Virtual leader and distance based formation control with funnel constraints

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***Abstract*— In this work we pursue the problem of distributed distance-based formation control with prescribed transient and steady state behavior under connectivity and collision avoidance constraints. In addition to the distance-based formation a subset of agents is enforced to reach a desired distance from a dynamic virtual leader with bounded velocity under prescribed transient and steady state constraints while preserving connectivity with the virtual leader and a desired safety distance. We show that the aforementioned objectives can be ensured when the communication graph is an undirected tree and a single agent has access to the virtual leader's state information. Under these conditions we propose a model-free control law for known nonlinear systems as also an adaptive controller when the nonlinear dynamics of the agents are considered unknown and approximated using neural networks. Simulation results verify the effectiveness of the proposed controller both for known and unknown dynamics.**

***Index Terms*— adaptive control, formation control, multi-agent systems, prescribed performance control**

I. Introduction

MULTI-agent systems have been widely considered in a plethora of applications such as in aerospace and agriculture as well as in industrial and underwater settings. In such applications the multi-agent team is often asked to move in a desired formation while tracking a desired trajectory in order to perform a variety of cooperative tasks under limited communication and/or limited actuation.

Distributed formation control [1] has been a well-studied subject of research for linear [2], [3], nonlinear [4], input-affine [5] and non-holonomic systems [6]. Of particular interest in this work is the problem of distributed distance-based formation control for undirected graphs. Distance-based formation has been mainly studied in the context of rigidity theory [7] and distributed control laws were designed to ensure a desired geometric shape for the multi-agent system [8]–[10]. Local asymptotic convergence to the desired formation has been shown in [11], where tree graph structures were considered while global asymptotic stability has been investigated in [12], [13] for systems with three and four agents. In addition to distance-based formation control, satisfaction of global objectives such as reference tracking has been considered in [14]–[16]. As shown in [17] the tracking objective leads always to formation tracking error unless agents have access or are capable of estimating the reference state. To that end, a reference estimator approach has been proposed in [18]. Unknown yet bounded references have been considered in [19] while in [20] more complex, second order nonlinear dynamics have been considered and neural network structures (NNs) have been introduced to estimate unknown nonlinearities of the dynamical agents.

In the majority of the aforementioned approaches simplified first and second order linear dynamics were considered while other transient constraints such as connectivity maintenance and collision avoidance were not addressed. In addition, convergence to the desired formation under the aforementioned control laws is unconstrained, i.e., it can be achieved with arbitrarily large overshoot and low speed. Prescribed performance control (PPC) on the other hand, initially proposed for reference tracking in [21], is a control framework that allow us to impose constraints on the desired transient and steady state behavior towards convergence to the pre-specified formation. In the context of multi-agent control, PPC has been considered indicatively in [22], [23] for position-based formation tracking with and without connectivity maintenance constraints respectively, in [24] for global connectivity maintenance, in [25] for leader-follower consensus of high order unknown nonlinear systems, and in [26] for consensus of second order linear systems with asymptotic stability guarantees. Closer to our work is the problem of distance-based formation control and distance-based formation tracking with prescribed performance, collision avoidance and connectivity maintenance constraints studied in [27]–[33]. Distance-based formation control for rigid bodies and tree graphs has been considered in [31] while rigidity based formation control and centroid trajectory tracking for single-integrator systems has been studied in [28]. In [27], [30] simultaneous leader-follower formation control and trajectory tracking for rigid graphs has been investigated for UAVs and underwater robots, respectively. In [29] the authors address the problem of fixed-time, rigidity-based formation control for nonholonomic systems while in [32] distance-based formation as well as trajectory tracking under collision avoidance and connectivity constraints is shown for a team of nonholonomic vehicles when a path communication graph is considered and the reference trajectory information is accessed only by the leader. In [33] the authors consider the distance-based formation control problem ensuring that the desired distances remain within a constant, known interval when non-minimally or non-infinitesimally

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rigid graphs that admit an appropriate decomposition involving a minimally rigid subgraph are considered.

Motivated by multi-agent applications under high-level objectives in which a set of collaborative tasks needs to be performed while a subset of agents need to reach desired areas within given time intervals [34], [35], in this work we consider the problem of asymptotically reaching a desired distance with respect to a dynamic virtual leader in addition to distance-based formation control under prescribed performance constraints. Contrary to existing approaches [27], [28], [30], [32], where unconstrained asymptotic reference tracking is considered as a secondary objective, here a known subset of agents is asked to asymptotically reach a desired distance from a virtual leader with bounded trajectory and bounded yet unknown dynamics under prescribed transient and steady state constraints. In addition, connected agents are enforced to reach a desired distance-based formation with pre-specified performance but without further restrictions on the geometric shape of the formation and consequently on the number of communication links as in rigidity-based approaches [27]–[30], [33]. By appropriately tuning the parameters of the time-varying functions encoding the desired transient and steady state performance we can ensure that the virtual leader is in a safety distance and within the sensing range of the selected agents while connected agents maintain connectivity and avoid collisions at all times. First, we consider known nonlinear dynamics and propose a control law that ensures the satisfaction of the time-varying constraints as well as ultimate boundedness of all closed loop signals using Lyapunov stability arguments. It can be shown that the aforementioned objectives are satisfied when the initial sensing graph is a tree and the set of selected agents responsible for asymptotically reaching a desired distance with respect to the virtual structure is a singleton. Finally, we consider the case of unknown nonlinear dynamics. Assuming that the unknown functions are continuous, we approximate the unknown nonlinearities using linearly parameterized NNs and propose a nonlinear adaptive controller that ensures the satisfaction of the objectives at all times as well as the ultimate boundedness of all closed-loop signals.

The remainder of the paper is organized as follows: Section II includes notation, preliminary information and the problem formulation. Section III introduces the proposed control scheme assuming that the dynamics of the agents are known and Section IV introduces the adaptive law when unknown nonlinearities are considered. Section V includes a simulation study while conclusions and future work are summarized in Section VI.

II. Preliminaries and Problem Formulation

$A \otimes B$ denotes the Kronecker product of A, B as defined in [36, Ch. 4]. The vector of all ones is denoted by $\mathbf{1}_d \in \mathbb{R}^d$ and the identity matrix of dimension n by I_n. The cardinality of a set $\mathcal{V}$ is denoted by $|\mathcal{V}|$. A matrix $E \in \{0,1\}^{m\times n}, m \geq n$, is called a *selection* matrix if it satisfies the following: 1) for every j there exists exactly one i such that $E(i,j) = 1$. In addition, if $E(i,j) = 1$, then $E(i,j') = 0$, for every $j \neq j'$. The minimum eigenvalue of a matrix $A \in \mathbb{R}^{n\times n}$ is denoted by $\lambda_{\min}(A)$ and $A \succ 0$ denotes that A is a positive definite matrix. Given $Q \succ 0$ the weighted Euclidean norm of $\mathbf{x} \in \mathbb{R}^n$ is given by $\|\mathbf{x}\|_Q := \sqrt{\mathbf{x}^T Q \mathbf{x}}$. If $Q = I_n$, we write $\|\mathbf{x}\|$. The block diagonal matrix of $A_1, \ldots, A_n$ is denoted by $\operatorname{diag}(A_1, \ldots, A_n)$.

A. Graph Theory

An *undirected* graph G is defined as a pair $G = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \ldots, R\} \subset \mathbb{N}$ is a finite set of nodes and $\mathcal{E} \subseteq \{(r, r') \in \mathcal{V} \times \mathcal{V} : r \neq r'\}$. The *adjacency matrix* $\widetilde{A}$ of G is the $R \times R$ symmetric matrix whose elements $\tilde{a}_{rr'}, r, r' \in \mathcal{V}$ are defined as follows: $\tilde{a}_{rr'} = 1$, if $(r, r') \in \mathcal{E}$ and $\tilde{a}_{rr'} = 0$, otherwise. The degree $\tilde{d}_r$ of a node r is given by $\tilde{d}_r = \sum_{r'\in\mathcal{V}} \tilde{a}_{rr'}$. Then, the *degree matrix* $\widetilde{\Delta}$ of G is defined as the diagonal matrix $R \times R$, containing the degrees of the nodes of G on the diagonal, i.e., $\widetilde{\Delta} = \operatorname{diag}(\tilde{d}_1, \ldots, \tilde{d}_R)$. Based on these matrices, we can define the *Laplacian* matrix L of G as $L = \widetilde{\Delta} - \widetilde{A}$. A *path* is a sequence of edges connecting two distinct vertices. A graph is *connected*, if there exists a path between any pair of vertices. Given a numbering $k \in \mathcal{M} = \{1, \ldots, M\}$ of the edges $\epsilon_k \in \mathcal{E}$, after assigning an orientation to each edge in G we may define the *incidence matrix* $D = [\hat{d}_{rk}]$ of G as follows: $\hat{d}_{rk} = 1$, if the node r is the head of edge k, $\hat{d}_{rk} = -1$, if r is the tail of edge k and $\hat{d}_{rk} = 0$, otherwise. The *edge Laplacian* matrix of G is given by $L_e = D^T D$.

B. Prescribed Performance Control

Prescribed Performance Control [21] is a control framework ensuring that the tracking error $e : \mathbb{R}_{\geq 0} \to \mathbb{R}$ remains at all times within a bounded region determined by a-priori known time-varying functions that impose a prescribed transient and steady state performance. Specifically, in PPC $-g(t) < e(t) < g(t)$ should hold for every $t \geq 0$, where $g(t)$ is a smooth, bounded, and monotonically decreasing function satisfying $\lim_{t\to\infty} g(t) = g_\infty > 0$. In the following $g(t)$ is chosen as $g(t) = (g_0 - g_\infty)e^{-lt} + g_\infty$, where g_0, g_∞, l are positive parameters chosen such that $|e(0)| < g_0$ and $g_\infty < g_0$. Note that the value of g_∞ determines the maximum allowable size of the tracking error at steady state and can be chosen arbitrarily small while the parameter l determines a lower bound on the speed of convergence of the tracking error. Hence, by properly selecting the parameters of $g(t)$ we can impose a desired transient and steady state behavior of $e(t)$.

C. Problem Formulation

In this work we consider a multi-agent team of R agents whose dynamics are given by:

$$\dot{\mathbf{x}}_r = f_r(\mathbf{x}_r) + h_r(\mathbf{x}_r)\mathbf{w}_r + \mathbf{u}_r, \tag{1}$$

where $f_r : \mathbb{R}^n \to \mathbb{R}^n$, $h_r : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ are locally Lipschitz functions, $\mathbf{x}_r, \mathbf{u}_r \in \mathbb{R}^n$, is the state and input of the r-th agent, respectively, $\mathbf{w}_r \in \mathbb{R}^m$ is a piece-wise continuous and bounded disturbance acting on the r-th agent, and $r \in \mathcal{V} := \{1, \ldots, R\}$. We denote the stacked vector of the states,inputs and disturbances of all the agents in $\mathcal{V}$ by $\mathbf{x} = \begin{bmatrix}\mathbf{x}_1^T & \dots & \mathbf{x}_R^T\end{bmatrix}^T \in \mathbb{R}^{Rn}$, $\mathbf{u} = \begin{bmatrix}\mathbf{u}_1^T & \dots & \mathbf{u}_R^T\end{bmatrix}^T \in \mathbb{R}^{Rn}$, $\mathbf{w} = \begin{bmatrix}\mathbf{w}_1^T & \dots & \mathbf{w}_R^T\end{bmatrix}^T \in \mathbb{R}^{Rm}$, respectively. The dynamics of the multi-agent team are thus given as follows:

$$\dot{\mathbf{x}} = f(\mathbf{x}) + h(\mathbf{x})\mathbf{w} + \mathbf{u}, \tag{2}$$

where $f(\mathbf{x}) = \left[f_1^T(\mathbf{x}_1), \dots, f_R^T(\mathbf{x}_R)\right]^T$ and $h(\mathbf{x}) = \mathrm{diag}(h_1(\mathbf{x}_1), \dots, h_R(\mathbf{x}_R))$. Each agent r is assumed to be able to measure its state $\mathbf{x}_r$ and has limited sensing range $s_r < \infty$. In particular, each agent r is assumed capable of communicating with other agents within its sensing range s_r and exchanging position information. Here, the communication graph is modelled as an undirected, time-varying graph $G = (\mathcal{V}, \mathcal{E}(t))$, where $\mathcal{V}$ is the node set and $\mathcal{E}(t) \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set at time t, where $\epsilon = (r, r') \in \mathcal{E}(t) \Leftrightarrow \|\mathbf{x}_r(t) - \mathbf{x}_{r'}(t)\| \leq \min(s_r, s_{r'})$. In the remaining of this work, $\mathcal{E}$ will denote the edge set of G at $t = 0$, i.e., $\mathcal{E} := \mathcal{E}(0)$. Here, we make the following assumption on the communication graph G at $t = 0$:

Assumption 1. *The graph $G = (\mathcal{V}, \mathcal{E})$ is connected.*

Assumption 1 implies that $\|\mathbf{x}_{r_k}(0) - \mathbf{x}_{r_k'}(0)\| \leq d_{k,\max}$, where $d_{k,\max} = \min(s_{r_k}, s_{r_k'})$ and $\epsilon_k = (r_k, r_k') \in \mathcal{E}$, $k \in \mathcal{M} := \{1, \dots, M\}$, $|\mathcal{E}| := M$. Specifically, the agents are assumed to satisfy $\|\mathbf{x}_{r_k}(0) - \mathbf{x}_{r_k'}(0)\| < d_{k,\max}$, for every $k \in \mathcal{M}$. In addition, the agents are assumed to be collision-free at time $t = 0$, i.e., $\|\mathbf{x}_{r_k}(0) - \mathbf{x}_{r_k'}(0)\| > d_{k,\min}$, for every $\epsilon_k = (r_k, r_k') \in \mathcal{E}$, where $d_{k,\min} > 0$, i.e.,

$$d_{k,\min} < \|\mathbf{x}_{r_k}(0) - \mathbf{x}_{r_k'}(0)\| < d_{k,\max}, \ \forall k \in \mathcal{M}. \tag{3}$$

Remark 1. *Although the actual communication graph is time-varying, the proposed controller requires only knowledge of the initial communication graph $G(\mathcal{V}, \mathcal{E}(0))$. As will be shown in Theorem 1 and Theorem 2, whenever Assumption 1 is satisfied, the proposed controllers* (18) *and* (27) *ensure under mild conditions that the initial communication links in $\mathcal{E}(0)$ will not be lost and thus guarantee connectivity maintenance for every $t \geq 0$.*

Each edge $\epsilon_k \in \mathcal{E}$ is assigned a scalar parameter $d_k > 0$ which expresses the desired distance among agents $r_k, r_{k'}$ in the formation. In the following we assume that $d_{k,\min} < d_k < d_{k,\max}$, for every $k \in \mathcal{M}$. Given $d_k, k \in \mathcal{M}$, the desired formation is called feasible if the set $\Phi_F := \{\mathbf{x} \in \mathbb{R}^{Rn} : \|\mathbf{x}_{r_k} - \mathbf{x}_{r_k'}\| = d_k$, for every $k \in \mathcal{M}\}$ is non-empty.

In addition to the distance formation, a subset of agents $\mathcal{V}_r \subset \mathcal{V}$ with $\mathcal{V}_r \neq \emptyset$ needs to stay $\bar{d}_r$ close to a dynamic virtual leader whose state is denoted by $\mathbf{x}_V \in \mathbb{R}^n$, where $\bar{d}_r > 0$, for every $r \in \mathcal{V}_r$. The virtual leader might be a real or fictitious agent who has information on the trajectory that agents in $\mathcal{V}_r$ need to closely follow. As opposed to other approaches [28] which require the whole team to move in a formation towards a given direction, here only a subset of agents needs to closely follow the virtual leader. This could be beneficial in applications where only a subset of agents needs to reach close to a desired set of regions while the rest agents need only to stay close to their neighbors as for example in information gathering [30] or cooperative transportation [37]. Here, the dynamics of the virtual leader are unknown but it is assumed that the leader moves with a bounded velocity as depicted in the following assumption:

Assumption 2. *The trajectory of the virtual leader $\mathbf{x}_V : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ is continuous and bounded, i.e., $\|\mathbf{x}_V\| \leq \varrho$, where $\varrho > 0$. In addition, there exists $\varsigma > 0$, such that $\|\dot{\mathbf{x}}_V\| \leq \varsigma$, for every $t \geq 0$, where $\dot{\mathbf{x}}_V$ is a piecewise-continuous function of t.*

Let $\Phi_T := \{\mathbf{x} \in \mathbb{R}^{Rn} : \|\mathbf{x}_r - \mathbf{x}_V\| = \bar{d}_r, r \in \mathcal{V}_r\}$. We will refer to the distance formation task with respect to the leader as the *leader formation* problem hereafter. Note that the leader formation problem is feasible if $\Phi_T \neq \emptyset$. Throughout the paper we assume that the joint problem of distance-based formation and leader formation is feasible, i.e., $\Phi_F \cap \Phi_T \neq \emptyset$. Initially, we assume that:

$$\bar{d}_{r,\min} < \|\mathbf{x}_r(0) - \mathbf{x}_V(0)\| < s_r, \tag{4}$$

where $\bar{d}_{r,\min} < \bar{d}_r < s_r, r \in \mathcal{V}_r$, i.e., the virtual leader is within the sensing range of agent r and at a safety distance strictly greater than $\bar{d}_{r,\min}$. Based on the above we may state the problem considered in this work as follows:

Problem 1. *Consider a team of R agents with system dynamics defined by* (1) *and let Assumptions 1-2 hold. Given a set $\mathcal{V}_r \subset \mathcal{V}, \mathcal{V}_r \neq \emptyset$ and parameters $d_k, d_{k,\min}, d_{k,\max}, k \in \mathcal{M}$ and $\bar{d}_{r,\min}, \bar{d}_r, r \in \mathcal{V}_r$ such that $d_{k,\min} < d_k < d_{k,\max}$, $k \in \mathcal{M}$, $\bar{d}_{r,\min} < \bar{d}_r < s_r, r \in \mathcal{V}_r$, and $\Phi_T \cap \Phi_F \neq \emptyset$, design the absolute control input $\mathbf{u}_r(t), r \in \mathcal{V}$ using local information such that:*

$$\begin{aligned}
\|\mathbf{x}_{r_k}(t) - \mathbf{x}_{r_k'}(t)\| &\in (d_k - \sigma_k, d_k + \sigma_k), \ k \in \mathcal{M}, \ \textit{as } t \to \infty \\
\|\mathbf{x}_r(t) - \mathbf{x}_V(t)\| &\in (\bar{d}_r - \bar{\sigma}_r, \bar{d}_r + \bar{\sigma}_r), \ r \in \mathcal{V}_r, \ \textit{as } t \to \infty \\
\|\mathbf{x}_{r_k}(t) - \mathbf{x}_{r_k'}(t)\| &\in (d_{k,\min}, d_{k,\max}), \ k \in \mathcal{M}, \ \forall t \geq 0 \\
\|\mathbf{x}_r(t) - \mathbf{x}_V(t)\| &\in (\bar{d}_{r,\min}, s_r), \ r \in \mathcal{V}_r, \ \forall t \geq 0
\end{aligned}$$

where $\sigma_k, k \in \mathcal{M}$, $\bar{\sigma}_r, r \in \mathcal{V}_r$ are small, positive tuning parameters.

III. Control Design for Known Dynamics

In this section we introduce the distributed formation protocol for each agent assuming that agents $r \in \mathcal{V}_r$ have access to relative position information with respect to the virtual leader, i.e., to $\mathbf{x}_r - \mathbf{x}_V$. To simplify notation we will omit the dependence of different signals on t, whenever it is clear from context. Define the error functions $e_k(t), k \in \mathcal{M}$ and $\xi_r(t), r \in \mathcal{V}_r$ as follows:

$$e_k := \|\mathbf{x}_{r_k} - \mathbf{x}_{r_k'}\|^2 - d_k^2, \tag{5a}$$
$$\xi_r := \|\mathbf{x}_r - \mathbf{x}_V\|^2 - \bar{d}_r^2. \tag{5b}$$

Differentiating (5a)-(5b) we have:

$$\begin{aligned}
\dot{e}_k &= 2(\mathbf{x}_{r_k} - \mathbf{x}_{r_k'})^T(\dot{\mathbf{x}}_{r_k} - \dot{\mathbf{x}}_{r_k'}), \\
\dot{\xi}_r &= 2(\mathbf{x}_r - \mathbf{x}_V)^T(\dot{\mathbf{x}}_r - \dot{\mathbf{x}}_V).
\end{aligned}$$

Let the stacked vectors $\mathbf{e} := \begin{bmatrix} e_1 & \dots & e_M \end{bmatrix}^T$ and $\boldsymbol{\xi} := \begin{bmatrix} \xi_{r_1} & \dots & \xi_{r_\pi} \end{bmatrix}^T$, where $\pi = |\mathcal{V}_r|$. Then, the derivatives are found by:

$$\dot{\mathbf{e}} = F_1(\mathbf{x})(D^T \otimes I_n)\dot{\mathbf{x}}, \tag{6a}$$
$$\dot{\boldsymbol{\xi}} = F_2(\mathbf{x}, \mathbf{x}_V)((E^T \otimes I_n)\dot{\mathbf{x}} - \mathbf{1}_\pi \otimes \dot{\mathbf{x}}_V), \tag{6b}$$

where $D \in \mathbb{R}^{R\times M}$ is the incidence matrix of G, $E \in \{0,1\}^{R\times\pi}$ is a selection matrix with $E(r_i, i) = 1, E(r', i) = 0, r' \neq r_i$ and $E(r_i, j) = 0$, for $j \neq i$, $i \in \{1, \dots, \pi\}$ and $F_1(\mathbf{x}) \in \mathbb{R}^{M\times Mn}, F_2(\mathbf{x}, \mathbf{x}_V) \in \mathbb{R}^{\pi\times\pi n}$ are the matrices defined as follows:

$$F_1(\mathbf{x}) := 2\begin{bmatrix} (\mathbf{x}_{r_1} - \mathbf{x}_{r_1'})^T & \dots & \mathbf{0}_{1\times n} \\ \vdots & \ddots & \vdots \\ \mathbf{0}_{1\times n} & \dots & (\mathbf{x}_{r_M} - \mathbf{x}_{r_M'})^T \end{bmatrix},$$
$$F_2(\mathbf{x}, \mathbf{x}_V) := 2\begin{bmatrix} (\mathbf{x}_{r_1} - \mathbf{x}_V)^T & \dots & \mathbf{0}_{1\times n} \\ \vdots & \ddots & \vdots \\ \mathbf{0}_{1\times n} & \dots & (\mathbf{x}_{r_\pi} - \mathbf{x}_V)^T \end{bmatrix}.$$

In order to ensure the distance-based formation and leader formation objectives as also collision avoidance and connectivity maintenance among agents and the virtual leader, we will enforce the following constraints that prescribe a desired transient and steady state behavior:

$$-\eta_{k,\min} g_k(t) < e_k(t), < \eta_{k,\max} g_k(t) \tag{7a}$$
$$-\zeta_{r,\min}\gamma_r(t) < \xi_r(t) < \zeta_{r,\max}\gamma_r(t), \tag{7b}$$

for every $k \in \mathcal{M}$, $r \in \mathcal{V}_r$, where the functions $g_k : \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$, $\gamma_r : \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$, are smooth, positive, strictly decreasing functions defined as:

$$g_k(t) := (1 - g_{k,\infty})e^{-l_k^g t} + g_{k,\infty}, \tag{8a}$$
$$\gamma_r(t) := (1 - \gamma_{r,\infty})e^{-l_r^\gamma t} + \gamma_{r,\infty}, \tag{8b}$$

where $g_{k,\infty} := \frac{\bar{g}_{k,\infty}}{\max(\eta_{k,\min},\eta_{k,\max})} > 0$, $\gamma_{r,\infty} := \frac{\bar{\gamma}_{r,\infty}}{\max(\zeta_{r,\min},\zeta_{r,\max})} > 0$, $\bar{g}_{k,\infty}, l_k^g, \bar{\gamma}_{r,\infty}, l_r^\gamma \in \mathbb{R}_{>0}$ are positive parameters determining the desired transient and steady state behavior and $\eta_{k,\min}, \eta_{k,\max}, \zeta_{r,\min}, \zeta_{r,\max} \in \mathbb{R}_{\geq 0}$ are parameters chosen to ensure the satisfaction of the safety and connectivity maintenance constraints within the multi-agent system and between the multi-agent system and the virtual leader. In particular following [31] we choose $\eta_{k,\min}, \eta_{k,\max}, k \in \mathcal{M}$ and $\zeta_{r,\min}, \zeta_{r,\max}, r \in \mathcal{V}_r$ as follows:

$$\eta_{k,\min} := d_k^2 - d_{k,\min}^2, \qquad \eta_{k,\max} := d_{k,\max}^2 - d_k^2, \tag{9a}$$
$$\zeta_{r,\min} := \bar{d}_r^2 - \bar{d}_{r,\min}^2, \qquad \zeta_{r,\max} := s_r^2 - \bar{d}_r^2. \tag{9b}$$

Note that since $d_{k,\min} < d_k < d_{k,\max}$, $k \in \mathcal{M}$, $\bar{d}_{r,\min} < \bar{d}_r < s_r, r \in \mathcal{V}_r$, hold, then given (3)-(4) it follows that:

$$-\eta_{k,\min} g_k(0) < e_k(0) < \eta_{k,\max} g_k(0), \tag{10a}$$
$$-\zeta_{r,\min}\gamma_r(0) < \xi_r(0) < \zeta_{r,\max}\gamma_r(0), \tag{10b}$$

for every $k \in \mathcal{M}$ and $r \in \mathcal{V}_r$. Therefore, given the strictly decreasing property of $\gamma_r(t), r \in \mathcal{V}_r$, and $g_k(t), k \in \mathcal{M}$ if we can ensure that (7a),(7b) are satisfied for every $t \geq 0$, then, $\|\mathbf{x}_{r_k}(t) - \mathbf{x}_{r_k'}(t)\| \in (d_{k,\min}, d_{k,\max})$, and $\|\mathbf{x}_r(t) - \mathbf{x}_V(t)\| \in (\bar{d}_{r,\min}, s_r)$, for every $k \in \mathcal{M}, r \in \mathcal{V}_r, t > 0$.

Next, given $e_k, k \in \mathcal{M}$ and $\xi_r, r \in \mathcal{V}_r$, define the normalized errors with respect to the prescribed performance functions as:

$$\bar{e}_k(t) := \frac{e_k(t)}{g_k(t)} \in (-\eta_{k,\min}, \eta_{k,\max}), \tag{11a}$$
$$\bar{\xi}_r(t) := \frac{\xi_r(t)}{\gamma_r(t)} \in (-\zeta_{r,\min}, \zeta_{r,\max}). \tag{11b}$$

Using the transformation functions $T_k : (-\eta_{k,\min}, \eta_{k,\max}) \to \mathbb{R}, k \in \mathcal{M}$ and $\bar{T}_r : (-\zeta_{r,\min}, \zeta_{r,\max}) \to \mathbb{R}, r \in \mathcal{V}_r$ that are smooth and strictly increasing we may define the transformed errors $\varepsilon_k := T_k(\bar{e}_k), k \in \mathcal{M}$ and $\delta_r := \bar{T}_r(\bar{\xi}_r), r \in \mathcal{V}_r$, where:

$$T_k(\star) := \ln\left(\frac{1 + \frac{\star}{\eta_{k,\min}}}{1 - \frac{\star}{\eta_{k,\max}}}\right), \quad \bar{T}_r(\star) := \ln\left(\frac{1 + \frac{\star}{\zeta_{r,\min}}}{1 - \frac{\star}{\zeta_{r,\max}}}\right). \tag{12}$$

Considering the transformed errors $\varepsilon_k, k \in \mathcal{M}$ and $\delta_r, r \in \mathcal{V}_r$ it can been shown that if ε_k, δ_r are bounded for every k, r and $t \geq 0$, then $\bar{e}_k \in (-\eta_{k,\min}, \eta_{k,\max})$, and $\bar{\xi}_r \in (-\zeta_{r,\min}, \zeta_{r,\max})$ which in turn ensures that (7a), (7b) are satisfied. Differentiating the transformed errors with respect to time we have:

$$\dot{\varepsilon}_k = \mathcal{J}_k(\bar{e}_k, t)\big(\dot{e}_k + \alpha_k(t)e_k\big), \tag{13a}$$
$$\dot{\delta}_r = \bar{\mathcal{J}}_r(\bar{\xi}_r, t)\big(\dot{\xi}_r + \bar{\alpha}_r(t)\xi_r\big), \tag{13b}$$

for every $k \in \mathcal{M}$, $r \in \mathcal{V}_r$, where

$$\mathcal{J}_k(\bar{e}_k, t) := \frac{\partial T_k(\bar{e}_k)}{\partial \bar{e}_k}\frac{1}{g_k(t)}, \quad \bar{\mathcal{J}}_r(\bar{\xi}_r, t) := \frac{\partial \bar{T}_r(\bar{\xi}_r)}{\partial \bar{\xi}_r}\frac{1}{\gamma_r(t)}, \tag{14}$$

$$\alpha_k(t) := -\frac{\dot{g}_k(t)}{g_k(t)}, \qquad \bar{\alpha}_r(t) := -\frac{\dot{\gamma}_r(t)}{\gamma_r(t)}, \tag{15}$$

are the normalized Jacobians of the transformation functions and the normalized derivative of the performance functions corresponding to the distance-based formation and leader formation objective, respectively, which are all positive. Let $\boldsymbol{\varepsilon} := \begin{bmatrix}\varepsilon_1 & \dots & \varepsilon_M\end{bmatrix}^T$. Then, the derivative can be written in vector form as:

$$\dot{\boldsymbol{\varepsilon}} = \mathcal{J}(\bar{\mathbf{e}}, t)\big(\dot{\mathbf{e}} + \boldsymbol{\alpha}(t)\mathbf{e}\big), \quad \dot{\boldsymbol{\delta}} = \bar{\mathcal{J}}(\bar{\boldsymbol{\xi}}, t)\big(\dot{\boldsymbol{\xi}} + \bar{\boldsymbol{\alpha}}(t)\boldsymbol{\xi}\big), \tag{16}$$

where $\mathcal{J}(\bar{\mathbf{e}}, t) := \mathrm{diag}(\mathcal{J}_1(\bar{e}_1, t), \dots, \mathcal{J}_M(\bar{e}_M, t))$, $\boldsymbol{\alpha}(t) := \mathrm{diag}(\alpha_1(t), \dots, \alpha_M(t))$, $\mathbf{e} := \begin{bmatrix}e_1 & \dots & e_M\end{bmatrix}^T$, $\bar{\mathbf{e}} := \begin{bmatrix}\bar{e}_1 & \dots & \bar{e}_M\end{bmatrix}^T$, $\bar{\mathcal{J}}(\boldsymbol{\xi}, t) := \mathrm{diag}(\bar{\mathcal{J}}_{r_1}(\bar{\xi}_{r_1}, t), \dots, \bar{\mathcal{J}}_{r_\pi}(\bar{\xi}_{r_\pi}, t))$, $\bar{\boldsymbol{\alpha}}(t) := \mathrm{diag}(\bar{\alpha}_{r_1}(t), \dots, \bar{\alpha}_{r_\pi}(t))$, $\boldsymbol{\xi} := \begin{bmatrix}\xi_{r_1} & \dots & \xi_{r_\pi}\end{bmatrix}^T$, and $\bar{\boldsymbol{\xi}} := \begin{bmatrix}\bar{\xi}_{r_1} & \dots & \bar{\xi}_{r_\pi}\end{bmatrix}^T$.

Based on the above, we define $\mathbf{u}_r$ for every $r \in \mathcal{V}$ as follows:

$$\mathbf{u}_r = -2b_r\bar{\mathcal{J}}_r(\bar{\xi}_r, t)\delta_r(\mathbf{x}_r - \mathbf{x}_V) - \sum_{k\in\mathcal{N}_r}\hat{d}_{rk}\mathcal{J}_k(\bar{e}_k, t)\varepsilon_k\mathbf{b}_k^T, \tag{17}$$

where $b_r = 1$, if $r \in \mathcal{V}_r$ and $b_r = 0$, otherwise, $D = [\hat{d}_{rk}]$, $\mathbf{b}_k \in \mathbb{R}^n$ is the non-zero row vector at the k-th row of $F_1(\mathbf{x})$ and $\mathcal{N}_r = \{k \in \mathcal{E} : \epsilon_k = (r, r') \in \mathcal{E}, r' \in \mathcal{V}\}$ is the index set of the edges involving the r-th agent. Then, $\mathbf{u}$ can be written as:

$$\mathbf{u} = -(D\otimes I_n)F_1^T(\mathbf{x})\mathcal{J}(\bar{\mathbf{e}},t)\boldsymbol{\varepsilon} - (E\otimes I_n)F_2^T(\mathbf{x},\mathbf{x}_V)\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)\boldsymbol{\delta}. \tag{18}$$

Notice that the proposed control law (17) of the r-th agent requires only knowledge of the relative state information of the k-th edge, where $k\in\mathcal{N}_r$, as well as knowledge of $\mathbf{x}_r-\mathbf{x}_V$, if $r\in\mathcal{V}_r$.

Remark 2. *Contrary to existing approaches [27], [30], [32] the proposed framework offers the flexibility to prescribe a desired behavior for the satisfaction of each objective separately allowing for an explicit characterization of the system's response as well as for quantification of the system's performance with respect to each objective. On the other hand, the existence of multiple constraints may give rise to undesired behaviors due to potentially conflicting objectives rendering the analysis as well as the design of appropriate performance functions challenging.*

Under the proposed control law, we can show the satisfaction of (7a)-(7b) at all times as depicted in the following theorem:

Theorem 1. *Consider the multi-agent system* (2) *and let Assumptions 1-2 hold. Assume further that* $\Phi_T\cap\Phi_F\neq\emptyset$ *and* $\mathbf{x}(0)$ *satisfies* (3)-(4). *Given* $\emptyset\neq\mathcal{V}_r\subseteq\mathcal{V}$ *and parameters* $d_k, d_{k,\min}, d_{k,\max}, k\in\mathcal{M}$ *and* $\bar{d}_{r,\min},\bar{d}_r, r\in\mathcal{V}_r$ *such that* $d_{k,\min}<d_k<d_{k,\max}$, $k\in\mathcal{M}$, $\bar{d}_{r,\min}<\bar{d}_r<s_r, r\in\mathcal{V}_r$, *the control law* (18) *ensures that* (7a)-(7b) *are satisfied for every* $t\geq 0$, *as well as the boundedness of all closed-loop signals provided that:*

$$\begin{bmatrix} L_e & D^TE \\ E^TD & I_\pi \end{bmatrix} \succ 0, \tag{19}$$

where L_e *is the edge Laplacian of* G.

Proof. The proof is divided into two parts. In the first part we show that there exists a maximal solution $\mathbf{y}(t) = \begin{bmatrix}\bar{\mathbf{e}}^T(t) & \bar{\boldsymbol{\xi}}^T(t)\end{bmatrix}^T$, such that $\mathbf{y}(t)\in\Omega_y$, for every $t\in[0,\tau_{\max})$, where $\Omega_y:=\Omega_e\times\Omega_\xi$, $\tau_{\max}\in\mathbb{R}_{>0}$, and

$$\Omega_e := (-\eta_{1,\min},\eta_{1,\max})\times\ldots\times(-\eta_{M,\min},\eta_{M,\max}), \tag{20a}$$
$$\Omega_\xi := (-\zeta_{r_1,\min},\zeta_{r_1,\max})\times\ldots\times(-\zeta_{r_\pi,\min},\zeta_{r_\pi,\max}). \tag{20b}$$

In the second step we show the boundedness of the closed loop signals which leads to $\tau_{\max}=+\infty$.

Part I: By the choice of the parameters $d_k, d_{k,\min}, d_{k,\max}, k\in\mathcal{M}$ and $\bar{d}_{r,\min},\bar{d}_r, r\in\mathcal{V}_r$ the set Ω_y is open and non-empty. In addition, $\mathbf{y}(0)\in\Omega_y$, as shown in (10a)-(10b). Differentiating (11a)-(11b) we obtain the following:

$$\dot{\bar{\mathbf{e}}} = \bar{G}(t)(\dot{\mathbf{e}}+\boldsymbol{\alpha}(t)\mathbf{e}), \qquad \dot{\bar{\boldsymbol{\xi}}} = \Gamma(t)(\dot{\boldsymbol{\xi}}+\bar{\boldsymbol{\alpha}}(t)\boldsymbol{\xi}),$$

where $\bar{G}(t) = \mathrm{diag}(g_1(t),\ldots,g_M(t))^{-1}$ and $\Gamma(t) = \mathrm{diag}(\gamma_{r_1}(t),\ldots,\gamma_{r_\pi}(t))^{-1}$. Substituting (6a),(6b) to the aforementioned equations we have:

$$\dot{\bar{\mathbf{e}}} := P_2(\mathbf{y},t) = \bar{G}(t)\big(F_1(\mathbf{x})(D^T\otimes I_n)P_1(\mathbf{y},t)+\boldsymbol{\alpha}(t)\mathbf{e}\big),$$

$$\dot{\bar{\boldsymbol{\xi}}} := P_3(\mathbf{y},t) = \Gamma(t)\big(F_2(\mathbf{x},\mathbf{x}_V)((E^T\otimes I_n)P_1(\mathbf{y},t) - \mathbf{1}_\pi\otimes\dot{\mathbf{x}}_V)+\bar{\boldsymbol{\alpha}}(t)\boldsymbol{\xi}\big),$$

where $P_1(\mathbf{y},t) := f(\mathbf{x})+h(\mathbf{x})\mathbf{w}-(D\otimes I_n)F_1^T(\mathbf{x})\mathcal{J}(\bar{\mathbf{e}},t)\boldsymbol{\varepsilon} - (E\otimes I_n)F_2^T(\mathbf{x},\mathbf{x}_V)\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)\boldsymbol{\delta}$. Let $P(\mathbf{y},t) := \begin{bmatrix}P_2^T(\mathbf{y},t) & P_3^T(\mathbf{y},t)\end{bmatrix}^T$ and note that $\dot{\mathbf{y}}$ can be written as $\dot{\mathbf{y}}=P(\mathbf{y},t)$. It can be easily shown that $P(\mathbf{y},t)$ is locally Lipschitz continuous with respect to $\mathbf{y}$ in Ω_y and piecewise continuous with respect to t. Hence, invoking [38, Th. 54] we can conclude that there exists a unique maximal solution $\mathbf{y}(t), t\in[0,\tau_{\max})$ with $\tau_{\max}\in\mathbb{R}_{>0}$ such that $\mathbf{y}(t)\in\Omega_y$ for every $t\in[0,\tau_{\max})$.

Part II: Next consider the radially unbounded and positive-definite Lyapunov function:

$$V = \frac{1}{2}\boldsymbol{\varepsilon}^T\boldsymbol{\varepsilon}+\frac{1}{2}\boldsymbol{\delta}^T\boldsymbol{\delta}. \tag{21}$$

Differentiating V and after substitution of $P(\mathbf{x},t)$ we get:

$$\begin{aligned}\dot{V} &= -\boldsymbol{\varepsilon}^T\mathcal{J}(\bar{\mathbf{e}},t)F_1(\mathbf{x})(D^T\otimes I_n)(D\otimes I_n)F_1^T(\mathbf{x})\mathcal{J}(\bar{\mathbf{e}},t)\boldsymbol{\varepsilon}\\ &- \boldsymbol{\varepsilon}^T\mathcal{J}(\bar{\mathbf{e}},t)F_1(\mathbf{x})(D^T\otimes I_n)(E\otimes I_n)F_2^T(\mathbf{x},\mathbf{x}_V)\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)\boldsymbol{\delta}\\ &- \boldsymbol{\delta}^T\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)F_2(\mathbf{x},\mathbf{x}_V)(E^T\otimes I_n)(D\otimes I_n)F_1^T(\mathbf{x})\mathcal{J}(\bar{\mathbf{e}},t)\boldsymbol{\varepsilon}\\ &- \boldsymbol{\delta}^T\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)F_2(\mathbf{x},\mathbf{x}_V)(E^T\otimes I_n)(E\otimes I_n)F_2^T(\mathbf{x},\mathbf{x}_V)\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)\\ &\times\boldsymbol{\delta}+\boldsymbol{\varepsilon}^T\mathcal{J}(\bar{\mathbf{e}},t)\boldsymbol{\alpha}(t)\mathbf{e}-\boldsymbol{\delta}^T\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)F_2(\mathbf{x},\mathbf{x}_V)(\mathbf{1}_\pi\otimes\dot{\mathbf{x}}_V)\\ &+\boldsymbol{\delta}^T\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)\bar{\boldsymbol{\alpha}}(t)\boldsymbol{\xi}+(\boldsymbol{\varepsilon}^T\mathcal{J}(\bar{\mathbf{e}},t)F_1(\mathbf{x})(D^T\otimes I_n)\\ &+\boldsymbol{\delta}^T\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)F_2(\mathbf{x},\mathbf{x}_V)(E^T\otimes I_n))(f(\mathbf{x})+h(\mathbf{x})\mathbf{w}).\end{aligned}$$

Using the properties of the Kronecker product we may write $\dot{V}$ in a more compact form as follows:

$$\begin{aligned}\dot{V} &= -\boldsymbol{\kappa}^T(\mathbf{y},t)A(\mathbf{x},\mathbf{x}_V)\boldsymbol{\kappa}(\mathbf{y},t)+\boldsymbol{\varepsilon}^T\mathcal{J}(\bar{\mathbf{e}},t)\boldsymbol{\alpha}(t)\mathbf{e}\\ &- \boldsymbol{\delta}^T\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)F_2(\mathbf{x},\mathbf{x}_V)(\mathbf{1}_\pi\otimes\dot{\mathbf{x}}_V)+\boldsymbol{\delta}^T\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)\bar{\boldsymbol{\alpha}}(t)\boldsymbol{\xi}\\ &+\boldsymbol{\kappa}^T(\mathbf{y},t)G(\mathbf{x},\mathbf{x}_V)(f(\mathbf{x})+h(\mathbf{x})\mathbf{w}),\\ &= -\boldsymbol{\kappa}^T(\mathbf{y},t)A(\mathbf{x},\mathbf{x}_V)\boldsymbol{\kappa}(\mathbf{y},t)+\boldsymbol{\kappa}^T(\mathbf{y},t)\big(G(\mathbf{x},\mathbf{x}_V)(f(\mathbf{x})\\ &+h(\mathbf{x})\mathbf{w})+\mathbf{z}\big),\end{aligned}$$

where $\mathbf{z} := \begin{bmatrix}\mathbf{e}^T\boldsymbol{\alpha}(t) & \boldsymbol{\xi}^T\bar{\boldsymbol{\alpha}}(t)-(\mathbf{1}_\pi\otimes\dot{\mathbf{x}}_V)^TF_2^T(\mathbf{x},\mathbf{x}_V)\end{bmatrix}^T$, $\boldsymbol{\kappa}(\mathbf{y},t) := \begin{bmatrix}\boldsymbol{\varepsilon}^T\mathcal{J}(\bar{\mathbf{e}},t) & \boldsymbol{\delta}^T\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)\end{bmatrix}^T$ and:

$$A := \begin{bmatrix}F_1(\mathbf{x}) & \mathbf{0}\\ \mathbf{0} & F_2(\mathbf{x},\mathbf{x}_V)\end{bmatrix}(B\otimes I_n)\begin{bmatrix}F_1(\mathbf{x}) & \mathbf{0}\\ \mathbf{0} & F_2(\mathbf{x},\mathbf{x}_V)\end{bmatrix}^T, \tag{22a}$$

$$B := \begin{bmatrix}D^TD & D^TE\\ E^TD & E^TE\end{bmatrix} = \begin{bmatrix}L_e & D^TE\\ E^TD & I_\pi\end{bmatrix}, \tag{22b}$$

$$G := \begin{bmatrix}F_1(\mathbf{x}) & \mathbf{0}\\ \mathbf{0} & F_2(\mathbf{x},\mathbf{x}_V)\end{bmatrix}(\begin{bmatrix}D^T\\ E^T\end{bmatrix}\otimes I_n), \tag{22c}$$

where the latter equality in (22b) is obtained by virtue of Lemma 1 in the Appendix, $A := A(\mathbf{x},\mathbf{x}_V)$ and $G := G(\mathbf{x},\mathbf{x}_V)$. Note that due to $\mathbf{y}\in\Omega_y$ it follows that $\mathbf{e}(t),\boldsymbol{\xi}(t)$ are bounded for every $t\in[0,\tau_{\max})$. In addition, when $\mathbf{y}\in\Omega_y$, it follows that $0<d_{k,\min}<\|\mathbf{x}_{r_k}(t)-\mathbf{x}_{r'_k}(t)\|$ and $0<\bar{d}_{r,\min}<\|\mathbf{x}_r(t)-\mathbf{x}_V(t)\|$ for every $k\in\mathcal{M}, r\in\mathcal{V}_r, t\in[0,\tau_{\max})$. As a result $\begin{bmatrix}F_1(\mathbf{x}) & \mathbf{0}\\ \mathbf{0} & F_2(\mathbf{x},\mathbf{x}_V)\end{bmatrix}$ is of full row rank. In addition, owing to Assumption 2 as well as the boundedness of $\|\mathbf{x}_{r_k}(t)-\mathbf{x}_{r'_k}(t)\|, k\in\mathcal{M}$ and $\|\mathbf{x}_r(t)-\mathbf{x}_V(t)\|, r\in$

$\mathcal{V}_r$, whenever $\mathbf{y} \in \Omega_y$, the matrix $\begin{bmatrix} F_1(\mathbf{x}) & \mathbf{0} \\ \mathbf{0} & F_2(\mathbf{x}, \mathbf{x}_V) \end{bmatrix}$ is bounded for every $t \in [0, \tau_{\max})$. Therefore, $A \succ 0$ due to $B \succ 0$. Let $\widetilde{\lambda}_{\min}(A) := \inf_{\substack{\mathbf{x}(t), \mathbf{x}_V(t) \\ t \in [0, \tau_{\max})}} \lambda_{\min}(A(\mathbf{x}(t), \mathbf{x}_V(t))) > 0$. Then, after employing the Cauchy-Schwarz inequality we obtain:

$$\dot{V} \leq -\|\boldsymbol{\kappa}(\mathbf{y}, t)\| \big(\widetilde{\lambda}_{\min}(A)\|\boldsymbol{\kappa}(\mathbf{y}, t)\| - \|G(\mathbf{x}, \mathbf{x}_V)(f(\mathbf{x}) + h(\mathbf{x})\mathbf{w}) + \mathbf{z}\|\big).$$

By Assumption 1 there always exist a path connecting every agent $r \in \mathcal{V}$ with at least one agent $r \in \mathcal{V}_r$. Hence, whenever $\mathbf{y} \in \Omega_y$ and thus $\|\mathbf{x}_{r_k}(t) - \mathbf{x}_{r_k'}(t)\| < d_{k,\max} < \infty, k \in \mathcal{M}$, and $\|\mathbf{x}_r(t) - \mathbf{x}_V(t)\| < s_r < \infty, r \in \mathcal{V}_r$, it follows that $\|\mathbf{x}_r\| < \infty, r \in \mathcal{V}$ owing to Assumption 2. Therefore, by the aforementioned analysis, the boundedness of $g_k(t), \dot{g}_k(t), k \in \mathcal{M}$, $\gamma_r(t), \dot{\gamma}_r(t), r \in \mathcal{V}_r$, $\mathbf{w}(t)$, Assumption 2, and the Extreme Value Theorem there exists a constant $\mu \in (0, +\infty)$, independent of t that satisfies $\|G(\mathbf{x}, \mathbf{x}_V)(f(\mathbf{x}) + h(\mathbf{x})\mathbf{w}) + \mathbf{z}\| \leq \mu$, for every $t \in [0, \tau_{\max})$. This ensures that $\dot{V} \leq \|\boldsymbol{\kappa}(\mathbf{y}, t)\|(\mu - \widetilde{\lambda}_{\min}(A)\|\boldsymbol{\kappa}(\mathbf{y}, t)\|)$. Notice further that $\boldsymbol{\kappa}(\mathbf{y}, t) = K(t)\boldsymbol{\chi}$, where $\boldsymbol{\chi} = \begin{bmatrix} \boldsymbol{\varepsilon}^T & \boldsymbol{\delta}^T \end{bmatrix}^T$ and $K(t) := \mathrm{diag}(\mathcal{J}(\bar{\mathbf{e}}, t), \bar{\mathcal{J}}(\bar{\boldsymbol{\xi}}, t)) \succ 0$. Recall that $\mathcal{J}(\bar{\mathbf{e}}, t), \bar{\mathcal{J}}(\bar{\boldsymbol{\xi}}, t)$ are diagonal matrices with elements $\mathcal{J}_k(\bar{e}_k, t) = \frac{\partial T_k(\bar{e}_k)}{\partial \bar{e}_k} \frac{1}{g_k(t)}, k \in \mathcal{M}$, $\bar{\mathcal{J}}_r(\bar{\xi}_r, t) = \frac{\partial \bar{T}_r(\bar{\xi}_r)}{\partial \bar{\xi}_r} \frac{1}{\gamma_r(t)}, r \in \mathcal{V}_r$ satisfying:

$$\mathcal{J}_k(\bar{e}_k, t) \geq \frac{4}{\eta_{k,\min} + \eta_{k,\max}}, \quad \bar{\mathcal{J}}_r(\bar{\xi}_r, t) \geq \frac{4}{\zeta_{r,\min} + \zeta_{r,\max}},$$

owing to the fact that $g_k(t) \leq 1, \gamma_r(t) \leq 1$, for every $t \geq 0$, $k \in \mathcal{M}, r \in \mathcal{V}_r$. Hence, we can deduce that $\|\boldsymbol{\kappa}(\mathbf{y}, t)\| \geq \beta\|\boldsymbol{\chi}\|$, where $\beta := \min\{\beta_F, \beta_T\}$, $\beta_F := \min_{k\in\mathcal{M}}\{\frac{4}{\eta_{k,\min}+\eta_{k,\max}}\}$ and $\beta_T := \min_{r\in\mathcal{V}_r}\{\frac{4}{\zeta_{r,\min}+\zeta_{r,\max}}\}$. This leads to $\dot{V} \leq \|\boldsymbol{\kappa}(\mathbf{y}, t)\|(\mu - \beta\widetilde{\lambda}_{\min}(A)\|\boldsymbol{\chi}\|)$ which in turn implies that:

$$\dot{V} < 0, \quad \forall \boldsymbol{\chi} \in \{\boldsymbol{\chi} : \|\boldsymbol{\chi}\| > \frac{\mu}{\beta\widetilde{\lambda}_{\min}(A)}\}. \tag{23}$$

Note that if the latter condition holds, then $\dot{V} < 0$ implies that $\|\boldsymbol{\chi}(t)\| < \|\boldsymbol{\chi}(0)\|$. Therefore, we can conclude that $\|\boldsymbol{\chi}(t)\| \leq \chi_{\max} := \max\{\|\boldsymbol{\chi}(0)\|, \frac{\mu}{\beta\widetilde{\lambda}_{\min}(A)}\}$, for every $t \in [0, \tau_{\max})$, thus $\|\boldsymbol{\varepsilon}(t)\| \leq \chi_{\max}$ and $\|\boldsymbol{\delta}(t)\| \leq \chi_{\max}$, for every $t \in [0, \tau_{\max})$. Using the inverse logarithmic functions for every $t \in [0, \tau_{\max})$ we have:

$$-\eta_{k,\min} < -\eta_k^l \leq \bar{e}_k(t) \leq \eta_k^u < \eta_{k,\max}, \quad k \in \mathcal{M},$$
$$-\zeta_{r,\min} < -\zeta_r^l \leq \bar{\xi}_r(t) \leq \zeta_r^u < \zeta_{r,\max}, \quad r \in \mathcal{V}_r,$$

where $\eta_k^l := \frac{\exp(-\chi_{\max})-1}{\exp(-\chi_{\max})+1}\eta_{k,\min}$, $\eta_k^u := \frac{\exp(\chi_{\max})-1}{\exp(\chi_{\max})+1}\eta_{k,\max}$, $\zeta_r^l := \frac{\exp(-\chi_{\max})-1}{\exp(-\chi_{\max})+1}\zeta_{r,\min}$, and $\zeta_r^u := \frac{\exp(\chi_{\max})-1}{\exp(\chi_{\max})+1}\zeta_{r,\max}$. Let $\Omega_e' := [\eta_1^l, \eta_1^u] \times \ldots \times [\eta_M^l, \eta_M^u]$, $\Omega_\xi' := [\zeta_{r_1}^l, \zeta_{r_1}^u] \times \ldots \times [\zeta_{r_\pi}^l, \zeta_{r_\pi}^u]$, $\Omega_y' := \Omega_e' \times \Omega_\xi'$ which is compact and non-empty satisfying $\Omega_{\bowtie}' \subset \Omega_{\bowtie}$, for every $\bowtie \in \{e, \xi, y\}$. Assuming that $\tau_{\max} < +\infty$ [38, Pr. C.3.6] implies that there exists a $t \in [0, \tau_{\max})$ such that $\mathbf{y}(t) \notin \Omega_y'$ which leads to contradiction. Hence, $\tau_{\max} = +\infty$. This concludes the proof. ■

In Theorem 1 the satisfaction of (7a)-(7b) is established under the condition that $B \succ 0$, which ensures that $\dot{V}$ is strictly negative outside a ball whose radius depends on B, as depicted in (23). A necessary and sufficient condition for (19) to hold can now be expressed in terms of the graph topology and the number of informed agents π as shown in the following proposition:

Proposition 1. *Let Assumption 1 hold. Then, the matrix $B \in \mathbb{R}^{(M+\pi)\times(M+\pi)}$ defined in* (22b) *is positive definite if and only if G is a tree and $\pi = 1$.*

Proof. By [39, Sec. A.5.5] B is positive definite if and only if $L_e - D^T E E^T D \succ 0$, $I_\pi \succ 0$, where $L_e = D^T D$ is the edge Laplacian of G. By definition of E, $D^T E = \begin{bmatrix} \mathbf{d}_{r_1} & \ldots & \mathbf{d}_{r_\pi} \end{bmatrix}$, where $\mathbf{d}_\chi \in \mathbb{R}^M$ is the χ-th column of D^T. In addition, by definition, $L_e = \sum_{r\in\mathcal{V}} \mathbf{d}_r\mathbf{d}_r^T$. Then, $L_e - D^T E E^T D = \sum_{r\in\mathcal{V}} \mathbf{d}_r\mathbf{d}_r^T - \sum_{r\in\mathcal{V}_r} \mathbf{d}_r\mathbf{d}_r^T = \sum_{r\in\mathcal{V}\setminus\mathcal{V}_r} \mathbf{d}_r\mathbf{d}_r^T$, i.e., $L_e - D^T E E^T D$ is the sum of $R - \pi$ positive semi-definite matrices, thus positive semi-definite. For an arbitrary, non-zero vector $\mathbf{p} \in \mathbb{R}^M$, it thus holds that $\mathbf{p}^T(L_e - D^T E E^T D)\mathbf{p} = 0 \Leftrightarrow \sum_{r\in\mathcal{V}\setminus\mathcal{V}_r} \|\mathbf{p}^T\mathbf{d}_r\|_2^2 = 0 \Leftrightarrow \|\mathbf{p}^T\mathbf{d}_r\|_2 = 0, \forall r \in \mathcal{V}\setminus\mathcal{V}_r \Leftrightarrow \mathbf{p} \perp \mathbf{d}_r, r \in \mathcal{V}\setminus\mathcal{V}_r$, or equivalently $\mathbf{p} \in \mathrm{null}(C)$, where $C \in \mathbb{R}^{(R-\pi)\times M}$ is the matrix with rows $\mathbf{d}_r^T, r \in \mathcal{V}\setminus\mathcal{V}_r$. Hence, a necessary and sufficient condition for B to be positive definite is $\mathrm{null}(C) = \{\mathbf{0}_M\}$.

First assume that B is positive definite. By Assumption 1, G is connected, thus $M \geq R-1$. In addition, $\pi \geq 1$, thus $M + \pi \geq R$ holds. If $M > R - \pi$, then by the rank-nullity theorem C has always a non-trivial nullspace, thus B is positive semi-definite which leads to contradiction. Hence, $M + \pi = R$ should hold which under Assumption 1 and the constraint $\pi \geq 1$ is true only when $M = R - 1$ and $\pi = 1$ and the result follows. Next assume that G is a tree and $\pi = 1$. Then, $M + \pi = R$ which implies that $\mathrm{rank}(C) \leq R - 1$. Observe further that $\mathbf{d}_r^T, r \in \mathcal{V}\setminus\mathcal{V}_r$ are rows of the incidence matrix D as well. Since the sum of rows of D is equal to $\mathbf{0}_{1\times(R-1)}$, it follows that $\mathbf{d}_{r'}^T = -\sum_{r\in\mathcal{V}\setminus\mathcal{V}_r} \mathbf{d}_r^T$, where $r' \in \mathcal{V}_r$. Therefore, $\mathrm{rank}(C) = \mathrm{rank}(D)$. If $\mathrm{rank}(C) < R - 1$, D will have a non-trivial nullspace which contradicts the fact that G is a tree [11, Lem. 1]. Hence, $\mathrm{rank}(C) = R - 1$. Then, by the rank-nullity theorem $\mathrm{null}(C) = \{\mathbf{0}_M\}$ and the result follows. ■

Proposition 1 suggests that the distance-based formation and leader formation objectives can be achieved under the proposed control law (18) when the communication graph is a tree and a single agent has access to the virtual leader's state. Although powerful this result is related to the choice of the Lyapunov function and thus can not be viewed as a necessary and sufficient condition for the satisfaction of the desired objectives.

Remark 3. *The proposed control law can be easily adapted for systems of the form $\dot{\mathbf{x}}_r = f_r(\mathbf{x}_r) + \phi_r(\mathbf{x}_r)\hat{\mathbf{u}}_r + h_r(\mathbf{x}_r)\mathbf{w}_r$, where $\phi_r : \mathbb{R}^n \to \mathbb{R}^{n\times v}$ is a locally Lipschitz function with full row rank for every $\mathbf{x}_r \in \mathbb{R}^n$. Specifically, the results of Theorem 1 remain valid when the modified control law is chosen as $\hat{\mathbf{u}}_r = \phi_r^\dagger(\mathbf{x}_r)\mathbf{u}_r$, where $\phi_r^\dagger : \mathbb{R}^n \to \mathbb{R}^{v\times n}$ is the Moore-Penrose matrix of $\phi_r(\cdot)$ and $\mathbf{u}_r \in \mathbb{R}^n$ is the same control law defined in* (17).

Remark 4. *Under the conditions of Theorem 1 the satisfaction* *of the funnel constraints 7a-(7b) follows for every $t \geq 0$. These functions prescribe a desired behavior for the system impacting its actual performance. In particular, smaller values of l_k^g, l_r^γ may induce a more oscillatory behavior for the system while larger values ensure a faster convergence. In addition, smaller values of $\bar{g}_{k,\infty}, \bar{\gamma}_{r,\infty}$ guarantee a better steady state performance for the system ensuring convergence to a small neighborhood of the origin.*

IV. Adaptive Formation Tracking control

In this section we design a control law to satisfy the control objectives of Problem 1 when the nonlinear terms $f_r(\mathbf{x}_r), r \in \mathcal{V}$ are unknown and $\mathbf{w}_r \equiv \mathbf{0}$, for every $r \in \mathcal{V}$. The proposed control law allows for online approximation of the unknown nonlinear terms providing the multi-agent system the ability to adapt its behavior according to changes in the environment towards ensuring the satisfaction of the objectives. This is very beneficial for real world applications where agents evolve in highly uncertain environments and may be subject to faults calling for a plug-and-play option of new agents without compromising the satisfaction of the task as well as the performance of the multi-agent system. Here, we approximate the unknown terms $f_r(\cdot)$ by:

$$f_r(\mathbf{x}_r) = Y_r(\mathbf{x}_r)\boldsymbol{\theta}_r + \mathbf{w}_{f,r}(\mathbf{x}_r),$$

where $\mathbf{w}_{f,r} : \mathbb{R}^n \to \mathbb{R}^n$ is the approximation error, $Y_r : \mathbb{R}^n \to \mathbb{R}^{n \times p_r}$ is a matrix of regressor vector terms that is locally Lipschitz continuous in $\mathbf{x}_r$ and $\boldsymbol{\theta}_r \in \mathbb{R}^{p_r}$ is a constant but unknown vector of parameters minimizing $\|f_r(\mathbf{x}_r) - Y_r(\mathbf{x}_r)\boldsymbol{\theta}_r\|$ over $\mathbf{x}_r \in \mathcal{B}_r$, where $\mathcal{B}_r \subset \mathbb{R}^n$ is a compact set to be determined later in this section. Then, the nonlinear term $f(\mathbf{x})$ in (2) can be written as:

$$f(\mathbf{x}) = Y(\mathbf{x})\boldsymbol{\theta} + \mathbf{w}_f(\mathbf{x}), \tag{24}$$

where $Y(\mathbf{x}) := \mathrm{diag}(Y_1(\mathbf{x}_1), \ldots, Y_R(\mathbf{x}_R))$, $\boldsymbol{\theta} := \begin{bmatrix}\boldsymbol{\theta}_1^T & \ldots & \boldsymbol{\theta}_R^T\end{bmatrix}^T$ and $\mathbf{w}_f(\mathbf{x}) := \begin{bmatrix}\mathbf{w}_{f,1}^T(\mathbf{x}_1) & \ldots & \mathbf{w}_{f,R}^T(\mathbf{x}_R)\end{bmatrix}^T$.

Remark 5. *When only partial information of $f_r(\mathbf{x}_r), r \in \mathcal{V}$ is available one may proceed with the design of the adaptive control law approximating only the unknown part of $f_r(\mathbf{x}_r)$. The results of Theorem 2 below remain valid thanks to the boundedness of the known part of $f_r(\mathbf{x}_r)$ within the compact set $\mathcal{B}_r$, for every $r \in \mathcal{V}$, whenever* (7a)-(7b) *are satisfied.*

Let $\hat{\boldsymbol{\theta}}_r$ be an estimate of the real parameter vector $\boldsymbol{\theta}_r$ and define $\widetilde{\boldsymbol{\theta}}_r := \hat{\boldsymbol{\theta}}_r - \boldsymbol{\theta}_r$, for every $r \in \mathcal{V}$. Define the control law of the r-th agent as:

$$\widetilde{\mathbf{u}}_r = \mathbf{u}_r - Y_r(\mathbf{x}_r)\hat{\boldsymbol{\theta}}_r, \tag{25}$$

where $\mathbf{u}_r$ is the control law defined in (17) and $\hat{\boldsymbol{\theta}}_r$ is a time-varying estimate of $\boldsymbol{\theta}_r$ that evolves according to:

$$\dot{\hat{\boldsymbol{\theta}}}_r = -\Delta_r Y_r^T(\mathbf{x}_r)\mathbf{u}_r, \tag{26}$$

where $\Delta_r \in \mathbb{R}^{p_r \times p_r}$ is a positive definite square matrix to be tuned and $\mathbf{u}_r$ is the control law defined in (17). Notice that (25) requires knowledge of the absolute state $\mathbf{x}_r \in \mathbb{R}^n$ in addition to the relative state information mentioned in Section III. The proposed controller and adaptive law can be written in stacked form as:

$$\widetilde{\mathbf{u}} = \mathbf{u} - Y(\mathbf{x})\hat{\boldsymbol{\theta}}, \tag{27}$$

$$\begin{aligned}\dot{\hat{\boldsymbol{\theta}}} &= \Delta Y^T(\mathbf{x})\big((D \otimes I_n)F_1^T(\mathbf{x})\mathcal{J}(\bar{\mathbf{e}}, t)\boldsymbol{\varepsilon} \\ &\quad + (E \otimes I_n)F_2^T(\mathbf{x}, \mathbf{x}_V)\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}}, t)\boldsymbol{\delta}\big),\end{aligned} \tag{28}$$

where $\hat{\boldsymbol{\theta}} := \begin{bmatrix}\hat{\boldsymbol{\theta}}_1^T & \ldots & \hat{\boldsymbol{\theta}}_R^T\end{bmatrix}^T$ and $\Delta := \mathrm{diag}(\Delta_1, \ldots, \Delta_R)$. Under the proposed control and update law we can show the satisfaction of the funnel constraints as depicted in the following theorem:

Theorem 2. *Consider the multi-agent system* (2) *with unknown nonlinear terms $f(\mathbf{x})$ and $\mathbf{w} \equiv \mathbf{0}$, and let Assumptions 1-2 hold. Assume that $\Phi_T \cap \Phi_F \neq \emptyset$ and $\mathbf{x}(0)$ satisfies* (3)-(4). *Let further* (19) *hold. Given $\emptyset \neq \mathcal{V}_r \subseteq \mathcal{V}$ and parameters $d_k, d_{k,\min}, d_{k,\max}, k \in \mathcal{M}$ and $\bar{d}_{r,\min}, \bar{d}_r, r \in \mathcal{V}_r$ such that $d_{k,\min} < d_k < d_{k,\max}$, $k \in \mathcal{M}$, $\bar{d}_{r,\min} < \bar{d}_r < s_r, r \in \mathcal{V}_r$, the control law* (27) *and update law* (28) *ensure that* (7a)-(7b) *are satisfied for every $t \geq 0$ and all closed-loop signals are bounded.*

Proof. As in Theorem 1 the proof is divided into two parts. In the first part we show the existence of a maximal solution within $[0, \tau_{\max})$, where $\tau_{\max} \in \mathbb{R}_{>0}$ and in the second term we show that $\tau_{\max} = +\infty$.

Part I: By the choice of the parameters $d_k, d_{k,\min}, d_{k,\max}, k \in \mathcal{M}$ and $\bar{d}_{r,\min}, \bar{d}_r, r \in \mathcal{V}_r$ and given the initial conditions $\mathbf{x}(0)$ the set $\Omega_y = \Omega_e \times \Omega_\xi$, where $\Omega_\bowtie, \bowtie \in \{e, \xi\}$ are defined as in (20a)-(20b), is non-empty and open. Let $\mathbf{y} := \begin{bmatrix}\bar{\mathbf{e}}^T & \bar{\boldsymbol{\xi}}^T\end{bmatrix}^T$. Given the proposed controller and update law, we may define the dynamic equation of $\mathbf{y}$ as $\dot{\mathbf{y}} = P'(\mathbf{y}, t)$, where $P'(\mathbf{y}, t) := \begin{bmatrix}P_2'^T(\mathbf{y}, t) & P_3'^T(\mathbf{y}, t)\end{bmatrix}^T$ and

$$\begin{aligned}P_1'(\mathbf{y}, t) &:= f(\mathbf{x}) - Y(\mathbf{x})\hat{\boldsymbol{\theta}} - (D \otimes I_n)F_1^T(\mathbf{x})\mathcal{J}(\bar{\mathbf{e}}, t)\boldsymbol{\varepsilon} \\ &\quad - (E \otimes I_n)F_2^T(\mathbf{x}, \mathbf{x}_V)\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}}, t)\boldsymbol{\delta}, \\ P_2'(\mathbf{y}, t) &:= \bar{G}(t)\big(F_1(\mathbf{x})(D^T \otimes I_n)P_1'(\mathbf{y}, t) + \boldsymbol{\alpha}(t)\mathbf{e}\big), \\ P_3'(\mathbf{y}, t) &:= \Gamma(t)\big(F_2(\mathbf{x}, \mathbf{x}_V)((E^T \otimes I_n)P_1'(\mathbf{y}, t) - \\ &\quad - \mathbf{1}_\pi \otimes \dot{\mathbf{x}}_V) + \bar{\boldsymbol{\alpha}}(t)\boldsymbol{\xi}\big),\end{aligned}$$

and where $\bar{G}(t) = \mathrm{diag}(g_1(t), \ldots, g_M(t))^{-1}$ and $\Gamma(t) = \mathrm{diag}(\gamma_{r_1}(t), \ldots, \gamma_{r_\pi}(t))^{-1}$. It can be easily verified that $P'(\mathbf{y}, t)$ is locally Lipschitz continuous with respect to $\mathbf{y}$ in Ω_y and piecewise continuous with respect to t. Hence, invoking [38, Th. 54] we can conclude that there exists a unique maximal solution $\mathbf{y}(t), t \in [0, \tau_{\max})$ with $\tau_{\max} \in \mathbb{R}_{>0}$ such that $\mathbf{y}(t) \in \Omega_y$ for every $t \in [0, \tau_{\max})$.

Part II: In Part I we have shown the existence of a maximal solution $\mathbf{y}(t)$ satisfying $\mathbf{y}(t) \in \Omega_y$, for every $t \in [0, \tau_{\max})$. Next, we will show that $\tau_{\max} = +\infty$. We start by showing the boundedness of the transformed errors $\boldsymbol{\varepsilon}, \boldsymbol{\delta}$. To that end, consider the Lyapunov function:

$$V' = V + \frac{1}{2}\widetilde{\boldsymbol{\theta}}^T \Delta^{-1} \widetilde{\boldsymbol{\theta}},$$

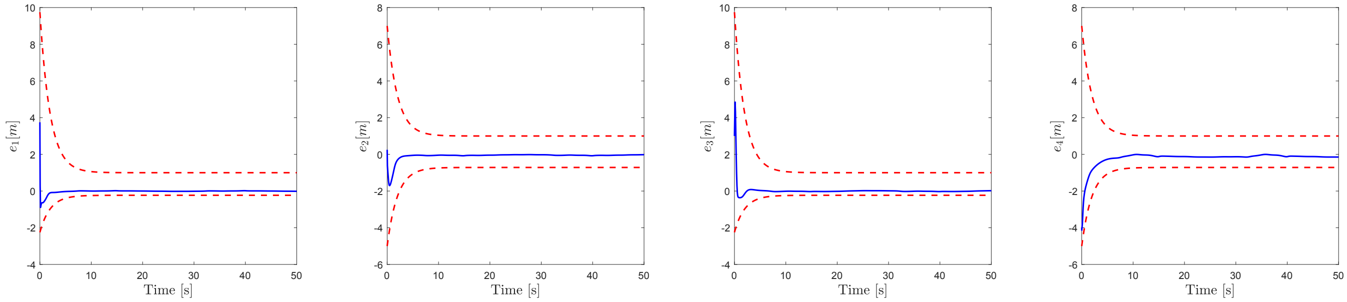

Fig. 1: Evolution of distance errors $e_k, k \in \mathcal{M}$ for known nonlinear dynamics.

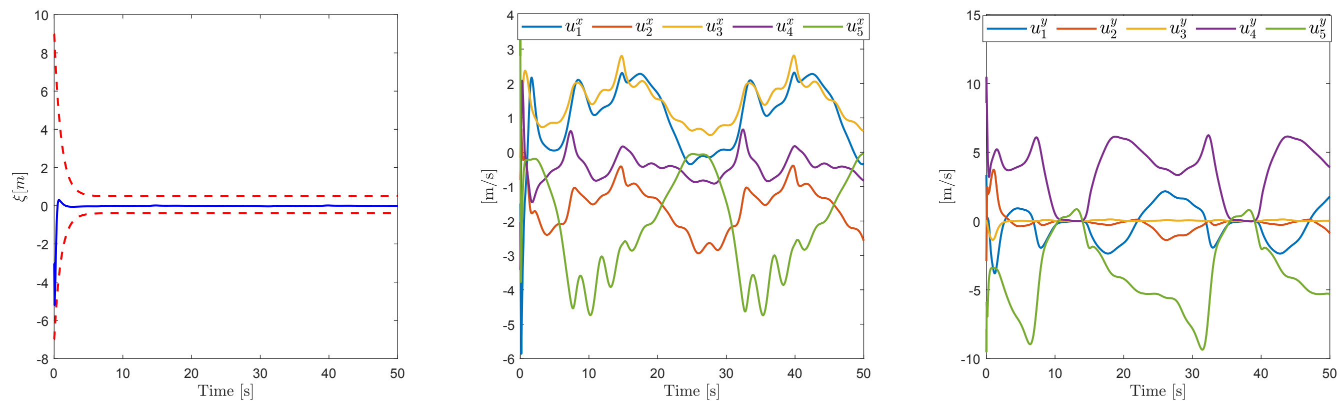

Fig. 2: Evolution of ξ and the resulting control signals along the x and y axis for known nonlinear dynamics.

where V is defined in (21) and $\widetilde{\boldsymbol{\theta}} = \begin{bmatrix}\widetilde{\boldsymbol{\theta}}_1^T & \dots & \widetilde{\boldsymbol{\theta}}_R^T\end{bmatrix}^T$. Differentiating V' and after substituting (16), (6a)-(6b) we have:

$$\begin{aligned}\dot{V}' &= (\boldsymbol{\varepsilon}^T\mathcal{J}(\bar{\mathbf{e}},t)F_1(\mathbf{x})(D^T\otimes I_n) + \boldsymbol{\delta}^T\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)F_2(\mathbf{x},\mathbf{x}_V)\\ &\quad\times(E^T\otimes I_n))(f(\mathbf{x})+\mathbf{u}) + \boldsymbol{\varepsilon}^T\mathcal{J}(\bar{\mathbf{e}},t)\boldsymbol{\alpha}(t)\mathbf{e}\\ &\quad-\boldsymbol{\delta}^T\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)F_2(\mathbf{x},\mathbf{x}_V)(\mathbf{1}_\pi\otimes\dot{\mathbf{x}}_V) + \boldsymbol{\delta}^T\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)\bar{\boldsymbol{\alpha}}(t)\boldsymbol{\xi}\\ &\quad+\widetilde{\boldsymbol{\theta}}^T\Delta^{-1}\dot{\hat{\boldsymbol{\theta}}}.\end{aligned}$$

Substituting (27) and (28) to $\dot{V}'$ we have:

$$\begin{aligned}\dot{V}' &= (\boldsymbol{\varepsilon}^T\mathcal{J}(\bar{\mathbf{e}},t)F_1(\mathbf{x})(D^T\otimes I_n) + \boldsymbol{\delta}^T\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)F_2(\mathbf{x},\mathbf{x}_V)\\ &\quad\times(E^T\otimes I_n))(f(\mathbf{x})-Y(\mathbf{x})\hat{\boldsymbol{\theta}}) + \boldsymbol{\varepsilon}^T\mathcal{J}(\bar{\mathbf{e}},t)\boldsymbol{\alpha}(t)\mathbf{e}\\ &\quad-\boldsymbol{\varepsilon}^T\mathcal{J}(\bar{\mathbf{e}},t)F_1(\mathbf{x})(D^T\otimes I_n)(D\otimes I_n)F_1^T(\mathbf{x})\mathcal{J}(\bar{\mathbf{e}},t)\boldsymbol{\varepsilon}\\ &\quad-\boldsymbol{\varepsilon}^T\mathcal{J}(\bar{\mathbf{e}},t)F_1(\mathbf{x})(D^T\otimes I_n)(E\otimes I_n)F_2^T(\mathbf{x},\mathbf{x}_V)\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)\boldsymbol{\delta}\\ &\quad-\boldsymbol{\delta}^T\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)F_2(\mathbf{x},\mathbf{x}_V)(E^T\otimes I_n)(D\otimes I_n)F_1^T(\mathbf{x})\mathcal{J}(\bar{\mathbf{e}},t)\boldsymbol{\varepsilon}\\ &\quad-\boldsymbol{\delta}^T\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)F_2(\mathbf{x},\mathbf{x}_V)(E^T\otimes I_n)(E\otimes I_n)F_2^T(\mathbf{x},\mathbf{x}_V)\\ &\quad\times\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)\boldsymbol{\delta}-\boldsymbol{\delta}^T\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)F_2(\mathbf{x},\mathbf{x}_V)(\mathbf{1}_\pi\otimes\dot{\mathbf{x}}_V)+\boldsymbol{\delta}^T\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)\\ &\quad\times\bar{\boldsymbol{\alpha}}(t)\boldsymbol{\xi}+\widetilde{\boldsymbol{\theta}}^TY^T(\mathbf{x})\big((D\otimes I_n)F_1^T(\mathbf{x})\mathcal{J}(\bar{\mathbf{e}},t)\boldsymbol{\varepsilon}\\ &\quad+(E\otimes I_n)F_2^T(\mathbf{x},\mathbf{x}_V)\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)\boldsymbol{\delta}\big).\end{aligned}$$

Adding and subtracting $Y(\mathbf{x})\hat{\boldsymbol{\theta}}$ in (24) we get:

$$f(\mathbf{x}) = -Y(\mathbf{x})\widetilde{\boldsymbol{\theta}} + Y(\mathbf{x})\hat{\boldsymbol{\theta}} + \mathbf{w}_f(\mathbf{x}).$$

Substituting the right-hand side of the aforementioned equation to $\dot{V}'$ and after simple calculations we have:

$$\begin{aligned}\dot{V}' &= (\boldsymbol{\varepsilon}^T\mathcal{J}(\bar{\mathbf{e}},t)F_1(\mathbf{x})(D^T\otimes I_n) + \boldsymbol{\delta}^T\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)F_2(\mathbf{x},\mathbf{x}_V)\\ &\quad\times(E^T\otimes I_n))\mathbf{w}_f(\mathbf{x}) + \boldsymbol{\varepsilon}^T\mathcal{J}(\bar{\mathbf{e}},t)\boldsymbol{\alpha}(t)\mathbf{e} + \boldsymbol{\delta}^T\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)\bar{\boldsymbol{\alpha}}(t)\boldsymbol{\xi}\\ &\quad-\boldsymbol{\varepsilon}^T\mathcal{J}(\bar{\mathbf{e}},t)F_1(\mathbf{x})(D^T\otimes I_n)(D\otimes I_n)F_1^T(\mathbf{x})\mathcal{J}(\bar{\mathbf{e}},t)\boldsymbol{\varepsilon}\\ &\quad-\boldsymbol{\varepsilon}^T\mathcal{J}(\bar{\mathbf{e}},t)F_1(\mathbf{x})(D^T\otimes I_n)(E\otimes I_n)F_2^T(\mathbf{x},\mathbf{x}_V)\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)\boldsymbol{\delta}\\ &\quad-\boldsymbol{\delta}^T\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)F_2(\mathbf{x},\mathbf{x}_V)(E^T\otimes I_n)(D\otimes I_n)F_1^T(\mathbf{x})\mathcal{J}(\bar{\mathbf{e}},t)\boldsymbol{\varepsilon}\\ &\quad-\boldsymbol{\delta}^T\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)F_2(\mathbf{x},\mathbf{x}_V)(E^T\otimes I_n)(E\otimes I_n)F_2^T(\mathbf{x},\mathbf{x}_V)\\ &\quad\times\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)\boldsymbol{\delta}-\boldsymbol{\delta}^T\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)F_2(\mathbf{x},\mathbf{x}_V)(\mathbf{1}_\pi\otimes\dot{\mathbf{x}}_V).\end{aligned}$$

Let $\boldsymbol{\kappa}(\mathbf{y},t) := \begin{bmatrix}\boldsymbol{\varepsilon}^T\mathcal{J}(\bar{\mathbf{e}},t) & \boldsymbol{\delta}^T\bar{\mathcal{J}}(\bar{\boldsymbol{\xi}},t)\end{bmatrix}^T$, $\mathbf{z} := \begin{bmatrix}\mathbf{e}^T\boldsymbol{\alpha}(t) & \boldsymbol{\xi}^T\bar{\boldsymbol{\alpha}}(t)-(\mathbf{1}_\pi\otimes\dot{\mathbf{x}}_V)^TF_2^T(\mathbf{x},\mathbf{x}_V)\end{bmatrix}^T$ and matrices A, B, G be defined as in (22a)-(22c). Then, $\dot{V}'$ can be written in compact form as:

$$\begin{aligned}\dot{V}' &= -\boldsymbol{\kappa}^T(\mathbf{y},t)A(\mathbf{x},\mathbf{x}_V)\boldsymbol{\kappa}(\mathbf{y},t)+\boldsymbol{\kappa}^T(\mathbf{y},t)(G(\mathbf{x},\mathbf{x}_V)\mathbf{w}_f(\mathbf{x})\\ &\quad+\mathbf{z})\\ &\leq -\boldsymbol{\kappa}^T(\mathbf{y},t)A(\mathbf{x},\mathbf{x}_V)\boldsymbol{\kappa}(\mathbf{y},t)+\|G(\mathbf{x},\mathbf{x}_V)\mathbf{w}_f(\mathbf{x})+\mathbf{z}\|\\ &\quad\times\|\boldsymbol{\kappa}(\mathbf{y},t)\|.\end{aligned}$$

By Assumption 1 there exists a path connecting every agent $r \in \mathcal{V}$ with at least one agent $r \in \mathcal{V}_r$. Hence, whenever $\mathbf{y} \in \Omega_y$ and thus $\|\mathbf{x}_{r_k}(t) - \mathbf{x}_{r'_k}(t)\| < d_{k,\max} < \infty, k \in \mathcal{M}$, and $\|\mathbf{x}_r(t) - \mathbf{x}_V(t)\| < s_r < \infty, r \in \mathcal{V}_r$, it follows that $\|\mathbf{x}_r\| < \infty$, for every $r \in \mathcal{V}$ owing to Assumption

2, i.e., for every $r \in \mathcal{V}$ there exists a positive constant $\mathfrak{b}_r$, such that $\mathbf{x}_r(t) \in \mathcal{B}_r := \{\mathbf{x}_r \in \mathbb{R}^n : \|\mathbf{x}_r\| \leq \mathfrak{b}_r\}$, for every $t \in [0, \tau_{\max})$. Thus, by the boundedness of $g_k(t), \dot{g}_k(t), k \in \mathcal{M}$, $\gamma_r(t), \dot{\gamma}_r(t), r \in \mathcal{V}_r$, Assumption 2, continuity of $\mathbf{w}_f(\mathbf{x})$ and the Extreme Value Theorem there exists a constant $\mu' \in (0, +\infty)$ independent of t that satisfies $\|G(\mathbf{x}, \mathbf{x}_V)\mathbf{w}_f(\mathbf{x}) + \mathbf{z}\| \leq \mu'$, for every $t \in [0, \tau_{\max})$. Following similar analysis as in Theorem 1 and due to (19) it can be shown that $A(\mathbf{x}, \mathbf{x}_V)$ is a positive definite matrix for every $t \in [0, \tau_{\max})$ and thus $\dot{V} < 0$ whenever $\|\boldsymbol{\chi}\| > \frac{\mu'}{\beta \widetilde{\lambda}_{\min}(A)}$, where $\widetilde{\lambda}_{\min}(A) := \inf_{\substack{\mathbf{x}(t), \mathbf{x}_V(t) \\ t \in [0, \tau_{\max})}} \lambda_{\min}(A(\mathbf{x}(t), \mathbf{x}_V(t))) > 0$ and $\boldsymbol{\chi} = \begin{bmatrix} \boldsymbol{\varepsilon}^T & \boldsymbol{\delta}^T \end{bmatrix}^T$. This in turn leads to $\|\boldsymbol{\chi}(t)\| \leq \chi_{\max} := \max\{\|\boldsymbol{\chi}(0)\|, \frac{\mu'}{\beta \widetilde{\lambda}_{\min}(A)}\}$, for every $t \in [0, \tau_{\max})$. Using the inverse functions of $T_k(\star), k \in \mathcal{M}$, $\bar{T}_r(\star), r \in \mathcal{V}_r$ we can determine the sets $\Omega'_{\bowtie} \subset \Omega_{\bowtie}$, for every $\bowtie \in \{e, \xi\}$ which are compact, non-empty and satisfy $\bar{\mathbf{e}}(t) \in \Omega'_e$ and $\bar{\boldsymbol{\xi}}(t) \in \Omega'_\xi$, for every $t \in [0, \tau_{\max})$. If $\tau_{\max} < +\infty$, [38, Pr. C.3.6] implies that there exists a $t \in [0, \tau_{\max})$ such that $\mathbf{y}(t) \notin \Omega'_y := \Omega'_e \times \Omega'_\xi$ which leads to contradiction. Hence, $\tau_{\max} = +\infty$ and this concludes the proof. ∎

Theorem 2 establishes the satisfaction of the funnel constraints (7a)-(7b) for unknown nonlinear systems approximated online by the potentially nonlinear term $Y_r(\mathbf{x}_r)\boldsymbol{\theta}_r$ independent of the choice of the regressor terms.

Remark 6. *The regressor terms can be chosen freely among several function classes such as sigmoids, B-splines or sifted sigmoids [21]. While there is no restriction on their design other than Lipschitz continuity, these terms need to be tuned offline allowing for a sufficient level of approximation of a wide range of nonlinear behaviors within the compact sets $\mathcal{B}_r, r \in \mathcal{V}$. In practice, smaller approximation errors lead to a better transient and steady state performance for the system while larger deviations from the real values of $f_r(\mathbf{x}_r), r \in \mathcal{V}$ may induce an oscillatory behavior within the funnel.*

Remark 7. *The control laws* (18), (27) *can be directly applied to time-varying formation and tracking control under slight modifications on the choice of the parameters $\eta_{k,\min}, \eta_{k,\max}, k \in \mathcal{M}$ and $\zeta_{r,\min}, \zeta_{r,\max}, r \in \mathcal{V}_r$, provided that the distance parameter functions and their derivatives are bounded. In particular, assume the following: 1) the time-varying formation parameters $d_k : \mathbb{R}_{\geq 0} \to [\underline{\mathfrak{d}}_k, \overline{\mathfrak{d}}_k], k \in \mathcal{M}$ and $\bar{d}_r : \mathbb{R}_{\geq 0} \to [\underline{\mathfrak{h}}_r, \overline{\mathfrak{h}}_r], r \in \mathcal{V}_r$ are continuously differentiable with $d_{k,\min} < \underline{\mathfrak{d}}_k \leq \overline{\mathfrak{d}}_k < d_{k,\max}$ and $\bar{d}_{r,\min} < \underline{\mathfrak{h}}_r \leq \overline{\mathfrak{h}}_r < s_r$, for every $k \in \mathcal{M}$ and $r \in \mathcal{V}_r$, respectively, and 2) there exist positive constants $\mathfrak{d}'_k$ and $\mathfrak{h}'_r$ for every $k \in \mathcal{M}$ and $r \in \mathcal{V}_r$, respectively, such that $|\dot{d}_k(t)| \leq \mathfrak{d}'_k$ and $|\dot{\bar{d}}_r(t)| \leq \mathfrak{h}'_r$, for every $t \geq 0$. Then, setting $\eta_{k,\min} := \underline{\mathfrak{d}}_k^2 - d_{k,\min}^2$, $\eta_{k,\max} := d_{k,\max}^2 - \overline{\mathfrak{d}}_k^2$, $\zeta_{r,\min} := \underline{\mathfrak{h}}_r^2 - \bar{d}_{r,\min}^2$ and $\zeta_{r,\max} := s_r^2 - \overline{\mathfrak{h}}_r^2$, we can ensure the satisfaction of the funnel constraints* (7a)-(7b) *as well as boundedness of all closed loop signals.*

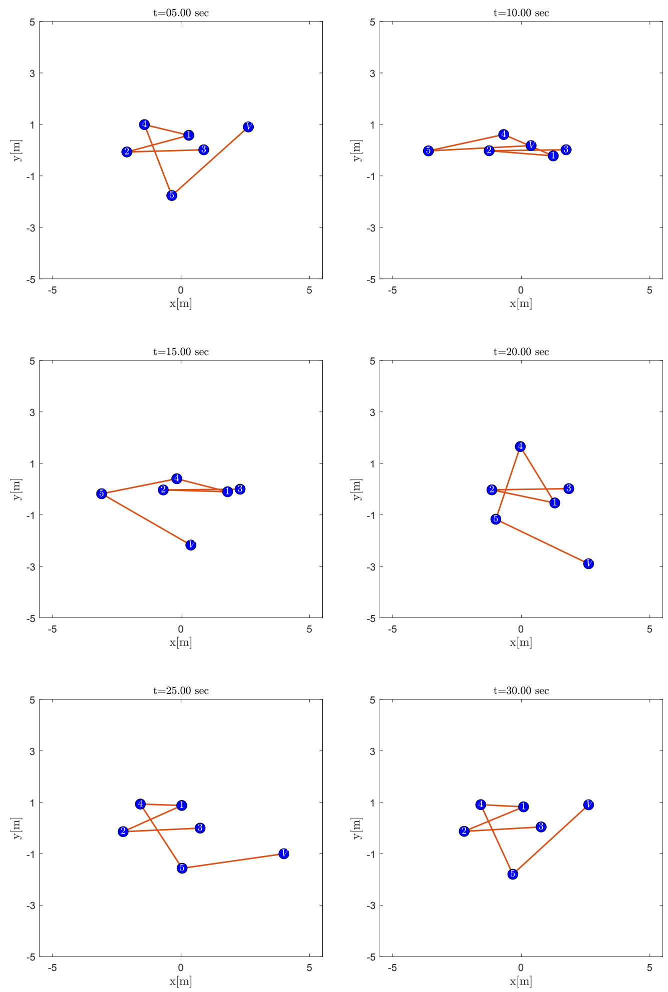

Fig. 3: Evolution of the multi-agent system and the virtual leader at selected time instants when the nonlinear dynamics are known.

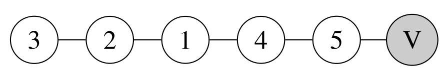

Fig. 4: Graph topology for the scenario considered in Section V.

V. Numerical Example

In order to demonstrate the applicability of the proposed controllers we will consider a team of $R = 5$ agents whose dynamics are given by (1), where $f_r(\mathbf{x}_r), h_r(\mathbf{x}_r), \mathbf{w}_r(t), r \in \mathcal{V}$ are given by $f_r(\mathbf{x}_r) = \begin{bmatrix} -x_r + 0.1y_r^2 + \sin(x_r y_r) & -(1 + x_r^2)y_r - y_r^3 \end{bmatrix}^T$, $h_r(\mathbf{x}_r) = \begin{bmatrix} 1 + x_r^2 & 0 \\ 0 & 1 + y_r^2 \end{bmatrix}$, $\mathbf{w}_r(t) = 0.05\sin(2t)\mathbf{1}_2$, and $\mathbf{x}_r = \begin{bmatrix} x_r & y_r \end{bmatrix}^T$. Here we consider the tree communication graph G shown in Figure 4 with agent 5 being the only agent with access to the virtual leader's state information. In particular we set $\epsilon_1 = (1,2), \epsilon_2 = (2,3), \epsilon_3 = (1,4)$ and $\epsilon_4 = (4,5)$. We choose $d_{k,\min} = 2, d_{k,\max} = 4$ for every $k \in \mathcal{M}$ and $\bar{d}_{5,\min} = 3, s_5 = 5$. The desired formation parameters are chosen as $d_1 = d_3 = 2.5$, $d_2 = d_4 = 3$ and $\bar{d}_5 = 4$. The virtual leader's

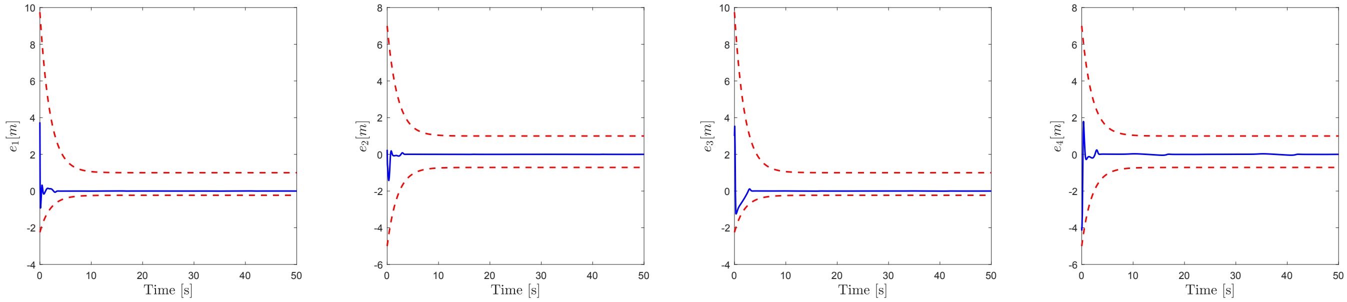

Fig. 5: Evolution of distance errors $e_k, k \in \mathcal{M}$ for unknown nonlinear dynamics.

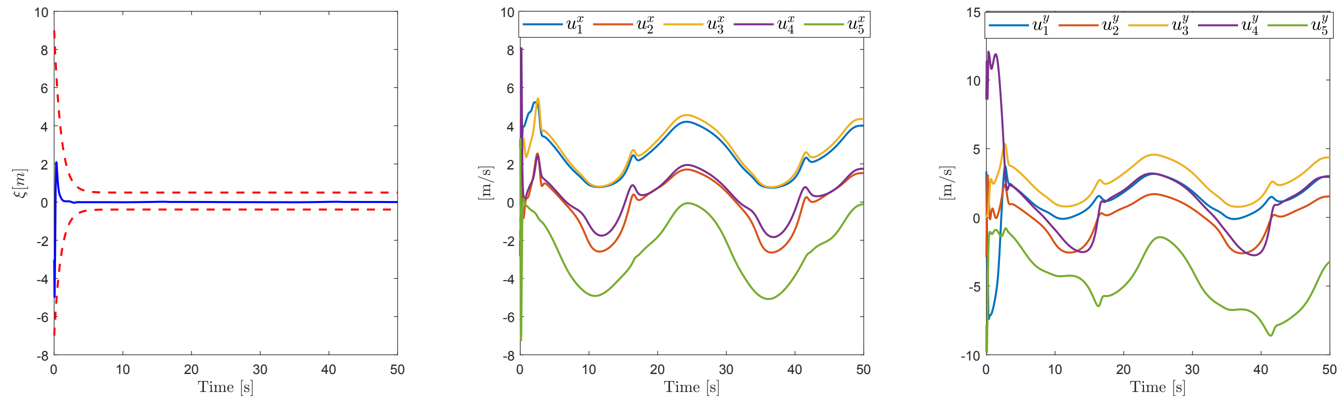

Fig. 6: Evolution of ξ and the resulting control signals along the x and y axis for unknown nonlinear dynamics.

trajectory is given by $x_V(t) = 2 + 2\cos(\omega t)$ and $y_V(t) = -1 + 2\sin(\omega t)$ for every t, where $\omega = \frac{2\pi}{25}$, and the initial conditions of the agents are chosen as $\mathbf{x}_1(0) = \begin{bmatrix}-1 & -1\end{bmatrix}^T$, $\mathbf{x}_2(0) = \begin{bmatrix}0 & 2\end{bmatrix}^T$, $\mathbf{x}_3(0) = \begin{bmatrix}3 & 2.5\end{bmatrix}^T$, $\mathbf{x}_4(0) = \begin{bmatrix}0 & 3\end{bmatrix}^T$, and $\mathbf{x}_5(0) = \begin{bmatrix}1 & 1\end{bmatrix}^T$. The parameters of $g_k(t), k \in \mathcal{M}$ are chosen as $l_k^g = 0.5$ and $\bar{g}_{k,\infty} = 1$, for every k while the parameters of $\gamma_5(t)$ as chosen as $l_5^\gamma = 1$, $\bar{\gamma}_{5,\infty} = 0.5$. Note that the initial conditions of the multi-agent system ensure that (3)-(4) are satisfied at $t = 0$. In the case of unknown nonlinear dynamics the terms $f_r(\mathbf{x}_r)$ are approximated using single layer neural networks with $p_r = 50$ nodes for every $r \in \mathcal{V}$. Specifically, $Y_r(\mathbf{x}_r)$ are determined by sigmoid functions $\vartheta_{vr}(\mathbf{x}_r) = \frac{1}{1+\exp\left(-\mathbf{s}_{vr}^T\mathbf{x}_r - l_{vr}\right)}, v = 1, \ldots, 100$ whose parameters are chosen offline by trial and error and considered known during the formation task. The gain matrices Δ_r are chosen as $\Delta_r = 0.2I_{p_r}$ for every r and $\hat{\boldsymbol{\theta}}_r(0) = \mathbf{0}, r \in \mathcal{V}$ indicating that there is no initial knowledge about the system dynamics. In Figures 1-2 and 5-6 the evolution of the error signals $e_k, k \in \mathcal{M}$ and ξ_5 as well as the resulting control signals are shown for known and unknown dynamics, respectively. Irrespective of the amount of knowledge we have for the system the proposed controllers ensure that the error signals remain within their corresponding funnels. In addition, it should be noted that when approximate knowledge of the unknown dynamical terms is considered in the control law through the approximation term $Y_r^T(\mathbf{x}_r)\hat{\boldsymbol{\theta}}_r$, then lower steady-state errors are observed at the cost of a slightly higher control effort. Finally, the evolution of the multi-agent system and the virtual leader at selected time instants is shown in Figures 3 and 7 for known and unknown dynamics, respectively.

VI. Conclusions

In this work a model-free prescribed performance control law is designed to ensure a desired distance-based formation of communicating agents as well as of a selected set of agents with respect to a dynamic virtual leader. In addition, an adaptive law is proposed when the nonlinear dynamics of the multi-agent team are considered unknown. Future work will consider more complex communication topologies and higher relative degree dynamics.

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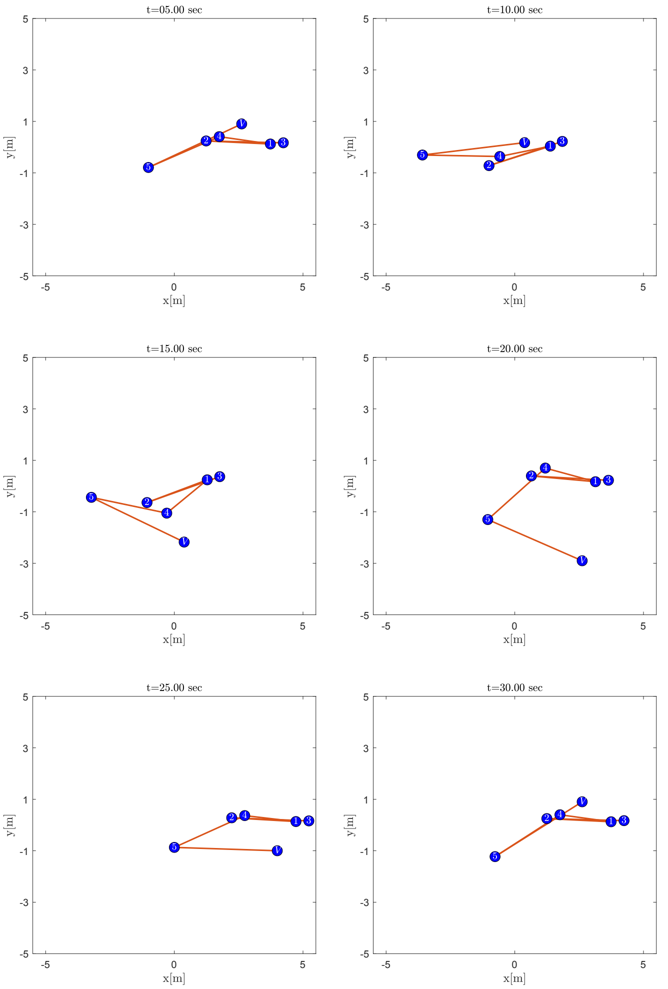

Fig. 7: Evolution of the multi-agent system and the virtual leader at selected time instants when the nonlinear dynamics are unknown.

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Appendix

Lemma 1. *Consider a matrix* $E \in \{0,1\}^{m \times n}$ *with* $m \geq n$, $E(i,j) = 1$ *at exactly one* $i \in \{1, \ldots, m\}$ *for each column* j, *i.e., if* $E(i,j) = 1$, *then* $E(i,j') = 0$, *for every* $j' \neq j$ *with* $j, j' \in \{1, \ldots, n\}$, *and* $E(i',j) = 0$, *for every* $i' \neq i$, *where* $i, i' \in \{1, \ldots, m\}$. *Then,* $E^T E = I_n$.

Proof. Let $E^T = [a_{ij}], E = [b_{ij}]$ and $E^T E = [c_{ij}]$. By matrix multiplication it is known that $c_{ij} = \sum_{k=1}^{m} a_{ik} b_{kj}$. Let $b_{k_1 j} = 1$. Then, by the properties of E we have $b_{kj} = 0,\ k \in \{1, \ldots, m\}$ for every $k \neq k_1$ and $b_{k_1, j'} = 0,\ j' \in \{1, \ldots, n\}$, $j \neq j'$. The latter implies that $a_{j' k_1} = 0$, for every $j \neq j'$. Then, $c_{ij} = a_{ik_1} b_{k_1 j} = a_{ik_1}$. If $i \neq j$, then $a_{ik_1} = 0$, thus $c_{ij} = 0$. Finally, if $i = j$, then $a_{jk_1} = b_{k_1 j} = 1$ and thus $c_{ii} = 1$. ■

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