Event-based vehicle coordination using nonlinear unidirectional controllers

Steffen Linsenmayer, Dimos V. Dimarogonas, and Frank Allgöwer

***Abstract*—This article presents a framework to control vehicle platoons with event-based communication and nonlinear controllers. The overall goal is to achieve a platoon that moves in a desired formation with a desired velocity and the convergence to this formation should be exponential while Zeno behavior has to be excluded. The set of admissible controllers for this problem is specified by the properties that they need to guarantee. These properties will be of a form such that they can be checked locally by every vehicle itself and heterogeneous controllers as well as heterogeneous possibly nonlinear dynamics of the vehicles in the platoon are allowed. The framework is shown to work with several communication networks and the set of networks will be characterized. Modifications that are necessary to cope with additive disturbances are described and a simulation example that shows the benefits of being able to use the framework in different networks is given.**

***Index Terms*—Decentralized control, nonlinear control system, event-triggered control, cooperative control.**

I. Introduction

INCREASING road safety and capacity are often seen as two contradictory goals in traffic systems. A concept that is devoted to achieve both goals at the same time is platooning. This already motivates why improvements in control and communication for platoons of vehicles is of major interest. Since it becomes more and more standard to connect vehicles with a communication network one goal is to use this communication for control purposes while keeping the network load moderate. In this article the main tool to reduce the network load is event-based communication instead of periodic time-triggered communication. The design of a class of event-triggering rules, that trigger the communication such that desired properties, in our case exponential convergence, are guaranteed is the control objective. We allow the platooning controllers to be nonlinear satisfying certain assumptions. The control and communication framework strongly depends on the communication network that describes which vehicle sends its information to other vehicles. Therefore the other main focus will be on deducing the class of communication networks that can be tackled within these event-triggering rules and controllers.

The topic of platooning control has a large history in research: In [1] the design of longitudinal vehicle controllers for platooning was considered. An early research direction that came up with platooning investigations was the analysis of string stability for platoons with countably infinite number of vehicles as in [2]. An overview about many investigations and extensions for two widely used control architectures is given in [3]. One of the outcomes of that article is that it indicates to use nonlinear controllers when only information from the predecessor is known for a vehicle. This combination of nonlinear controller and state information from the front neighbor is one that can be tackled with our approach as well. In general many investigations on what influence the communication network has on the performance of the platoon have been made, for example in [4], [5]. Those works show the potential benefit of our approach, since the work at hand is not devoted to a specific network. In [5], additional state information through wireless communication is used in a stochastic approach to deduce stochastic guarantees. Recently researchers investigated the effect of communication constraints on the platoon. For example in [6] a modeling framework and string stability analysis is presented that accounts for communication delays and constraints. In contrast to the just mentioned references, the focus of the article at hand is on the sampling strategy. Whereas in [5] time-triggered sampling is used and in [6] general uncertain transmission intervals are covered, the paper at hand studies the use of event-based sampling.

The use of event-triggered controllers initially started with the work in [7] and experienced increasing relevance through [8]. In this work an event-triggering rule is presented that guarantees asymptotic stability of a nonlinear system and exclusion of Zeno behavior (see [9]), i.e., the occurrence of infinitely many events in finite time, under the condition that a controller with certain properties exists. The specification of the controllers used by an assumption that they have to fulfill is a similar approach which we use here. The general concept was enhanced to perform event-triggered control for distributed systems in [10] and [11]. A new approach, pointing out the benefits of event-based communication, in multi-agent systems was given in [12]. In this work each agent transmits its state information to neighbors only when the difference of its current state and the last transmitted one crosses a time-dependent threshold. The concept of time-dependent triggering functions gives the possibility that one vehicle decides solely on its own absolute measurements when to trigger the next event, whereas for state dependent event-triggering rules the

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agent has to know at least some continuous relative measure to neighboring agents. In the proposed setup every vehicle uses its own continuous state information to decide when to send information. This preserves the possibility of immediately reacting, which is a benefit when comparing to self-triggered control, a concept that does not employ continuous measurements but always computes the next transmission instant a priori.

Due to the benefits just described the work in [12] emerged the usage of event-based communication for example in leader-following pinning control [13] and in platooning control [14]. In that paper, firstly a linear event-triggered symmetric bidirectional controller is analyzed. An analysis of the nonlinear predecessor-following controller is given as well. The result states an event-triggering rule, that guarantees boundedness of the state error. Furthermore there is a statement, that guarantees convergence of the state error if the trigger error converges. However, in [14] there is no tool to derive an event-triggering rule that has this convergence property while Zeno behavior can be excluded. This gap is closed in [15] where an event-triggering rule is derived that provides a similarity to the linear case in [14] since the condition says that the event-triggering rule must have a slower decrease than the system dynamics. The work in [15] tackled vehicles with double-integrator dynamics and a nonlinear predecessor-following controller.

The goal of the work at hand, that builds on preliminary results presented in [15], is to show that the tools developed there are not restricted to this specific configuration. The dynamics of the vehicles are now allowed to contain nonlinear effects that are possibly heterogeneous, i.e., the vehicles in a platoon do not need to have the same system dynamics. As a second extension there is no special controller that is designed and analyzed but we present a general assumption that all controllers need to fulfill such that this framework can be applied. The third generalization is that we do not restrict the communication network to predecessor information only but work out a general class of communication networks for which this approach can be used. The fact that this latter extension is not only a theoretical completion but provides the possibility to apply controllers that perform better while reducing the network load is illustrated in the simulation result that is given. Furthermore some comments on robustness and necessary modifications of the framework are given here as well that were not made previously. To sum these points up, this article provides a general framework for event-based coordination of heterogeneous vehicles with a typical class of nonlinear vehicle dynamics using possibly heterogeneous, nonlinear controllers and various communication topologies and contains the work in [15] as a special case.

The quantification on how fast the system decreases is done using the concept of input-to-state exponential stability, being defined in the next section together with some general graph theoretic concepts. This is followed by a precise statement of the problem that is tackled. In Section IV the set of controllers and communication architectures that can be used in the framework are defined and the analysis of convergence and Zeno exclusion can be found in Section V. Before presenting a simulation example in Section VII some comments on robustness and how the framework needs to be modified in this case are given in Section VI. The paper ends with a conclusion given in Section VIII.

II. Preliminaries

In this section we will introduce some concepts and general notation for the remainder of the paper. The concept that will be used to describe the communication capabilities along the platoon are directed graphs, often called digraphs. A digraph $\mathcal{G} = (V, E)$ consists of a set of vertices V and a set of edges E. The elements of E are ordered pairs of vertices (v_i, v_j) where v_i is the head and v_j is the tail of the edge. Using the edge set E we can define the neighbor set of an agent i as the set of all vehicles of which vehicle i can receive information from, i.e., $\mathcal{N}_i = \{j | (v_j, v_i) \in E\}$. If the digraph contains no cycles and there exists a root vertex $v_r \in V$ such that there exists a path from v_r to every $v_i \in V \setminus \{v_r\}$ the graph is called a rooted out-branching. This definition can be found together with other results on graph theory in [16].

We will furthermore use the notion that functions belong to class $\mathcal{K}$, respectively $\mathcal{K}_\infty$ or $\mathcal{KL}$. A function $\gamma : [0, \infty) \to [0, \infty)$ belongs to class $\mathcal{K}$ if it is continuous, strictly increasing and $\gamma(0) = 0$. If $\gamma(r) \to \infty$ for $r \to \infty$ the function γ is said to belong to class K_∞. A function $\beta : [0, \infty) \times [0, \infty) \to [0, \infty)$ is said to be of class $\mathcal{KL}$, when $\beta(\cdot, s)$ belongs to class $\mathcal{K}$ for every fixed s and $\beta(r, \cdot)$ is decreasing to zero for every fixed r, see [17]. Throughout the paper $\|\cdot\|$ denotes the Euclidean vector norm.

One last definition that is given in this section is that of input-to-state exponential stability (ISES), introduced in [18] and playing a central role in this paper.

Definition 1: A system $\dot{x}(t) = f(t, x(t), u(t))$ is ISES, if there exist $k \geq 1$, $\lambda > 0$ and $\gamma \in \mathcal{K}_\infty$ such that the solution of the system $x(t) := x(t, x(t_0), u(\cdot))$ with initial condition $x(t_0)$ at t_0 satisfies for all $t \geq t_0$

$$\|x(t)\| \leq \max\left\{k\mathrm{e}^{-\lambda(t-t_0)}\|x(t_0)\|, \gamma\left(\sup_{t_0 \leq \nu \leq t} \|u(\nu)\|\right)\right\}.$$

The difference between ISES and ISS is the restriction of the class $\mathcal{KL}$ function appearing in the definition of ISS to be an exponentially decreasing function in the case of ISES.

III. Problem Setup

In this paper the control of a 1-D platoon consisting of N vehicles is investigated. The platoon is assumed to be equipped with the possibility to communicate information from certain vehicles to specified other vehicles. Thus, when introducing the set of vehicles

$$P = \{1, 2, \dots, N\} \tag{1}$$

we can model the platoon together with the possible communication links as a digraph

$$G = (P, E) \tag{2}$$

where E is the edge set as introduced in the foregoing section. From this graph the neighbor sets of all vehicles can be deduced. Throughout the paper we assume that there is one additional communication link, that is a link from a fictitious reference trajectory, called vehicle 0, to the first vehicle in the platoon, i.e.,

$$0 \in \mathcal{N}_1 \text{ and } 0 \notin \mathcal{N}_i, \forall i \neq 1.$$

The trajectory of this fictitious lead vehicle 0 is given as a reference with constant velocity,

$$p_0(t) = v_0 t + p_{0,0}, \quad \forall t \geq 0 \tag{3}$$

where $p_{0,0}, v_0 \in \mathbb{R}$ and in general $p_i(t) \in \mathbb{R}$ represents the position of vehicle i at time t. The control goal is now that every vehicle in the platoon moves with the desired velocity v_0 and that the spacing in between two following vehicles equals the constant desired gap Δ. This can be summarized using the desired trajectory p_i^* as

$$\begin{aligned} \dot{p}_i^*(t) &= \quad v_0, & \forall i \in P; \\ p_i^*(t) - p_j^*(t) &= (j-i)\Delta, & \forall i, j \in P \cup \{0\}\,. \end{aligned} \tag{4}$$

From (4) it can be seen that the desired distance to the preceding vehicle is identical for all vehicles. In [14] and [15] these distances were not assumed to be equal. Since both scenarios can be covered with the same concepts we investigate the scenario described in (4) to simplify the notation. The dynamics of the vehicles of the platoon are given as

$$\ddot{p}_i(t) = u_i(t) - h_i(\dot{p}_i(t)) \quad i \in P, \forall t \geq 0 \tag{5}$$

where $u_i(t)$ represents the control input and h_i represents driving resistances that depend on the current velocity of the vehicle, e.g. due to air drag. Thus, the study at hand does not cover general nonlinear systems but a common model for nonlinear vehicle dynamics. The right hand side is assumed to be smooth enough to guarantee existence and uniqueness of the solution. This results in possibly nonlinear and heterogeneous system dynamics.

In our approach the communication along the edges in E is assumed to be event-based. This means that vehicle i communicates its state information along all the edges in E that start at vertex i only at the discrete time instances t_k^i with $k \in \mathbb{N}_0$. These time instances are determined through a certain event-triggering rule that will be specified later. Due to that fact, information from neighboring vehicles is not available in a continuous fashion and thus every agent needs to run an estimator in between these instances. This estimation, based on the last transmitted state information is specified as

$$\begin{bmatrix} p_{j,est}(t) \\ \dot{p}_{j,est}(t) \end{bmatrix} = f_{est}\left(p_j(t_k^j), \dot{p}_j(t_k^j)\right), \quad \forall t \in [t_k^j, t_{k+1}^j), \tag{6}$$

where the elements of the sequence $(t^j)_{k \in \mathbb{N}_0}$ are the instants when state information from vehicle j is sent to its neighboring vehicles, f_{est} is a function describing the estimator that is run to estimate the position, $p_{j,est}(t)$, and velocity, $\dot{p}_{j,est}(t)$, of vehicle j. Possible choices for f_{est} will be given later. Using this estimation we can now give the structure of the event-triggering rule that runs locally at every vehicle to decide when new state information has to be sent out. Those events are determined as soon as the inequality, dependent on positive real numbers c_i and α_i,

$$\left\| \begin{bmatrix} p_{i,est}(t) - p_i(t) \\ \dot{p}_{i,est}(t) - \dot{p}_i(t) \end{bmatrix} \right\| := \left\| \begin{bmatrix} e_i(t) \\ e_{d,i}(t) \end{bmatrix} \right\| \leq \sigma_i(c_i, \alpha_i, t), \tag{7}$$

where σ_j is an event-triggering function that will be defined later, is violated. We call $\begin{bmatrix} e_i(t) & e_{d,i}(t) \end{bmatrix}^\top$ the transmitted error for agent i. Since it is no hard assumption that the reference information is known to vehicle 0 in a continuous fashion, due to the fact that it is no real state measurement but can be provided from a pure simulation we will assume $[e_0(t), e_{d,0}(t)]^\top = [0, 0]^\top$.

To consider the event-based communication we are restricted to apply controllers that only employ the estimated information from vehicles that are in their neighbor set since they only have access to the state information at the discrete time instances. Thus the controllers that will be derived are of the form

$$u_i(t) = \tilde{f}_{ctrl,i}\left(p_i(t), \dot{p}_i(t), p_{j,est}(t), \dot{p}_{j,est}(t), i-j, \Delta\right), \tag{8}$$

for all $j \in \mathcal{N}_i, t \geq 0$ where $\tilde{f}_{ctrl,i}$ is assumed to be smooth enough to guarantee existence and uniqueness of solutions of (5) for all $i \in P$ and we assume that the index of the neighboring agents j is known. The goal of the paper is to design $\sigma_i(c_i, \alpha_i, t), f_{est}$ and $\tilde{f}_{ctrl,i}$ such that the desired constant spacing and constant velocity is achieved and the triggering instances of every vehicle have no finite accumulation points, i.e., Zeno behavior is excluded. For one specific communication architecture and homogeneous vehicles with double integrator dynamics this problem is solved in [15]. Here we want to deal with more general communication architectures and possibly nonlinear and heterogeneous system dynamics.

IV. Admissible Controller and Communication Architecture

In this section we want to derive a set of event-triggered controllers and communication architectures that are suitable to achieve the control goals and will be analyzed in the subsequent section.

A. Admissible Controller

The class of controllers will be derived in an emulation based fashion, meaning that we first pose a controller that employs continuous information from neighboring vehicles and then substitute this continuous information by the estimation that we have available in our scenario. The controller that is employing continuous information is given as

$$\begin{aligned} u_{c,i}(t) = f_{ctrl,i}\Bigg(&\sum_{j \in \mathcal{N}_i} p_i(t) - p_j(t) - (j-i)\Delta, \\ &\sum_{j \in \mathcal{N}_i} \dot{p}_i(t) - \dot{p}_j(t), \dot{p}_i(t) \Bigg), \ \forall t \geq 0. \end{aligned} \tag{9}$$

This controller formulation differs from the general controller from (8) by taking neighbor information into account in a relative fashion, i.e., as a difference to the own state. Thus, (9) is a special case of (8) and this is due to that we are interested in a constant spacing and synchronized velocity. As indicated earlier to cope with the event-triggered communication neighboring information can only be used in its estimated version. Thus $p_j(t)$ and $\dot{p}_j(t)$ need to be replaced by $p_{j,est}(t) = p_j(t) + e_j(t)$ and $\dot{p}_{j,est}(t) = \dot{p}_j(t) + e_{d,j}(t)$, i.e., for all $t \geq 0$

$$\begin{aligned} u_i(t) = f_{ctrl,i}\Bigg(&\sum_{j\in\mathcal{N}_i} p_i(t) - p_{j,est}(t) - (j-i)\Delta, \\ &\sum_{j\in\mathcal{N}_i} \dot{p}_i(t) - \dot{p}_{j,est}(t), \dot{p}_i(t)\Bigg) \\ = f_{ctrl,i}\Bigg(&\sum_{j\in\mathcal{N}_i} p_i(t) - p_j(t) - e_j(t) - (j-i)\Delta, \\ &\sum_{j\in\mathcal{N}_i} \dot{p}_i(t) - \dot{p}_j(t) - e_{d,j}(t), \dot{p}_i(t)\Bigg). \end{aligned} \tag{10}$$

To analyze the convergence to the desired formation we introduce the state error of vehicle i, $x_i(t) = [\tilde{p}_i(t), \dot{\tilde{p}}_i(t)]^\top$ with

$$\begin{aligned} \tilde{p}_i(t) &= p_i(t) - p_i^*(t) = p_i(t) - (p_0(t) - i\Delta) \\ \dot{\tilde{p}}_i(t) &= \dot{p}_i(t) - \dot{p}_i^* = \dot{p}_i(t) - v_0. \end{aligned} \tag{11}$$

Thus the control goal is to drive the state error x_i to zero for every vehicle i. This can also be denoted as driving the state error of the platoon $x(t) := [x_1^\top(t), \dots, x_N^\top(t)]^\top$ to zero. With this notation we know $\ddot{\tilde{p}}_i(t) = \ddot{p}_i(t)$ and the event-triggered closed-loop system composed of (5) and (10) can be written as

$$\begin{aligned} \ddot{\tilde{p}}_i(t) = &- h_i(\dot{\tilde{p}}_i(t) + v_0) \\ &+ f_{ctrl,i}\Bigg(\sum_{j\in\mathcal{N}_i} \tilde{p}_i(t) - \tilde{p}_j(t) - e_j(t), \\ &\sum_{j\in\mathcal{N}_i} \dot{\tilde{p}}_i(t) - \dot{\tilde{p}}_j(t) - e_{d,j}(t), \dot{\tilde{p}}_i(t) + v_0\Bigg) \end{aligned} \tag{12}$$

where we applied the auxiliary computation:

$$\begin{aligned} &p_i(t) - p_j(t) = \tilde{p}_i(t) + p_i^*(t) - \tilde{p}_j(t) - p_j^*(t) \\ &= \tilde{p}_i(t) - \tilde{p}_j(t) + p_0 - i\Delta - p_0 + j\Delta = \tilde{p}_i - \tilde{p}_j + (j-i)\Delta \end{aligned}$$

In the following an assumption that is assumed to hold throughout the paper is introduced. It is a locally verifiable condition on the controller $f_{ctrl,i}$ in the sense that all quantities and functions that are necessary to check the assumption, i.e., the local vehicle dynamics, the local controller, and the neighbor set, are available at the individual agents.

Assumption 1: Assume that

$$\begin{aligned} &f_{ctrl,i}\Bigg(\sum_{j\in\mathcal{N}_i} \tilde{p}_i(t) - \tilde{p}_j(t) - e_j(t), \\ &\qquad \sum_{j\in\mathcal{N}_i} \dot{\tilde{p}}_i(t) - \dot{\tilde{p}}_j(t) - e_{d,j}(t), \dot{\tilde{p}}_i(t) + v_0\Bigg) \\ &= f_{rel,i}\Bigg(\sum_{j\in\mathcal{N}_i} \tilde{p}_i(t) - \tilde{p}_j(t) - e_j(t), \\ &\qquad \sum_{j\in\mathcal{N}_i} \dot{\tilde{p}}_i(t) - \dot{\tilde{p}}_j(t) - e_{d,j}(t)\Bigg) + f_{abs,i}\left(\dot{\tilde{p}}_i(t) + v_0\right) \end{aligned} \tag{13}$$

where $f_{rel,i}$ is globally Lipschitz in both arguments with Lipschitz constants L_1, L_2 and the condition

$$\|f_{abs,i}\left(\dot{\tilde{p}}_i(t) + v_0\right) - h_i\left(\dot{\tilde{p}}_i(t) + v_0\right)\| \leq L_3 \|\dot{\tilde{p}}_i(t)\|$$

is fulfilled. Furthermore assume that $f_{ctrl,i}$ renders the system (12) ISES with respect to the input

$$w_i(t) = \left[\sum_{j\in\mathcal{N}_i} \tilde{p}_j(t) + e_j(t) \quad \sum_{j\in\mathcal{N}_i} \dot{\tilde{p}}_j(t) + e_{d,j}(t)\right]^\top,$$

i.e.,

$$\|x_i(t)\| \leq \max\left\{k_i \mathrm{e}^{-\lambda_i(t-t_0)}\|x_i(t_0)\|, \gamma_i(\sup_{t_0\leq\nu\leq t}\|w_i(\nu)\|)\right\}$$

with $k_i > 1$, $\lambda_i > 0$ and the ISES gain being bounded by a linear function, i.e., $\gamma_i(r) = c_{\gamma,i} r$.

Remark 1: Arguably, Assumption 1 is a somewhat restrictive assumption. Nevertheless, in Section VII a controller will be presented that satisfies the assumption. Furthermore compared to other ISS stabilizing assumptions, with an ISES stabilizing assumption it is more likely to be able to compute the convergence rate that is used in the design of the triggering rule.

B. Communication architecture

Next, the communication architecture, used to control the platoon will be defined. We will give a remark on what makes this class special and why it fits in this framework. Furthermore some well known architectures that fulfill Assumption 2 will be provided.

Assumption 2: Assume that the communication graph $G = (P, E)$ is a rooted out-branching where the first node is the root, i.e., $v_r = 1$.

It is known that every graph that is a rooted out-branching induces a topological ordering. This means one can order the vertices of the graph such that for all edges $(v_i, v_j) \in E$ it holds that $i < j$, where the interpretation is that the information flow is unidirectional. We will throughout the paper assume that this topological ordering coincides with the physical ordering in the platoon. This assumption on the ordering is only introduced to simplify the notation. The more general case could be covered with the same methods.

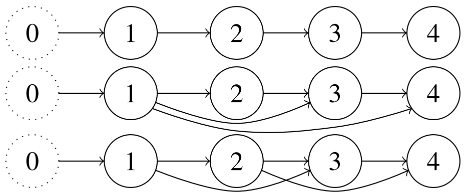

Fig. 1. Three different communication architectures that satisfy Assumption 2.

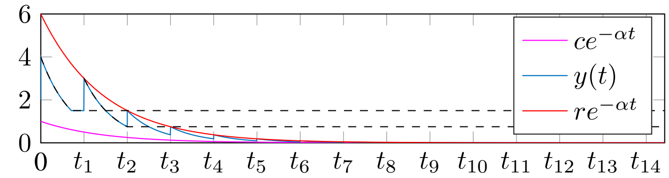

Fig. 2. Figure to express the idea of the proof

Due to the definition in [16] given in Section II one of the key properties of a rooted out-branching is that it does not contain (directed) cycles. This is the main reason why we choose to investigate communication architectures that are a rooted out-branching. If we had allowed for directed cycles we would have had to impose strong conditions on the ISES gain in Assumption 1 to be able to draw conclusions on the convergence of the whole platoon to the desired formation. This is due to the fact that we would have had to use small-gain theorems as in [19] and [20]. Additionally, for the probably most common communication architecture that contains cycles, the so called symmetric bidirectional architecture, the study in [3] suggests to use linear controllers. This is then the special case that is already covered by [14].

Examples for possible communication architectures are given in Fig. 1. The architecture on the left describes the classical predecessor-following communication (PF) whereas the one in the middle shows a situation where information from the predecessor and first vehicle is available (FPF). Both scenarios are investigated in [4] to analyze the disturbance propagation along the string. With these two architectures being the most common ones that satisfy Assumption 2 one can think also of a communication as depicted on the right where one also uses the benefits of communication by not only using nearest neighbor interaction but still keeps the communication radius quite small.

V. Convergence and Zeno exclusion

By introducing Assumption 1 in the previous section it is quite clear that ISES systems play an important role in our approach. From standard ISS systems it is known that they have the property that converging input signals generate a converging state [21]. With the same approach it is straightforward to show that for ISES systems with γ being linear, i.e., the system class used in this work, an exponentially converging input signal generates an exponentially converging state. The next result, initially presented in [15], further specifies this behavior by showing that an exponentially converging input signal with rate α generates an exponentially converging state with the same rate.

Proposition 1: Consider a nonlinear system with state x and input u that is ISES with $\gamma(r) = c_\gamma(r)$ being a linear function, i.e.,

$$\|x(t)\| \leq \max\left\{k\mathrm{e}^{-\lambda(t-t_0)}\|x(t_0)\|, c_\gamma \sup_{t_0\leq\nu\leq t}\|u(\nu)\|\right\}. \quad (14)$$

Assume the input satisfies

$$\|u(t)\| \leq c\mathrm{e}^{-\alpha t}, t \geq 0$$

with $0 < \alpha < \lambda$, $c > 0$. Then it holds that

$$\|x(t)\| \leq r\mathrm{e}^{-\alpha t}, t \geq 0$$

with

$$r = \max\left\{kc_\gamma c\mathrm{e}^{\alpha\tau}, k\|x(0)\|\right\}$$

where

$$\tau = \max\left\{\frac{1}{\lambda}\ln\frac{k\|x(0)\|}{c_\gamma c}, \frac{\ln k}{\lambda-\alpha}\right\}.$$

Proof: Define first the function $y : \mathbb{R} \to \mathbb{R}$ with $y(0) = \|x(0)\|$ and

$$y(t) = \max\left\{ky(m\tau)\mathrm{e}^{-\lambda(t-m\tau)}, c_\gamma c\mathrm{e}^{-\alpha m\tau}\right\} \quad (15)$$

for all $t \in (m\tau, (m+1)\tau]$ with $m \in \mathbb{N}_0$. To illustrate why we introduce $y(t)$ Fig. 2 shows the bound on the norm of $u(t)$ in magenta. Furthermore the dotted black line starting at $t = 0$ represents the bound we derive by using (14) with $t_0 = 0$. If we reuse the bound (14) after $\tau > 0$ we derive the bound represented by the second dotted black line. Therefore iterative use of the bound (14) leads to the blue line. Notice, that the blue line is exactly the function $y(t)$. Thus we know $\|x(t)\| \leq y(t)$ for all $t \geq 0$. It remains to explain the red line. This line represents an upper bound on $y(t)$, and therefore also on $\|x(t)\|$ which is stated as an exponentially decreasing function $r\mathrm{e}^{-\alpha t}$. Therefore to prove the proposition we need to show that under the given assumptions on α and r it holds that $y(t) \leq r\mathrm{e}^{-\alpha t}$ for all $t \geq 0$.

For the proof of the existence of such an upper bound the choice of τ is crucial. Therefore we start with computing a lower bound on τ_1 such that $y(\tau_1) = c_\gamma c$, i.e.,

$$\begin{aligned} k\|x(0)\|\mathrm{e}^{-\lambda\tau_1} &\leq c_\gamma c \\ \Leftrightarrow \qquad \tau_1 &\geq \frac{1}{\lambda}\ln\frac{k\|x(0)\|}{c_\gamma c} \end{aligned} \quad (16)$$

where we assumed $t_0 = 0$ without loss of generality. The next step is to compute a value for $\tau_2 := t_2 - t_1 = t_2 - \tau_1$ that guarantees $y(t_2) = c_\gamma c\mathrm{e}^{-\alpha t_1}$ under the assumption $y(t_1) = c_\gamma c$, i.e.,

$$\begin{aligned} k\underbrace{y(t_1)}_{c_\gamma c}\mathrm{e}^{-\lambda\tau_2} &= c_\gamma c\mathrm{e}^{-\alpha t_1} \\ \Leftrightarrow \qquad \tau_2 &= \frac{\ln k + \alpha t_1}{\lambda}. \end{aligned} \quad (17)$$

If this procedure is iterated we deduce the condition

$$\begin{aligned} \tau_1 &\geq \frac{1}{\lambda}\ln\frac{k\|x(0)\|}{c_\gamma c} \\ \tau_{i+1} &= \frac{\ln k + \alpha\tau_i}{\lambda}, \ \forall i \in \mathbb{N}. \end{aligned} \quad (18)$$

In the next step of the proof we compute one global τ that guarantees the demanded bounds from above, i.e., $y(t_i) = c_\gamma c\mathrm{e}^{-\alpha t_{i-1}}$ instead of different τ_i. To compute such a τ we analyze the conditions from (18). We will demand the sequence of τ_i to be nonincreasing and show that this can be stated as a condition on τ_1, i.e.,

$$\begin{aligned} &\tau_{i+1} \leq \tau_i \\ \Leftrightarrow \quad &\frac{\ln k + \alpha\tau_i}{\lambda} \leq \frac{\ln k + \alpha\tau_{i-1}}{\lambda}. \end{aligned} \tag{19}$$

Thus, the condition

$$\begin{aligned} \tau_2 = \frac{\ln k + \alpha\tau_1}{\lambda} &\leq \tau_1 \\ \Leftrightarrow \quad \tau_1 &\geq \frac{\ln k}{\lambda - \alpha} \end{aligned} \tag{20}$$

guarantees the monotonicity. As a consequence of (16) and (20) we choose

$$\tau = \max\left\{\frac{1}{\lambda}\ln\frac{k\|x(0)\|}{c_\gamma c}, \frac{\ln k}{\lambda - \alpha}\right\} \tag{21}$$

to conclude $y(m\tau) = c_\gamma c\mathrm{e}^{-(m-1)\alpha\tau} = c_\gamma c\mathrm{e}^{\alpha\tau}\mathrm{e}^{-\alpha m\tau}$ and therefore $y(m\tau^+) \leq kc_\gamma c\mathrm{e}^{\alpha\tau}\mathrm{e}^{-\alpha m\tau}$ with $y(m\tau^+) = \lim_{t\to m\tau, t>m\tau} y(t)$. Up to now this bound holds only for $m \geq 1$. In the case of $m = 0$ we have the condition $y(0^+) \leq k\|x(0)\|$. Thus with

$$r = \max\left\{kc_\gamma c\mathrm{e}^{\alpha\tau}, k\|x(0)\|\right\} \tag{22}$$

and $\alpha < \lambda$ we conclude

$$\|x(t)\| \leq r\mathrm{e}^{-\alpha t}. \tag{23}$$

■

The next result will focus on the convergence of the whole platoon. It will give an indication on how the inputs of the interconnection, that is the transmitted errors, should be bounded such that the local controller due to Assumption 1 and the communication architecture due to Assumption 2 are sufficient to guarantee exponential convergence of the platoon to the desired formation.

Corollary 2: Let Assumptions 1 and 2 hold. Assume furthermore that

$$\left\|\begin{bmatrix} e_i(t) \\ e_{d,i}(t) \end{bmatrix}\right\| \leq c_i\mathrm{e}^{-\alpha_i t},\ t \geq 0$$

with $\alpha_i < \lambda_i$ and $\alpha_i \geq \alpha_j$ for $i < j$. Then the state error of the whole platoon converges to zero with rate α_N, i.e., there exists $r > 0$ such that

$$\|x(t)\| \leq r\mathrm{e}^{-\alpha_N t},\ t \geq 0.$$

Proof: We know by definition that $\tilde{p}_0 = \dot{\tilde{p}}_0 = 0$ since the fictitious vehicle defines the reference trajectory. Furthermore we assumed $e_0 = e_{d,0} = 0$. By Assumption 2 we know that vehicle 1 is the root of the platoon without the reference vehicle and therefore vehicle 0 is the only neighboring node of vehicle 0. From these observations we see that the (disturbance) input to vehicle 1, i.e. $w_1(t)$ in the notation of Assumption 1, equals 0 for all times. Thus we can conclude immediately from this assumption and Proposition 1 above that there exists $r_1 > 0$ such that

$$\|x_1(t)\| \leq r_1\mathrm{e}^{-\lambda_1 t}.$$

By Assumption 2 and the one on the topological ordering one concludes that the only neighbor of vehicle 2 can be vehicle 1 and thus the disturbance input to vehicle 2 can be formulated as $w_2(t) = [\tilde{p}_1(t) + e_1(t), \dot{\tilde{p}}_1(t) + e_{d,1}(t)]^\top$. Thus its norm can be bounded by

$$\begin{aligned} \|w_2(t)\| &\leq \left\|\begin{bmatrix} \|\tilde{p}_1(t)\| + \|e_1(t)\| \\ \|\dot{\tilde{p}}_1(t)\| + \|e_{d,1}(t)\| \end{bmatrix}\right\| \leq \left\|\begin{bmatrix} c_1\mathrm{e}^{-\alpha_1 t} + r_1\mathrm{e}^{-\lambda_1 t} \\ c_1\mathrm{e}^{-\alpha_1 t} + r_1\mathrm{e}^{-\lambda_1 t} \end{bmatrix}\right\| \\ &= \sqrt{2}\left(c_1\mathrm{e}^{-\alpha_1 t} + r_1\mathrm{e}^{-\lambda_1 t}\right). \end{aligned} \tag{24}$$

Using the conditions $\alpha_1 < \lambda_1$ and $\alpha_2 \leq \alpha_1$ one can conclude existence of $\tilde{r}_2 > 0$ such that $\|w_2(t)\| \leq \tilde{r}_2\mathrm{e}^{-\alpha_2 t}$. Using this knowledge together with the fact that Assumption 1 holds one can use Proposition 1 to show existence of $r_2 > 0$ such that

$$\|x_2(t)\| \leq r_2\mathrm{e}^{-\alpha_2 t}.$$

With the same arguments as above one can derive, for all $i \in P$,

$$\begin{aligned} \|w_i(t)\| &= \left\|\begin{bmatrix} \sum_{j\in\mathcal{N}_i} \tilde{p}_j(t) + e_j(t) \\ \sum_{j\in\mathcal{N}_i} \dot{\tilde{p}}_j(t) + e_{d,j}(t) \end{bmatrix}\right\| \\ &\leq \sqrt{2}\sum_{j\in\mathcal{N}_i} c_j\mathrm{e}^{-\alpha_j t} + r_j\mathrm{e}^{-\lambda_j t} \end{aligned} \tag{25}$$

where $w_i(t)$ is the disturbance input to vehicle i. Thus, with the condition that $\alpha_j \geq \alpha_i$ for all $j \in \mathcal{N}_i$ due to the conditions in this corollary and the topological ordering we can conclude existence of $\tilde{r}_i > 0$ such that

$$\|w_i(t)\| \leq \tilde{r}_i\mathrm{e}^{-\alpha_i t}$$

and therefore with Proposition 1 and the fact that $\alpha_i < \lambda_i$ we know that $r_i > 0$ exists such that

$$\|x_i(t)\| \leq r_i\mathrm{e}^{-\alpha_i t}$$

for all $i \in P$. Thus we can finally conclude existence of $r > 0$ such that $\|x(t)\| \leq r\mathrm{e}^{-\alpha_N t}$ for $t \geq 0$. ■

Remark 2: The assumption that the topological and the physical ordering coincide is used in this result. If this was not the case one would need an auxiliary function $o : P \to P$ that maps the index of the vehicle, according to the physical ordering, to the value in the topological ordering. The condition in the previous result needs to be modified to $\alpha_i \geq \alpha_j$ for $o(i) < o(j)$ in this case.

The preceding corollary emphasizes that the transmitted error should be bounded such that

$$\|g_i(t)\| := \left\|\begin{bmatrix} e_i(t) \\ e_{d,i}(t) \end{bmatrix}\right\| \leq c_i\mathrm{e}^{-\alpha_i t},\ t \geq 0$$

to guarantee exponential convergence to the desired formation. This clearly indicates to use the event-triggering rule

$$\sigma_i(c_i, \alpha_i, t) = c_i\mathrm{e}^{-\alpha_i t},\ t \geq 0. \tag{26}$$

In the main theorem below we will show that with this event-triggering rule not only exponential convergence of the formation but also exclusion of Zeno behavior will be guaranteed under the assumptions made and another assumption on the estimators that are used to generate $p_{i,est}$ and $\dot{p}_{i,est}$. The assumption will be the following one.

Assumption 3: Assume the estimator is designed such that

$$\begin{bmatrix} p_{i,est}(t_k^i) \\ \dot{p}_{i,est}(t_k^i) \end{bmatrix} = \begin{bmatrix} p_i(t_k^i) \\ \dot{p}_i(t_k^i) \end{bmatrix} \tag{27}$$

and the norm of the estimated acceleration exponentially converges to zero with the same rate as the event-triggering rule, i.e., there exists $l_i > 0$ for all $i \in P$ such that

$$\|\ddot{p}_{i,est}(t)\| \leq l_i \mathrm{e}^{-\alpha_i t}. \tag{28}$$

One example for an estimator that fulfills Assumption 3 is given by $p_{i,est}(t) = p_i(t_k^i) + (t - t_k^i)\dot{p}_i(t_k^i)$, i.e., every agent runs a double-integrator simulation to estimate the state based on the last transmitted state information.

With the event-triggering rule just defined, the assumption on the estimator and the preceding corollary, we are now able to state the main theorem.

Theorem 3: Let Assumptions 1, 2 and 3 hold. Assume furthermore that the event-triggering rule is based on σ_i as in (26) with $c_i > 0$, $\alpha_i < \lambda_i$ and $\alpha_i \geq \alpha_j$ for $i < j$. Then the state error of the whole platoon converges to zero with rate α_N, i.e., there exists $r > 0$ such that

$$\|x(t)\| \leq r\mathrm{e}^{-\alpha_N t}, \ t \geq 0$$

and the inter-event intervals are uniformly lower bounded, i.e., Zeno behavior is excluded.

Proof: The proof of the convergence result is essentially given by Corollary 2 and the design of the event-triggering rule (26). Thus it remains to show the Zeno exclusion. We analyze the norm of the derivative of the transmitted error $g_i(t) := [e_i(t), e_{d,i}(t)]^\top$ in between two arbitrary triggering instances t_k^i and t_{k+1}^i. We can compute with the definition of the transmitted error that

$$\begin{aligned} \|\dot{g}_i(t)\| &= \left\| \begin{bmatrix} \dot{e}_i(t) \\ \dot{e}_{d,i}(t) \end{bmatrix} \right\| = \left\| \begin{bmatrix} \dot{p}_{i,est}(t) - \dot{p}_i(t) \\ \ddot{p}_{i,est}(t) - \ddot{p}_i(t) \end{bmatrix} \right\| \\ &= \left\| \begin{bmatrix} e_{d,i}(t) \\ \ddot{p}_{i,est}(t) - \ddot{p}_i(t) \end{bmatrix} \right\| \\ &\leq \sigma_i(t) + \|\ddot{p}_{i,est}(t)\| + \|\ddot{p}_i(t)\| \end{aligned} \tag{29}$$

where we can make use of Assumption 1 and 3 to conclude

$$\begin{aligned} \|\dot{g}_i(t)\| \leq\ & c_i \mathrm{e}^{-\alpha_i t} + l_i \mathrm{e}^{-\alpha_i t} + L_3 \|\dot{\tilde{p}}_i(t)\| \\ & + L_1 \left(|\mathcal{N}_i| \|\tilde{p}_i(t)\| + \| \sum_{j \in \mathcal{N}_i} \tilde{p}_j(t) + e_j(t) \| \right) \\ & + L_2 \left(|\mathcal{N}_i| \|\tilde{p}_i(t)\| + \| \sum_{j \in \mathcal{N}_i} \dot{\tilde{p}}_j(t) + e_{d,j}(t) \| \right) \\ \leq\ & (c_i + l_i + 2\max\{L_1, L_2\} \tilde{r}_i \\ & + (L_3 + L_1|\mathcal{N}_i| + L_2|\mathcal{N}_i|)\, r_i)\, \mathrm{e}^{-\alpha_i t} \end{aligned} \tag{30}$$

where $|\mathcal{N}_i|$ denotes the number of neighboring agents of i and we used the computation and notation of Corollary 2 in the last inequality. From the previous calculation we thus know that there exists $\tilde{c}_i > 0$ such that

$$\|\dot{g}_i(t)\| \leq \tilde{c}_i \mathrm{e}^{-\alpha_i t}, \ \forall t \in (t_k^i, t_{k+1}^i)$$

and therefore

$$\|g_i(t)\| \leq \tilde{c}_i \mathrm{e}^{-\alpha_i t_k^i}(t - t_k^i), \ \forall t \in [t_k^i, t_{k+1}^i).$$

The condition for the next triggering instance is $\|g_i(t_{k+1}^i)\| = c_i \mathrm{e}^{-\alpha_i t_{k+1}^i}$, i.e., the next triggering is defined by the equality

$$\begin{aligned} \tilde{c}_i \mathrm{e}^{-\alpha_i t_k^i}(t_{k+1}^i - t_k^i) &= c_i \mathrm{e}^{-\alpha_i t_{k+1}^i} \\ \tilde{c}_i (t_{k+1}^i - t_k^i) &= c_i \mathrm{e}^{-\alpha_i (t_{k+1}^i - t_k^i)} \end{aligned} \tag{31}$$

and thus the uniform lower bound on the inter-event intervals, independent of k, can be computed by the intersection of the linear function $\tilde{c}_i(t_{k+1}^i - t_k^i)$ starting at 0 and the exponentially decreasing function $c_i \mathrm{e}^{-\alpha_i(t_{k+1}^i - t_k^i)}$ starting at $c_i > 0$. It can be deduced that this uniform lower bound is greater than 0 and therefore Zeno behavior is excluded. ∎

Note that Zeno exclusion is understood in the sense that no vehicle initiates infinitely many transmissions in finite time. This does not prevent the possibility of two transmissions from different vehicles at the same time. Such problems need to be accounted for by certain network protocols and are relevant issues for further research.

VI. Discussion on Robustness

In Section VII, it will be shown that one approach to cover the nonlinearity in the system dynamics, as quite usual with vehicle dynamics, is a feedback linearizing approach. Since such an approach is sensitive to model mismatches this section presents some brief comments on how the approach needs to be modified if we allow for bounded disturbances to act on each vehicle's dynamics in a way that is represented by the extended model

$$\dot{x}_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_i + \begin{bmatrix} 0 \\ -1 \end{bmatrix} h_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i + Dd_i \tag{32}$$

with $u_i = f_{ctrl,i}$ as in (13) and $h_i(\dot{p}_i)$ as in the previous case. We assume that $D \in \mathbb{R}^{2\times 1}$ and $d_i \in [d_{i,min}, d_{i,max}]$. One possible interpretation of such disturbances is that a controller was designed to cancel out the nonlinearity h_i completely but due to model mismatches this induces a disturbing force acting on the system. Thus a disturbance would act on the acceleration of the vehicle resulting in a possibly non-vanishing disturbance d_i that enters the system via $D = \begin{bmatrix} 0 & 1 \end{bmatrix}^\top$.

To be able to deal with such a scenario one needs to strengthen Assumption 1 such that a robust control design is guaranteed. This can be done by enforcing the controller $f_{ctrl,i}$ to be designed such that all the conditions given in Assumption 1 are fulfilled and furthermore the closed loop of each individual vehicle is ISES with the additional disturbance input d_i, i.e.,

$$\begin{aligned} \|x_i(t)\| \leq \max\Bigg\{ & k_i \mathrm{e}^{-\lambda_i(t-t_0)}\|x_i(t_0)\|, \gamma_i\left(\sup_{t_0 \leq \nu \leq t} \|w_i(\nu)\|\right), \\ & \gamma_{d,i}\left(\sup_{t_0 \leq \nu_d \leq t} \|d_i(\nu_d)\|\right)\Bigg\} \end{aligned} \tag{33}$$

with $k_i > 1$, $\lambda_i > 0$ and $\gamma_{i/d,i}$ being a linear function, i.e., $\gamma_{i/d,i}(r) = c_{\gamma,i/d,i} r$.

A weaker result than Proposition 1 guarantees that bounded inputs generate bounded states for ISS and therefore also ISES systems. Thus with the very same methods as in the proof of Corollary 2 one can show that with such a robust controller and bounded disturbances the state error, i.e., the deviation of the desired formation and velocity is bounded for the whole platoon.

Observe that to achieve this boundedness one needs to guarantee that the transmitted error is bounded as well, whereas convergence to zero is not necessary. This motivates to modify the event-triggering rule here as it has been done in [12] and [14] to

$$\sigma_i(c_{i,o}, c_i, \alpha_i, t) = c_{i,o} + c_i \mathrm{e}^{-\alpha_i t}. \tag{34}$$

with $c_i \geq 0$ and $c_{i,o} > 0$. With this event-triggering rule one can now proceed in a similar way as in the proof of Theorem 3 to exclude Zeno behavior, i.e., one analyzes $\|\dot{g}_i\|$. Using the bounded disturbances, transmitted errors and resulting bounded states it is straightforward to show that also the norm of this derivative is uniformly bounded. Using the same computations as before one arrives at the result that a uniform lower bound on the inter-event intervals is given by the intersection of a linear function with finite slope starting at 0 and the constant $c_{i,o}$. A similar computation has been done in [14] for the case of non-vanishing transmitted errors only.

Thus, the main conclusion of the above discussion is that when considering disturbances acting on the vehicles the control design needs to be robust and the bound on the transmitted error has to be upper bounded by a positive offset. This is translated here to the assumption that the closed-loop system is ISES to the additional disturbance input and the event-triggering rule was complemented with a positive constant. This section provided short comments on robustness issues and sketches necessary modifications to the general approach. To perform a thorough robust analysis the topic of string stability is of great interest. We will comment on that aspect in the concluding section.

VII. Simulation Example

A. Simulation scenario

We investigate a setup containing $N = 10$ vehicles with dynamics as in (5) with $h_i(\cdot) = 0.95^i \arctan(\cdot)$ as an example. Two different communication architectures are considered, one being PF and the other one FPF, i.e., the left and middle architecture depicted in Fig. 1. In the PF case the controller is chosen as

$$\begin{aligned} u_{i,PF}(t) &= -f(p_i(t) - p_{i-1,est}(t) + \Delta) \\ &\quad -g(\dot{p}_i(t) - \dot{p}_{i-1,est}(t)) + h_i(\dot{p}_i(t)) \\ &= -f(\tilde{p}_i(t) - \tilde{p}_{i-1}(t) - e_{i-1}(t)) \\ &\quad -g(\dot{\tilde{p}}_i(t) - \dot{\tilde{p}}_{i-1}(t) - e_{d,i-1}(t)) + h_i(\dot{\tilde{p}}_i(t) + v_0) \end{aligned} \tag{35}$$

for all $i \in P \setminus \{1\}$. The chosen nonlinear functions f and g shown in Fig. 3, are odd, globally Lipschitz, sector

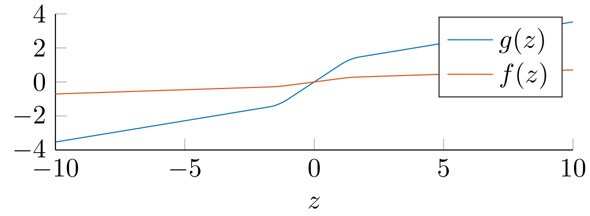

Fig. 3. Nonlinear functions $f(z), g(z)$ in the Simulation example

nonlinearities and the appearance of h_i in (35) indicates a feedback linearization approach. We will comment on the controller for $i = 1$ later. With the controller just introduced we can derive the closed loop equation of each vehicle when using the PF communication architecture as

$$\ddot{\tilde{p}}_i(t) = -f(\tilde{p}_i(t) - w_{i,1}(t)) - g(\dot{\tilde{p}}_i(t) - w_{i,2}(t)) \tag{36}$$

with

$$w_i(t) = \begin{bmatrix} \sum_{j\in\mathcal{N}_i} \tilde{p}_j(t) + e_j(t) \\ \sum_{j\in\mathcal{N}_i} \dot{\tilde{p}}_j(t) + e_{d,j}(t) \end{bmatrix} =: \begin{bmatrix} w_{i,1}(t) \\ w_{i,2}(t) \end{bmatrix}.$$

In [15] it was shown that this closed loop equation satisfies Assumption 1 and by simulations the value for $\lambda_i = 0.05$ was verified.

The controller using the FPF communication architecture follows the same approach, i.e.,

$$\begin{aligned} &u_{i,FPF}(t) \\ &= -f\left(p_i(t) - \frac{1}{2}\left(p_{i-1,est}(t) + p_{1,est} + (i-1)\Delta + \Delta\right)\right) \\ &\quad -g\left(\dot{p}_i(t) - \frac{1}{2}\left(\dot{p}_{i-1,est}(t) + \dot{p}_{1,est}(t)\right)\right) + h_i\left(\dot{p}_i(t)\right) \\ &= -f\left(\tilde{p}_i(t) - \frac{1}{2}\left(\tilde{p}_{i-1}(t) + e_{i-1}(t) + \tilde{p}_1(t) + e_1(t)\right)\right) \\ &\quad -g\left(\dot{\tilde{p}}_i(t) - \frac{1}{2}\left(\dot{\tilde{p}}_{i-1}(t) + e_{d,i-1}(t) + \dot{\tilde{p}}_1(t) + e_{d,1}(t)\right)\right) \\ &\quad +h_i\left(\dot{\tilde{p}}_i(t) + v_0\right) \end{aligned} \tag{37}$$

for all $i \in P \setminus \{1\}$ with the same nonlinear functions f and g, shown in Fig. 3. Thus, the closed loop equation for each vehicle with FPF architecture can be written as

$$\ddot{\tilde{p}}_i(t) = -f(\tilde{p}_i(t) - \frac{1}{2} w_{i,1}(t)) - g(\dot{\tilde{p}}_i(t) - \frac{1}{2} w_{i,2}(t)) \tag{38}$$

with $w_i(t)$ as before. Therefore Assumption 1 is fulfilled with the FPF controller as well. The only difference is the factor $\frac{1}{2}$ in the ISES gain $c_{\gamma,i}$ but for this gain it is only important that it is a linear function which is the case for both controllers.

The controller for the first vehicle is chosen to be the same in both scenarios and uses the information from the fictitious reference vehicle, i.e.,

$$\begin{aligned} u_{1,FPF} = u_{1,PF} &= -f(p_1(t) - p_0(t) + \Delta) \\ &\quad -g(\dot{p}_1(t) - \dot{p}_0(t)) - h_1(\dot{p}_1(t)) \end{aligned} \tag{39}$$

and we can therefore also conclude $\lambda_1 = 0.05$.

As an estimator every agent runs the double-integrator estimator introduced in Section V and the event-triggering rule

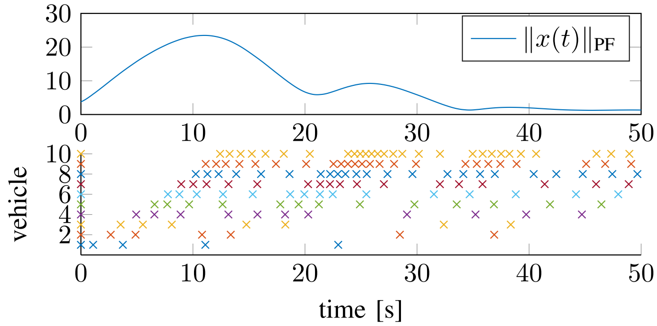

Fig. 4. Norm of the state error and inter-execution times with PF architecture

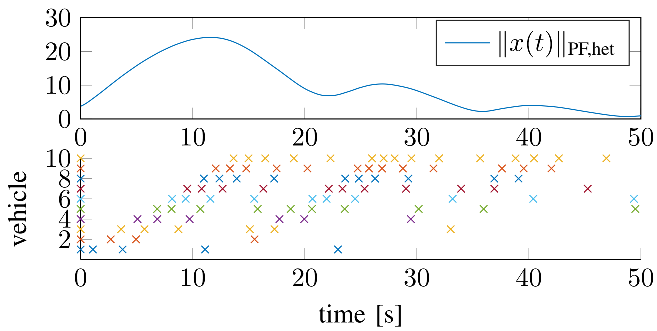

Fig. 5. Norm of the state error and inter-execution times with heterogeneous event-triggering rule (PF)

has the structure as in (26) with $c_i = 1$ and $\alpha_i = 0.045$ except for one modification that appears later. It can be seen that with the bounds on λ_i from the different controllers the assumptions of Theorem 3 are fulfilled.

B. Simulation results

The simulation results in this section are, as mentioned earlier, for a platoon of $N = 10$ vehicles. The vehicles start from velocity 0 and with nonzero spacing errors. The desired velocity is $v_0 = 1$. In Fig. 4 simulation results are shown for the case that the PF controller is used. In the upper subplot the norm of the state error is shown. From Theorem 3 it is guaranteed that this norm converges to zero and the result indicates that this holds true. In the lower subplot every time instance when a vehicle sends information to its following vehicle is marked. When having a closer look on these time instances one can see, that for vehicles that are quite far behind the inter-event times are quite dense in the beginning. One possibility to tackle this behavior is to use heterogeneous event-triggering rules where the value of α_i decreases along the platoon according to $\alpha_i = 0.045 \cdot 0.85^{i-1}$. This idea was already introduced in [15] and still fulfills all assumptions of Theorem 3. In Fig. 5 it can be seen that the inter-event times of the vehicles that are at the end of the platoon are now not as dense anymore in the beginning. The price to pay for this benefit is a transient behavior that is a little bit worse than with the original event-triggering rule. This indicates that the design of the event-triggering rule induces a trade-off between performance and communication.

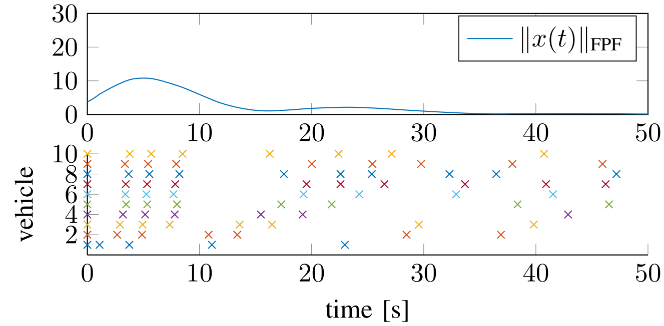

Fig. 6. Norm of the state error and inter-execution times with FPF architecture

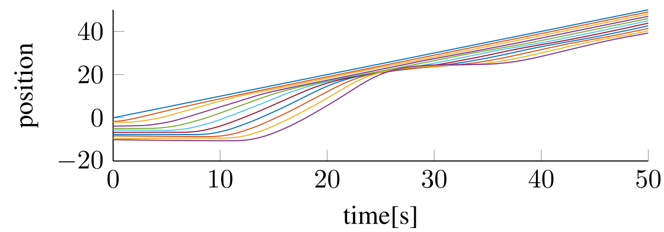

Fig. 7. Position of all vehicles over time with PF architecture

In Fig. 6 the results using the FPF architecture are given. One can see that this architecture helps to break up the trade-off that was described before. The results indicate that on the one hand the performance is immensely improved while on the other hand the inter-event times are much larger as well. This clearly shows that the extension towards other communication architectures presented in this work is a large advantage. The superiority of the FPF controller over the PF controller becomes even more clear when investigating Fig. 7 and Fig. 8. In these results one gets a feeling for the meaning of the state error in the platooning scenario. The two figures both show the positions of all vehicles, including the fictitious leader, over time. One can see that with the PF architecture in this simulation collisions between the last vehicles would occur whereas using the FPF architecture no collisions between the vehicles occur. Note that the possibility of collisions is not due to the event-based sampling concept but due to the fact that the controllers do not run mechanisms for collision avoidance. Thus a worse performance, i.e. a larger deviation from the desired distance, is related to a higher possibility for collisions.

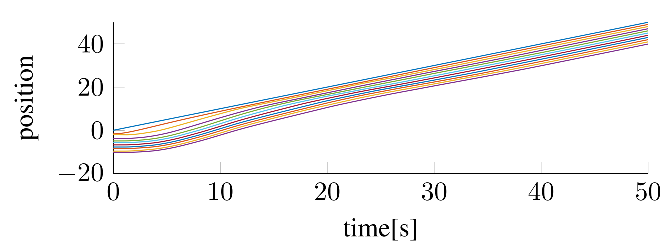

Fig. 8. Position of all vehicles over time with FPF architecture

VIII. Conclusion

In this article we presented a framework for decentralized control of platoons of vehicles using event-based communication. The approach presented is general in the sense that the vehicle dynamics are not restricted to have double-integrator dynamics, and in fact can be nonlinear and heterogeneous. The same holds for the controllers that are applied. The class of controllers that can be used is specified by an assumption that needs to be fulfilled by every controller and that can be checked locally. The other key part of the paper was the characterization of the communication architecture. It was deduced that communication architectures that can be represented as a rooted out-branching can be controlled with the same techniques and event-triggering rules as in the special case of [15]. This is an important extension as already indicated in the simulation results. In particular when looking to future developments this is important since a natural question arising from the paper is if it is possible to guarantee string stability and for that reason it is important to be able to deal with more complex communication architectures. Another point that remains open is how to tackle network induced imperfections such as communication delays and constraints that can lead to lost packets as well as more complex reference profiles. These points are important issues for further research.

References

[1] J. K. Hedrick, D. McMahon, V. Narendran, and D. Swaroop, "Longitudinal vehicle concontrol desing for IVHS systems," in *Proc. American Control Conf. (ACC)*, Boston, MA, USA, 1991, pp. 3107–3112.

[2] D. Swaroop and J. Hedrick, "String stability of interconnected systems," *IEEE Trans. Autom. Control*, vol. 41, no. 3, pp. 349–357, 1996.

[3] H. Hao and P. Barooah, "Stability and robustness of large platoons of vehicles with double-integrator models and nearest neighbor interaction," *Int. J. Robust and Nonlinear Control*, vol. 23, pp. 2097–2122, 2013.

[4] P. Seiler, A. Pant, and K. Hedrick, "Disturbance propagation in vehicle strings," *IEEE Trans. Autom. Control*, vol. 49, no. 10, pp. 1835–1841, 2004.

[5] L. Y. Wang, A. Syed, G. Yin, A. Pandya, and H. Zhang, "Coordinated vehicle platoon control: Weighted and constrained consensus and communication network topologies," in *Proc. 51st IEEE Conf. Decision and Control (CDC)*, Maui, HI, USA, 2012, pp. 4057–4062.

[6] S. Öncü, N. van de Wouw, W. P. M. H. Heemels, and H. Nijmeijer, "String stability of interconnected vehicles under communication constraints," in *Proc. 51st IEEE Conf. Decision and Control (CDC)*, Maui, HI, USA, 2012, pp. 2459–2464.

[7] K.-E. Årzén, "A simple event-based PID controller," in *Proc. of the 14th IFAC World Congress*, Beijing, P.R. China, 1999, pp. 423–428.

[8] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks," *IEEE Trans. Autom. Control*, vol. 52, no. 9, pp. 1680–1685, 2007.

[9] K. H. Johansson, M. Egerstedt, J. Lygeros, and S. Sastry, "On the regularization of zeno hybrid automata," *Syst. Contr. Lett.*, vol. 38, no. 3, pp. 141–150, 1999.

[10] X. Wang and M. D. Lemmon, "Event-triggering in distributed networked control systems," *IEEE Trans. Autom. Control*, vol. 56, no. 3, pp. 586–601, 2011.

[11] D. V. Dimarogonas, E. Frazzoli, and K. H. Johansson, "Distributed event-triggered control for multi-agent systems," *IEEE Trans. Autom. Control*, vol. 57, no. 5, pp. 1291–1297, 2012.

[12] G. S. Seyboth, D. V. Dimarogonas, and K. H. Johansson, "Event-based broadcasting for multi-agent average consensus," *Automatica*, vol. 49, no. 1, pp. 245–252, 2013.

[13] A. Adaldo, F. Alderisio, D. Liuzza, G. Shi, D. Dimarogonas, M. di Bernardo, and K. Johansson, "Event-triggered pinning control of switching networks," *IEEE Trans. Control Netw. Syst.*, vol. 2, no. 2, pp. 204–213, 2015.

[14] S. Linsenmayer and D. V. Dimarogonas, "Event-triggered control for vehicle platooning," in *Proc. American Control Conf. (ACC)*, Chicago, IL, USA, 2015, pp. 3101–3106.

[15] S. Linsenmayer, D. V. Dimarogonas, and F. Allgöwer, "Nonlinear event-triggered platooning control with exponential convergence," in *Proc. 5th IFAC Workshop on Distributed Estimation and Control in Networked Systems (NecSys)*, Philadelphia, PA, USA, 2015, pp. 138 – 143.

[16] M. Mesbahi and M. Egerstedt, *Graph theoretic methods in multiagent networks*. Princeton University Press, 2010.

[17] E. D. Sontag, "Smooth stabilization implies coprime factorization," *IEEE Trans. Autom. Control*, vol. 34, no. 4, pp. 435–443, 1989.

[18] L. Grüne, E. D. Sontag, and F. R. Wirth, "Asymptotic stability equals exponential stability, and ISS equals finite energy gain — if you twist your eyes," *Syst. Contr. Lett.*, vol. 38, no. 2, pp. 127–134, 1999.

[19] Z. P. Jiang, A. R. Teel, and L. Praly, "Small-gain theorem for ISS systems and applications," *Mathematics of Control, Signals and Systems*, vol. 7, no. 2, pp. 95–120, 1994.

[20] S. Dashkovskiy, B. S. Rüffer, and F. R. Wirth, "An ISS small gain theorem for general networks," *Mathematics of Control, Signals, and Systems*, vol. 19, no. 2, pp. 93–122, 2007.

[21] E. D. Sontag, "Remarks on stabilization and input-to-state stability," in *Proc. 28th IEEE Conf. Decision and Control (CDC)*, Tampa, FL, USA, 1989, pp. 1376–1378.

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