Transient Control of Linear Multi-Agent Systems with Leader-Follower Configuration

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***Abstract*— This paper presents a distributed control framework for leader-follower multi-agent systems with general linear dynamics to achieve consensus alongside prescribed transient performance. The multi-agent system is in a leader-follower configuration with only a set of agents selected as leaders and controlled via external inputs. In particular, we propose a distributed control framework comprising a prescribed performance controller for the leaders, while the followers are governed solely by a consensus protocol. When the decay rate of the performance functions is within a sufficient bound, we show that this distributed control law achieves consensus for the entire multi-agent system, with the guarantee that the trajectories of the consensus errors remain bounded by a time-varying prescribed transient performance function. Finally, the proposed results are illustrated through numerical examples.**

I. Introduction

Distributed control of multi-agent systems attracted profound research interest in the last decades due to the broad applications of multi-robot coordination, including cooperative control of unmanned aerial vehicles, distributed sensor networks, and attitude synchronization of satellites [1]. The majority of existing results primarily focus on achieving global tasks such as consensus [2], [3], formation control [4], [5], connectivity maintenance [6], and collision avoidance [7]. In various applications, the transient behaviors of multi-agent systems during dynamic evolution should also adhere to certain performance requirements, including those concerning bounded convergence time and rate, or in terms of overshoot.

In order to deal with such transient constraints, prescribed performance control (PPC) approaches were originally introduced in [8], and were shown to be a promising tool in prescribing the evolution of the system's output (or tracking error) within predefined regions. Built upon the results in [8], agreement protocols were designed for multi-agent systems to achieve average consensus with additionally desired prescribed performance constraints; see [9] for multi-agent systems with single integrator dynamics and [10] with double-integrator dynamics. The work in [11] proposed a PPC-based robust formation control approach for single-integrator agents under unknown external disturbances. A similar approach called funnel control was developed first in [12] for the distributed control of multi-agent systems.

In most existing works, including those referenced earlier, the designed transient control laws are implemented across all agents in the network, which can often be redundant and costly in many scenarios. Motivated by this, we investigate in this paper the problem of transient control of multi-agent systems with leader-follower configurations. In this framework, the set of agents with advanced capabilities are selected as *leaders* to guide the entire network to achieve global tasks while satisfying prescribed performance requirements. In particular, the selected *leader* agents are controlled via external inputs alongside a consensus protocol, while the remaining agents serve solely as *followers*, adhering to the consensus protocols without any authority over knowledge or control for additional objectives, such as the transient behavior under consideration. Leader-follower multi-agent systems can be found in various application domains, including multi-vehicle platooning and multi-robot coordination, where tasks are assigned and controllers are designed only for leader robots/vehicles to efficiently achieve collision avoidance, connectivity maintenance, or even more complex spatiotemporal tasks while satisfying transient constraints. There has been some recent research on multi-agent systems in such a leader-follower setup, e.g., in [13]–[17]. However, these results are focused either on network controllability [13]–[15], or on leader selection problems to guarantee controllability [16] and achieve optimal system performance [17].

In this paper, we propose a distributed transient control framework for leader-follower multi-agent systems interacting over an arbitrary connected undirected graph, where each agent is governed by general linear dynamics. In this framework, we design control laws for only the leader agents by leveraging PPC-based ideas, so that they can guide the entire leader-follower multi-agent system to achieve consensus as well as certain time-varying performance bounds, with the followers only equipped with a consensus protocol. Compared to the standard leader-follower setting studied in the literature, e.g., [18], [19], in which the multi-agent system only has one leader who is treated as a reference for the followers, we consider a different leader-follower framework which allows arbitrary number of leaders with external inputs in order to achieve additional transient control objectives. Note that unlike the existing work on transient constraints for standard leader-follower multi-agent systems [18], the followers in our framework are guided solely through dynamic couplings with the controlled leaders, without any additional

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control mechanisms or knowledge of the transient control objective, which makes the problem in hand much more challenging. Preliminary results on leader-follower formation control with prescribed performance guarantees are presented in [20], [21] for single- and double-integrator systems. This paper generalizes the results in [20], [21] in the following aspects. First, the agent dynamics are extended to general linear systems of any order, but not limited to any specific first- or second-order forms as in [20], [21]. Particularly for second-order systems, unlike the backstepping approach proposed in [20] that is limited to double integrator dynamics, our proposed method can be directly applied to achieve the control objective in a unified framework for linear system dynamics of any structure. Note that the linear dynamics may also represent the linearized dynamics of originally nonlinear agents. Moreover, our result relaxes those achieved in [20], [21] by allowing for a larger maximum decay rate of the desired performance functions (cf. Remark 3.4).

The remainder of this paper is organized as follows. Section II presents the preliminaries and problem formulation. The main results are presented in Section III and are illustrated through numerical examples in Section IV. Concluding remarks and future directions are given in Section V.

II. Preliminaries and Problem Formulation

Notation: We denote the set of real and nonnegative real numbers by $\mathbb{R}$ and $\mathbb{R}_+$, respectively. We simply denote $v = [v_1, \ldots, v_m]^\top \in \mathbb{R}^{mn}$ as the stacked vector of $v_1, \ldots, v_m \in \mathbb{R}^n$. For a symmetric matrix A, $A > 0$ and $A \geq 0$ respectively means that A is positive definite and positive semidefinite. Given n sets $X_1 \ldots, X_n$, $\Pi_{i=1,\ldots,n} X_i = \{(x_1, \ldots, x_n) \mid x_i \in X_i \text{ for all } i = 1, \ldots, n\}$ denotes the Cartesian product over X_i, $i = 1, \ldots, n$. We denote by I_n the identity matrix of size n and by diag $(A_1, \ldots, A_n)$ the block-diagonal matrix with matrices $A_i, i = 1, \ldots, n$, on its diagonal. The Kronecker product of matrices $A \in \mathbf{R}^{m\times n}$ and $B \in \mathbf{R}^{p\times q}$ is denoted as $A \otimes B$, which satisfies the following properties: $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$, $(A \otimes B)^\top = A^\top \otimes B^\top$, and $A \otimes B + A \otimes C = A \otimes (B + C)$.

A. Graph theory

Consider N agents that interact over a communication topology described by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \ldots, N\}$ represents the set of vertices indexed by the agents, and $\mathcal{E} = \{(i, j) \in \mathcal{V} \times \mathcal{V} \mid j \in \mathcal{N}_i\}$ denotes the set of edges indexed by $e_1, \ldots, e_M$. Here, $j \in \mathcal{N}_i$ represents the neighboring agents that can communicate with agent i, and $M = |\mathcal{E}|$ denotes the number of edges. By randomly assigning an orientation to each edge of $\mathcal{G}$, the *incidence matrix* can be defined by $D = D(\mathcal{G}) = [d_{ij}] \in \mathbb{R}^{N\times M}$ [22]. The rows and columns of D are indexed, respectively, by the vertices and the edges of the graph $\mathcal{G}$ with $d_{ij} = 1$ if the vertex i is the head of the edge (i, j), $d_{ij} = -1$ if i is the tail of the edge (i, j), and $d_{ij} = 0$ otherwise. Given an undirected graph $\mathcal{G}$, the graph Laplacian can be described as $L = DD^\top$ and the edge Laplacian is denoted by $L_e = D^\top D$ [23]. Note that if $\mathcal{G}$ is connected, the eigenvalues of the Laplacian L can be listed in an increasing order $0 = \lambda_1 < \lambda_2 \leq \ldots \leq \lambda_N$ [22].

B. System description

Consider a multi-agent system with N agents, each of which is modeled by the general linear dynamics:

$$\dot{x}_i = Ax_i + B\nu_i, \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $x_i \in \mathbb{R}^n$ denotes the state, and $\nu_i \in \mathbb{R}^m$ is the control input. We consider that the multi-agent system consists of *leader* and *follower* agents, with the agents interacting over communication graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with vertices $\mathcal{V} = \{1, 2, \ldots, N\}$. We assume that, without loss of generality, the first N_F agents are followers while the last N_L agents are leaders with vertex sets $\mathcal{V}_F := \{1, \ldots, N_F\}$, $\mathcal{V}_L := \{N_F + 1, \ldots, N_F + N_L\}$, and $N = N_F + N_L$. The follower agents are governed by a consensus-type protocol (cf. [22] and [1]), while the leaders are governed by a consensus protocol with an additional control input. Thus, the dynamics of each agent $i \in \mathcal{V}$ in (1) can be rewritten as:

$$\dot{x}_i = Ax_i + B(K \sum_{j\in\mathcal{N}_i} (x_i - x_j) - c_i K' u_i), \tag{2}$$

where $u_i \in \mathbb{R}^n$ is the external control input with $c_i = 1$ if $i \in \mathcal{V}_L$ and $c_i = 0$ if $i \in \mathcal{V}_F$, and $K, K' \in \mathbb{R}^{m\times n}$ are feedback gain matrices to be determined.

By stacking (2), the dynamics of the leader-follower multi-agent system are written as:

$$\Sigma : \dot{x} = (I_N \otimes A + L \otimes BK)x - (C \otimes BK')u, \tag{3}$$

where $x = [x_1, \ldots, x_N]^\top \in \mathbb{R}^{nN}$, $C = \begin{bmatrix} 0_{N_F\times N_L} \\ I_{N_L} \end{bmatrix}$, $u = [u_{N_F+1}, \ldots, u_{N_F+N_L}]^\top \in \mathbb{R}^{nN_L}$, and L is the graph Laplacian.

Let us next define the *edge state* as the relative state vector:

$$\bar{x}_k := x_i - x_j, (i, j) = e_k \in \mathcal{E}, k = 1, \ldots, M. \tag{4}$$

Note that an edge state represents the difference between two nodes incident to an edge. Let us denote $\bar{x} = [\bar{x}_1, \ldots, \bar{x}_M]^\top \in \mathbb{R}^{nM}$ as the stack vector of edge states. It can be verified that $(L \otimes I_n)x = (D \otimes I_n)\bar{x}$ and $(D^\top \otimes I_n)x = \bar{x}$, where D is the incidence matrix of the graph $\mathcal{G}$.

C. Problem statement

This work aims to design control strategies for the leaders so that the leader-follower multi-agent system can achieve consensus while guaranteeing transient constraints. To solve this problem, we employ the idea of prescribed performance control (PPC) [8] based on the edge-based representation of the system dynamics. In particular, the control objective requires that $x_i - x_j \to 0$ for all $(i, j) \in \mathcal{E}$, or equivalently $\bar{x}(t) \to 0$, as $t \to \infty$, while guaranteeing that the relative distance evolves within a predefined region described as

$$\|\bar{x}_k(t)\|^2 < \rho_k(t), \forall k = 1, \ldots, M, \tag{5}$$

where $\rho_k : \mathbb{R}_+ \to \mathbb{R}_+ \backslash \{0\}$, $k = 1, \ldots, M$, are so-called performance functions which capture the desired predefined region for the relative distance to evolve in.

III. Leader-Follower Consensus Control with Prescribed Performance Guanrantees

A. Prescribed performance bounds

The aim of prescribed performance control [8], [9] is to design control inputs such that the evolution of the state distances remains bounded by a positive performance function as in (5).

Let us define the squared relative distance error as $\eta_k := \|\bar{x}_k\|^2$. The problem of guaranteeing consensus and transient performance in (5) can be rewritten as:

$$\eta_k(t) < \rho_k(t), \forall k = 1, \ldots, M. \tag{6}$$

where $\rho_k : \mathbb{R}_+ \to \mathbb{R}_+ \backslash \{0\}, k = 1, \ldots, M$ are positive, smooth and strictly decreasing performance functions. In the sequel, let us consider performance functions in the form of:

$$\rho_k(t) = \left(\rho_k^0 - \rho_k^\infty\right) e^{-l_k t} + \rho_k^\infty \tag{7}$$

with ρ_k^0, ρ_k^∞ and l_k being positive parameters.

Define the modulated error as $\hat{\eta}_k = \frac{\eta_k}{\rho_k}$ and the corresponding prescribed performance region as $\mathcal{D}_{\hat{\eta}_k} \triangleq \{\hat{\eta}_k : \hat{\eta}_k \in [0, 1)\}$.

Then the modulated error is transformed through a function $T_{\hat{\eta}_k}$ that defines the smooth and strictly increasing mapping $T_{\hat{\eta}_k} : \mathcal{D}_{\hat{\eta}_k} \to \mathbb{R}_+, T_{\eta_k}(0) = 0$. Here, one example choice is $T_{\hat{\eta}_k}(\hat{\eta}_k) = \ln\left(\frac{1+\hat{\eta}_k}{1-\hat{\eta}_k}\right)$.

The transformed error is then defined as $\varepsilon_k(\hat{\eta}_k) = T_{\hat{\eta}_k}(\hat{\eta}_k)$. Differentiating $\varepsilon_k(\hat{\eta}_k)$ w.r.t. time, we derive

$$\dot{\varepsilon}_k(\hat{\eta}_k) = \mathcal{J}_{T_{\hat{\eta}_k}}(\hat{\eta}_k, t)\left(\dot{\eta}_k + \alpha_k(t)\eta_k\right) \tag{8}$$

where $\mathcal{J}_{T_{\hat{\eta}_k}}(\hat{\eta}_k, t) \triangleq \frac{\partial T_{\hat{\eta}_k}(\hat{\eta}_k)}{\partial \hat{\eta}_k} \frac{1}{\rho_k(t)} > 0$ and $\alpha_k(t) \triangleq -\frac{\dot{\rho}_k(t)}{\rho_k(t)} > 0$ are the normalized Jacobian of the transformed function $T_{\hat{\eta}_k}$ and the normalized derivative of the performance function, respectively.

The basic idea of prescribed performance control is to design a controller such that the transformed error $\varepsilon_k(\hat{\eta}_k)$ is bounded, which in turn results in the satisfaction of (6). We present our controller design method in the following subsection.

B. Proposed controller

By decomposing the incidence matrix D into the rows corresponding to the followers and the rows corresponding to the leaders, we get $D = \begin{bmatrix} D_F^\top & D_L^\top \end{bmatrix}^\top$. By left multiplying $D^\top \otimes I_n$ on both sides of (3), the dynamics of the multi-agent system in edge space can be written as

$$\Sigma_e : \dot{\bar{x}} = (I_M \otimes A + L_e \otimes BK)\bar{x} - (D_L^\top \otimes BK')u, \tag{9}$$

where $\bar{x} = [\bar{x}_1, \ldots, \bar{x}_M]^\top \in \mathbb{R}^{nM}$ and $L_e \in \mathbb{R}^{M \times M}$ is the edge Laplacian matrix.

For the edge dynamics (9), we use the following PPC-based local control strategy:

$$u_i = -\sum_{k \in \Phi_i} \bar{x}_k \mathcal{J}_{T_{\hat{\eta}_k}}(\hat{\eta}_k, t)\, \varepsilon_k(\hat{\eta}_k), \quad i \in \mathcal{V}_L \tag{10}$$

where $\Phi_i = \{k \mid (i,j) = e_k, j \in \mathcal{N}_i\}$, i.e., the set of all the edges that include leader agent $i \in \mathcal{V}_L$ as a node, $\mathcal{J}_{T_{\hat{\eta}_k}}$ and ε_k are defined as in (8). Let us define $X = \text{diag}(\bar{x}_k)$. Then the stack input vector is

$$u = -(D_L \otimes I_n) X \mathcal{J}_{T_{\hat{\eta}}} \varepsilon, \tag{11}$$

where $\mathcal{J}_{T_{\hat{\eta}}} := \text{diag}(\mathcal{J}_{T_{\hat{\eta}_k}}) \in \mathbb{R}^{M \times M}$, $\hat{\eta} = [\hat{\eta}_1, \ldots, \hat{\eta}_M]^\top \in \mathbb{R}^M$, $\varepsilon \triangleq \varepsilon(\hat{\eta}) \in \mathbb{R}^M$. Then, by inserting input (11) to the edge dynamics (9), we have

$$\Sigma_e : \dot{\bar{x}} = (I_M \otimes A + L_e \otimes BK)\bar{x} + (D_L^\top D_L \otimes BK') X \mathcal{J}_{T_{\hat{\eta}}} \varepsilon. \tag{12}$$

In the following result, we prove that by applying the control strategy as in (10) to the leaders, the target consensus can be achieved with guaranteed transient behaviors using Lyapunov-like methods.

Theorem 3.1: Consider the leader-follower multi-agent system interacting over a connected undirected communication graph with dynamics (3) and control law (11). Given the predefined performance functions as in (5), for all initial conditions such that $\bar{x}_k(0)$ are within the performance bounds (5) for all $k = 1, \ldots, M$, if for any $\gamma \geq l = \max_{k=1,\ldots,M}(l_k)$, where l is the largest decay rate of $\rho_k(t)$, the following condition holds:

$$\Gamma = \begin{bmatrix} \mathcal{B} + \mathcal{B}^\top & -\mathcal{A} - \frac{\gamma}{2}(I_{nM} - 2\mathcal{B}^\top \mathcal{P}) \\ * & -\gamma(\mathcal{P}\mathcal{A} + \mathcal{A}^\top \mathcal{P}) \end{bmatrix} \geq 0, \tag{13}$$

for some $\mathcal{P} = \mathcal{P}^\top > 0$, where $\mathcal{A} \triangleq I_M \otimes A + L_e \otimes BK$ and $\mathcal{B} \triangleq -D_L^\top D_L \otimes BK'$, then the closed-loop system achieves $\bar{x}(t) \to 0$ while satisfying the performance bounds as in (5).

Proof: To prove the theorem, we omit the arguments t and $\hat{\eta}$ from $\varepsilon(\hat{\eta})$ and $\mathcal{J}_{T_{\hat{\eta}}}(\hat{\eta}, t)$ for the compactness of notation. First, let us consider a candidate Lyapunov-like function

$$V(\varepsilon, \bar{x}) = \frac{1}{2}\varepsilon^\top \varepsilon + \gamma \bar{x}^\top \mathcal{P} \bar{x}, \tag{14}$$

where $\mathcal{P} = \mathcal{P}^\top > 0$. Then, the derivative of (14) with respect to time is $\dot{V}(\varepsilon, \bar{x}) = \varepsilon^\top \dot{\varepsilon} + 2\gamma \bar{x}^\top \mathcal{P} \dot{\bar{x}}$. By stacking (8), we have

$$\dot{\varepsilon} = \mathcal{J}_{T_{\hat{\eta}}}(\dot{\eta} + \alpha(t)\eta), \tag{15}$$

where $\alpha(t) := \text{diag}(\alpha_k(t)) \in \mathbb{R}^{M \times M}$, $\eta = [\eta_1, \ldots, \eta_M]^\top \in \mathbb{R}^M$. Note that by definition, we have $\eta_k := \|\bar{x}_k\|^2 = \bar{x}_k^\top \bar{x}_k$, and thus $\dot{\eta}_k = 2\bar{x}_k^\top \dot{\bar{x}}_k$. Then, we can further obtain that $\eta = X^\top \bar{x}$ and $\dot{\eta} = 2X^\top \dot{\bar{x}}$ with $X = \text{diag}(\bar{x}_k)$. By inserting (15) and the edge dynamics in (12) into $\dot{V}(\varepsilon, \bar{x}) = \varepsilon^\top \dot{\varepsilon} + 2\gamma \bar{x}^\top \mathcal{P} \dot{\bar{x}}$, we have

$$\begin{aligned}
\dot{V}(\varepsilon, \bar{x}) =& \varepsilon^\top \dot{\varepsilon} + 2\gamma \bar{x}^\top \mathcal{P} \dot{\bar{x}} = \varepsilon^\top \mathcal{J}_{T_{\hat{\eta}}}(\dot{\eta} + \alpha(t)\eta) + 2\gamma \bar{x}^\top \mathcal{P} \dot{\bar{x}} \\
=& 2\varepsilon^\top \mathcal{J}_{T_{\hat{\eta}}} X^\top (I_N \otimes A + L_e \otimes BK)\bar{x} \\
&+ 2\varepsilon^\top \mathcal{J}_{T_{\hat{\eta}}} X^\top (D_L^\top D_L \otimes BK') X \mathcal{J}_{T_{\hat{\eta}}} \varepsilon \\
&+ \varepsilon^\top \mathcal{J}_{T_{\hat{\eta}}} \alpha(t) X^\top \bar{x} \\
&+ 2\gamma \bar{x}^\top \mathcal{P}(I_N \otimes A + L_e \otimes BK)\bar{x} \\
&+ 2\gamma \bar{x}^\top \mathcal{P}(D_L^\top D_L \otimes BK') X \mathcal{J}_{T_{\hat{\eta}}} \varepsilon.
\end{aligned} \tag{16}$$

Let us denote $\mathcal{A} \triangleq I_N \otimes A + L_e \otimes BK$ and $\mathcal{B} \triangleq -D_L^\top D_L \otimes BK'$, then (16) becomes

$$\begin{aligned}\dot{V}(\varepsilon, \bar{x}) =& 2\varepsilon^\top \mathcal{J}_{T_{\hat{\eta}}} X^\top \mathcal{A}\bar{x} - \varepsilon^\top \mathcal{J}_{T_{\hat{\eta}}} X^\top (\mathcal{B}+\mathcal{B}^\top) X \mathcal{J}_{T_{\hat{\eta}}} \varepsilon \\ &+ \varepsilon^\top \mathcal{J}_{T_{\hat{\eta}}} \alpha(t) X^\top \bar{x} + \gamma \bar{x}^\top (\mathcal{P}\mathcal{A} + \mathcal{A}^\top \mathcal{P})\bar{x} \\ &- 2\gamma \bar{x}^\top \mathcal{P}\mathcal{B} X \mathcal{J}_{T_{\hat{\eta}}} \varepsilon. \end{aligned} \tag{17}$$

By adding and subtracting $\gamma \varepsilon^\top \mathcal{J}_{T_{\hat{\eta}}} X^\top \bar{x}$ on the right hand side of (17), we obtain

$$\begin{aligned}\dot{V}(\varepsilon, \bar{x}) =& -\varepsilon^\top \mathcal{J}_{T_{\hat{\eta}}} (\gamma I_M - \alpha(t)) X^\top \bar{x} \\ &+ \varepsilon^\top \mathcal{J}_{T_{\hat{\eta}}} X^\top (2\mathcal{A} - 2\gamma \mathcal{B}^\top \mathcal{P} + \gamma I_{nM})\bar{x} \\ &- \varepsilon^\top \mathcal{J}_{T_{\hat{\eta}}} X^\top (\mathcal{B}+\mathcal{B}^\top) X \mathcal{J}_{T_{\hat{\eta}}} \varepsilon \\ &+ \gamma \bar{x}^\top (\mathcal{P}\mathcal{A} + \mathcal{A}^\top \mathcal{P})\bar{x} \\ =& -\varepsilon^\top \mathcal{J}_{T_{\hat{\eta}}} (\gamma I_M - \alpha(t)) X^\top \bar{x} \\ &- y^\top \begin{bmatrix} \mathcal{B}+\mathcal{B}^\top & -\mathcal{A} - \frac{\gamma}{2}(I_{nM} - 2\mathcal{B}^\top \mathcal{P}) \\ * & -\gamma(\mathcal{P}\mathcal{A} + \mathcal{A}^\top \mathcal{P}) \end{bmatrix} y \\ =& -\varepsilon^\top \mathcal{J}_{T_{\hat{\eta}}} (\gamma I_M - \alpha(t)) X^\top \bar{x} - y^\top \Gamma y, \end{aligned} \tag{18}$$

where $y = \begin{bmatrix} \varepsilon^\top \mathcal{J}_{T_{\hat{\eta}}} X^\top & \bar{x}^\top \end{bmatrix}^\top$. Using (7) and $\alpha_k(t) \triangleq -\frac{\dot{\rho}_k(t)}{\rho_k(t)} > 0$, we have that $\alpha_k(t) \triangleq -\frac{\dot{\rho}_k(t)}{\rho_k(t)} = l_k \frac{\rho_k(t) - \rho_k^\infty}{\rho_k(t)} < l_k$. Then, $(\gamma I_M - \alpha(t))$ is a diagonal positive definite matrix if $\gamma \geq l = \max_{k=1,\ldots,M}(l_k) > \bar{\alpha} = \sup \alpha_k(t)$. Moreover, by the definitions of the modulated error $\hat{\eta}_k$ and transformed error ε_k in Sec. III-A, we know that $\hat{\eta}_k = \frac{\eta_k}{\rho_k} \geq 0$ and $\varepsilon_k(\hat{\eta}_k) = T_{\hat{\eta}_k}(\hat{\eta}_k) = \ln\left(\frac{1+\hat{\eta}_k}{1-\hat{\eta}_k}\right) \geq 0$. Also note that $\mathcal{J}_{T_{\hat{\eta}_k}}(\hat{\eta}_k, t) \triangleq \frac{\partial T_{\hat{\eta}_k}(\hat{\eta}_k)}{\partial \hat{\eta}_k} \frac{1}{\rho_k(t)} > 0$, and $\bar{x}_k^\top \bar{x}_k \geq 0$ hold. Then, by setting $\gamma := \theta + \bar{\alpha}$ with θ being any positive constant such that (13) holds, we obtain that

$$-\varepsilon^\top \mathcal{J}_{T_{\hat{\eta}}} (\gamma I_M - \alpha(t)) X^\top \bar{x} \leq -\theta \varepsilon^\top \mathcal{J}_{T_{\hat{\eta}}} X^\top \bar{x} \leq 0.$$

Therefore, we obtain that $-\varepsilon^\top \mathcal{J}_{T_{\hat{\eta}}} (\gamma I_M - \alpha(t)) X^\top \bar{x} < 0$, and $\Gamma \geq 0$ follows by condition (13), which implies that $\dot{V} \leq 0$ holds.

Note that $\dot{V} \leq 0$ also implies $V(\varepsilon(\hat{\eta}), \bar{x}) \leq V(\varepsilon(\hat{\eta}(0)), \bar{x}(0))$. Let us define the set $\mathcal{D}_{\hat{\eta}} := \Pi_{k=1,\ldots,M} \mathcal{D}_{\hat{\eta}_k}$. Hence if $\eta(0)$ is chosen within the region (6), which implies $\hat{\eta}(0) \in \mathcal{D}_{\hat{\eta}}$ and $\bar{x}(0)$ is within the region (5), then we get that $V(\varepsilon(\hat{\eta}(0)), \bar{x}(0))$ is finite, which further implies that $V(\varepsilon(\hat{\eta}), \bar{x})$ is bounded for all time. Therefore $\varepsilon(\hat{\eta}), \bar{x}$ are bounded and the boundedness of the transformed error $\varepsilon(\hat{\eta})$ implies that $\hat{\eta}(t)$ evolves within the prescribed performance bounds (6) for all time. Then, by calculating the derivative of $\dot{V}(\varepsilon, \bar{x})$ in (17), we can deduce the boundedness of $\ddot{V}(\varepsilon(\hat{\eta}), \bar{x})$ based on the boundedness of $\varepsilon(\hat{\eta}), \dot{\varepsilon}(\hat{\eta})$, $\bar{x}$ and $\dot{\bar{x}}$. The boundedness of $\ddot{V}(\varepsilon(\hat{\eta}), \bar{x})$ implies the uniform continuity of $\dot{V}(\varepsilon(\hat{\eta}), \bar{x})$, which in turn implies that $\dot{V}(\varepsilon(\hat{\eta}), \bar{x}) \to 0$ as $t \to \infty$ by applying Barbalat's Lemma [24]. This implies $\bar{x} \to 0$ as $t \to \infty$, which means that the target consensus problem is achieved with (5) satisfied. ■

The proof of Theorem 3.1 can be interpreted as follows: we first leverage a candidate Lyapunov function to show that the transformed error $\varepsilon(\hat{\eta})$ is bounded. The boundedness of $\varepsilon(\hat{\eta})$ implies that the modulated errors $\hat{\eta}_k$ are constrained within the desired regions $\mathcal{D}_{\hat{\eta}_k}$. This further implies that the distance error η_k evolves within the desired performance bounds (6), and equivalently, the relative distance $\|\bar{x}_k(t)\|$ evolves within the predefined region as in (5). Finally, the convergence of $\bar{x}_k(t) \to 0$ is proved by showing $\dot{V}(\varepsilon(\hat{\eta}), \bar{x}) \to 0$ as $t \to \infty$.

Note that the condition (13) is not part of the control law. We should mention that Theorem 3.1 provides a sufficient but not necessary condition for the leader-follower multi-agent systems to achieve consensus while guaranteeing transient behaviors. The feasibility of the linear matrix inequality (LMI) condition in (13) is determined not only by the matrices L_e and D_L which characterize the leader-follower graph topology, but also by the system's matrices A, B and gain matrices K, K'. In the sequel, we provide some guidelines for the design of K, K', as stated in Corollary 3.2 and Remark 3.3, which can enhance our chances of obtaining a feasible LMI condition (13). In particular, an equivalent condition to (13) is presented under an assumption on the stabilizability property of the system.

Consider the LMI condition in (13). For the sake of simple presentation, let us rewrite Γ as $\Gamma = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, where $A_{11} = \mathcal{B} + \mathcal{B}^\top; A_{12} = -\mathcal{A} - \frac{\gamma}{2}(I_{nM} - 2\mathcal{B}^\top \mathcal{P}); A_{21} = A_{12}^\top; A_{22} = -\gamma(\mathcal{P}\mathcal{A} + \mathcal{A}^\top \mathcal{P})$ with $\mathcal{A} \triangleq I_M \otimes A + L_e \otimes BK$ and $\mathcal{B} \triangleq -D_L^\top D_L \otimes BK'$.

Corollary 3.2: Consider a multi-agent system (3) with a connected communication graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Suppose (A, B) is stabilizable, and let $K := -cB^\top P$, with $P = P^\top > 0$ being a solution to the following condition:

$$PA + A^\top P - PBB^\top P < 0, \tag{19}$$

and $c > \frac{1}{2\lambda_2} > 0$, where λ_2 is the smallest non-zero eigenvalue of the graph Laplacian L. Then, the condition $\Gamma \geq 0$ in (13) holds if and only if $A_{11} - A_{12}A_{22}^{-1}A_{12}^\top \geq 0$, where A_{22} is positive definite with $\mathcal{P} := I_M \otimes P$.

Proof: We start by proving A_{22} is positive definite. Consider an arbitrary non-zero vector $\xi \in \mathbb{R}^{Mn}$. We have $\xi^\top A_{22} \xi = -2\gamma \xi^\top (\mathcal{P}\mathcal{A} + \mathcal{A}^\top \mathcal{P})\xi = -2\gamma \xi^\top (I_M \otimes (PA + A^\top P) + L_e \otimes (PBK + K^\top B^\top P))\xi$. Recall that since $\mathcal{G}$ is connected, the eigenvalues of the Laplacian L can be listed in an increasing order $0 = \lambda_1 < \lambda_2 \leq \ldots \leq \lambda_N$ [22]. Note that the non-zero eigenvalues of L_e are identical to those of L [25]. Then, the eigenvalues of L_e can be also listed in an increasing order $0 \leq \bar{\lambda}_1 \leq \ldots \leq \bar{\lambda}_p \leq \ldots \leq \bar{\lambda}_M$, where $\bar{\lambda}_p = \lambda_2$ denotes the smallest non-zero eigenvalue of L_e, where $1 \leq p \leq M$. Thus, there exists an orthogonal matrix $U \in \mathbb{R}^{M \times M}$ such that $U^\top L_e U = \Lambda = \mathrm{diag}(\bar{\lambda}_i)$. Let us introduce the state transformation $\xi = (U \otimes I_N)\delta$. Then:

$$\begin{aligned} &\xi^\top A_{22} \xi \\ =& -2\gamma \delta^\top (I_M \otimes (PA + A^\top P) \\ &+ U^\top L_e U \otimes (PBK + K^\top B^\top P))\delta \\ =& -2\gamma \delta^\top (I_M \otimes (PA + A^\top P) + \Lambda \otimes (PBK + K^\top B^\top P))\delta \\ =& -2\gamma \delta^\top \mathrm{diag}(PA + A^\top P + \bar{\lambda}_i (PBK + K^\top B^\top P))\delta. \end{aligned}$$

By letting $K = -cB^\top P$, with $P = P^\top > 0$ satisfying (19) and $c > \frac{1}{2\lambda_2} > 0$, we obtain that $PA + A^\top P + \bar{\lambda}_i(PBK + K^\top B^\top P) = PA + A^\top P - 2c\bar{\lambda}_i PBB^\top P \leq PA + A^\top P - 2c\lambda_2 PBB^\top P \overset{(19)}{<} 0$. This implies that $\xi^\top A_{22}\xi > 0$, and thus $A_{22} > 0$. Therefore, by using Schur complements [26], we can conclude that $\Gamma \geq 0$ in (13) holds if and only if $A_{11} - A_{12}A_{22}^{-1}A_{12}^\top \geq 0$. ■

Remark 3.3: As shown in [27], the matrix pair (A, B) being stabilizable is a necessary condition for the consensusability of multi-agent systems with homogeneous general linear agents. Moreover, it is known [28] that given a stabilizable pair (A, B), the following algebraic Riccati equation (ARE)

$$PA + A^\top P - PBB^\top P + I_n = 0 \tag{20}$$

has a (unique) solution $P = P^\top > 0$. Thus, one can always find a symmetric matrix $P > 0$ satisfying (19). Therefore, in the case that all agents are stabilizable and equipped with a standard consensus protocol, Corollary 3.2 provides us with a proper choice of the gain matrix K and the candidate Lyapunov function as in (14) with $\mathcal{P} := I_M \otimes P$. Given the properly chosen gain matrix K and $\mathcal{P} := I_M \otimes P$, one can further search for a gain matrix K' by solving the linear matrix inequality in (13) with any arbitrary $\gamma \geq l = \max_{k=1,\ldots,M}(l_k)$ as in Theorem 3.1 using computational tools, e.g., `YALMIP` [29] or `MATLAB LMI Toolbox`.

Remark 3.4: Let us remark that a prescribed performance control law is proposed in [20] for leader-follower MAS with agent dynamics being either single- or double-integrators. The results we propose here generalize the results in [20] to general linear agent dynamics of any order, but not limited to single-integrators, double-integrators, or any structural high-order linear systems. The linear dynamics can also be considered as the linearized dynamics of some originally nonlinear agents. Note that when dealing with second-order systems, the result proposed in [20, Section III-B] is built upon a *backstepping approach* where PPC-based control laws are designed separately for each dimension of second-order systems, and thus can only handle double integrator agent dynamics. Instead, the control law proposed here can be directly applied to achieve the desired control objective in a unified framework for linear system dynamics of any structure. Moreover, in the case that the dynamics of the agents are single or double integrators, the condition (13) in Theorem 3.1 boils down to

$$\Gamma = \begin{bmatrix} D_L^\top D_L & \frac{1}{2}(L_e - \frac{\gamma}{2}(I_M - 2D_L^\top D_L)) \\ * & \gamma L_e \end{bmatrix} \geq 0,$$

which relaxes the results in [20, Thm. 1 & Thm. 2] by allowing a larger allowable decay rate γ that is larger than the one achieved in [20, Thm. 1 & Thm. 2].

IV. Case Study

In this section, we illustrate the proposed theoretical results through a simple numerical example on aircraft consensus flying. We consider a network of 3 agents with communication graph $\mathcal{G}$ as shown in Fig. 1, where leaders and followers are marked in grey and white nodes, respectively. The dynamics of each agent is governed by an aircraft phugoid model that simplifies the F-16 longitudinal motion, borrowed from [30]. The longitudinal model is linearized at the steady state with $V = 60$ m/s and $\sigma = \pi/12$ rad, where V represents the longitudinal velocity and σ represents the inclination angle. Let us denote the state vector by $x_i = [\tilde{V}_i, \tilde{\sigma}_i]^\top$, where $\tilde{V}_i$ and $\tilde{\sigma}_i$ represent the deviations from the corresponding steady states. The dynamics of the i-th agent is described as

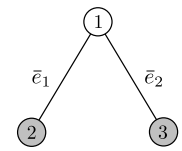


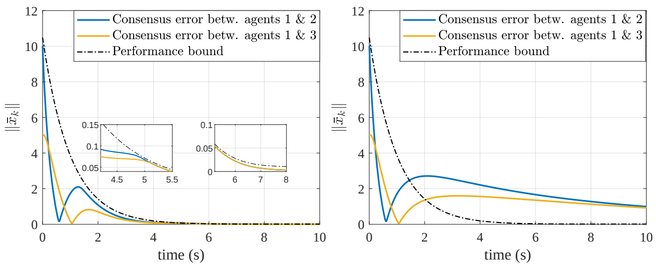
Fig. 1: Communication topology of the three-agent system.

$$\begin{bmatrix} \dot{\tilde{V}}_i \\ \dot{\tilde{\sigma}}_i \end{bmatrix} = \begin{bmatrix} -0.0240 & -9.5135 \\ 0.0042 & 0.0471 \end{bmatrix} \begin{bmatrix} \tilde{V}_i \\ \tilde{\sigma}_i \end{bmatrix} + \begin{bmatrix} 0.0622 & 2.8622 \\ -0.0079 & 0.0431 \end{bmatrix} u_i, \tag{21}$$

where $u_i = [\delta_e, \delta_t]^\top$ with δ_e being the elevator deflection angle and δ_t the thrust lever deflection angle. The states of the agents are initialized as $x_1(0) = [5, 0.2]^\top$, $x_2(0) = [-5, -0.1]^\top$, and $x_3(0) = [10, 0.3]^\top$.

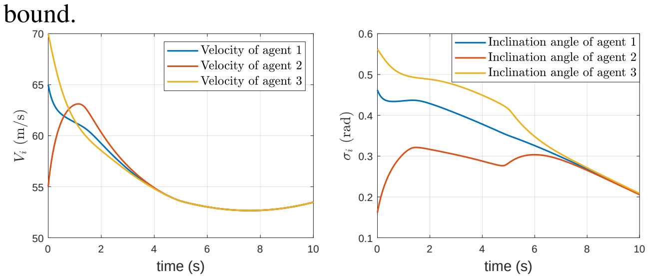
The control objective is to achieve consensus for the leader-follower multi-agent systems while ensuring the performance constraints in (5) with $\rho_k(t) = \left(\rho_k^0 - \rho_k^\infty\right) e^{-l_k t} + \rho_k^\infty$, where we choose, without loss of generality, the same parameters for the prescribed performance functions as $\rho_k^0 = 600$, and $\rho_k^\infty = 0.1$, for both edges $k = 1, 2$. To choose the decay rate l_k of the prescribed performance functions, we utilize the results in Theorem 3.1 and Corollary 3.2 as follows. First note that for the communication graph of the network, one can calculate that the smallest non-zero eigenvalue of the Laplacian L is $\lambda_2 = 1$, and then we select $c = 0.6 > \frac{1}{2\lambda_2}$. As we consider here only the leaders know the prescribed performance bounds and apply the control law in (10), the agents' dynamics can be written as $\dot{x}_i = Ax_i + B(K\sum_{j\in\mathcal{N}_i}(x_i - x_j) - c_i K' u_i)$ as in (2), where $c_1 = 0$ and $c_2 = c_3 = 1$. Since the system (21) is stabilizable, by solving the ARE (20) as stated in Remark 3.3, one can get $P = P^\top = [0.8, -38.7; -38.7, 2998] > 0$ that satisfies condition (19) and the gain matrix $K = -cB^\top P = [-0.2145, 15.6552; -0.4273, -11.0499]$. Then, by further letting $\mathcal{P} = I_2 \otimes P$ as suggested in Corollary 3.2, the matrix inequality (13) is feasible with the decay rate being $\gamma = 8$, where a gain matrix $K' = [-0.9418, 0.0060; -0.8755, -0.0125]$ was found by solving the linear matrix inequality in (13) using `MATLAB LMI Toolbox`. Hence, according to Theorem 3.1, it suffices to choose $l_k = 2 < \gamma = 8$ for both edges $k = 1, 2$.

The simulation results when applying the PPC law in (10) is shown in Figs. 2 and 3. Fig. 2 shows the evolution of the consensus errors of the edges with the black dashed line representing the performance bound. Note that Fig. 2a shows the consensus errors with PPC law, while Fig. 2b without PPC law. One can readily see that the evolution of



(a) With PPC law (b) Without PPC law

Fig. 2: Consensus errors evolution between neighboring agents. The black dashed line represents the performance bound.



(a) velocity (b) inclination angle

Fig. 3: Evolution of the agents' velocity (a) and inclination angle (b).

the consensus errors without PPC law violates the prescribed performance bounds, which is improved by applying the PPC law (10). Fig. 3 depicts the evolution of the state trajectories of the agents, which also shows that the agents successfully achieved consensus.

V. Conclusions

This paper developed distributed control laws for general leader-follower multi-agent systems with general linear dynamics. A prescribed performance control law is designed and implemented for leaders with followers solely obeying a consensus protocol. We proved that under the proposed distributed control law, the closed-loop multi-agent system achieves consensus along with the desired prescribed performance guarantees when the decay rate of the performance functions is within a sufficient bound. Future research directions include investigating leader selection problems and transient control of leader-follower multi-agent systems under more complex spatiotemporal tasks.

References

[1] K.-K. Oh, M.-C. Park, and H.-S. Ahn, "A survey of multi-agent formation control," *Automatica*, vol. 53, pp. 424–440, 2015.

[2] W. Ren and R. W. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Transactions on automatic control*, vol. 50, no. 5, pp. 655–661, 2005.

[3] Z. Li, Z. Duan, G. Chen, and L. Huang, "Consensus of multiagent systems and synchronization of complex networks: A unified viewpoint," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 57, no. 1, pp. 213–224, 2009.

[4] T. Balch and R. C. Arkin, "Behavior-based formation control for multirobot teams," *IEEE transactions on robotics and automation*, vol. 14, no. 6, pp. 926–939, 1998.

[5] X. Fang, L. Xie, and D. V. Dimarogonas, "Simultaneous distributed localization and formation tracking control via matrix-weighted position constraints," *Automatica*, vol. 175, p. 112188, 2025.

[6] M. M. Zavlanos and G. J. Pappas, "Distributed connectivity control of mobile networks," *IEEE Transactions on Robotics*, vol. 24, no. 6, pp. 1416–1428, 2008.

[7] A. Dorri, S. S. Kanhere, and R. Jurdak, "Multi-agent systems: A survey," *Ieee Access*, vol. 6, pp. 28 573–28 593, 2018.

[8] C. P. Bechlioulis and G. A. Rovithakis, "Robust adaptive control of feedback linearizable mimo nonlinear systems with prescribed performance," *IEEE Transactions on Automatic Control*, vol. 53, no. 9, pp. 2090–2099, 2008.

[9] Y. Karayiannidis, D. V. Dimarogonas, and D. Kragic, "Multi-agent average consensus control with prescribed performance guarantees," in *51st Conf. Decis. Control*. IEEE, 2012, pp. 2219–2225.

[10] L. Macellari, Y. Karayiannidis, and D. V. Dimarogonas, "Multi-agent second order average consensus with prescribed transient behavior," *IEEE Transactions on Automatic Control*, vol. 62, no. 10, pp. 5282–5288, 2016.

[11] F. Mehdifar, C. P. Bechlioulis, F. Hashemzadeh, and M. Baradarannia, "Prescribed performance distance-based formation control of multi-agent systems," *Automatica*, vol. 119, p. 109086, 2020.

[12] A. Ilchmann, E. P. Ryan, and S. Trenn, "Tracking control: Performance funnels and prescribed transient behaviour," *Systems & Control Letters*, vol. 54, no. 7, pp. 655–670, 2005.

[13] H. G. Tanner, "On the controllability of nearest neighbor interconnections," in *2004 43rd IEEE conference on decision and control (CDC)*, vol. 3. IEEE, 2004, pp. 2467–2472.

[14] A. Rahmani, M. Ji, M. Mesbahi, and M. Egerstedt, "Controllability of multi-agent systems from a graph-theoretic perspective," *SIAM Journal on Control and Optimization*, vol. 48, no. 1, pp. 162–186, 2009.

[15] D. Goldin and J. Raisch, "Controllability of second order leader-follower systems," *IFAC Proceedings Volumes*, vol. 43, no. 19, pp. 233–238, 2010.

[16] A. Y. Yazicioğlu and M. Egerstedt, "Leader selection and network assembly for controllability of leader-follower networks," in *2013 American Control Conference*. IEEE, 2013, pp. 3802–3807.

[17] K. Fitch and N. E. Leonard, "Optimal leader selection for controllability and robustness in multi-agent networks," in *2016 European control conference (ECC)*. IEEE, 2016, pp. 1550–1555.

[18] I. Katsoukis and G. A. Rovithakis, "Output feedback leader-follower with prescribed performance guarantees for a class of unknown nonlinear multi-agent systems," in *2016 24th Mediterranean Conference on Control and Automation (MED)*. IEEE, 2016, pp. 1077–1082.

[19] S.-L. Dai, S. He, X. Chen, and X. Jin, "Adaptive leader–follower formation control of nonholonomic mobile robots with prescribed transient and steady-state performance," *IEEE Transactions on Industrial Informatics*, vol. 16, no. 6, pp. 3662–3671, 2019.

[20] F. Chen and D. V. Dimarogonas, "Leader–follower formation control with prescribed performance guarantees," *IEEE Transactions on Control of Network Systems*, vol. 8, no. 1, pp. 450–461, 2020.

[21] ——, "Further results on leader-follower multi-agent formation control with prescribed performance guarantees," in *2020 59th IEEE Conference on Decision and Control (CDC)*. IEEE, 2020, pp. 4023–4030.

[22] M. Mesbahi, *Graph theoretic methods in multiagent networks*. Princeton University Press, 2010.

[23] D. Zelazo and M. Mesbahi, "Edge agreement: Graph-theoretic performance bounds and passivity analysis," *IEEE Transactions on Automatic Control*, vol. 56, no. 3, pp. 544–555, 2010.

[24] I. Barbalat *et al.*, "Systemes d'équations différentielles d'oscillations non linéaires," *Rev. Math. Pures Appl*, vol. 4, no. 2, pp. 267–270, 1959.

[25] R. A. Horn and C. R. Johnson, *Matrix analysis*. Cambridge university press, 2012.

[26] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear matrix inequalities in system and control theory*. SIAM, 1994.

[27] C.-Q. Ma and J.-F. Zhang, "Necessary and sufficient conditions for consensusability of linear multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 55, no. 5, pp. 1263–1268, 2010.

[28] E. D. Sontag, *Mathematical control theory: deterministic finite dimensional systems*. Springer Science & Business Media, 2013, vol. 6.

[29] J. Lofberg, "Yalmip: A toolbox for modeling and optimization in matlab," in *2004 IEEE international conference on robotics and automation (IEEE Cat. No. 04CH37508)*. IEEE, 2004, pp. 284–289.

[30] B. L. Stevens, F. L. Lewis, and E. N. Johnson, *Aircraft control and simulation: dynamics, controls design, and autonomous systems*. John Wiley & Sons, 2015.