Dynamic Event-Triggered and Self-Triggered Control for Multi-Agent Systems

Xinlei Yi, Kun Liu, Dimos V. Dimarogonas and Karl H. Johansson

Abstract— **We propose two novel dynamic event-triggered control laws to solve the average consensus problem for first-order continuous-time multi-agent systems over undirected graphs. Compared with most existing triggering laws, the proposed laws involve internal dynamic variables, which play an essential role in guaranteeing that the triggering time sequence does not exhibit Zeno behavior. Moreover, some existing triggering laws are special cases of ours. For the proposed self-triggered algorithm, continuous agent listening is avoided as each agent predicts its next triggering time and broadcasts it to its neighbors at the current triggering time. Thus, each agent only needs to sense and broadcast at its triggering times, and to listen to and receive incoming information from its neighbors at their triggering times. It is proved that the proposed triggering laws make the state of each agent converge exponentially to the average of the agents' initial states if and only if the underlying graph is connected. Numerical simulations are provided to illustrate the effectiveness of the theoretical results.**

Index Terms—**Consensus, dynamic event-triggered control, multi-agent systems, self-triggered control.**

I. INTRODUCTION

The average consensus problem involves a group of agents in a network who seeks the average of a set of network-wide measurements or states. It has been widely investigated because its many applications in sensor networks, mobile robots, autonomous underwater vehicles, and unmanned air vehicles, e.g., [1] and the references therein. In these papers, agents have continuous-time dynamics and actuation. However, in practice, typically agents communicate with their neighbors and take actions at discrete time points. There are also many papers that study agents with discrete-time dynamics or continuous-time dynamics but discontinuous information transmission, e.g., [2], [3]. In these papers, time-triggered sampling is used to determine when agents should establish communication with its neighbors, and is often implemented by periodic sampling. A nice feature of such a model is that analysis and design becomes rather straightforward and the vast literature on sample-data control can be used [4]. Drawbacks are that agents need to take action in a synchronous manner, which is often hard to implement when the number of agents is large, and it is not energy-efficient to communicate when the state has not changed much.

Event-triggered sampling has been proposed for single-agent systems [5]–[7]. The concept was originally extended to multi-agent systems in [8]. In event-triggered multi-agent systems actuation updates and inter-agent communications occur only when some specific events are triggered, for instance, a measure of the state error exceeds a specified threshold. The control is often constant between any two consecutive triggering times. Many researchers studied event-triggered control for multi-agent systems recently [8]–[18]. A key challenge is how to design triggering laws to determine the corresponding triggering times, while excluding Zeno behavior, i.e., infinite number of triggers in a finite time interval [19].

To overcome the drawback of continuous monitoring of the triggering law, self-triggered control were proposed for single-agent systems [20]–[22]. Many researchers have investigated self-triggered control for multi-agent systems [8], [14], [17]. For self-triggered single-agent systems, the next triggering time is determined at the previous triggering instance. However, the self-triggered approaches for multi-agent systems mentioned above are not in accordance with this. Although continuous sensing of each agent's own and neighbors' states is avoided in these papers, continuous listening is still needed since the triggering times are determined during runtime and not known in advance. To overcome this drawback, some researchers introduced local clock variables in the self-triggering policy [23], others combined event-triggered control with periodic sampling [11], [13], [24], and some proposed cloud-supported algorithms [25]. By introducing an internal dynamic variable, a new class of event-triggering mechanisms was presented in [26] and later extended to a discrete-time setting in [27]. The idea of using internal dynamic variables in event- and self-triggered control can also be found in [23], [24], [28], [29]. In this paper, we make essential modifications to the dynamic event-triggering mechanism for single-agent systems in [26] and extend it to multi-agent systems.

The main contribution of this paper in the introduction and convergence analysis of dynamic event- and self-triggered control laws for multi-agent systems. The control laws are truly distributed in the sense that they do not require any a priori knowledge of global network parameters. We prove that the proposed dynamic triggering laws yield consensus exponentially fast, and we show that they are free from Zeno

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behavior by verifying that the triggering time sequence of each agent is divergent. We show also that the triggering laws in [9], [10] are special cases of our event-triggered law. The main disadvantage of the event-triggered law is that continuous sensing and listening are needed. To overcome this, we present a self-triggered control law. The main idea to avoid continuous listening is that each agent predicts its next triggering time and broadcasts it to its neighbors at the current triggering time. As a result, each agent only needs to sense and broadcast at its triggering times, and to listen to and receive incoming information from its neighbors at their triggering times. This is to say that, in terms of avoiding continuous listening, our self-triggered algorithm improves the ones in [8], [14], [17] and other papers using a similar approach. Although continuous sensing, broadcasting, listening, and receiving are also avoided in [11], [13], [24] by combining event-triggered control with periodic sampling, the additional periodic sensing and listening are still needed. Moreover, it is not clear how to show that the average inter-event time is strictly larger than the required sampling period. Our self-triggered control law is reminiscent of the event-triggered cloud access in [25]. The main difference is that we do not need the cloud to store data and we use different analysis techniques.

The rest of this paper is organized as follows. Section II introduces the necessary preliminaries. The main average consensus convergence results on dynamic event- and self-triggered control (Theorems 1 and 2) are stated in Sections III and IV, respectively. Simulations are given in Section V. Finally, the paper is concluded in Section VI.

Notations: $\mathbb{R}$ and $\mathbb{R}^n$ denote the set of real numbers and n-dimensional column vectors. $\|\cdot\|$ represents the Euclidean norm for vectors or the induced 2-norm for matrices. $\mathbf{1}_n$ denotes the column vector with each component being 1 and dimension n. I_n is the n-dimensional identity matrix. $\rho_2(\cdot)$ indicates the minimum positive eigenvalue for matrices having positive eigenvalues. Given two symmetric matrices M, N, $M \geq N$ means $M-N$ is positive semi-definite. $|S|$ is the cardinality of a set S.

II. PRELIMINARIES

In this section, we present some definitions from algebraic graph theory [30] and the considered multi-agent system model.

A. Algebraic Graph Theory

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ denote a weighted undirected graph with the set of agents (vertices or nodes) $\mathcal{V} = \{v_1, \dots, v_n\}$, the set of links (edges) $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and the (weighted) adjacency matrix $A = A^\top = (a_{ij})$ with nonnegative elements a_{ij}. A link of $\mathcal{G}$ is denoted by $(v_i, v_j) \in \mathcal{E}$ and exists if $a_{ij} > 0$, i.e., if agents v_i and v_j can communicate with each other. It is assumed that $a_{ii} = 0$ for all $i \in \mathcal{I}$, where $\mathcal{I} = \{1, \dots, n\}$. Let $\mathcal{N}_i = \{j \in \mathcal{I} \mid a_{ij} > 0\}$ and $\deg_i = \sum\limits_{j=1}^{n} a_{ij}$ denote the neighbor index set and weighted degree of agent v_i, respectively. The degree matrix of $\mathcal{G}$ is $\mathrm{Deg} = \mathrm{diag}([\deg_1, \dots, \deg_n])$. The Laplacian matrix is $L = (L_{ij}) = \mathrm{Deg} - A$. A path of length k between agent v_i and agent v_j is a subgraph with distinct agents $v_{i_0} = v_i, \dots, v_{i_k} = v_j \in \mathcal{V}$ and edges $(v_{i_l}, v_{i_{l+1}}) \in \mathcal{E},\ l = 0, \dots, k-1$. An undirected graph is connected if there exists at least one path between any two agents.

For a connected graph we have the following results.

Lemma 1: ([16], [30]) If a graph $\mathcal{G}$ is connected, then its Laplacian matrix L is positive semi-definite, i.e., $z^\top L z \geq 0$ for any $z \in \mathbb{R}^n$. Moreover, $z^\top L z = 0$ if and only if $z = a\mathbf{1_n}$ for some $a \in \mathbb{R}$. Finally, we have $0 \leq \rho_2(L) K_n \leq L$, where $K_n = I_n - \frac{1}{n}\mathbf{1_n}\mathbf{1_n}^\top$.

B. System Model

We consider a set of n agents modelled as single integrators

$$\dot{x}_i(t) = u_i(t),\ i \in \mathcal{I},\ t \geq 0, \tag{1}$$

where $x_i(t) \in \mathbb{R}$ is the state and $u_i(t) \in \mathbb{R}$ is the control input.

The classical distributed consensus protocol is given by $u_i(t) = -\sum_{j=1}^{n} L_{ij} x_j(t)$ [1]. To implement such a consensus protocol, continuous-time state information from neighbors is needed. However, it is often impractical to require continuous communication in physical applications. In order to avoid continuous communication in our setting, each agent broadcasts its state information only at discrete time instances $\{t_k^i\}_{k=1}^{\infty}$ and uses the following event-triggered consensus protocol

$$u_i(t) = -\sum_{j=1}^{n} L_{ij} \hat{x}_j(t), \tag{2}$$

where $\hat{x}_j(t) \equiv x_j(t_k^j),\ t \in [t_k^j, t_{k+1}^j)$. We call the increasing time sequences $\{t_k^i\}_{k=1}^{\infty}$ and $\{t_{k+1}^i - t_k^i\}_{k=1}^{\infty}$ the triggering times and the inter-event times of agent i, respectively. Note that the control protocol (2) only updates at the triggering times and is constant between any two consecutive triggering times. To simplify notation, let $x(t) = [x_1(t), \dots, x_n(t)]^\top$, $\hat{x}(t) = [\hat{x}_1(t), \dots, \hat{x}_n(t)]^\top$, $e_i(t) = \hat{x}_i(t) - x_i(t)$, and $e(t) = [e_1(t), \dots, e_n(t)]^\top = \hat{x}(t) - x(t)$.

Our goal in this paper is to propose methods to determine the triggering times such that average consensus is reached, while avoiding continuous exchange of information, continuous update of actuators, and Zeno behavior.

III. DYNAMIC EVENT-TRIGGERED CONTROL LAW

In this section, we propose a dynamic event-triggered control law to achieve average consensus. We first give the following well-known lemma, e.g., [8].

Lemma 2: Consider the multi-agent system (1)–(2). Suppose that the underlying graph $\mathcal{G}$ is undirected. The average of all agents' states $\bar{x}(t) = \frac{1}{n}\sum_{i=1}^{n} x_i(t)$ is constant, i.e., $\bar{x}(t) \equiv \bar{x}(0), \forall t \geq 0$.

We next introduce a static event-triggered control law to determine the triggering times and show that it achieves average consensus.

Proposition 1: Consider the multi-agent system (1)–(2). Suppose that the underlying graph $\mathcal{G}$ is undirected. Given the first triggering time $t_1^i = 0$, agent v_i determines the triggering time sequence $\{t_k^i\}_{k=2}^{\infty}$ by

$$t_1^i=0, t_{k+1}^i=\max_{r\geq t_k^i}\Big\{r: e_i^2(t)\leq \frac{\sigma_i}{2L_{ii}}\hat{q}_i(t), \forall t\in[t_k^i, r]\Big\}, \quad (3)$$

where $\sigma_i \in (0,1)$ is a design parameter that can be arbitrarily chosen, and

$$\hat{q}_i(t) = -\frac{1}{2}\sum_{j=1}^{n} L_{ij}(\hat{x}_j(t)-\hat{x}_i(t))^2 \geq 0. \quad (4)$$

Then, average consensus is achieved exponentially if and only if $\mathcal{G}$ is connected.

Proof: The necessity is straightforward so we only prove sufficiency here. Consider the Lyapunov function candidate

$$V(x(t)) = \frac{1}{2}x^\top(t)K_n x(t) = \frac{1}{2}\sum_{i=1}^{n}[x_i(t)-\bar{x}(0)]^2, \quad (5)$$

where $K_n = I_n - \frac{1}{n}\mathbf{1_n}\mathbf{1_n}^\top$ and the last equality holds from Lemma 2. The derivative of $V(x(t))$ along the trajectories of (1)–(2) satisfies

$$\begin{aligned}\dot{V}(x(t)) &= \sum_{i=1}^{n}[x_i(t)-\bar{x}(0)]\dot{x}_i(t) = \sum_{i=1}^{n}x_i(t)\sum_{j=1}^{n}-L_{ij}\hat{x}_j(t)\\
&= -\sum_{i=1}^{n}(\hat{x}_i(t)-e_i(t))\sum_{j=1}^{n}L_{ij}\hat{x}_j(t)\\
&= -\sum_{i=1}^{n}\hat{q}_i(t) - \sum_{i=1}^{n}\sum_{j=1}^{n}e_i(t)L_{ij}\hat{x}_j(t) \quad (6)\\
&= -\sum_{i=1}^{n}\hat{q}_i(t) - \sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}e_i(t)L_{ij}(\hat{x}_j(t)-\hat{x}_i(t))\\
&\leq -\sum_{i=1}^{n}\hat{q}_i(t) - \sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}L_{ij}e_i^2(t)\\
&\quad -\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}L_{ij}\frac{1}{4}(\hat{x}_j(t)-\hat{x}_i(t))^2\\
&= -\sum_{i=1}^{n}\frac{1}{2}\hat{q}_i(t) + \sum_{i=1}^{n}L_{ii}e_i^2(t), \quad (7)\end{aligned}$$

where the equalities (6) and (7) hold since

$$\sum_{i=1}^{n}\hat{q}_i(t) = -\sum_{i=1}^{n}\frac{1}{2}\sum_{j=1}^{n}L_{ij}(\hat{x}_j(t)-\hat{x}_i(t))^2 = \hat{x}^\top(t)L\hat{x}(t),$$

and the inequality holds since $ab \leq a^2 + \frac{1}{4}b^2,\ \forall a,b\in\mathbb{R}$.

Then, from (7) and (3), we have

$$\begin{aligned}\dot{V}(x(t)) &\leq -\sum_{i=1}^{n}\frac{1}{2}\hat{q}_i(t) + \sum_{i=1}^{n}L_{ii}e_i^2(t)\\
&\leq -\frac{1}{2}(1-\sigma_{\max})\hat{x}^\top(t)L\hat{x}(t), \quad (8)\end{aligned}$$

where $\sigma_{\max} = \max\{\sigma_1,\dots,\sigma_n\} < 1$. Noting that

$$\begin{aligned}x^\top(t)Lx(t) &= (\hat{x}(t)+e(t))^\top L(\hat{x}(t)+e(t))\\
&\leq 2\hat{x}^\top(t)L\hat{x}(t) + 2e^\top(t)Le(t)\\
&\leq 2\hat{x}^\top(t)L\hat{x}(t) + \frac{\|L\|\sigma_{\max}}{\min_i\{L_{ii}\}}\sum_{i=1}^{n}\hat{q}_i(t)\\
&= \Big(2+\frac{\|L\|\sigma_{\max}}{\min_i\{L_{ii}\}}\Big)\hat{x}^\top(t)L\hat{x}(t), \quad (9)\end{aligned}$$

where the first inequality holds since L is positive semi-definite and $2a^\top Lb \leq a^\top La + b^\top Lb, \forall a,b\in\mathbb{R}^n$, and the second inequality holds since $a^\top La \leq \|L\|\|a\|^2, \forall a\in\mathbb{R}^n$, and (3). We then have

$$\begin{aligned}\dot{V}(x(t)) &\leq -\frac{(1-\sigma_{\max})\min_i\{L_{ii}\}}{4\min_i\{L_{ii}\}+2\|L\|\sigma_{\max}}x^\top(t)Lx(t)\\
&\leq -\frac{(1-\sigma_{\max})\min_i\{L_{ii}\}\rho_2(L)}{2\min_i\{L_{ii}\}+\|L\|\sigma_{\max}}V(x(t)),\end{aligned}$$

where the last inequality holds due to Lemma 1. Hence,

$$V(x(t)) \leq V(x(0))\exp\Big(-\frac{(1-\sigma_{\max})\min_i\{L_{ii}\}\rho_2(L)t}{2\min_i\{L_{ii}\}+\|L\|\sigma_{\max}}\Big), \quad \forall t\geq 0. \quad (10)$$

This implies that the multi-agent system (1)–(2) reaches consensus exponentially, as the underlying graph $\mathcal{G}$ is connected. ■

Remark 1: We refer to (3) as a static triggering law since it does not involve any extra dynamic variables more than $x_i(t), \hat{x}_i(t)$ and $\hat{x}_j(t), j\in\mathcal{N}_i$. The triggering law is distributed since each agent's control action only depends on its own state information and its neighbors' state information, without any a prior knowledge of any global parameters, such as the eigenvalues of the Laplacian matrix.

Remark 2: If we consider the same graph as in [8], i.e., $a_{ij} = 1$ if $(i,j)\in\mathcal{E}$, then $L_{ii} = |\mathcal{N}_i|$. Since $a(1-a|\mathcal{N}_i|) \leq \frac{1}{4|\mathcal{N}_i|}$, $\forall a \in (0, \frac{1}{|\mathcal{N}_i|})$ and $(\sum_{j=1}^{n}(\hat{x}_j(t)-\hat{x}_i(t)))^2 \leq 2|\mathcal{N}_i|\sum_{j=1}^{n}(\hat{x}_j(t)-\hat{x}_i(t))^2$, we have $\frac{\sigma_i a(1-a|\mathcal{N}_i|)}{|\mathcal{N}_i|}(\sum_{j=1}^{n}(\hat{x}_j(t)-\hat{x}_i(t)))^2 \leq \frac{\sigma_i}{2|\mathcal{N}_i|}\hat{q}_i(t)$. In other words, the distributed triggering law (6) proposed in [9] is a special case of the static triggering law (3).

Remark 3: The main purpose of using event-triggered control is to reduce the overall need of actuation updates and communication between agents, so it is essential to exclude Zeno behavior. However, in [13] it is argued that the distributed triggering law (6) in [9] "does not discard the possibility of an infinite number of events happening in a finite time period". Zeno behavior may also not be excluded under the static triggering law (3). In the following, in order to explicitly exclude Zeno behavior, we replace the static triggering law (3) by a dynamic one.

Inspired by [26], we propose the following internal dynamic variable χ_i to agent v_i:

$$\dot{\chi}_i(t) = -\beta_i\chi_i(t) + \delta_i(\frac{\sigma_i}{2}\hat{q}_i(t) - L_{ii}e_i^2(t)),\ i\in\mathcal{I} \quad (11)$$

where $\chi_i(0) > 0$, $\beta_i > 0$, $\delta_i\in[0,1]$, and $\sigma_i\in[0,1)$ are design parameters that can be arbitrarily chosen. This dynamics leads to the event-triggered control law and convergence result stated in the following theorem.

Theorem 1: Consider the multi-agent system (1)–(2). Suppose that the underlying graph $\mathcal{G}$ is undirected. Given $\theta_i > \frac{1-\delta_i}{\beta_i}$ and the first triggering time $t_1^i = 0$, agent v_i determines the triggering time sequence $\{t_k^i\}_{k=2}^{\infty}$ by

$$t_{k+1}^i = \max_{r\geq t_k^i}\Big\{r: \theta_i\Big(L_{ii}e_i^2(t) - \frac{\sigma_i}{2}\hat{q}_i(t)\Big) \leq \chi_i(t), \forall t\in[t_k^i, r]\Big\} \quad (12)$$

with $\hat{q}_i(t)$ and $\chi_i(t)$ defined in (4) and (11), respectively. Then, (i) average consensus is achieved exponentially if and only if $\mathcal{G}$ is connected; and (ii) there is no Zeno behavior.

Proof: (i) The necessity is straightforward so we only prove sufficiency here. From (11) and (12), we have $\dot{\chi}_i(t) \geq -\beta_i\chi_i(t) - \frac{\delta_i}{\theta_i}\chi_i(t),\ \forall t \geq 0$. Thus

$$\chi_i(t) \geq \chi_i(0)e^{-(\beta_i+\frac{\delta_i}{\theta_i})t} > 0,\ \forall t \geq 0. \tag{13}$$

Consider the Lyapunov function candidate

$$F(x(t),\chi(t)) = V(x(t)) + \sum_{i=1}^{n}\chi_i(t),$$

where $V(x(t))$ is defined in (5) and $\chi(t) = [\chi_1(t),\dots,\chi_n(t)]^\top$. Then the derivative of $F(x(t),\chi(t))$ along the trajectories of the multi-agent system (1)–(2) and with (11) satisfies

$$\begin{aligned}
&\dot{F}(x(t),\chi(t)) = \dot{V}(x(t)) + \sum_{i=1}^{n}\dot{\chi}_i(t)\\
\leq& -\sum_{i=1}^{n}\frac{1}{2}\hat{q}_i(t) + \sum_{i=1}^{n}L_{ii}e_i^2(t) - \sum_{i=1}^{n}\beta_i\chi_i(t)\\
&+\sum_{i=1}^{n}\delta_i(\frac{\sigma_i}{2}\hat{q}_i(t) - L_{ii}e_i^2(t))\\
=& -\sum_{i=1}^{n}\frac{1}{2}(1-\sigma_i)\hat{q}_i(t) - \sum_{i=1}^{n}\beta_i\chi_i(t)\\
&+\sum_{i=1}^{n}(\delta_i-1)(\frac{\sigma_i}{2}\hat{q}_i(t) - L_{ii}e_i^2(t))\\
\leq& -\sum_{i=1}^{n}\frac{1}{2}(1-\sigma_i)\hat{q}_i(t) - \sum_{i=1}^{n}\beta_i\chi_i(t) + \sum_{i=1}^{n}\frac{1-\delta_i}{\theta_i}\chi_i(t)\\
=& -\sum_{i=1}^{n}\frac{1}{2}(1-\sigma_i)\hat{q}_i(t) - \sum_{i=1}^{n}\Big(\beta_i - \frac{1-\delta_i}{\theta_i}\Big)\chi_i(t)\\
\leq& -(1-\sigma_{\max})\sum_{i=1}^{n}\frac{1}{2}\hat{q}_i(t) - k_1\sum_{i=1}^{n}\chi_i(t)\\
=& -\frac{1}{2}(1-\sigma_{\max})\hat{x}^\top(t)L\hat{x}(t) - k_1\sum_{i=1}^{n}\chi_i(t),
\end{aligned}$$

where $k_1 = \min_{i\in\mathcal{I}}\{\beta_i - \frac{1-\xi_i}{\theta_i}\} > 0$. Similar to the derivation of (9), we have

$$\begin{aligned}
&x^\top(t)Lx(t) \leq 2\hat{x}^\top(t)L\hat{x}(t) + 2\|L\|\|e(t)\|^2\\
\leq& 2\hat{x}^\top(t)L\hat{x}(t) + \frac{\|L\|\sigma_{\max}}{\min_i\{L_{ii}\}}\sum_{i=1}^{n}\hat{q}_i(t)\\
&+\frac{2\|L\|}{\min_i\{\theta_iL_{ii}\}}\sum_{i=1}^{n}\chi_i(t)\\
=&\Big(2+\frac{\|L\|\sigma_{\max}}{\min_i\{L_{ii}\}}\Big)\hat{x}^\top(t)L\hat{x}(t) + \frac{2\|L\|}{\min_i\{\theta_iL_{ii}\}}\sum_{i=1}^{n}\chi_i(t)\\
\leq& k_2\hat{x}^\top(t)L\hat{x}(t) + \frac{2\|L\|}{\min_i\{\theta_iL_{ii}\}}\sum_{i=1}^{n}\chi_i(t),
\end{aligned}$$

where $k_2 = \max\left\{2+\frac{\|L\|\sigma_{\max}}{\min_i L_{ii}},\ \frac{2(1-\sigma_{\max})\|L\|}{k_d\min_i\{\theta_iL_{ii}\}}\right\}$. Then,

$$\begin{aligned}
&-\frac{1}{2}(1-\sigma_{\max})\hat{x}^\top(t)L\hat{x}(t)\\
&\leq -\frac{1}{2k_2}(1-\sigma_{\max})x^\top(t)Lx(t) + \frac{k_1}{2}\sum_{i=1}^{n}\chi_i(t).
\end{aligned}$$

Thus,

$$\begin{aligned}
&\dot{F}(x(t),\chi(t))\\
\leq& -\frac{1}{2k_2}(1-\sigma_{\max})x^\top(t)Lx(t) - \frac{k_1}{2}\sum_{i=1}^{n}\chi_i(t)\\
\leq& -\frac{\rho_2(L)}{2k_2}(1-\sigma_{\max})x^\top(t)K_nx(t) - \frac{k_1}{2}\sum_{i=1}^{n}\chi_i(t)\\
=& -\frac{\rho_2(L)}{k_2}(1-\sigma_{\max})V(t) - \frac{k_1}{2}\sum_{i=1}^{n}\chi_i(t)\\
\leq& k_3F(x(t),\chi(t)),
\end{aligned}$$

where $k_3 = \min\left\{\frac{\rho_2(L)}{k_2}(1-\sigma_{\max}),\ \frac{k_1}{2}\right\}$. Hence,

$$V(x(t)) < F(x(t),\chi(t)) \leq F(x(0),\chi(0))e^{-k_3t},\ \forall t \geq 0. \tag{14}$$

This implies that (1)–(2) reaches average consensus exponentially.

(ii) Next, we prove that by contradiction there is no Zeno behavior. Suppose there exists Zeno behavior. Then there exists an agent v_i, such that $\lim_{k\to+\infty}t_k^i = T_0$, where T_0 is a positive constant.

From (14), we know that there exists a positive constant $M_0 > 0$ such that $|x_i(t)| \leq M_0$ for all $t \geq 0$ and $i = 1,\dots,n$. Then, we have $|u_i(t)| \leq 2M_0L_{ii},\ \forall t \geq 0$. Let $\varepsilon_0 = \frac{\sqrt{\chi_i(0)}}{4\sqrt{\theta_iL_{ii}^3}M_0}e^{-\frac{1}{2}(\beta_i+\frac{\delta_i}{\theta_i})T_0} > 0$. Then from the property of limits, there exists a positive integer $N(\varepsilon_0)$ such that

$$t_k^i \in [T_0-\varepsilon_0, T_0],\ \forall k \geq N(\varepsilon_0). \tag{15}$$

Noting that $\hat{q}_i(t) \geq 0$ and (13) holds, we can conclude that one sufficient condition to guarantee that the inequality in (12) holds is

$$|\hat{x}_i(t) - x_i(t)| \leq \sqrt{\frac{\chi_i(0)}{\theta_iL_{ii}}}e^{-\frac{1}{2}(\beta_i+\frac{\delta_i}{\theta_i})t}. \tag{16}$$

Again, noting that $|\dot{x}_i(t)| = |u_i(t)| \leq 2M_0L_{ii}$ and $|\hat{x}_i(t_k^i) - x_i(t_k^i)| = 0$ for any triggering time t_k^i, we can conclude that one sufficient condition to guarantee that the above inequality holds is

$$(t-t_k^i)2M_0L_{ii} \leq \frac{\sqrt{\chi_i(0)}}{\sqrt{\theta_iL_{ii}}}e^{-\frac{1}{2}(\beta_i+\frac{\delta_i}{\theta_i})t}. \tag{17}$$

Now suppose that the $N(\varepsilon_0)$-th triggering time of v_i, $t^i_{N(\varepsilon_0)}$, has been determined. Let $t^i_{N(\varepsilon_0)+1}$ and $\tilde{t}^i_{N(\varepsilon_0)+1}$ denote the next triggering time determined by (12) and (17), respectively. Then

$$t^i_{N(\varepsilon_0)+1} - t^i_{N(\varepsilon_0)} \geq \tilde{t}^i_{N(\varepsilon_0)+1} - t^i_{N(\varepsilon_0)}$$

$$=\frac{\sqrt{\chi_i(0)}}{2\sqrt{\theta_i L_{ii}^3}M_0}e^{-\frac{1}{2}(\beta_i+\frac{\delta_i}{\theta_i})\tilde{t}^i_{N(\varepsilon_0)+1}}$$
$$\geq\frac{\sqrt{\chi_i(0)}}{2\sqrt{\theta_i L_{ii}^3}M_0}e^{-\frac{1}{2}(\beta_i+\frac{\delta_i}{\theta_i})T_0}=2\varepsilon_0, \quad (18)$$

which contradicts (15). Therefore, Zeno behavior is excluded. ■

Remark 4: We refer to (12) as a dynamic triggering law since it involves the extra dynamic variable $\chi_i(t)$. Similar to the static triggering law (3), it is also truly distributed in its implementation as no global network parameters are needed. The static triggering law (3) can be seen as a limit case of the dynamic triggering law (12) when θ_i grows large. Thus, from the analysis in Remark 2, we can conclude that the distributed triggering law (6) proposed in [9] is a special case of the dynamic triggering law (12).

Remark 5: If we choose $\xi_i = 0$ in (11) and $\sigma_i = 0$ in (12), then $\chi_i(t) = \chi_i(0)e^{-\beta_i t}$ and now the inequality in (12) becomes $|e_i(t)| \leq \frac{\sqrt{\chi_i(0)}}{\sqrt{\theta_i L_{ii}}}e^{-\frac{\beta_i}{2}t}$. This is the triggering function (7) proposed in [10] with $c_0 = 0, c_1 = \frac{\sqrt{\eta_i(0)}}{\sqrt{\theta_i L_{ii}}}, \alpha = \frac{\beta_i}{2}$. However, we do not need the constraint $\alpha < \rho_2(L)$ to hold, which necessary for the analysis in [10].

Remark 6: If we choose β_i large enough, then $k_3 = \frac{(1-\sigma_{\max})\min_i\{L_{ii}\}\rho_2(L)}{2\min_i\{L_{ii}\}+\|L\|\sigma_{\max}}$. Hence, in this case, from (10) and (14), we know that the trajectories of the multi-agent system (1)–(2) under static triggering law (3) and dynamic triggering law (12) have the same guaranteed decay rate given by (10).

Remark 7: Intuitively, from (13), one can conclude that the larger $\chi_i(0)$ the larger the inter-event time. This is also consistent with the definition of ε_0 in the proof. How do those design parameters $\chi_i(0), \beta_i, \xi_i, \sigma_i, \theta_i$ affect the inter-event times and decay rate in general is unclear. We leave this as future study.

Remark 8: In addition to the event-triggered control laws in [9], [10], with some modifications, the laws in [11]–[14], [16], [17], can most likely also be extended to dynamic cases. We leave this for future study.

IV. SELF-TRIGGERED CONTROL LAW

When applying the dynamic triggering law (12) in Theorem 1, agent v_i needs to continuously sense its own state since it has to continuously check the triggering law (12) and continuously listen to $x_j(t^j_k), k = 1, 2, \dots, j \in \mathcal{N}_i$, since it does not know the triggering times of its neighbors, $t^j_k, k = 1, 2, \dots, j \in \mathcal{N}_i$, in advance. The way to avoid continuous sensing is straightforward since the control input of each agent is piecewise constant so the state of each agent can be easily predicted as shown in (19) below. The challenge is to avoid continuous listening. If every agent $v_i \in \mathcal{V}$, at its current triggering time t^i_k, can predict its next triggering time t^i_{k+1} and broadcast it to its neighbors, then at time t^i_k agent v_i knows agent v_j's, $j \in \mathcal{N}_i$, latest triggering time $t^j_{k_j(t^i_k)}$, which is before t^i_k, and its next triggering time $t^j_{k_j(t^i_k)+1}$ which is after t^i_k. In this case, agent v_i only needs to listen to and receive information at $\{t^j_k\}^\infty_{k=1}, j \in \mathcal{N}_i$, since it knows these time instants in advance. Moreover, each agent only needs to broadcast at its own triggering times, and to listen to incoming information from its neighbors at their triggering times. Inspired by the reasoning, in the following we propose an algorithm such that at time t^i_k each agent v_i could determine t^i_{k+1} in advance. The idea is illustrated as follows.

Let $t^j_{k_j(t)} = \max\{t^j_k : t^j_k \leq t\}$. From $\dot{x}_i(t) = u_i(t) = -\sum_{j=1}^n L_{ij}x_j(t^j_{k_j(t)}) = -\sum_{j=1}^n L_{ij}u_{ij}(t)$ with $u_{ij}(t) = x_j(t^j_{k_j(t)}) - x_i(t^i_{k_i(t)})$, we have

$$x_i(t) = x_i(t^i_k) - \int_{t^i_k}^t \sum_{j=1}^n L_{ij}u_{ij}(s)ds, \ t \in [t^i_k, t^i_{k+1}). \quad (19)$$

Thus for $t \in [t^i_k, t^i_{k+1})$, we have

$$|e_i(t)| = |x_i(t^i_k) - x_i(t)| = \left|\sum_{j=1}^n \int_{t^i_k}^t L_{ij}u_{ij}(s)ds\right|. \quad (20)$$

Here we need to highlight that $u_{ij}(t)$ may not be constant for all $t \in [t^i_k, t^i_{k+1})$ since $x_j(t^j_{k_j(t)})$ may not be constant in the same interval due to that agent v_j may trigger at some time instants in this interval. So at time t^i_k, we do not know the value of $|e_i(t)|$ for all $t \in (t^i_k, t^i_{k+1})$ in advance. However, if at time t^i_k we could estimate the upper bound of $u_{ij}(t)$, then we could estimate the upper bound of $|e_i(t)|$. Consequently, we could estimate t^i_{k+1} at time t^i_k.

In order to estimate the upper bound of $u_{ij}(t)$, we first need to simplify the dynamic triggering law (12) in Theorem 1. Just as Remark 5 pointed out, if we choose $\xi_i = 0$ in (11) and $\sigma_i = 0$ in (12), then $\chi_i(t) = \chi_i(0)e^{-\beta_i t}$, so the inequality in (12) becomes $|e_i(t)| \leq \alpha_i e^{-\frac{\beta_i}{2}t}$ with $\alpha_i = \frac{\sqrt{\chi_i(0)}}{\sqrt{\theta_i L_{ii}}} > 0$. Here α_i can be chosen as any positive real number, since $\chi_i(0)$ can be chosen as any positive real number. Then from Theorem 1 and the reasoning above, we derive the following corollary.

Corollary 1: Consider the multi-agent system (1)–(2). Suppose that the underlying graph $\mathcal{G}$ is undirected and connected. Given $\alpha > 0$, $\beta > 0$ and the first triggering time $t^i_1 = 0$, agent v_i determines the triggering time sequence $\{t^i_k\}^\infty_{k=2}$ by

$$t^i_{k+1} = \max_{r \geq t^i_k}\left\{r : |e_i(t)| \leq \frac{\alpha}{\sqrt{L_{ii}}}e^{-\frac{\beta}{2}t}, \ \forall t \in [t^i_k, r]\right\}. \quad (21)$$

Then, (i) average consensus is achieved exponentially if and only if $\mathcal{G}$ is connected; and (ii) there is no Zeno behavior.

Remark 9: The design parameters α and β can be distributively chosen for each agent in the above corollary, so α and β in (21) could be replaced by α_i and β_i. Their effects on inter-event times and decay rate are not clear. The reason that we require every agent to choose the same design parameters here is that it gives a simpler self-triggered control law in the following.

Next, let us estimate $|x_i(t) - x_j(t)|$ which will be used later. Consider again $V(x(t))$ defined in (5) and similar to the derivation process to get (7), we have

$$\dot{V}(x(t)) = \sum_{i=1}^n [x_i(t) - \bar{x}(0)]\dot{x}_i(t)$$

$$
\begin{aligned}
&= -\sum_{i=1}^{n} x_i(t) \sum_{j=1}^{n} L_{ij}(x_j(t) + e_j(t)) \\
&\leq -\sum_{i=1}^{n} \frac{1}{2} q_i(t) + \sum_{i=1}^{n} L_{ii} e_i^2(t), \qquad (22)
\end{aligned}
$$

where $q_i(t) = -\frac{1}{2}\sum_{j=1}^{n} L_{ij}(x_j(t) - x_i(t))^2 \geq 0$.From (22) and (21), we have

$$
\begin{aligned}
\dot{V}(x(t)) \leq& -\sum_{i=1}^{n} \frac{1}{2} q_i(t) + \sum_{i=1}^{n} L_{ii} e_i^2(t) \\
\leq& -\rho_2(L)V(x(t)) + n\alpha^2 e^{-\beta t}.
\end{aligned}
$$

Then $\frac{dV(x(t))e^{\rho_2(L)t}}{dt} \leq n\alpha^2 e^{(\rho_2(L)-\beta)t}$. Hence, $V(x(t)) \leq V(x(0))e^{-\rho_2(L)t} + \frac{n\alpha^2}{\rho_2(L)-\beta}(e^{-\beta t} - e^{-\rho_2(L)t})$ if $\rho_2(L) \neq \beta$, and $V(x(t)) \leq V(x(0))e^{-\rho_2(L)t} + n\alpha^2 t e^{-\rho_2(L)t}$ if $\rho_2(L) = \beta$. Hence, $V(x(t)) \leq k_4 e^{-\rho_2(L)t} + k_5(t)e^{-\beta t},\ \forall t \geq 0$, where

$$
k_4 = \begin{cases} V(x(0)) - \frac{n\alpha^2}{\rho_2(L)-\beta}, & \text{if } \rho_2(L) \neq \beta, \\ V(x(0)), & \text{if } \rho_2(L) = \beta, \end{cases}
$$

$$
k_5(t) = \begin{cases} \frac{n\alpha^2}{\rho_2(L)-\beta}, & \text{if } \rho_2(L) \neq \beta,\ \forall t \geq 0, \\ n\alpha^2 t, & \text{if } \rho_2(L) = \beta,\ \forall t \geq 0. \end{cases}
$$

Then, from (5), we have $\sum_{i=1}^{n} |x_i(t) - \bar{x}(0)|^2 = 2V(x(t)) \leq 2(k_4 e^{-\rho_2(L)t} + k_5(t)e^{-\beta t}),\ \forall t \geq 0$. Thus,

$$
\begin{aligned}
|x_i(t) - x_j(t)| \leq& |x_i(t) - \bar{x}(0)| + |x_j(t) - \bar{x}(0)| \\
\leq& \sqrt{2(|x_i(t) - \bar{x}(0)|^2 + |x_j(t) - \bar{x}(0)|^2)} \\
\leq& f(t),\ \forall t \geq 0, \qquad (23)
\end{aligned}
$$

where $f(t) = 2\sqrt{k_4 e^{-\rho_2(L)t} + k_5(t)e^{-\beta t}}$.

Let us now estimate the upper bound of $u_{ij}(t)$ as follows

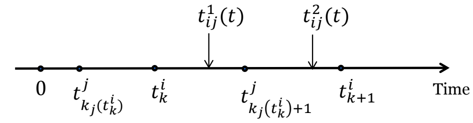
$$
\begin{aligned}
&|u_{ij}(t)| = |x_j(t^j_{k_j(t)}) - x_i(t^i_{k_i(t)})| \\
=& |x_j(t^j_{k_j(t)}) - x_j(t) + x_j(t) - x_i(t) + x_i(t) - x_i(t^i_{k_i(t)})| \\
\leq& \Big(\frac{\alpha}{\sqrt{L_{ii}}} + \frac{\alpha}{\sqrt{L_{jj}}}\Big)e^{-\frac{\beta}{2}t} + f(t),\ \forall t \geq 0. \qquad (24)
\end{aligned}
$$

Finally, let us estimate the upper bound of $e_i(t)$. At time t^i_k, agent v_i already knows $t^j_{k_j(t^i_k)}$ and $x_j(t^j_{k_j(t^i_k)})$, for $j \in \mathcal{N}_i$. If at time t^i_k, agent v_i also knows $t^j_{k_j(t^i_k)+1}$, then at time t^i_k it knows that $u_{ij}(t)$ is constant for $t \in [t^i_k, t^j_{k_j(t^i_k)+1}]$. For simplicity, we introduce the following notations. For $t \in [t^i_k, t^i_{k+1})$, denote

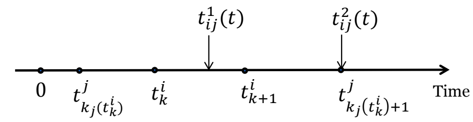
$$
t^1_{ij}(t) = \min\{t,\ t^j_{k_j(t^i_k)+1}\},\ t^2_{ij}(t) = \max\{t,\ t^j_{k_j(t^i_k)+1}\}.
$$

Fig. 1 illustrates the relation between $t^i_k, t^i_{k+1}, t \in [t^i_k, t^i_{k+1})$, $t^j_{k_j(t^i_k)}$, $t^j_{k_j(t^i_k)+1}$, $t^1_{ij}(t)$, and $t^2_{ij}(t)$. From the definition of $u_{ij}(t)$ and $t^1_{ij}(t)$, we know that $u_{ij}(t)$ is a constant for $t \in [t^i_k, t^1_{ij}(t)]$. And for $t > t^1_{ij}(t)$, $u_{ij}(t)$ can be upper bounded by (24). Thus, from (20), for $t \in [t^i_k, t^i_{k+1})$ we have

$$
|e_i(t)| = \Bigg|\sum_{j=1}^{n} L_{ij}\Bigg\{\int_{t^i_k}^{t^1_{ij}} u_{ij}(s)ds + \int_{t^j_{k_j(t^i_k)+1}}^{t^2_{ij}} u_{ij}(s)ds\Bigg\}\Bigg| \leq g_i(t),
$$



(a) The case of $t^j_{k_j(t^i_k)+1} < t^i_{k+1}$.



(b) The case of $t^j_{k_j(t^i_k)+1} \geq t^i_{k+1}$.

Fig. 1: Illustration of the relation between t^i_k, t^i_{k+1}, $t \in [t^i_k, t^i_{k+1})$, $t^j_{k_j(t^i_k)}$, $t^j_{k_j(t^i_k)+1}$, $t^1_{ij}(t)$, and $t^2_{ij}(t)$.

where $g_i(t) = \big|\sum_{j=1}^{n} L_{ij}(t^1_{ij} - t^i_k)u_{ij}(t^i_k)\big| - \sum_{j=1, j\neq i}^{n} L_{ij}\int_{t^j_{k_j(t^i_k)+1}}^{t^2_{ij}} \big[\big(\frac{\alpha}{\sqrt{L_{ii}}} + \frac{\alpha}{\sqrt{L_{jj}}}\big)e^{-\frac{\beta}{2}s} + f(s)\big]ds$.

Hence, a sufficient condition to guarantee (21), i.e., $|e_i(t)| \leq \frac{\alpha}{\sqrt{L_{ii}}}e^{-\frac{\beta}{2}t},\ \forall t \in [t^i_k, t^i_{k+1})$, is $g_i(t) \leq \frac{\alpha}{\sqrt{L_{ii}}}e^{-\frac{\beta}{2}t},\ \forall t \in [t^i_k, t^i_{k+1})$. Since $\frac{\alpha}{\sqrt{L_{ii}}}e^{-\frac{\beta}{2}t}$ decreases with respect to t, $g_i(t)$ increases with respect to t during $[t^i_k, t^i_{k+1})$ and $g_i(t^i_k) = 0$. Then given t^i_k, agent v_i can estimate t^i_{k+1} by solving

$$
g_i(t) = \frac{\alpha}{\sqrt{L_{ii}}}e^{-\frac{\beta}{2}t},\ t \geq t^i_k. \qquad (25)
$$

In other words, if at time t^i_k agent v_i knows $t^j_{k_j(t^i_k)}$, $t^j_{k_j(t^i_k)+1}$, $x_j(t^j_{k_j(t^i_k)})$, and L_{jj} for all $j \in \mathcal{N}_i$, then it can estimate its next triggering time t^i_{k+1} by (potentially numerically) solving (25). In conclusion, we propose the following algorithm.

Self-triggered control law:

1: Choose $\alpha > 0$ and $\beta > 0$;
2: Agent $v_i \in \mathcal{V}$ sends L_{ii} to its neighbors;
3: Initialize $t^i_1 = 0$ and $k = 1$;
4: At time $s = t^i_k$, agent v_i senses its own state $x_i(t^i_k)$, and broadcasts $\{t^i_k, x_i(t^i_k)\}$ to its neighbors, and updates its control input $u_i(t^i_k)$ by (2), and determines t^i_{k+1} by (25)[1], and broadcasts it to its neighbors[2];
5: At the triggering times of neighbors $\mathcal{N}_i$ between $[t^i_k, t^i_{k+1}]$, agent v_i receives triggering information from its neighbors[3] and updates its control input $u_i(\cdot)$ by (2);
6: Agent v_i resets $k := k + 1$, and goes back to Step 4.

Our main result for self-triggered control of multi-agent systems follows from the derivation above and is given in the following theorem.

Theorem 2: Consider the multi-agent system (1)–(2). Suppose that the underlying graph $\mathcal{G}$ is undirected and connected. If all agents follow the self-triggered control law, then, (i)

[1]Agent v_i uses $t^j_{k_j(t^i_k)}$ instead of $t^j_{k_j(t^i_k)+1}$ to determine t^i_{k+1} from (25) when $t^i_k = t^j_{k_j(t^i_k)}$.

[2]We assume that all these actions are done instantaneously.

[3]In other words, agent v_i only listens to coming information at its neighbors' triggering times. Thus continuous listening is avoided.

average consensus is achieved exponentially if and only if $\mathcal{G}$ is connected; and (ii) there is no Zeno behavior.

Remark 10: Self-triggered control approaches are also proposed in [8], [14], [17]. However, one potential drawback of these papers and other papers using a similar approach is that continuous listening is still needed. As verified above, continuous sensing, broadcasting, listening, and receiving are voided under the self-triggered algorithm proposed in this paper. Although these are also avoided in [11], [13], [24] by combining event-triggered control with periodic sampling, periodic sensing and listening are still needed. It is not clear in these cases if the average inter-event time in general is strictly larger than the required sampling period.

Remark 11: It follows from the proof above that for the self-triggered control law, the global parameters $V(x(0))$, n, and $\rho_2(L)$ are needed, which is obviously a drawback.

V. SIMULATIONS

In this section, a numerical example is given to demonstrate the presented results. Consider a connected network of four agents with the Laplacian matrix

$$L = \begin{bmatrix} 3.4 & -3.4 & 0 & 0 \\ -3.4 & 9.8 & -2.1 & -4.3 \\ 0 & -2.1 & 3.2 & -1.1 \\ 0 & -4.3 & -1.1 & 5.4 \end{bmatrix}.$$

We choose an arbitrary initial state $x(0) = [6.2945, 8.1158, -7.4603, 8.2675]^\top$. The average is $\bar{x}(0) = 3.8044$. Fig. 2 (a) shows the state evolutions under the static triggering law (3) with $\sigma_i = 0.5$. Fig. 2 (c) shows the corresponding triggering times for each agent. Fig. 2 (b) shows the state evolution under the dynamic triggering law (12) with $\sigma_i = 0.5$, $\chi_i(0) = 10$, $\beta_i = 1$, $\delta_i = 1$, and $\theta_i = 1$. Fig. 2 (d) shows the corresponding triggering times. Fig. 3 (a) shows the state evolution (1)–(2) under the self-triggered control law with $\alpha = 10$ and $\beta = 1$. Fig. 3 (c) shows the corresponding triggering times. Finally, Fig. 3 (b) shows the state evolution (1)–(2) under the triggering law (2) in [11] (which is a representative algorithm that combines event-triggered control with periodic sampling) with $\sigma_i = \frac{1}{2\lambda_n^2} = 0.0028$ and $h = \frac{1}{20\lambda_n} = 0.0037$. Fig. 3 (d) shows the corresponding triggering times for each agent.

It can be seen that consensus is achieved for all triggering laws. Moreover, just as Theorems 1 and 2 predict, we note that there is no Zeno behavior under the dynamic event-triggered law (12) or under the self-triggered law. It can also be seen that inter-triggering times under the dynamic triggering law are in general larger than that determined by the self-triggered law. Note that the event-triggered control with periodic sampling in [11] requires more sampling in this example. Although there is no Zeno behavior under the static triggering law (3) in the simulations, it is still not clear if this could be proved.

VI. CONCLUSION

In this paper, we presented dynamic event-triggered and self-triggered control law for multi-agent systems. We showed that, some existing triggering laws are special cases of the proposed dynamic triggering laws and if the communication graph is undirected and connected, consensus is achieved exponentially. In addition, Zeno behavior was excluded by proving that the triggering time sequence of each agent is divergent. Moreover, each agent only needs to broadcast at its own triggering times, and listen to incoming information from its neighbors at their triggering times. Thus continuous listening is avoided by the proposed triggering laws. Future research directions include considering the influence of parameters in the proposed dynamic triggering laws.

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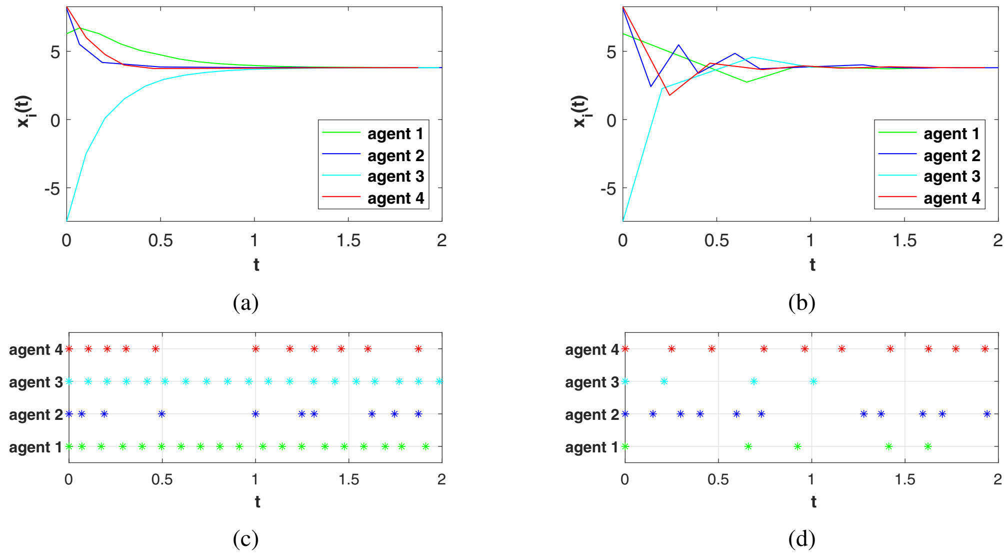

Fig. 2: The state evolutions ((a) and (b)) and triggering times ((c) and (d)) under event-triggering laws (3) and (12).

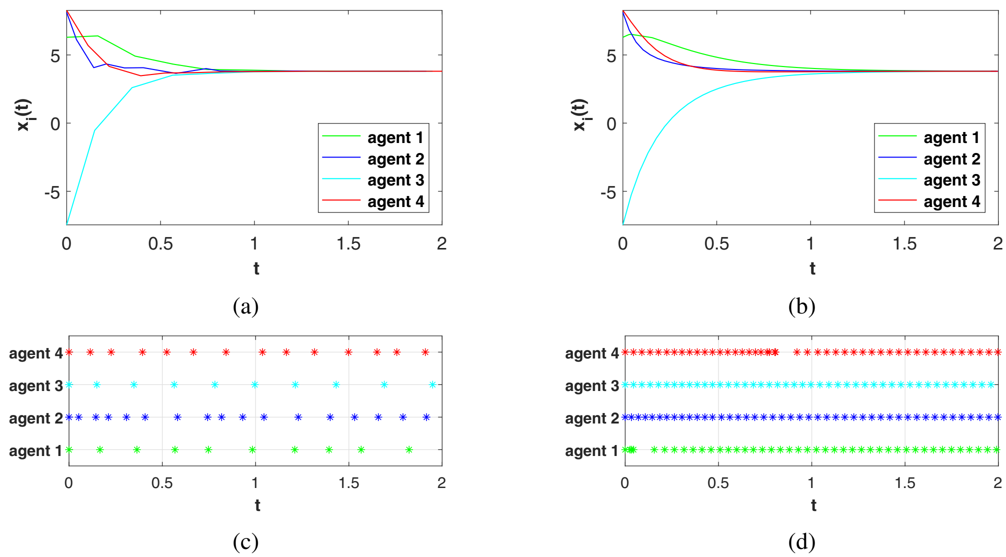

Fig. 3: The state evolutions ((a) and (b)) and triggering times ((c) and (d)) under the self-triggered control law proposed in this paper and the triggering law (2) in proposed in [11].

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