributed nonholonomic feedback control strategy that is discontinuous and time invariant. These type of controllers have in general better convergence properties than time-varying ones. An experimental comparison between these two types of nonholonomic controllers that supports our preference to time-invariant strategies has appeared in [14]. In that reference, it was deduced that time varying controllers were too slow and oscillatory for most practical situations. On the other hand, time-invariant controllers achieved a significantly better performance. Clearly, this is the best we can hope for regarding the nonholonomic feedback strategy (either smooth and time-varying or nonsmooth and time-invariant), as it is a well known fact that that nonholonomic systems do not satisfy the Brocket's necessary smooth feedback stabilization condition ([2]). Another distinction of this work is that we considered merely bidirectional communication topology, whereas directed graphs are taken into account in [17]. The extension of the proposed framework to directed graphs is a topic of ongoing research. The stability of the proposed scheme is analyzed using tools from algebraic graph theory and nonsmooth stability theory.

The rest of the paper is organized as follows: section II describes the system and the problem that is treated in this paper. Assumptions regarding the communication topology between the agents are presented and modelled in terms of an undirected graph. Section III begins with some background on algebraic graph theory and nonsmooth analysis that is used in the sequel and proceeds with the introduction of the distributed nonsmooth time invariant feedback control strategy that drives the multi-agent team to a common configuration in the state space as well as the corresponding stability analysis. Some computer simulation results are included in section IV while section V summarizes the results of this paper and indicates current research efforts.

II. SYSTEM AND PROBLEM DEFINITION

Consider a system of N nonholonomic point agents operating in the same workspace $W \subset \mathbb{R}^2$. Let $q_i = [x_i, y_i]^T \in$

The authors are with the Control Systems Lab, Department of Mechanical Engineering, National Technical University of Athens, 9 Heroon Polytechniou Street, Zografou 15780, Greece ddimar,kkyria@mail.ntua.gr

$\mathbb{R}^2$ denote the position of agent i (see figure 1). The configuration space is spanned by $q = [q_1, \ldots, q_N]^T$. Each of the N mobile agents has a specific orientation θ_i with respect to the global coordinate frame. The orientation vector of the agents is represented by $\theta = [\theta_1 \ldots \theta_N]$. The configuration of each agent is represented by $p_i = \begin{bmatrix} q_i & \theta_i \end{bmatrix} \in \mathbb{R}^2 \times (-\pi, \pi]$. Agent motion is described by the following nonholonomic kinematics:

$$\begin{aligned} \dot{x}_i &= u_i \cos\theta_i \\ \dot{y}_i &= u_i \sin\theta_i \quad , i \in \mathcal{N} = [1, \ldots, N] \\ \dot{\theta}_i &= \omega_i \end{aligned} \tag{1}$$

where u_i, ω_i denote the translational and rotational velocity of agent i, respectively. These are considered as the control inputs of the multi-agent system.

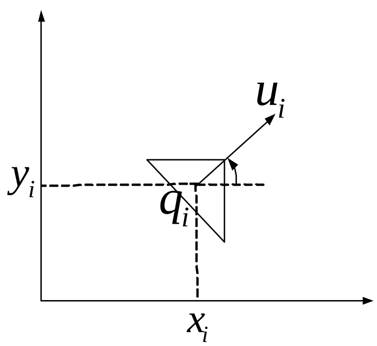

Fig. 1. Nonholonomic agent

The design objective in this paper is to construct feedback controllers that lead the multi-agent system to agreement, i.e. all agents should converge to a common point in the state space. Each agent is assigned with a specific subset N_i of the rest of the team, called agent i's *communication set*, that includes the agents with which it can communicate in order to achieve the desired objective. Following the literature on cooperative control [22],[28], inter-agent communication can be encoded in terms of a *communication graph*:

Definition 1: The *communication graph* $G = \{V, E\}$ is an undirected graph that consists of a set of vertices $V = \{1, ..., N\}$ indexed by the team members, (ii) a set of edges, $E = \{(i,j) \in V \times V | i \in N_j\}$ containing pairs of nodes that represent inter-agent communication specifications.

Each agent has only knowledge of the state of agents that belong to its communication set at each time instant. This fact highlights the distributed nature of the approach. In this paper, we assume that the communication graph is static, i.e. the neighboring set N_i of agent i is constant. The case of switching interconnection topology is a topic of ongoing research.

We also assume that the communication graph is undirected, in the sense that in the sense that

$$i \in N_j \Leftrightarrow j \in N_i, \forall i, j \in \mathcal{N}, i \neq j$$

It is obvious that $(i,j) \in E$ iff $i \in N_j \Leftrightarrow j \in N_i$.

As an example, the next figure represents the communication graph of a team of seven agents with corresponding communication sets:

$$N_1 = \{2, 6\}, N_2 = \{1, 5\}, N_3 = \{6, 7\}$$

$$N_4 = \{5\}, N_5 = \{2, 4, 7\}, N_6 = \{1, 3, 7\}, N_7 = \{3, 5, 6\}$$

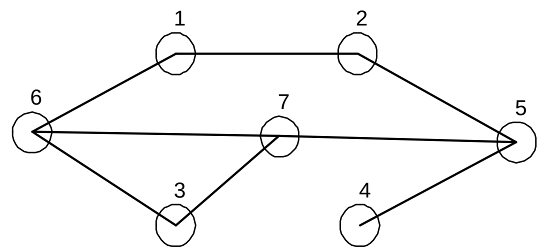

Fig. 2. The communication graph of a seven-agent team

Hence, the problem treated in this paper can be stated as follows: "*under the preceding assumptions, derive a set of distributed control laws that drive the team of agents from any initial configuration to a common configuration in the state space*".

III. Control Strategy and Stability Analysis

In this section, the proposed feedback control strategy and the corresponding stability analysis of the system are presented. The mathematical tools required for this analysis are discussed in the next two subsections.

A. Tools from Algebraic Graph Theory

In this subsection we review some tools from algebraic graph theory that we shall use in the stability analysis of the next sections. The following can be found in any standard textbook on algebraic graph theory(e.g. [1],[10]).

For an undirected graph G with n vertices the *adjacency matrix* $A = A(G) = (a_{ij})$ is the $n \times n$ matrix given by

$$a_{ij} = \begin{cases} 1, \text{if } (i,j) \in E \\ 0, \text{otherwise} \end{cases}$$

If there is an edge connecting two vertices i, j, i.e. $(i,j) \in E$, then i, j are called *adjacent*. A *path* of length r from a vertex i to a vertex j is a sequence of $r+1$ distinct vertices starting with i and ending with j such that consecutive vertices are adjacent. If there is a path between any two vertices of the graph G, then G is called *connected* (otherwise it is called *disconnected*). The *degree* d_i of vertex i is defined as the number of its neighboring vertices, i.e.

$$d_i = \{\# j : (i,j) \in E\}$$

Let Δ be the $n \times n$ diagonal matrix of d_i's. The (combinatorial) *Laplacian* of G is the symmetric positive semidefinite matrix $\mathcal{L} = \Delta - A$. The Laplacian captures many interesting topological properties of the graph. Of particular interest in our case is the fact that for a connected graph, the

Laplacian has a single zero eigenvalue and the corresponding eigenvector is the vector of ones, $\overrightarrow{\mathbf{1}}$.

As an example, the Laplacian matrix of the communication graph in figure 2 is given by:

$$\mathcal{L} = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & -1 & 0 \\ -1 & 2 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 3 & 0 & -1 \\ -1 & 0 & -1 & 0 & 0 & 3 & -1 \\ 0 & 0 & -1 & 0 & -1 & -1 & 3 \end{bmatrix}$$

B. Tools from Nonsmooth Analysis

In this subsection, we review some elements from nonsmooth analysis and Lyapunov theory for nonsmooth systems that we use in the stability analysis of the next sections.

For a differential equation with discontinuous right-hand sode we have the following definition:

Definition 2: [8] In the case when the state-space is finite dimensional, the vector function $x(.)$ is called a *Filippov solution* of $\dot{x} = f(x)$ if it is absolutely continuous and $\dot{x} \in K[f](x)$ almost everywhere where

$$K[f](x) \equiv \overline{co}\{\lim_{x_i \to x} f(x_i) | x_i \notin N\}$$

where N is a set of measure zero.

Lyapunov stability theorems have been extended for nonsmooth systems in [26],[3]. The following chain rule provides a calculus for the time derivative of the energy function in the nonsmooth case:

Theorem 1: [26] Let x be a Filippov solution to $\dot{x} = f(x)$ on an interval containing t and $V : \mathbb{R}^n \to \mathbb{R}$ be a Lipschitz and regular function. Then $V(x(t))$ is absolutely continuous, $(d/dt)V(x(t))$ exists almost everywhere and

$$\frac{d}{dt}V(x(t)) \in^{a.e.} \dot{\tilde{V}}(x) := \bigcap_{\xi \in \partial V(x(t))} \xi^T K[f](x(t))$$

where "a.e." stands for "almost everywhere".

In this theorem, ∂V is *Clarke's generalized gradient*. The definition of the generalized gradient and of the *regularity* of a function can be found in [4]. In the case we encounter in this paper, the candidate Lyapunov function function V we use is smooth and hence regular, while its generalized gradient is a singleton which is equal to its usual gradient everywhere in the state space: $\partial V(x) = \{\nabla V(x)\} \forall x$.

We shall use the following nonsmooth version of LaSalle's invariance principle to prove the convergence of the prescribed system:

Theorem 2: [26] Let Ω be a compact set such that every Filippov solution to the autonomous system $\dot{x} = f(x), x(0) = x(t_0)$ starting in Ω is unique and remains in Ω for all $t \geq t_0$. Let $V : \Omega \to \mathbb{R}$ be a time independent regular function such that $v \leq 0 \forall v \in \dot{\tilde{V}}$(if $\dot{\tilde{V}}$ is the empty set then this is trivially satisfied). Define $S = \{x \in \Omega | 0 \in \dot{\tilde{V}}\}$. Then every trajectory in Ω converges to the largest invariant set,M, in the closure of S.

C. Proposed control design

Denote the stack vector $q = [x, y]^T$ into the coefficients that correspond to the x, y directions of the agents respectively. We also use the function

$$\text{sgn}(x) = \begin{cases} 1, x \geq 0 \\ -1, x < 0 \end{cases}$$

The function $\arctan 2(x, y)$ that is also used in the sequel is the same as the arc tangent of the two variables x and y with the distinction that the signs of both arguments are used to determine the quadrant of the result. We also use $\arctan 2(0, 0) = 0$. Finally, the notation $(a)_i$ is used to denote the i-th element of a vector a.

Convergence of the agents to a common configuration is guaranteed by the following theorem:

Theorem 3: Assume that the communication graph is connected. Then the discontinuous time-invariant feedback control strategy:

$$u_i = -\text{sgn}\{\gamma_{xi}\cos\theta_i + \gamma_{yi}\sin\theta_i\} \cdot \left(\gamma_{xi}^2 + \gamma_{yi}^2\right)^{1/2} \quad (2)$$

$$\omega_i = -(\theta_i - \theta_{nh_i}) \quad (3)$$

where

$$\gamma_{xi} = (\mathcal{L}x)_i, \gamma_{yi} = (\mathcal{L}y)_i$$

and the "nonholonomic angle"

$$\theta_{nh_i} = \arctan 2(\gamma_{yi}, \gamma_{xi})$$

and where $\mathcal{L}$ denotes the Laplacian matrix of the communication graph, drives the agents to a common configuration in the state space.

Proof: We use the smooth positive semidefinite function

$$V = \sum_i \gamma_i$$

as a candidate Lyapunov function, where

$$\gamma_i = \frac{1}{2}\sum_{j \in N_i} \|q_i - q_j\|^2$$

First note that

$$\sum_i \nabla\gamma_i = \sum_i \begin{bmatrix} \frac{\partial\gamma_i}{\partial q_1} \\ \vdots \\ \frac{\partial\gamma_i}{\partial q_N} \end{bmatrix}$$

and

$$\frac{\partial\gamma_i}{\partial q_j} = \begin{cases} \sum_{j \in N_i}(q_i - q_j), i = j \\ -(q_i - q_j), j \in N_i, j \neq i \\ 0, j \notin N_i \end{cases}$$

so that

$$\sum_i \frac{\partial\gamma_i}{\partial q_j} = \frac{\partial\gamma_j}{\partial q_j} + \sum_{i \in N_j} \frac{\partial\gamma_i}{\partial q_j} =$$

$$\sum_{i \in N_j}(q_j - q_i) + \sum_{i \in N_j}(-(q_i - q_j)) =$$

$$2 \cdot \sum_{i \in N_j} q_j - 2 \cdot \sum_{i \in N_j} q_i = 2 \cdot d_j q_j - 2 \cdot \sum_{i \in N_j} q_i$$

and

$$\sum_i \nabla\gamma_i = \sum_i \begin{bmatrix} \frac{\partial\gamma_i}{\partial q_1} \\ \vdots \\ \frac{\partial\gamma_i}{\partial q_N} \end{bmatrix} = 2 \begin{bmatrix} d_1 \cdot q_1 \\ \vdots \\ d_N \cdot q_N \end{bmatrix}$$

$$-2 \begin{bmatrix} \sum\limits_{j\in N_1} q_j \\ \vdots \\ \sum\limits_{j\in N_N} q_j \end{bmatrix} = 2\left(\Delta \otimes I_2\right) - 2\left(A \otimes I_2\right) q \Rightarrow$$

$$\Rightarrow \sum_i \nabla\gamma_i = 2\left(\mathcal{L} \otimes I_2\right) q$$

The last equation is a direct consequence of the fact that the communication graph is undirected.

Since the proposed control law is discontinuous we use the concept of Theorem 1 for the time derivative of the candidate Lyapunov function. Since V is smooth we have

$$\partial V = \{\nabla V\} = \left\{\sum_i \nabla\gamma_i\right\}$$

so that

$$V = \sum_i \gamma_i \Rightarrow$$

$$\dot{\tilde{V}} = \left\{\sum_i \left(\nabla\gamma_i\right)^T\right\} \cdot K \begin{bmatrix} u_1 \cos\theta_1 \\ u_1 \sin\theta_1 \\ \vdots \\ u_N \cos\theta_N \\ u_N \sin\theta_N \end{bmatrix} =$$

$$2q^T\left(\mathcal{L} \otimes I_2\right) \begin{bmatrix} K[u_1]\cos\theta_1 \\ K[u_1]\sin\theta_1 \\ \vdots \\ K[u_N]\cos\theta_N \\ K[u_N]\sin\theta_N \end{bmatrix} =$$

$$2\left(\mathcal{L}x\right)^T \begin{bmatrix} K[u_1]\cos\theta_1 \\ \vdots \\ K[u_N]\cos\theta_N \end{bmatrix} + 2\left(\mathcal{L}y\right)^T \begin{bmatrix} K[u_1]\sin\theta_1 \\ \vdots \\ K[u_N]\sin\theta_N \end{bmatrix}$$

$$= \sum_i \left\{2K[u_i]\left(\left(\mathcal{L}x\right)_i \cos\theta_i + \left(\mathcal{L}y\right)_i \sin\theta_i\right)\right\}$$

But since $K[\mathrm{sgn}(x)]\,x = \{|x|\}$ the choice of control laws (2),(3) results in

$$\dot{\tilde{V}} = 2\sum_i \left\{-\left|\gamma_{xi}\cos\theta_i + \gamma_{yi}\sin\theta_i\right|\left(\gamma_{xi}^2 + \gamma_{yi}^2\right)^{1/2}\right\} \leq 0$$

Since the candidate Lyapunov function is quadratic in the agents' relative positions, its level sets are compact and invariant for the trajectories of the closed loop system. Specifically, we have

$$V \leq c \Rightarrow \|q_i - q_j\| \leq \sqrt{2c},\ \forall\,(i,j) \in E$$

Connectivity of the communication graph ensures that the maximum length of a path connecting two vertices of the graph is at most $N-1$. Hence $\|q_i - q_j\| \leq \sqrt{2c}\,(N-1)\,,\ \forall i,j \in \mathcal{N}$.

By the nonsmooth version of LaSalle's invariance principle(theorem 2), the trajectories of the system converge to the largest invariant set contained in the set

$$S = \left\{ \begin{array}{r} \left(\gamma_{xi} = \gamma_{yi} = 0\right) \vee \left(\gamma_{xi}\cos\theta_i + \gamma_{yi}\sin\theta_i = 0\right), \\ \forall i \in N \end{array} \right\}$$

However,using similar arguments as in [27], for each $i \in \mathcal{N}$, we have $|\omega_i| = \frac{\pi}{2}$ whenever $\gamma_{xi}\cos\theta_i + \gamma_{yi}\sin\theta_i = 0$, due to the proposed angular velocity control law. In particular, this choice of angular velocity renders the surface $\gamma_{xi}\cos\theta_i + \gamma_{yi}\sin\theta_i = 0$ repulsive for agent i, whenever i is not located at the desired equilibrium, namely when $\gamma_{xi} = \gamma_{yi} = 0$. Hence the largest invariant set E contained in S is

$$S \supset E = \{\gamma_{xi} = \gamma_{yi} = 0, \forall i \in \mathcal{N}\}$$

In addition $\left(\gamma_{xi} = \gamma_{yi} = 0\right)\forall i$ guarantees that the agents converge to a common configuration. This is easily derived by the fact that

$$\left(\gamma_{xi} = \gamma_{yi} = 0\right)\forall i \Rightarrow Lq = 0$$

where $L = \mathcal{L} \otimes I_2$. We can now compute

$$Lq = 0 \Rightarrow \mathcal{L}x = \mathcal{L}y = 0$$

where x, y the stack vectors of q in the x, y directions. The fact that the formation graph is connected implies that the Laplacian has a simple zero eigenvalue with corresponding eigenvector the vector of ones, $\overrightarrow{\mathbf{1}}$. This guarantees that both x, y are eigenvectors of $\mathcal{L}$ belonging to span$\{\overrightarrow{\mathbf{1}}\}$. Hence for all $i \in \mathcal{N}$, all q_i have a common vector value, implying that all agents converge to a common configuration at steady state. $\diamondsuit$

It must be stressed out that the proposed feedback control strategy (2),(3) is purely *decentralized*, since each agent requires information only of the states of agents within each neighboring set at each time instant. This is a consequence of the definitions of the terms $\gamma_{xi}, \gamma_{yi}, \theta_{nh_i}$ and the form of the Laplacian matrix $\mathcal{L}$ of the communication graph.

IV. SIMULATIONS

To verify the result of the previous paragraphs we provide some computer simulations of the proposed control framework (2),(3).

A. Four unicycles

In the first simulation, four nonholonomic agents starting from arbitrary initial position, navigate under the proposed control scheme. The communication sets in this simulation have been chosen as

$$N_1 = \{2,3,4\}, N_2 = \{1,3\}, N_3 = \{1,2\}, N_4 = \{1\}$$

It is easily verified that the corresponding communication graph is connected(see Figure 3). In Figure 4 Screenshots I-V show the evolution in time of the multi agent team. In the first screenshot, A-i denotes the initial position of agent

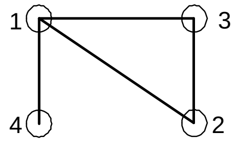

Fig. 3. The communication graph of the four-agent team in the first simulation

i respectively. In the last screenshot the agents converge to a common configuration. Figure 5 shows a plot of the functions γ_i of each agent with respect to time. One can observe that these functions tend to zero as the agents converge to a common point.

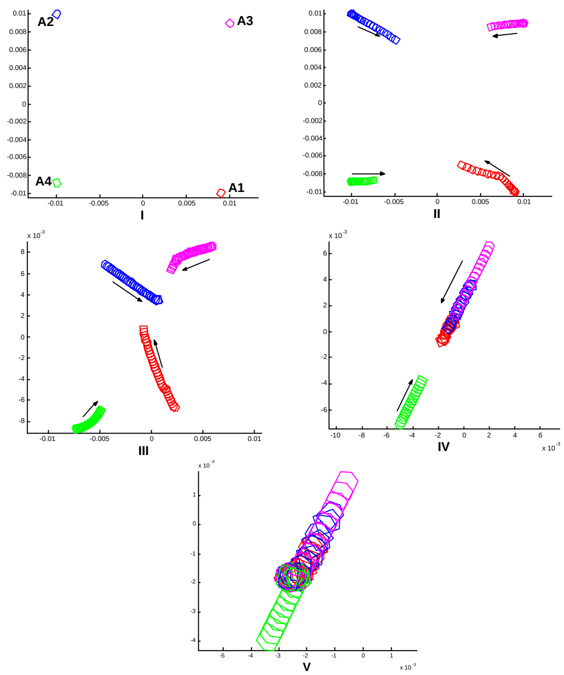

Fig. 4. Convergence to a common configuration for the four unicycles

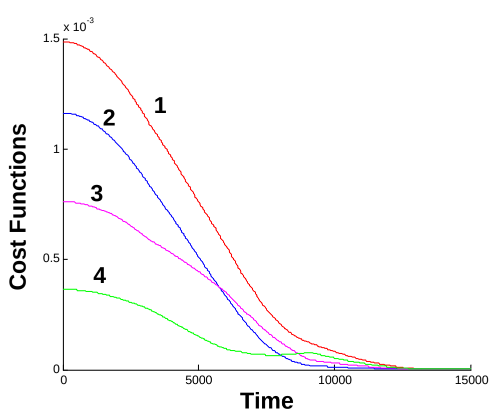

Fig. 5. Plots of the γ_i function for each agent

B. Six unicycles

In the second simulation, a team of six nonholonomic agents starting from arbitrary initial position, navigate under the proposed control scheme. The communication sets in this simulation have been chosen as $N_1 = \{2,3,4,5\}, N_2 = \{1,3\}, N_3 = \{1,2\}, N_4 = \{1\}, N_5 = \{1,6\}, N_6 = \{5\}$. It is easily verified that the corresponding communication graph is connected in this case as well. As in the previous

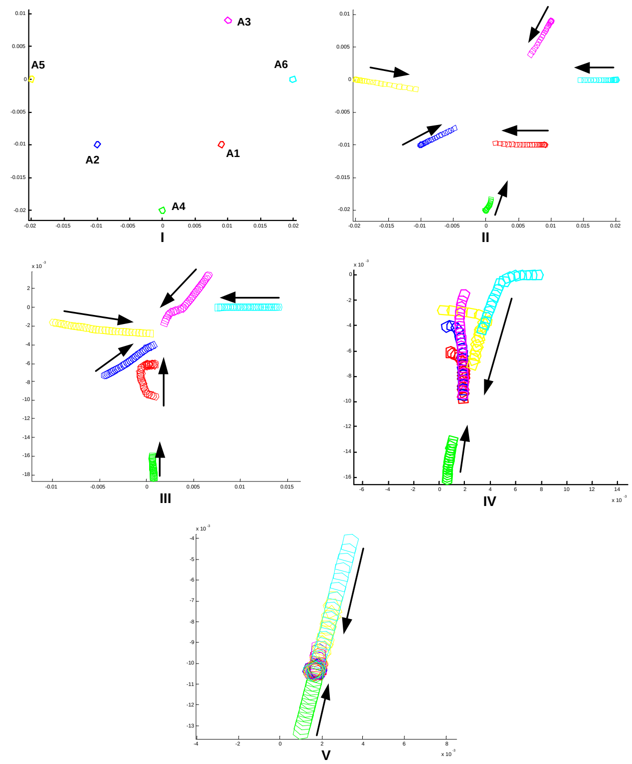

Fig. 6. Convergence to a common configuration for six unicycles

simulation, screenshots I-V in figure 6 show the evolution in time of the of the six unicycles under the proposed control strategy. In the first screenshot, A-i denotes the initial position of agent i respectively. In the last screenshot the agents converge to a common configuration. Figure 7 shows a plot of the functions γ_i of each agent.

V. Conclusions

In this contribution, a feedback control strategy that drives a system of multiple nonholonomic unicycles to agreement has been introduced. The problem treated in this work is similar to the problem solved in [17]. In that reference, the authors use a time varying periodic smooth controller, inspired by the work in [29], to solve the agreement problem. Inspired by our previous work on decentralized navigation of multiple nonholonomic agents [19], we have proposed in the current paper a distributed nonholonomic feedback control strategy that is discontinuous and time invariant. These type of controllers have in general better convergence properties than time-varying ones. Clearly, this is the best we can hope for regarding the nonholonomic feedback strategy (either

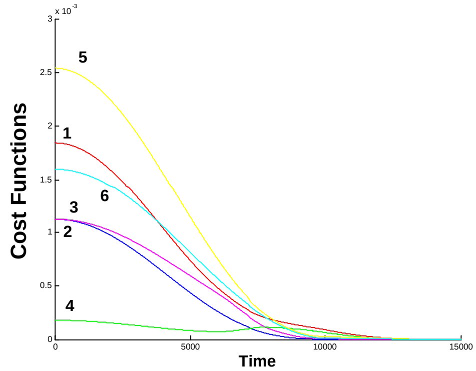

Fig. 7. Plots of the γ_i function for each agent in the second simulation

smooth and time-varying or nonsmooth and time-invariant), as it is a well known fact that that nonholonomic systems do not satisfy the Brocket's necessary smooth feedback stabilization condition ([2]). Another distinction of this work is that we considered merely bidirectional communication topology, whereas directed graphs are taken into account in [17]. The extension of the proposed framework to directed graphs is a topic of ongoing research. The stability of the proposed scheme was analyzed using tools from algebraic graph theory and nonsmooth stability theory.

Current research involves extending the proposed framework to directed graphs and switching interconnection topology. More general motion models such as three-dimensional kinematics are also currently pursued. As a parallel result of this work, formation convergence to arbitrary feasible formation configurations for multiple unicycles is also under investigation.

VI. Acknowledgements

The authors would like to acknowledge the contribution of the European Commission through contracts MICRON(IST-2001-33567) and I-SWARM (IST-2004-507006).

References

[1] B. Bollobás. *Modern Graph Theory*. Springer Graduate Texts in Mathematics # 184, 1998.

[2] R. W. Brockett. Control theory and singular riemannian geometry. In *New Directions in Applied Mathematics*, pages 11–27. Springer, 1981.

[3] F. Ceragioli. *Discontinuous Ordinary Differential Equations and Stabilization*. PhD thesis, Dept. of Mathematics, Universita di Firenze, 1999.

[4] F. Clarke. *Optimization and Nonsmooth Analysis*. Addison - Wesley, 1983.

[5] J. Cortes, S. Martinez, and F. Bullo. Robust rendezvous for mobile autonomous agents via proximity graphs in d dimensions. *IEEE Transactions on Automatic Control*, submitted for publication, 2004.

[6] D.V. Dimarogonas and K.J. Kyriakopoulos. A feedback stabilization and collision avoidance scheme for multiple independent nonholonomic non-point agents. *Joint International Symposium on Intelligent Control & 13th Mediterranean Conference on Control and Automation*, pages 820–825, 2005.

[7] D.V. Dimarogonas and K.J. Kyriakopoulos. Formation control and collision avoidance for multi-agent systems and a connection between formation infeasibility and flocking behavior. *44th IEEE Conf. Decision and Control*, pages 84–89, 2005.

[8] A. Filippov. *Differential equations with discontinuous right-hand sides*. Kluwer Academic Publishers, 1988.

[9] V. Gazi and K.M. Passino. Stability analysis of swarms. *IEEE Transactions on Automatic Control*, 48(4):692–696, 2003.

[10] C. Godsil and G. Royle. *Algebraic Graph Theory*. Springer Graduate Texts in Mathematics # 207, 2001.

[11] Project ISWARM. http://microrobotics.ira.uka.de/.

[12] A. Jadbabaie, J. Lin, and A.S. Morse. Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Transactions on Automatic Control*, 48(6):988–1001, 2003.

[13] M. Ji and M. Egerstedt. Connectedness preserving distibuted coordination control over dynamic graphs. *2005 American Control Conference*, pages 93–98.

[14] B. Kim and P. Tsiotras. Controllers for unicycle-type wheeled robots: Theoretical results and experimental validation. *IEEE Transactions on Robotics and Automation*, 18(3):294–307, 2002.

[15] G. Lafferriere, A. Williams, J. Caughman, and J.J.P. Veerman. Decentralized control of vehicle formations. *Systems and Control Letters*, 54(9):899–910, 2005.

[16] J. Lin, A.S. Morse, and B. D. O. Anderson. The multi-agent rendezvous problem. *42st IEEE Conf. Decision and Control*, pages 1508–1513, 2003.

[17] Z. Lin, B. Francis, and M. Maggiore. Necessary and sufficient graphical conditions for formation control of unicycles. *IEEE Transactions on Automatic Control*, 50(1):121–127, 2005.

[18] Z. Lin, B. Francis, and M. Maggiore. On the state agreement problem for multiple nonlinear dynamical systems. *16th IFAC World Congress*, 2005.

[19] S.G. Loizou, D.V. Dimarogonas, and K.J. Kyriakopoulos. Decentralized feedback stabilization of multiple nonholonomic agents. *2004 IEEE International Conference on Robotics and Automation*, pages 3012–3017.

[20] Project MICRON. http://wwwipr.ira.uka.de/ micron/.

[21] L. Moreau. Stability of continuous-time distributed consensus algorithms. *43rd IEEE Conf. Decision and Control*, pages 3998–4003, 2004.

[22] A. Muhammad and M. Egerstedt. Connectivity graphs as models of local interactions. *43rd IEEE Conf. Decision and Control*, pages 124–129, 2004.

[23] R. Olfati-Saber, J.A. Fax, and R.M. Murray. Consensus and cooperation in networked multi-agent systems. *Proceedings of the IEEE*, submitted for publication, 2006.

[24] R. Olfati-Saber and R.M. Murray. Consensus problems in networks of agents with switching topology and time-delays. *IEEE Transactions on Automatic Control*, 49(9):1520–1533, 2004.

[25] W. Ren, R. W. Beard, and E. M. Atkins. A survey of consensus problems in multi-agent coordination. *2005 American Control Conference*, pages 1859–1864.

[26] D. Shevitz and B. Paden. Lyapunov stability theory of nonsmooth systems. *IEEE Trans. on Automatic Control*, 49(9):1910–1914, 1994.

[27] H. Tanner and K.J. Kyriakopoulos. Backstepping for nonsmooth systems. *Automatica*, 39:1259–1265, 2003.

[28] H.G. Tanner and A. Kumar. Formation stabilization of multiple agents using decentralized navigation functions. *Robotics: Science and Systems*, 2005.

[29] H. Yamaguchi and J. W. Burdick. Asymptotic stabilization of multiple nonholonomic mobile robots forming group formations. *1998 IEEE International Conference on Robotics and Automation*, 1998.