Distributed Event-Triggered Communication and Control of Linear Multi-Agent Systems Under Tactile Communication

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***Abstract*—This note is concerned with the consensus of linear multi-agent systems under tactile communication. Motivated by the emerging tactile communication technology where extremely low-latency has to be supported, a distributed event-triggered communication and control scheme is proposed for the data reduction of each agent. Firstly, an event-triggered data reduction scheme is designed for the communication between neighbors. Under such a communication scheme, a distributed event-triggered output feedback controller is further implemented for each agent, which is updated asynchronously with the communication action. It is proven that the consensus of the underlying multi-agent systems is achieved asymptotically. Furthermore, it is shown that the proposed communication and control strategy fulfils the reduction of both the frequency of communication and controller updates as well as excluding Zeno behavior. A numerical example is given to illustrate the effectiveness of the proposed control strategy.**

***Index Terms*—Distributed event-triggered control; multi-agent systems; output feedback; tactile communication.**

I. Introduction

CURRENT technological trends involving the emergence of Internet of Things (IoT) enable the interconnection of numerous smart devices. In a natural evolution to IoT, Tactile Internet and the underlying tactile/haptic communication are believed to be a next evolutionary technological leap [1]. At the very core of the design of Tactile Internet is the '1ms Challenge' (a round-trip communication below 1ms) [2]. However, while most cutting-edge current networks (e.g., 5G) will deliver very high data rates, they will also provide communication delays of the order of 25 ms [1], which is unacceptable for many IoT services. It is well known that high network traffic is directly related to network congestion and hence large transmission latency that can cause instability of the overall system [3]. Data reduction methods are therefore of great importance for the Tactile Internet applications.

The state-of-the-art in Tactile Internet communications for data reduction is the perceptual deadband-based (PDB) schemes. Otanez et al. [4] were first to propose to use deadbands as a solution to reduce network traffic for networked control systems. By taking the human perceptual limitations into account, Hinterseer et al. [5] then proposed a PDB principle, which was inspired by a famous psychophysical law called 'Weber's law' [6]. In their work, the deadband threshold was adjusted for each transmitted sample (proportional to the most recently transmitted sample). As long as the perception error stays within the deadband, no transmissions occur, thereby resulting in a reduced communication sequence. It is shown that the PDB data reduction can lead to high reduction rates. However, stability of the global control loop, as stated in [7], is not guaranteed. In addition, all of the above papers considered the tactile communication within one single plant (called agent), however, the coordination of multiple agent systems under tactile communication was not investigated.

Looking at the PDB principle in tactile communication, we find that its idea essentially coincide with the event-triggered control (ETC) paradigm in control community, which has been widely studied in the coordination of multi-agent systems (MAS) in recent years. Several efforts were devoted to the ETC of MAS with single/double-integrator dynamics [9]–[12]. For MAS with linear dynamics, some researches have recently considered event-triggered consensus problem [13]–[18]. In [13]–[15], continuous communication of neighbors' states are required to check the triggering conditions. Then, in [16]–[18], the continuous requirement for communication is relaxed by adding additional assumptions [16] or by using the matrix exponential function e^{At} [17], [18], nevertheless the continuous controller update is still required. However, to meet the requirement of tactile communication, both the continuous requirements for communication and control have to be relaxed. Moreover, control theory provides guidance on how to design appropriate data reduction schemes such that the stability issue of tactile communication is tackled.

Motivated by the above discussion, this note investigates the distributed event-triggered communication and control (ETCC) of linear MAS under tactile communication, where output feedback is considered. Although event-triggered output synchronization is also considered in [19], additional reference generators are required for each agent and globally bounded synchronization is achieved. Moreover, while asynchronous operation of the output and input event detectors is considered in [20], the analysis is focusing on the single-agent case. The contribution of this note is summarized as follows. Firstly, an event-triggered data reduction scheme is proposed for the communication between neighbors. Under this communication scheme, a distributed event-triggered output feedback controller is further introduced to reduce the frequency of controller updates, and it is updated asynchronously with the communication action. Moreover, the consensus of the closed loop MAS is proved to be achieved asymptotically under the proposed communication and control strategy. Furthermore, it is shown that the integration of tactile communication and ETC is capable of reducing the frequency of communication and controller updates as well as excluding the so-called Zeno behavior.

The remainder of this note is organized as follows. In Section II, some necessary preliminaries on graph theory are provided and the problem is formulated. The main results on the distributed ETCC under tactile communication are presented in Section III. Section IV provides an illustrative example and Section V concludes the note.

II. Preliminaries and Problem Statement

Denote $\mathbb{R}$ as the set of real numbers, $\mathbb{Z}$ as the set of nonnegative integers, $\mathbb{R}^n$ as the $n-$dimension real vector space, $\mathbb{R}^{n\times m}$ as the $n \times m$ real matrix space. I_n is the identity matrix of order n and 1_n is the column vector of order n with all entries equal to one. Let $\|x\|$ and $\|A\|$ be the Euclidean norm of vector x and matrix A. For a symmetric matrix A, denote $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ as the smallest

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and largest eigenvalue of A, and $A \succ 0$ means that A is positive definite. For a complex number λ, let $\mathrm{Re}(\lambda)$, $|\lambda|$ be the real part and the modulus of λ, respectively. In addition, we use $\cap$ to denote the logical operator AND and $\cup$ the logical operator OR. The Kronecker product is denoted by $\otimes$.

A. Graph Theory

Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ be a directed graph (digraph) of order n with the set of nodes $\mathcal{V} = 1, 2, \ldots, N$, and $\mathcal{E} \subseteq \{(i, j) : i, j \in \mathcal{V}, j \neq i\}$ being the set of edges. If $(j, i) \in \mathcal{E}$, then node j is called a neighbor of node i and node j can receive information from node i. A directed path from node i_1 to node i_n is a sequence of ordered edges of the form $(i_k, i_{k+1}), k = 1, 2, \ldots, n-1$. A digraph contains a directed spanning tree if there exists a node called the root such that there exist directed paths from this node to every other node. The neighboring set of node i is denoted by $\mathcal{N}_i = \{j \in \mathcal{E} | (j, i) \in \mathcal{E}\}$ and $\mathcal{N}_i^+ = \mathcal{N}_i \cup \{i\}$.

The adjacency matrix is denoted by $\mathcal{A} = (a_{ij})_{N\times N}$ and is given by $a_{ij} = 1$, if $(j, i) \in \mathcal{E}$, otherwise $a_{ij} = 0$. Let $\mathcal{D} = (d_{ij})_{N\times N}$ represent the degree matrix which is a diagonal matrix with entries $d_i = \sum_{j=1, j\neq i}^{N} a_{ij}$. Then the Laplacian matrix of the digraph $\mathcal{G}$ is defined as $L = (l_{ij})_{N\times N} = \mathcal{D} - \mathcal{A}$.

Assumption 1: The digraph $\mathcal{G}$ contains a directed spanning tree.

Let

$$\tilde{L} = (\tilde{l}_{ij})_{(N-1)\times(N-1)} = \begin{pmatrix} l_{22} - l_{12} & \cdots & l_{2N} - l_{1N} \\ \ldots & \ddots & \ldots \\ l_{N2} - l_{12} & \cdots & l_{NN} - l_{1N} \end{pmatrix}. \tag{1}$$

Lemma 1: [21] Denote the eigenvalues of Laplacian matrix L and the matrix $\tilde{L}$, respectively by $\lambda_1, \lambda_2, \ldots, \lambda_N$ and $\mu_1, \mu_2, \ldots, \mu_{N-1}$, where $0 = |\lambda_1| \leqslant |\lambda_2| \leqslant \ldots \leqslant |\lambda_N|$ and $|\mu_1| \leqslant |\mu_2| \leqslant \ldots \leqslant |\mu_{N-1}|$. Then $\lambda_2 = \mu_1, \lambda_3 = \mu_2, \cdots, \lambda_N = \mu_{N-1}$.

Lemma 2: [18] Suppose that the matrix $A \in \mathbb{R}^{n\times n}$ is Hurwitz. Then, for all $t \geq 0$, it holds that $\|e^{At}\| \leq \|P_A\| \|P_A^{-1} c_A e^{a_A t}\|$, where P_A is a nonsingular matrix such that $P_A^{-1} A P_A = J_A$ with J_A being the Jordan canonical form of A, c_A is a positive constant determined by A, and $\max_i \mathrm{Re}(\lambda_i(A)) < a_A < 0$.

B. Problem Statement

Consider a MAS with N agents moving in the n dimensional Euclidean space, each of which is formulated by

$$\begin{aligned} \dot{x}_i(t) &= A x_i(t) + B u_i(t), \\ y_i(t) &= C x_i(t), \quad i = 1, 2, \ldots, N. \end{aligned} \tag{2}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{l\times n}$ are constant matrices; $x_i \in \mathbb{R}^n, u_i \in \mathbb{R}^m$ and $y_i \in \mathbb{R}^l$ are agent i's state, control input and measurement output, respectively.

Assumption 2: [22] The matrix pair (A, B) is stabilizable. That is, the following algebraic Riccati equation (ARE):

$$A^T P + PA - PBR^{-1}B^T P + Q = 0, \tag{3}$$

has a unique solution $P = P^T \succ 0$ for any given matrices $R = R^T \succ 0$ and $Q = Q^T \succ 0$.

Assumption 3: The matrix pair (A, C) is detectable.

An observer-based consensus protocol is proposed as

$$\dot{\hat{x}}_i(t) = A\hat{x}_i(t) + Bu_i(t) + F(y_i(t) - C\hat{x}_i(t)), \tag{4}$$

and

$$u_i(t) = -cK \sum_{j\in\mathcal{N}_i} \Big(\hat{x}_i(t) - \hat{x}_j(t)\Big), \tag{5}$$

where $c > 0$ is the coupling gain, $\hat{x}_i \in \mathbb{R}^n$ is the observer state, and $F \in \mathbb{R}^{n\times l}$ and $K \in \mathbb{R}^{m\times n}$ are the feedback gain matrices to be determined. Let the gain matrix $K = R^{-1}B^T P$, where P is the unique solution of ARE (3) for appropriately chosen R and Q. It was proven in [23] that under Assumptions 1-3, consensus of the closed loop MAS (2), (4) is achieved with the controller (5) if and only if the coupling gain $c > 1/(2\underline{\lambda}_R)$, where $\underline{\lambda}_R = \min_{2\leq i\leq N} \mathrm{Re}(\lambda_i)$.

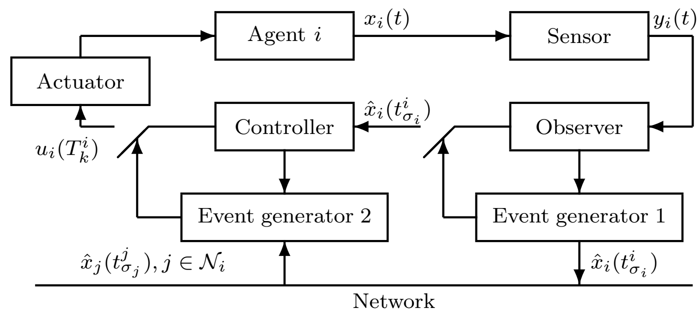

Fig. 1: Structure of the control system.

However, in order to be implemented, the controller is required to access the observer state continuously and update continuously. Although different ETC strategies regarding consensus of linear MAS were proposed recently, either continuous communication [13]–[15] or continuous controller update [17], [18] is required. In this note, we are interested in an implementation of distributed ETCC under tactile communication, which is able to relax the aforementioned continuous communication requirements.

III. Distributed ETCC under tactile communication

In this section, a distributed ETCC strategy is proposed for linear MAS. The information within one agent (from sensor to controller, from controller to actuator) and between neighboring agents are assumed to be transmitted in a tactile fashion, i.e., the transmission latency can be ignored. However, the data rate has to be reduced. By means of reducing data rate, a substantial reduction of latency may be achieved to support tactile communication.

The desired structure of the control system is shown in Fig. 1, where event generators are implemented both in the communication side and the controller side. The proposed design procedure can be divided into two major stages. In the first stage, an event-triggered data reduction scheme is designed for the communication between neighbors. In the second stage, a distributed event-triggered controller that depends only on the transmitted information of agent itself and its neighbors is proposed for each agent. The following subsections detail this procedure.

A. Event-triggered data reduction for communication

This subsection presents the design of the event-triggered data reduction scheme. For each agent i, let $t^i_{\sigma_i}, \sigma_i = 0, 1, 2, \ldots$ be the increasing sequence of communication time instants at which $\hat{x}_i$ is transmitted. Then, we define the communication measurement error for agent i as

$$e_i(t) = e^{A\left(t - t^i_{\sigma_i}\right)} \hat{x}_i(t^i_{\sigma_i}) - \hat{x}_i(t), \quad t \in [t^i_{\sigma_i}, t^i_{\sigma_i+1}),$$

where the communication time instant $t^i_{\sigma_i}$ is updated by

$$t^i_{\sigma_i+1} = \inf\Big\{t > t^i_{\sigma_i} : f(t, e_i(t)) \geq 0\Big\}, \tag{6}$$

where

$$f(t, e_i(t)) = \|e_i(t)\| - I_0 e^{-\alpha t} \tag{7}$$

with $I_0 > 0$ and $\alpha > 0$ is a constant to be determined. The condition $f(t, e_i(t)) \geq 0$ is called the communication function. Without loss of generality, we assume $t_0^i = 0, \forall i$.

Remark 1: For Tactile Internet communications, the PDB data reduction scheme is widely utilized for the transmission of haptic data [24]. However, stability of the global control loop, as stated in [7], is not guaranteed. Although reconstruction strategies on the receiver side are proposed in [5], [7], none of them are appropriate for the multi-agent coordination case. Therefore, in this note, a well-defined exponentially decaying function is adopted as a threshold for the communication function of each agent, which is crucial to ensure stability of the closed-loop MAS as well as to exclude the Zeno behavior.

B. Event-triggered data reduction for controller update

Before proceeding, the following notations are introduced. For each agent i, $\zeta_i(t)$ is defined as $\zeta_i(t) = e^{A\left(t-t_{\sigma_i}^i\right)}\hat{x}_i(t_{\sigma_i}^i), t \in [t_{\sigma_i}^i, t_{\sigma_i+1}^i)$ and $\zeta_j^i(t)$ is defined as $\zeta_j^i(t) = e^{A\left(t-t_{\sigma_j}^j\right)}\hat{x}_j(t_{\sigma_j}^j), t \in [t_{\sigma_j}^j, t_{\sigma_j+1}^j), j \in \mathcal{N}_i$.

The event-triggered controller for each agent i is designed as

$$u_i(t) = -cK \sum_{j \in \mathcal{N}_i} \left(\zeta_i(T_k^i) - \zeta_j^i(T_k^i)\right), \quad t \in [T_k^i, T_{k+1}^i), \tag{8}$$

where $T_k^i, k \in \mathbb{Z}$ is the controller update time sequence of agent i. From the above definitions, one has $\zeta_i(T_k^i) = e^{A\left(T_k^i - t_{\sigma_i}^i\right)}\hat{x}_i(t_{\sigma_i}^i), \sigma_i \triangleq \arg\min_{l \in \mathbb{Z}: T_k^i \geq t_l^i} \left\{T_k^i - t_l^i\right\}$, $\zeta_j^i(T_k^i) = e^{A\left(T_k^i - t_{\sigma_j}^j\right)}\hat{x}_j(t_{\sigma_j}^j), \sigma_j \triangleq \arg\min_{l \in \mathbb{Z}: T_k^i \geq t_l^j} \left\{T_k^i - t_l^j\right\}, \forall j \in \mathcal{N}_i$, where $t_{\sigma_i}^i, \hat{x}_i(t_{\sigma_i}^i)$ and $t_{\sigma_j}^j, \hat{x}_j(t_{\sigma_j}^j), j \in \mathcal{N}_i$ represent the latest communication time instant and the latest received information of agent i and its neighbors j before T_k^i, respectively.

Let $z_i(t) = \sum_{j \in \mathcal{N}_i} (\zeta_i(t) - \zeta_j^i(t))$. Define the controller measurement error of agent i as the combined state differences between the last triggering instant T_k^i and current time, which is

$$\begin{aligned} \hat{e}_i(t) &= \sum_{j \in \mathcal{N}_i} \left(\zeta_i(T_k^i) - \zeta_j^i(T_k^i)\right) - \sum_{j \in \mathcal{N}_i} \left(\zeta_i(t) - \zeta_j^i(t)\right) \\ &= z_i(T_k^i) - z_i(t). \end{aligned} \tag{9}$$

Combining $e_i(t)$ and $\hat{e}_i(t)$, the controller (8) can be rewritten as

$$u_i(t) = -cK \left\{ \sum_{j \in \mathcal{N}_i} \left(\hat{x}_i(t) + e_i(t) - (\hat{x}_j(t) + e_j(t))\right) + \hat{e}_i(t) \right\}. \tag{10}$$

Define the observation error of agent i as $\tilde{x}_i(t) = x_i(t) - \hat{x}_i(t)$, let $\tilde{x}(t) = \left[\tilde{x}_1^T(t), \ldots, \tilde{x}_N^T(t)\right]^T$. Then the dynamics of the observation error system is $\dot{\tilde{x}}(t) = (I_N \otimes (A - FC))\tilde{x}(t)$, which is globally asymptotically stable if and only if the matrix $A - FC$ is Hurwitz.

Let the gain matrix $K = R^{-1}B^T P$, where $P = P^T \succ 0$ is the unique solution of ARE (3) for appropriately chosen $R = R^T \succ 0$ and $Q = Q^T \succ 0$. Let $\hat{x}, e, \hat{e}$ be the concatenated vectors of $\hat{x}_i, e_i, \hat{e}_i$, respectively. Then, the dynamics of the observer (4) can be rewritten as

$$\begin{aligned} \dot{\hat{x}}(t) =& (I_N \otimes A)\hat{x}(t) - \left(cL \otimes BR^{-1}B^T P\right)(\hat{x}(t) + e(t)) \\ &- \left(cI_N \otimes BR^{-1}B^T P\right)\hat{e}(t) + (I_N \otimes FC)\tilde{x}(t). \end{aligned} \tag{11}$$

Let $\xi_i(t) = \hat{x}_i(t) - \hat{x}_1(t), \forall i$ and the disagreement vector $\xi(t) = \left[\xi_1^T(t), \xi_{2-N}^T(t)\right]^T$, where $\xi_{2-N}(t) \triangleq \left[\xi_2^T(t), \ldots, \xi_N^T(t)\right]^T \in \mathbb{R}^{(N-1)n}$. It follows that $\xi_1(t) \equiv 0$, and the vectors $\xi_{2-N}(t)$ satisfy

$$\begin{aligned} \dot{\xi}_{2-N}(t) =& (I_{N-1} \otimes A - c\tilde{L} \otimes BR^{-1}B^T P)\xi_{2-N}(t) \\ &- \left(c\tilde{L}W \otimes BR^{-1}B^T P\right)e(t) \\ &- \left(cW \otimes BR^{-1}B^T P\right)\hat{e}(t) + (W \otimes FC)\tilde{x}(t) \\ \triangleq& \Pi\xi_{2-N}(t) - G_1 e(t) - G_2\hat{e}(t) + G_3\tilde{x}(t), \end{aligned} \tag{12}$$

where $\Pi = I_{N-1} \otimes A - c\tilde{L} \otimes BR^{-1}B^T P$, $G_1 = c\tilde{L}W \otimes BR^{-1}B^T P$, $G_2 = cW \otimes BR^{-1}B^T P$, $G_3 = W \otimes FC$, $W = [-1_{N-1}, I_{N-1}] \in \mathbb{R}^{(N-1)\times N}$ and $\tilde{L}$ is defined in (1). It can be seen that the observer (11) achieves consensus, if and only if $\lim_{t\to\infty} \xi_{2-N}(t) = 0$.

Since Assumption 1 holds, it follows from Lemma 1 that $\tilde{L} \in \mathbb{R}^{(N-1)\times(N-1)}$ is a full-rank matrix. Moreover, the eigenvalues of $\tilde{L}$ have positive real parts. Choosing the coupling gain $c > 1/(2\underline{\lambda}_R)$, it is proven in [23] that the matrix Π is Hurwitz. Thus, there exists a positive definite matrix $\bar{P} = \bar{P}^T$ that satisfies the Lyapunov condition $\bar{P}\Pi + \Pi^T\bar{P} = -\bar{Q}$ for any given $\bar{Q} = \bar{Q}^T \succ 0$.

Now, we are ready to define the controller update time sequence. The controller update time instants T_k^i for each agent i are given by

$$T_{k+1}^i = \inf\left\{t > T_k^i : g\left(\hat{e}_i(t), z_i(t), t\right) \geq 0\right\}, \tag{13}$$

where

$$g\left(\hat{e}_i(t), z_i(t), t\right) = \|\hat{e}_i(t)\| - \left(\theta\gamma\|z_i(t)\| + \eta e^{-\alpha t}\right) \tag{14}$$

with constants $0 \leq \theta < 1$, $\gamma = \lambda_{\min}\left(\bar{Q}\right)/(2(\hat{l} + N\sqrt{\hat{l}})\left\|\bar{P}G_2\right\|)$, $\eta > 0$, $\hat{l} = \max_i\{d_i\} \leq N-1$ and α is defined in (7). The condition $g\left(\hat{e}_i(t), z_i(t), t\right) \geq 0$ is called the control function. Without loss of generality, we assume $T_0^i = 0, \forall i$.

From the definition of $z_i(t)$ and $\hat{e}_i(t)$, one can see that only the discrete communication time instants and the states transmitted at these communication time instants (determined by the communication function (7) of the agent itself and its neighbors) are required to implement the control function (14). When the controller measurement error $\hat{e}_i$ exceeds a certain threshold, that is, $g\left(\hat{e}_i(T_k^i), z_i(T_k^i), t\right) \geq 0$, an event is triggered for agent i. Agent i updates its controller using the latest communication instants and the latest received states of itself and its neighbors. Meanwhile, the controller measurement error $\hat{e}_i$ is reset to zero.

Remark 2: The communication function $f(t, e_i(t)) \geq 0$ can be seen as a trigger for communication and the control function $g\left(\hat{e}_i(t), z_i(t), t\right) \geq 0$ can be seen as a trigger for controller update. They work asynchronously. Note that no communication is required at the controller update time instants for each agent, which is different from the previous ETC strategies, where each agent updates its controller and communicates with its neighbors at the same time.

Remark 3: It is worth to mention that except for the controller update time instants, the control function $g\left(\hat{e}_i(t), z_i(t), t\right)$ will also be reset at the communication instants of itself and its neighbors. Therefore, $g\left(\hat{e}_i(t), z_i(t), t\right)$ can be discontinuous within two neighboring controller update instants, which brings additional difficulty in proving the exclusion of Zeno behavior. Furthermore, different from the control function proposed in [10], [17], in this paper, an extra term $\eta e^{-\alpha t}$ is introduced such that Zeno triggering is excluded.

C. Convergence result

Definition 1: The consensus of the closed loop MAS (2), (4) is said to be achieved asymptotically, if and only if for any initial condition, $\lim_{t\to\infty}\|\hat{x}_i(t) - \hat{x}_j(t)\| = 0, \quad \forall(i,j) \in \mathcal{E}$ and $\lim_{t\to\infty}\|x_i(t) - \hat{x}_i(t)\| = 0, \quad \forall i$.

Now, we are in the position to give the following result.

Theorem 1: Consider the MAS (2) with the observer (4), where the matrix F is chosen such that the matrix $(A-FC)$ is Hurwitz. Let controller be given in (8), where the coupling gain satisfies $c > 1/(2\underline{\lambda}_R)$. Suppose Assumptions 1-3 hold and that the communication function (7) and the control function (14) are applied with $0 < \alpha < \min\{((1-\theta)\lambda_{\min}(\bar{Q})-a)/2\lambda_{\max}(\bar{P}), -\max_i \mathrm{Re}(\lambda_i(A-FC))\}$, where $0 < a < (1-\theta)\lambda_{\min}(\bar{Q})$ is a positive constant that can be chosen arbitrarily small. Then, the consensus of the closed loop MAS (2), (4) is achieved asymptotically. Furthermore, Zeno behavior is excluded.

Proof. Consider the following Lyapunov function candidate $V(t) = \xi_{2-N}(t)^T(t)\bar{P}\xi_{2-N}(t)$. Differentiating $V(t)$ along the trajectories of (12), one has

$$\begin{aligned}\dot{V}(t) =& -\xi_{2-N}^T(t)\bar{Q}\xi_{2-N}(t) - 2\xi_{2-N}^T(t)\bar{P}G_1 e(t)\\ &-2\xi_{2-N}^T(t)\bar{P}G_2\hat{e}(t) + 2\xi_{2-N}^T(t)\bar{P}G_3\tilde{x}(t).\end{aligned} \tag{15}$$

According to (6), one has $\|e_i(t)\| \le I_0 e^{-\alpha t}, \forall i$, and thus $\|e(t)\| \le \sqrt{N} I_0 e^{-\alpha t}$. Besides, for agent i, an event for controller update is triggered at T_{k+1}^i when $g(\hat{e}_i(t), z_i(t)) \ge 0$. Thus, one has $g(\hat{e}_i(t), z_i(t)) < 0$ for $t \in [T_k^i, T_{k+1}^i)$. According to the definition of $z_i(t)$, one has

$$\begin{aligned}\|z_i(t)\| =& \Big\| \sum_{j\in\mathcal{N}_i} (\zeta_i(t) - \zeta_j^i(t)) \Big\|\\ \le& \Big\| \sum_{j\in\mathcal{N}_i} ((\hat{x}_i(t)-\hat{x}_1(t)) - (\hat{x}_j(t)-\hat{x}_1(t))) \Big\|\\ &+ \Big\| \sum_{j\in\mathcal{N}_i} (e_i(t)-e_j(t)) \Big\|\\ \le& \sum_{j\in\mathcal{N}_i} (\|\xi_i(t)\| + \|\xi_j(t)\|) + \sum_{j\in\mathcal{N}_i} (\|e_i(t)\| + \|e_j(t)\|).\end{aligned} \tag{16}$$

Letting $\hat{\gamma} = \lambda_{\min}(\bar{Q})/(2\|\bar{P}G_2\|)$, one further has

$$\begin{aligned}\|\hat{e}(t)\| \le& \sum_{i=1}^{N} \left(\theta\gamma \|z_i(t)\| + \eta e^{-\alpha t}\right)\\ \le& \theta\hat{\gamma}(\|\xi(t)\| + \|e(t)\|) + N\eta e^{-\alpha t}\\ \le& \theta\hat{\gamma}\|\xi_{2-N}(t)\| + (\theta\hat{\gamma} I_0 + \eta) N e^{-\alpha t}.\end{aligned} \tag{17}$$

Using the inequality $2xy \le ax^2 + 1/ay^2, \forall a > 0$ several times, (15) can then be rewritten as

$$\begin{aligned}\dot{V}(t) \le& -\lambda_{\min}(\bar{Q})\|\xi_{2-N}(t)\|^2 + 2\|\xi_{2-N}(t)\|\left\|\bar{P}G_1\right\|\|e(t)\|\\ &+ 2\|\xi_{2-N}(t)\|\left\|\bar{P}G_2\right\|\|\hat{e}(t)\| + 2\|\xi_{2-N}(t)\|\left\|\bar{P}G_3\right\|\|\tilde{x}(t)\|\\ \le& -\left((1-\theta)\lambda_{\min}(\bar{Q}) - a\right)\|\xi_{2-N}(t)\|^2\\ &+ \frac{2}{a}\left(\left\|\bar{P}G_1\right\| I_0 + \left\|\bar{P}G_2\right\|(\theta\hat{\gamma}I_0+\eta)\right)^2 N^2 e^{-2\alpha t}\\ &+ \frac{2}{a}\|PG_3\|^2\|\tilde{x}(t)\|^2\\ \le& -2\beta_1 V(t) + \beta_2 e^{-2\alpha t} + \beta_3\|\tilde{x}(t)\|^2,\end{aligned} \tag{18}$$

where $\beta_1 = \left((1-\theta)\lambda_{\min}(\bar{Q}) - a\right)/2\lambda_{\max}(\bar{P})$, $\beta_2 = 2\left(\left\|\bar{P}G_1\right\| I_0 + \left\|\bar{P}G_2\right\|(\theta\hat{\gamma}I_0+\eta)\right)^2 N^2/a$ and $\beta_3 = 2\|PG_3\|^2/a$. Choosing $a < (1-\theta)\lambda_{\min}(\bar{Q})$, β_1 is positive. Based on the comparison theorem in [25] and (18), one can get that the solution of $V(t)$ satisfies

$$V(t) \le e^{-2\beta_1 t}V(0) + \int_0^t e^{-2\beta_1(t-s)}(\beta_2 e^{-2\alpha s} + \beta_3\|\tilde{x}(s)\|^2)ds.$$

Define $\hat{C} = A - FC$ and let $P_{\hat{C}}$ and $P_{\hat{C}}^{-1}$ be the matrices such that $P_{\hat{C}}^{-1}\hat{C}P_{\hat{C}} = J_{\hat{C}}$, where $J_{\hat{C}}$ is the Jordan canonical form of the matrix $\hat{C}$. Then, it follows from Lemma 2 that for $0 \le s \le t$,

$$\begin{aligned}&\left\|e^{-2\beta_1(t-s)}(\beta_2 e^{-2\alpha s} + \beta_3\|\tilde{x}(s)\|^2)\right\| \le \beta_2 e^{-2\beta_1(t-s)}e^{-2\alpha s}\\ &\quad+ \beta_3\left(c_{\hat{C}}\|P_{\hat{C}}\|\left\|P_{\hat{C}}^{-1}\right\|\|\tilde{x}(0)\|\right)^2 e^{-2\beta_1(t-s)}e^{2a_{\hat{C}}s},\end{aligned} \tag{19}$$

where $\max_i \mathrm{Re}(\lambda_i(\hat{C})) < a_{\hat{C}} < 0$, $c_{\hat{C}}$ is a positive constant with respect to $\hat{C}$. Let $a_1 = V(0) + a_2 + a_3, a_2 = \beta_2/|2\alpha - 2\beta_1|$ and $a_3 = \beta_3(c_{\hat{C}}\|P_{\hat{C}}\|\|P_{\hat{C}}^{-1}\|\|\tilde{x}(0)\|)^2/|2a_{\hat{C}} + 2\beta_1|$, then one can further have $V(t) \le a_1 e^{-2\beta_1 t} + a_2 e^{-2\alpha t} + a_3 e^{2a_{\hat{C}}t}$. Since $\beta_1 > 0, \alpha > 0$ and $a_{\hat{C}} < 0$, one has $\lim_{t\to\infty} V(t) = 0$. From the definition of V, one can see that $V(t) = 0$ if and only if $\|\xi_{2-N}(t)\| = 0$, which is equivalent to $\|\hat{x}_i(t) - \hat{x}_j(t)\| = 0, \forall (i,j) \in \mathcal{E}$. Besides, one has $\lim_{t\to\infty}\|\tilde{x}(t)\| = 0$, which is equivalent to $\lim_{t\to\infty}\|x_i(t) - \hat{x}_i(t)\| = 0, \forall i$. Therefore, consensus of the closed loop MAS (2), (4) is achieved asymptotically.

In the following, we will show that Zeno behavior is excluded. Firstly, the communication function (7) is analysed.

From the definition of $\xi(t)$, one has $\|\xi(t)\| = \|\xi_{2-N}(t)\| \le \sqrt{V(t)/\lambda_{\min}(\bar{P})} = b_1 e^{-\beta_1 t} + b_2 e^{-\alpha t} + b_3 e^{a_{\hat{C}}t}$, where $b_1 = \sqrt{a_1/\lambda_{\min}(\bar{P})}$, $b_2 = \sqrt{a_2/\lambda_{\min}(\bar{P})}$ and $b_3 = \sqrt{a_3/\lambda_{\min}(\bar{P})}$.

Let $u(t)$ be the column stack vector of $u_i(t)$. Then one has

$$\begin{aligned}\|(I_N\otimes B)u(t)\| \le& \left\|(cL\otimes BR^{-1}B^TP)(\hat{x}(t) + e(t))\right\|\\ &+ \left\|cI_N\otimes BR^{-1}B^TP\right\|\|\hat{e}(t)\|\\ \le& c\|L\|\left\|BR^{-1}B^TP\right\|(\|\xi(t)\| + \|e(t)\|)\\ &+ c\left\|BR^{-1}B^TP\right\|\|\hat{e}(t)\|\\ \le& d_1 e^{-\beta_1 t} + d_2 e^{-\alpha t} + d_3 e^{a_{\hat{C}}t}\end{aligned} \tag{20}$$

where $d_1 = c(\|L\| + \theta\gamma)\left\|BR^{-1}B^TP\right\|b_1$, $d_2 = c\left(\|L\|I_0 + (\theta\gamma I_0 + \eta)\left\|BR^{-1}B^TP\right\|\right)\sqrt{N} + d_1 b_2/b_1$ and $d_3 = c(\|L\| + \theta\gamma)\left\|BR^{-1}B^TP\right\|b_3$. Furthermore, $\forall t \in [t_{\sigma_i}^i, t_{\sigma_i+1}^i)$, one has $\dot{e}_i(t) = Ae^{A(t-t_{\sigma_i}^i)}\hat{x}_i(t_{\sigma_i}^i) - (A\hat{x}_i(t) + Bu_i(t) + F(y_i(t) - C\hat{x}_i(t))) = Ae_i(t) - Bu_i(t) - FC\tilde{x}_i(t)$ and $\|u_i(t)\| \le \|u(t)\|$. Thus, one has $\|\dot{e}_i(t)\| \le k_1 e^{-\beta_1 t} + k_2 e^{-\alpha t} + k_3 e^{a_{\hat{C}}t}$, where $k_1 = d_1, k_2 = d_2 + \|A\|I_0$, $k_3 = d_3 + c_{\hat{C}}\|P_{\hat{C}}\|\|P_{\hat{C}}^{-1}\|\|FC\|\|\tilde{x}_i(0)\|$. Denote the latest communication time of agent i by $\hat{t}_i^*$, then the next communication time will not occur before $\|e_i(t)\| = I_0 e^{-\alpha t}$. Thus, a lower bound on the inter-communication time of the agent $i, \forall i$ is given by $\tau_i = t - \hat{t}_i^*$ that solves the equation $(k_1 e^{-\beta_1\hat{t}_i^*} + k_2 e^{-\alpha\hat{t}_i^*} + k_3 e^{a_{\hat{C}}\hat{t}_i^*})\tau_i = I_0 e^{-\alpha t}$, which is equivalent to

$$\left(k_1 e^{(-\beta_1+\alpha)\hat{t}_i^*} + k_2 + k_3 e^{\left(a_{\hat{C}}+\alpha\right)\hat{t}_i^*}\right)\tau_i = I_0 e^{-\alpha\tau_i}. \tag{21}$$

Since $\beta_1 = ((1-\theta)\lambda_{\min}(\bar{Q}) - a)/2\lambda_{\max}(\bar{P})$, one has $\alpha < \min\{\beta_1, -\max_i\mathrm{Re}(\lambda_i(\hat{C}))\}$, then there exists a positive constant $\alpha < \beta_1$ and $-\max_i\mathrm{Re}(\lambda_i(\hat{C})) > -a_{\hat{C}} > \alpha > 0$. Thus, it is concluded that the solution τ_i of (21) is greater or equal to τ^*, which is given by $(k_1 + k_2 + k_3)\tau^* = I_0 e^{-\alpha\tau^*}$ for all agent i, which is strictly positive. Therefore, Zeno behavior is excluded for the communication function (7).

Next, the control function (14) is analysed. Note that the control function $g(\hat{e}_i(t), z_i(t), t)$ is not necessary continuous within two neighboring controller update instants (as stated in Remark 3). Define $\hat{t}_{\hat{\sigma}_i}^i = \cup_{j\in\mathcal{N}_i^+} t_{\sigma_j}^j, \hat{\sigma}_i \in \mathcal{Z}$ as the increasing sequence of communication time instants of agent i and its neighbors, i.e., $0 = \hat{t}_0^i < \hat{t}_1^i < \hat{t}_2^i < \cdots, \forall i$. Let $\hat{T}_{\hat{k}}^i = \hat{t}_{\hat{\sigma}_i}^i \cup T_k^i, \hat{k} \in \mathcal{Z}$, where $0 = \hat{T}_0^i < \hat{T}_1^i < \hat{T}_2^i < \cdots, \forall i$. Then the set $\{\hat{T}_{\hat{k}}^i\}$ can be seen as the jumping set of function $g(\hat{e}_i(t), z_i(t), t)$. Within two neighboring instants of $\hat{T}^i_{\hat{k}}$, $g\left(\hat{e}_i\left(t\right), z_i(t), t\right)$ is continuous. Define $\tau'_i = T^i_{k+1} - T^i_k$ as the time interval between two neighboring controller update instants of agent i, then for each $t \in [T^i_k, T^i_{k+1})$,

i) If $T^i_k \in \{\hat{t}^i_{\hat{\sigma}_i}\}$ and $T^i_{k+1} \in \{\hat{t}^i_{\hat{\sigma}_i}\}$, one has $\tau'_i = T^i_{k+1} - T^i_k \geq \hat{t}^i_{\hat{\sigma}_i+1} - \hat{t}^i_{\hat{\sigma}_i}$. Since Zeno behavior is excluded for the communication function (6), one has that τ'_i is strictly positive in this case;

ii) If $T^i_k \in \{\hat{t}^i_{\hat{\sigma}_i}\}, T^i_{k+1} \notin \{\hat{t}^i_{\hat{\sigma}_i}\}$, there must exist a $\hat{\sigma}^*_i \in \mathcal{Z}$ such that $T^i_{k+1} \in (t^i_{\hat{\sigma}^*_i}, t^i_{\hat{\sigma}^*_i+1})$. If $t^i_{\hat{\sigma}^*_i} \geq \hat{t}^i_{\hat{\sigma}_i+1}$, one has $\tau'_i \geq \hat{t}^i_{\hat{\sigma}_i+1} - \hat{t}^i_{\hat{\sigma}_i}$. Otherwise, $t^i_{\hat{\sigma}^*_i} = \hat{t}^i_{\hat{\sigma}_i}$, which means T^i_k and T^i_{k+1} are neighboring instants of $\hat{T}^i_{\hat{k}}$, thus $g\left(\hat{e}_i\left(t\right), z_i(t), t\right)$ is continuous for $t \in [T^i_k, T^i_{k+1})$. Taking the derivative of $\hat{e}_i(t)$ on t, one has $\|\dot{\hat{e}}_i(t)\| \leq \|Az_i(t)\| \leq m_1 e^{-\beta_1 t} + m_2 e^{-\alpha t} + m_3 e^{a\hat{c}t}$, where $m_1 = \|A\|\,(\hat{l} + N\sqrt{\hat{l}})b_1, m_2 = \|A\|\,(\hat{l} + N\sqrt{\hat{l}})(b_2 + \sqrt{N}I_0)$ and $m_3 = \|A\|\,(\hat{l} + N\sqrt{\hat{l}})b_3$. Besides, from (14), one observes that the next controller update time will not occur before $\|\hat{e}_i\left(t\right)\| = \eta e^{-\alpha t}$. Similar to the following analysis of the communication function (7), one can get that $\tau'_i, \forall i$ is greater than or equal to the solution $\hat{\tau}^*$ of $(m_1 + m_2 + m_3)\,\hat{\tau}^* = \eta e^{-\alpha\hat{\tau}^*}$, which is strictly positive;

iii) For the cases $T^i_k \notin \{\hat{t}^i_{\hat{\sigma}}\}, T^i_{k+1} \in \{\hat{t}^i_{\hat{\sigma}}\}$ and $T^i_k \notin \{\hat{t}^i_{\hat{\sigma}}\}, T^i_{k+1} \notin \{\hat{t}^i_{\hat{\sigma}}\}$, one can also get that $\tau'_i, \forall i$ is strictly positive similar to the analysis of i) and ii).

Since there is a strictly positive lower bound on the neighboring controller update time instants in all cases, one can conclude that Zeno behavior is excluded for the control function (14). ■

Remark 4: The observer given in (4) can only reconstruct the state x_i for each agent i asymptotically, and the convergence rate of the observation error $\tilde{x}_i$ is constrained by $A - FC$. However, it was shown in [26] that it is possible to reconstruct the state x_i in finite-time $h > 0$ if a finite-time observer (FTO) is utilized. As for the robustness problem of the FTO given in [26], a possible modification is that one can use the FTO first, such that the state x_i can be reconstructed in time h, and then switch to the observer (4).

IV. Simulation Results

In this section, a numerical example is given to verify the theoretical results. A network of 6 agents with communication graph $\mathcal{G}$ is shown in Fig. 2. One can calculate that $\underline{\lambda}_R = 1$, then we choose $c = 1 > 1/(2\underline{\lambda}_R)$. The initial state $x_i(0)$ of each agent i is chosen randomly from the box $[-5, 5] \times [-5, 5]$, and the initial state of the observer $\hat{x}_i(0)$ of each agent i is chosen to be $[0, 0]^T, \forall i$.

The system matrices are chosen as $A = [-2, 1; 0.1, 0.2], B = [0.8, 0.5]^T$ and $C = [1, 0]$. Given $Q = 5I_N$ and $R = 2I_N$, one can get $K = [0.6917, 1.8780]$ by solving the ARE (3). The feedback gain matrix F is chosen as $F = [-0.5, 2]^T$ such that $A - FC$ is Hurwitz. Given $\bar{Q} = 5I_{2N-2}$, then one can get $\lambda_{\max}(\bar{P}) = 6.7513$ by solving the Lyapunov function $\Pi^T\bar{P} + \bar{P}\Pi = -\bar{Q}$. Choosing $\theta = 0.4$ and $a = 0.001$, one has $\beta_1 = 0.444$. Then, we can choose $\alpha = 0.4 < \min\{\beta_1, -\max_i \mathrm{Re}\left(\lambda_i\left(A - FC\right)\right)\}$.

The simulation results for the MAS (2), (4) with controller (8) are shown in Figs. 3-5. The state trajectories are plotted in Fig. 3, where x_{i1} and x_{i2} are the state components of agent i. The evolutions of controller (8) for each agent i are plotted in Fig. 4. As an example, the communication/controller update time instants of agent 1 (labeled as t^1_σ/T^1_k) and the inter-communication/controller update interval of agent 1 (labeled as $\Delta(t^1_\sigma)/\Delta(T^1_k)$) is presented in Fig. 5. One can see that Zeno behavior is excluded for both the communication and controller update. In Fig. 6, the state trajectories under the controller (2) proposed in [18] are depicted, and the evolutions of controller (2) are plotted in Fig. 7.

TABLE I summarises the simulation results for the controller (8) and the controller (2) of [18]. The amount of communication times and controller update times within the simulation time intervals $[0, 10]$, $[0, 20]$ and $[0, 30]$ are given for each agent and the overall communication times are calculated. Moreover, the minimum inter-communication time (MIC) and the minimum inter-controller update time (MICU) are also stated for each agent (since the controller proposed in [18] is updated continuously, no UT or MICU is given, and thus the improvement in our case is straightforward). One can see that during the time interval $[0, 10]$, the overall communication times for the two controllers are the same. During the time intervals $[0, 20]$ and $[0, 30]$, the overall communication times of controller (8) are slightly more than that of controller (2) proposed in [18]. Therefore, it is concluded that the proposed distributed ETCC strategy reduces significantly the controller update times without significantly increasing the communication times.

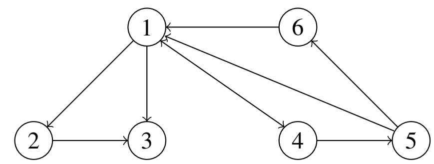

Fig. 2: Communication graph among the agents.

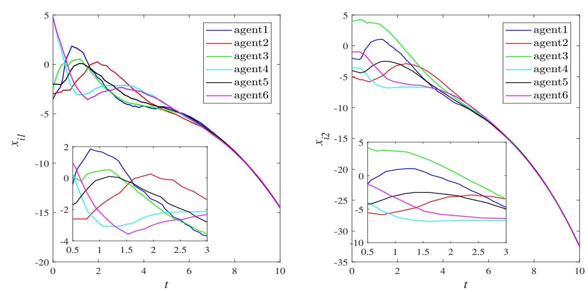

Fig. 3: The evolution of x_{i1}, x_{i2} under controller (8).

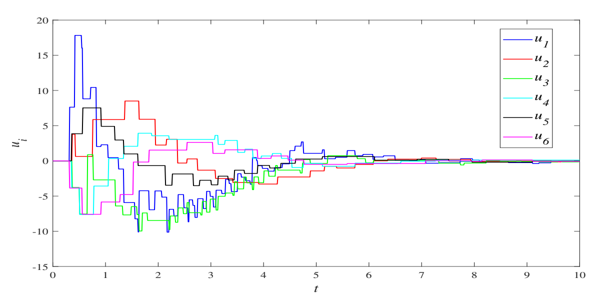

Fig. 4: The evolution of controller (8).

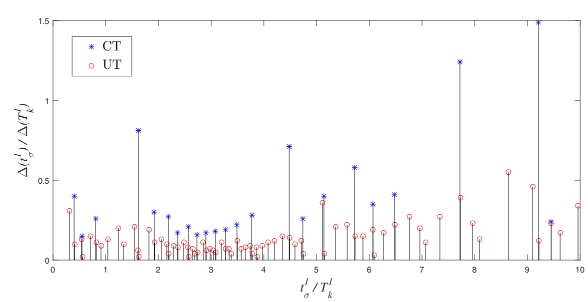

Fig. 5: The communication/controller update time instants of agent 1.

TABLE I: The amount of CT/UT and the MIC/MICU

	CT/MIC	CT/MIC [18]	CT/MIC	CT/MIC [18]	CT/MIC	CT/MIC [18]	UT/MICU	UT/MICU	UT/MICU
Time Interval	[0, 10]	[0, 10]	[0, 20]	[0, 20]	[0, 30]	[0, 30]	[0, 10]	[0, 20]	[0, 30]
Agent 1	23/0.15	23/0.14	36/0.15	35/0.14	44/0.15	46/0.14	73/0.02	106/0.02	129/0.02
Agent 2	22/0.17	22/0.17	34/0.17	30/0.17	41/0.17	37/0.17	26/0.06	40/0.06	52/0.06
Agent 3	28/0.13	28/0.13	38/0.13	38/0.13	45/0.13	45/0.13	56/0.02	81/0.02	97/0.02
Agent 4	18/0.26	18/0.25	23/0.26	25/0.25	32/0.26	29/0.25	25/0.06	39/0.04	50/0.04
Agent 5	11/0.34	12/0.40	21/0.34	20/0.40	30/0.34	29/0.40	25/0.12	36/0.12	49/0.12
Agent 6	16/0.21	15/0.25	28/0.21	22/0.25	40/0.21	30/0.25	19/0.22	36/0.07	50/0.07
Overall CT	118	118	180	170	232	216	-	-	-

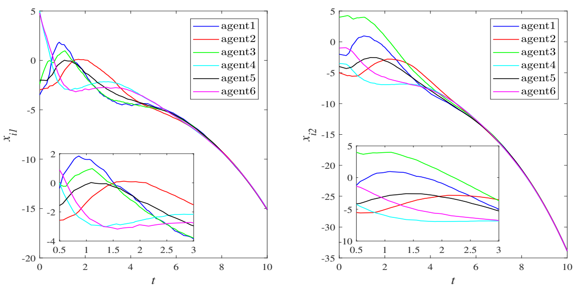

Fig. 6: The evolution of x_{i1}, x_{i2} under controller (2) proposed in [18].

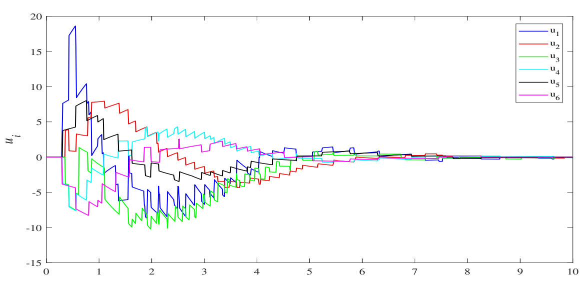

Fig. 7: The evolution of controller (2) proposed in [18].

V. Conclusion

In this note, the distributed ETCC of linear MAS was investigated under tactile communication. Firstly, in the communication side, an event-triggered data reduction scheme was proposed for each agent. Then, in the control side, a distributed event-triggered controller was implemented to each agent to further reduce the controller update. It was proven that the consensus of the closed loop MAS is achieved asymptotically. It was also shown that the proposed ETCC strategy is capable of reducing both the frequency of communication and controller update as well as excluding Zeno behaviour. In the future, more general systems, such as nonlinear systems, and communication constraints, such as quantization, disturbance and packet loss should be taken into account.

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