Event-Triggered Output Feedback Control for Linear Systems under Tactile Communication

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***Abstract*— This paper investigates an event-triggered output feedback control strategy of linear systems under tactile communication, for which two different frameworks are considered. Motivated by the emerging tactile communications technology where latencies are very small but at the price of limited message sizes, a perception-based deadband principle is proposed for the data reduction of communication. In each framework, under an assumption that the deadband factor is upper bounded with respect to the system model, it is proven that global asymptotic stability of the closed loop system is achieved. Then, an event-triggered output feedback controller under tactile communication is further introduced. It is shown that the designed controller is capable of reducing the frequency of controller updates as well as excluding Zeno behavior. Numerical examples are given to illustrate the effectiveness of the proposed control algorithm.**

I. INTRODUCTION

Motivated by recent technological advances on computing and communication ability, event-triggered control (ETC) has attracted special attention in the control community in recent years. The essential idea of ETC is to measure the next sample and update the controller if and only if a certain measurement error becomes large enough [1]. In this way, a substantial reduction of the amount of transmissions may be achieved compared to conventional time-triggered setups.

In most of the ETC strategies, a state or data dependent trigger function implicitly determines the time instants at which state or data is transmitted over the communication network and the controller is updated [2]–[8]. However, continuous measuring may still be needed for the detection of the trigger function [2]–[6]. Besides, any event-triggered controllers should ensure the exclusion of Zeno behavior, which is crucial in practical applications as the hardware cannot generate transmissions that are arbitrarily close in time. Although periodic ETC [7], [8] is proposed as a solution to these problems, more frequent communication may be required as the price to pay.

The Tactile Internet and the underlying tactile/haptic communication are believed to be a next evolutionary technological leap [9]. At the very core of the design of Tactile Internet is the ‘1ms Challenge’ (a round-trip latency of 1ms is supported) [10]. Aiming to decrease the latency, several techniques have been proposed, such as lossy compression [11], deadband-based data reduction [12], network coding and software defined networking [13], etc. These approaches do not take into account the human perceptual limitations. To this end, Hintenseer et al. [14] propose a perception-based deadband (PD) principle for tactile data, where the deadband is adjusted for each transmitted sample. As long as the perception error stays within the deadband, no transmissions are triggered, thereby resulting in a reduced discrete communication sequence. Looking at the ETC strategy in control and the PD principle in tactile communication, we find that their ideas essential coincide. Moreover, if we implement both the PD principle in communication side and the ETC strategy in control side, further reduction of communication or/and control action may be achieved.

Motivated by the above discussion, this paper investigates the event-triggered output feedback control of linear systems under tactile communication. To the best of our knowledge, this paper is the first attempt to integrate ETC and tactile communication. It is concluded that such integration fulfils the reduction of both the communication and control as well as excluding Zeno behavior. The contribution of this paper is summarized as follows. Linear systems of two different frameworks are controlled through output feedback under tactile communication. Firstly, a PD principle is proposed for tactile communication. Under an assumption that the deadband factor is upper bounded with respect to the system model, it is proven that global asymptotic stability of the closed loop system is achieved. Then, ETC is further implemented asynchronously with the communication action. It is shown that the designed event-triggered output feedback controller reduces the frequency of controller update and additionally excludes Zeno behavior.

Notation: Denote $\mathbb{R}$ as the set of real numbers, $\mathscr{Z}$ as the set of nonnegative integers, $\mathbb{R}^n$ as the $n-$dimension real vector space and $\mathbb{R}^{n\times m}$ as the $n\times m$ real matrix space. I_n is the identity matrix of order n and 0_n is the square matrix of order n with all zeros. We denote $\lambda_i(A)$ as the ith eigenvalue of matrix A, $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ as the smallest and largest eigenvalue of matrix A, respectively. Let $|\lambda|$ be the absolute value of a real number λ, $\|x\|$ and $\|A\|$ be the Euclidean norm of vector x and matrix A, respectively. For a complex number, $\mathrm{Re}(\cdot)$ denotes its real part. $P \succ 0$ means that P is a positive definite matrix and P^T is the transpose of P.

II. PRELIMINARIES AND PROBLEM STATEMENT

A. PD Data reduction with tactile communication

The communication strategy considered in this paper is based on the exploitation of limitations of human tactile

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perception. In 19th century, Ernst Heinrich Weber observed that the human perception of change in a given stimulus appearing to be lawfully related to the initial stimulus magnitude. This relationship, which can be well approximated by Weber's law [15], is expressed as:

$$\Delta I/I = k \quad \text{or} \quad \Delta I = kI \tag{1}$$

where I represents the initial stimulus intensity, ΔI represents the difference threshold and $k > 0$ is a constant. Weber's Law says that the Just Noticeable Difference (JND), the smallest change that can be perceived (i.e., ΔI), is a constant proportion of the initial stimulus.

Based on Weber's Law, a PD principle is introduced in [14], where data are sent over the communication channel only if the the difference between the last sent value $x(t')$ and the current value $x(t)$, where $t > t'$, exceeds the JND,

$$\begin{aligned} &\text{If: } |x(t') - x(t)| \leq p|x(t')|, \quad \text{deviation is not perceived,} \\ &\text{Else: } |x(t') - x(t)| > p|x(t')|, \quad \text{transmit new value,} \end{aligned} \tag{2}$$

where $p > 0$ is called the deadband factor, and which corresponds to the constant k in Weber's law (1).

B. Problem Statement

Consider a continuous-time linear plant described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t). \end{aligned} \tag{3}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{l\times n}$ are constant matrices; $x \in \mathbb{R}^n, u \in \mathbb{R}^m$ and $y \in \mathbb{R}^l$ are the system state, input and output, respectively.

The observer of the state is

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + F(y(t) - C\hat{x}(t)), \tag{4}$$

and the control input is

$$u(t) = -K\hat{x}(t), \tag{5}$$

where $\hat{x} \in \mathbb{R}^n$ is the observer state; $K \in \mathbb{R}^{m\times n}, F \in \mathbb{R}^{n\times l}$ are the gain matrices, respectively.

Defining now the observation error as $\tilde{x}(t) \triangleq x(t) - \hat{x}(t)$, then straightforward computation gives the dynamics of the observation error

$$\dot{\tilde{x}}(t) = (A - FC)\tilde{x}(t), \tag{6}$$

which means $\tilde{x}(t) = e^{(A-FC)t}\tilde{x}(0)$.

Assumption 1: The matrix pair (A,B) in (3) is stablizable, and the matrix pair (A,C) in (6) is detectable.

Lemma 1: [6] Suppose that $A \in \mathscr{R}^{n\times n}$ is Hurwitz. Then, for all $t \geq 0$, it holds that $\left\|e^{At}\right\| \leq \|P_A\| \left\|P_A^{-1} c_A e^{a_A t}\right\|$, where P_A is a nonsingular matrix such that $P_A^{-1} A P_A = J_A$ with J_A being the Jordan canonical form of A, c_A is a positive constant determined by A, and $\max_i \mathrm{Re}(\lambda_i(A)) < a_A < 0$.

With Assumption 1, it is sufficient to design the gain matrices F and K such that $(A-FC)$, $(A-BK)$ are Hurwitz. Thus, the closed loop system (3)-(5) is globally asymptotically stable. However, in order to be implemented, the observer is required to continuously access the sensor output, and the controller is required to continuously access the observer state and update the controller accordingly.

In this paper, we are interested in an implementation of tactile communication and ETC, which is able to relax the continuous requirements for communication and controller update. Two different frameworks of the closed loop system are considered. In Framework I, the sensor and the observer are assumed to be co-located, which means the observer has access to the sensor output at all time. The deadband principle is implemented in the observer, therefore, the observer state is available to the controller only at relevant discrete communication instants. In Framework II, the sensor and the observer are not co-located. In this case, the deadband principle is implemented in the observer and the sensor output. Hence, both the sensor output and the observer state are available in a resulting discrete fashion.

III. MAIN RESULT

A. Framework I

In Framework I, the sensor and the controller are co-located. The closed loop system is described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \\ \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + F(y(t) - C\hat{x}(t)) \end{aligned} \tag{7}$$

and the controller under tactile communication is given by

$$u(t) = -K\hat{x}(\hat{t}_{\hat{\sigma}}), \quad t \in [\hat{t}_{\hat{\sigma}}, \hat{t}_{\hat{\sigma}+1}), \tag{8}$$

where $\hat{t}_{\hat{\sigma}}, \hat{\sigma} \in \mathscr{Z}$ is the communication time sequence of the observer, at which a new observer state is perceived and transmitted.

The communication time sequence $\hat{t}_{\hat{\sigma}}$ is updated by

$$\hat{t}_{\hat{\sigma}+1} = \inf\left\{t > \hat{t}_{\hat{\sigma}} : f_1\left(\hat{x}(t), \hat{x}(\hat{t}_{\hat{\sigma}})\right) \geq 0\right\}, \tag{9}$$

where

$$f_1\left(\hat{x}(t), \hat{x}(\hat{t}_{\hat{\sigma}})\right) = \begin{cases} \|\hat{x}(\hat{t}_{\hat{\sigma}}) - \hat{x}(t)\| - p\,\|\hat{x}(\hat{t}_{\hat{\sigma}})\|, \|\hat{x}(\hat{t}_{\hat{\sigma}})\| \geq I_0 \\ \|\hat{x}(\hat{t}_{\hat{\sigma}}) - \hat{x}(t)\| - I_0 e^{-\alpha t}, \|\hat{x}(\hat{t}_{\hat{\sigma}})\| < I_0, \end{cases}$$

$I_0 > 0$ and α is a positive constant to be determined. The condition $f_1\left(\hat{x}(t), \hat{x}(\hat{t}_{\hat{\sigma}})\right) \geq 0$ is called the **deadband condition**. Different from (2), here we introduce a constant I_0 in the deadband condition. Intuitively, I_0 corresponds to the smallest intensity of a given physical quantity that can be reliably detected. In fact, it's often the case that for very small values of I, Weber's law starts to break down (the JND is no longer proportional to the initial stimulus) [16]. Without loss of generality, we assume $\hat{t}_0 = 0$.

If $\|\hat{x}(\hat{t}_{\hat{\sigma}})\| \geq I_0$, the deadband condition becomes the usual formulation of Weber's law. In this case, denote the **deadband** at time $\hat{t}_{\hat{\sigma}}$ to be the set of observer state that do not fulfil the deadband condition $\mathscr{F}_1(\hat{t}_{\hat{\sigma}}) = \{\hat{x}(t)\,|\,\|\hat{x}(\hat{t}_{\hat{\sigma}}) - \hat{x}(t)\| \leq p\,\|\hat{x}(\hat{t}_{\hat{\sigma}})\|, t \in [\hat{t}_{\hat{\sigma}}, \hat{t}_{\hat{\sigma}+1})\}$. Let $\mathscr{F}_1 = \{\mathscr{F}_1(\hat{t}_0), \mathscr{F}_1(\hat{t}_1), \mathscr{F}_1(\hat{t}_2), \ldots\}$. Motivated by [17], the following definition of $\theta-$cover is utilized here.

Definition 1: We say that $\mathscr{F}_1(\hat{t}_{\hat{\sigma}})$ is a θ-cover if there exists a constant $\theta > 0$, such that $\sin(\mathrm{ang}(g,d)) < \theta$ holds for every vector pair (g,d), where $g \in \mathscr{F}_1(\hat{t}_{\hat{\sigma}}), d = \hat{x}(\hat{t}_{\hat{\sigma}})$, $\mathrm{ang}(g,d)$ is the relative angle between vector g and d.

Definition 2: We say that $\mathscr{F}_1$ is a $\theta-$cover if for $\forall \mathscr{E} \in \mathscr{F}_1$, $\mathscr{E}$ is a $\theta-$cover.

The definition of $\theta-$cover can be seen as the geometrical interpretation of the deadband. Similar to the analysis of Section III. C of [14], one can further have

$$p = \sin \alpha_{\max} < \theta. \tag{10}$$

Define the observer tactile error as the observer state difference between the last communication instant $\hat{t}_{\hat{\sigma}}$ and current time t, which is $\hat{e}(t) = \hat{x}(\hat{t}_{\hat{\sigma}}) - \hat{x}(t)$. Then the control input can be rewritten as

$$u(t) = -K(\hat{x}(t) + \hat{e}(t)). \tag{11}$$

Since (A,B) is stabilizable, then the following algebraic Riccati equation (ARE):

$$A^T P + PA - PBR^{-1}B^T P + Q = 0. \tag{12}$$

has a unique solution $P = P^T \succ 0$ for any given matrices $R = R^T \succ 0$ and $Q = Q^T \succ 0$. Let $K = R^{-1}B^T P/2$, then dynamics of the observer can be rewritten as

$$\dot{\hat{x}}(t) = A\hat{x}(t) - BR^{-1}B^T P/2(\hat{x}(t) + \hat{e}(t)) + FC\tilde{x}(t). \tag{13}$$

Assumption 2: $\mathscr{F}_1$ is a $\theta-$cover for

$$\theta = \frac{\lambda_{\min}(Q)}{\lambda_{\max}(PBR^{-1}B^T P) + \lambda_{\min}(Q)}, \tag{14}$$

where matrices P, R, Q are defined in (12).

Remark 1: Assumption 2 gives an upper bound on the deadband factor p according to (10). This is reasonable since the deadband cannot be arbitrarily large.

Theorem 1: Consider the closed loop system given by (7) with control input (8). Suppose Assumptions 1-2 hold and that the deadband condition (9) holds with $\alpha < \min\{-\max_i \mathrm{Re}(\lambda_i(A-BK)), -\max_i \mathrm{Re}(\lambda_i(A-FC))\}$. Then, the origin of the closed loop system is globally asymptotically stable. Moreover, Zeno behavior is excluded.

Proof. Consider the following Lyapunov function

$$V(t) = \hat{x}^T P \hat{x}. \tag{15}$$

Differentiating $V(t)$ along the trajectories of (13), one has

$$\begin{aligned}\dot{V}(t) =& \hat{x}^T(t)(A^T P + PA)\hat{x}(t) - \hat{x}^T(t)PBR^{-1}B^T P\hat{x}(t) \\ & - \hat{x}^T(t)PBR^{-1}B^T P\hat{e}(t) + 2\hat{x}^T(t)PFC\tilde{x}(t) \\ \leq & -\hat{x}^T(t)Q\hat{x}(t) + \lambda_{\max}\left(PBR^{-1}B^T P\right)\|\hat{x}(t)\|\,\|\hat{e}(t)\| \\ & + 2\hat{x}^T(t)PFC\tilde{x}(t).\end{aligned} \tag{16}$$

According to the deadband condition (9), one has i) If $\|\hat{x}(\hat{t}_{\hat{\sigma}})\| \geq I_0$, then for $\forall t \in [\hat{t}_{\hat{\sigma}}, \hat{t}_{\hat{\sigma}+1})$,

$$\begin{aligned}&\|\hat{e}(t)\| = \|\hat{x}(\hat{t}_{\hat{\sigma}}) - \hat{x}(t)\| \leq p\,\|\hat{x}(\hat{t}_{\hat{\sigma}})\| \\ &\Rightarrow (1-p)\,\|\hat{x}(\hat{t}_{\hat{\sigma}})\| \leq \|\hat{x}(t)\| \leq (1+p)\,\|\hat{x}(\hat{t}_{\hat{\sigma}})\|,\end{aligned} \tag{17}$$

which implies that $\|\hat{e}(t)\| \leq p\,\|\hat{x}(\hat{t}_{\hat{\sigma}})\| \leq p/(1-p)\,\|\hat{x}(t)\|, \forall t \geq 0$. ii) If $\|\hat{x}(\hat{t}_{\hat{\sigma}})\| < I_0$, we have $\|\hat{e}(t)\| \leq I_0 e^{-\alpha t}$ for $t \in [\hat{t}_{\hat{\sigma}}, \hat{t}_{\hat{\sigma}+1})$. Combining i) and ii), one can get $\|\hat{e}(t)\| \leq p/(1-p)\,\|\hat{x}(t)\| + I_0 e^{-\alpha t}, \forall t \geq 0$. Then (16) can be rewritten as

$$\begin{aligned}\dot{V}(t) \leq & -\left(\lambda_{min}(Q) - \frac{p}{1-p}\lambda_{\max}\left(PBR^{-1}B^T P\right)\right)\|\hat{x}(t)\|^2 \\ & + \lambda_{\max}\left(PBR^{-1}B^T P\right)\|\hat{x}(t)\|\,I_0 e^{-\alpha t} + 2\hat{x}^T(t)PFC\tilde{x}(t).\end{aligned}$$

According to (10), one has $p < \theta$, then one can further have $\lambda_{\min}(Q) - p\lambda_{\max}\left(PBR^{-1}B^T P\right)/(1-p) := r > 0$. Using the inequality $2xy \leq ax^2 + y^2/a, \forall a > 0$, the derivative $\dot{V}$ can then be rewritten as

$$\begin{aligned}\dot{V}(t) \leq & -(1-\kappa)r\|\hat{x}(t)\|^2 \\ & + \frac{1}{\kappa r}(\lambda_{\max}^2\left(PBR^{-1}B^T P\right)I_0^2 e^{-2\alpha t} + \|PFC\|^2\|\tilde{x}(t)\|^2).\end{aligned}$$

According to Assumption 1, the matrix $A - FC$ is Hurwitz, which implies $\|\tilde{x}(t)\| = \left\|e^{((A-FC))t}\tilde{x}(0)\right\| \to 0$ as $t \to \infty$. Furthermore, one has $e^{-2\alpha t} \to 0$ as $t \to \infty$ for all $\alpha > 0$. It follows from the input-to-state stability argument in Chapter 4 [18] that $V(t) \to 0$ as $t \to \infty$, which implies that the observer (13) is globally asymptotically stable. Besides, the observation error is globally asymptotically stable. We thus conclude that the closed loop system (7) is globally asymptotically stable.

Next, we will show that Zeno behavior is excluded. From the definition of $\hat{e}(t)$, one has

$$\begin{aligned}\left\|\dot{\hat{e}}(t)\right\| =& \left\|\dot{\hat{x}}(t)\right\| \leq \left\|A - BR^{-1}B^T P/2\right\|\,\|\hat{x}(t)\| \\ & + \left\|BR^{-1}B^T P/2\right\|\,\|\hat{e}(t)\| + \|FC\|\,\|\tilde{x}(t)\|, \forall t \in [\hat{t}_{\hat{\sigma}}, \hat{t}_{\hat{\sigma}+1}).\end{aligned}$$

i) If $\|\hat{x}(\hat{t}_{\hat{\sigma}})\| \geq I_0$, we have

$$\left\|\dot{\hat{e}}(t)\right\| \leq \left\|\hat{A}\right\|(1+p)\,\|\hat{x}(\hat{t}_{\hat{\sigma}})\| + p\left\|\hat{B}\right\|\,\|\hat{x}(\hat{t}_{\hat{\sigma}})\| + \|FC\|\,\|\tilde{x}(0)\|,$$

where $\hat{A} = A - BR^{-1}B^T P/2$ and $\hat{B} = BR^{-1}B^T P/2$. It is proved that the observer (13) is globally asymptotically stable, then one can conclude that there exists a constant $\hat{x}^* > 0$, such that $\|\hat{x}(t)\| \leq \hat{x}^*, \forall t \geq 0$. Thus, $\left\|\dot{\hat{e}}(t)\right\| \leq \left(\left\|\hat{A}\right\|(1+p) + p\left\|\hat{B}\right\|\right)\hat{x}^* + \|FC\|\,\|\tilde{x}(0)\| := u^*$. Denote the latest communication time of the observer by $\hat{t}^*$ and let $\tau = t - \hat{t}^*$ be the inter-communication time, then one has $\tau \geq p\,\|\hat{x}(\hat{t}_{\hat{\sigma}})\|/u^* \geq pI_0/u^*$.

ii) If $\|\hat{x}(\hat{t}_{\hat{\sigma}})\| < I_0$, we have $\|\hat{e}(t)\| \leq I_0 e^{-\alpha t}$. The solution of $\hat{x}(t)$ can be obtained as

$$\hat{x}(t) = e^{\hat{A}t}\hat{x}(0) + \int_0^t e^{\hat{A}(t-s)}\left(-\hat{B}\hat{e}(s) + FC\tilde{x}(s)\right)ds. \tag{18}$$

Let $P_{\hat{A}}$ and $P_{\hat{A}}^{-1}$ be the matrices such that $P_{\hat{A}}^{-1}\hat{A}P_{\hat{A}} = J_{\hat{A}}$, where $J_{\hat{A}}$ is the Jordan canonical form of the matrix $\hat{A}$. Define $\hat{C} = A - FC$ and let $P_{\hat{C}}$ and $P_{\hat{C}}^{-1}$ be the matrices such that $P_{\hat{C}}^{-1}\hat{C}P_{\hat{C}} = J_{\hat{C}}$, where $J_{\hat{C}}$ is the Jordan canonical form of the matrix $\hat{C}$. It follows from Lemma 1 that for $0 \leq s \leq t$,

$$\begin{aligned}&\left\|e^{\hat{A}(t-s)}\left(-\hat{B}\hat{e}(s) + FC\tilde{x}(s)\right)\right\| \\ \leq & c_{\hat{A}}\left\|P_{\hat{A}}\right\|\left\|P_{\hat{A}}^{-1}\right\|\left\|\hat{B}\right\| I_0 e^{a_{\hat{A}}(t-s)}e^{-\alpha t} \\ & + c_{\hat{A}}c_{\hat{C}}\left\|P_{\hat{A}}\right\|\left\|P_{\hat{A}}^{-1}\right\|\left\|P_{\hat{C}}\right\|\left\|P_{\hat{C}}^{-1}\right\|\|FC\|\,\|\tilde{x}(0)\|\,e^{a_{\hat{A}}(t-s)}e^{a_{\hat{C}}t},\end{aligned} \tag{19}$$

where $\max_i \mathrm{Re}(\lambda_i(\hat{A})) < a_{\hat{A}} < 0, \max_i \mathrm{Re}(\lambda_i(\hat{C})) < a_{\hat{C}} < 0$ and $a_{\hat{A}} \neq a_{\hat{C}}$, $c_{\hat{A}}, c_{\hat{C}}$ are positive constants with respect to $\hat{A}$ and $\hat{C}$, respectively.

Let $a_1 = c_{\hat{A}} \|P_{\hat{A}}\| \left\|P_{\hat{A}}^{-1}\right\| \|\hat{x}(0)\|$, $a_2 = c_{\hat{A}} \|P_{\hat{A}}\| \left\|P_{\hat{A}}^{-1}\right\| \|\hat{B}\| I_0 / |a_{\hat{A}} + \alpha|$ and $a_3 = c_{\hat{A}} c_{\hat{C}} \|P_{\hat{A}}\| \left\|P_{\hat{A}}^{-1}\right\| \|P_{\hat{C}}\| \left\|P_{\hat{C}}^{-1}\right\| \|FC\| \|\tilde{x}(0)\| / |a_{\hat{A}} - a_{\hat{C}}|$. It follows from (18) and (19) that $\|\hat{x}(t)\| \leq (a_1 + a_2 + a_3) e^{a_{\hat{A}} t} + a_2 e^{-\alpha t} + a_3 e^{a_{\hat{C}} t}$. Then one has

$$\begin{aligned} \|\hat{e}(t)\| &\leq \|\hat{A}\| \|\hat{x}(t)\| + \|\hat{B}\| \|\hat{e}(t)\| + \|FC\| \|\tilde{x}(t)\| \\ &\leq k_1 e^{a_{\hat{A}} t} + k_2 e^{-\alpha t} + k_3 e^{a_{\hat{C}} t}, \end{aligned} \tag{20}$$

where $k_1 = \|\hat{A}\| (a_1 + a_2 + a_3)$, $k_2 = \|\hat{A}\| a_2 + \|\hat{B}\| I_0$ and $k_3 = \|\hat{A}\| a_3 + c_{\hat{C}} \|P_{\hat{C}}\| \left\|P_{\hat{C}}^{-1}\right\| \|FC\| \|\tilde{x}(0)\|$. Denote the latest communication time of the observer by $\hat{t}^*$, then the next communication time will not occur before $\|\hat{e}(t)\| = I_0 e^{-\alpha t}$. Thus, a lower bound on the inter-communication time is given by $\tau = t - \hat{t}^*$ that solves the equation $\left(k_1 e^{a_{\hat{A}} \hat{t}^*} + k_2 e^{-\alpha \hat{t}^*} + k_3 e^{a_{\hat{C}} \hat{t}^*}\right) \tau = I_0 e^{-\alpha t}$, which is equivalent to $\left(k_1 e^{(a_{\hat{A}} + \alpha) \hat{t}^*} + k_3 e^{(a_{\hat{C}} + \alpha) \hat{t}^*} + k_2\right) \tau = I_0 e^{-\alpha \tau}$. Because $\alpha < \min\{-\max_i \mathrm{Re}(\lambda_i(A - BK)), -\max_i \mathrm{Re}(\lambda_i(A - FC))\} = \min\{-\max_i \mathrm{Re}(\lambda_i(\hat{A})), -\max_i \mathrm{Re}(\lambda_i(\hat{C}))\}$, there must exist negative constants $\max_i \mathrm{Re}(\lambda_i(\hat{A})) < a_{\hat{A}} < -\alpha < 0$ and $\max_i \mathrm{Re}(\lambda_i(\hat{C})) < a_{\hat{C}} < -\alpha < 0$. Since $\alpha < -a_{\hat{A}}$ and $\alpha < -a_{\hat{C}}$, it is conclude that the solutions $\tau(\hat{t}^*)$ of $\left(k_1 e^{(a_{\hat{A}} + \alpha) \hat{t}^*} + k_3 e^{(a_{\hat{C}} + \alpha) \hat{t}^*} + k_2\right) \tau = I_0 e^{-\alpha \tau}$ are greater or equal to τ, which is given by $(k_1 + k_2 + k_3) \tau = I_0 e^{-\alpha \tau}$ for all $\hat{t}^* \geq 0$, and which is strictly positive.

Since there is a positive lower bound τ on the inter-communication time in both cases, one can thus conclude that Zeno behavior is excluded. ■

Next, ETC is introduced to reduce the controller updates as well. For the closed loop system (7), the event-triggered controller under tactile communications is redesigned as

$$u(t) = -K\hat{x}(T_k), \quad t \in [T_k, T_{k+1}), \tag{21}$$

where $T_k, k \in \mathscr{Z}$ is the controller update time sequence.

Define the measurement error as the observer state difference between the last event-triggered instant T_k and the last communication instant $\hat{t}_{\hat{\sigma}}$, which is

$$\delta(t) = \hat{x}(T_k) - \hat{x}(\hat{t}_{\hat{\sigma}}), \quad t \in [T_k, T_{k+1}). \tag{22}$$

Combining the definition of $\hat{e}(t)$ and $\delta(t)$, the observer (13) can be rewritten as

$$\dot{\hat{x}}(t) = A\hat{x}(t) - BR^{-1}B^T P/2(\hat{x}(t) + \hat{e}(t) + \delta(t)) + FC\tilde{x}(t). \tag{23}$$

The event-triggered time instants are defined as:

$$T_{k+1} = \inf\{t > T_k : g_1(\delta(t), \hat{x}(\hat{t}_{\hat{\sigma}})) \geq 0\}, \tag{24}$$

where

$$\begin{aligned} g_1(\delta(t), \hat{x}(\hat{t}_{\hat{\sigma}})) &= \|\delta(t)\| - \sqrt{\varepsilon_1 \gamma_1} \|\hat{x}(\hat{t}_{\hat{\sigma}})\|, \\ \gamma_1 &= \hat{\gamma}_1 / \frac{1}{2a} \left(\frac{1}{1-p}\right)^2 \lambda_{\max}\left(PBR^{-1}B^T P\right) \\ \hat{\gamma}_1 &= r - a\lambda_{\max}\left(PBR^{-1}B^T P\right)/2 \end{aligned} \tag{25}$$

with constants $0 < a < 2r/\lambda_{\max}\left(PBR^{-1}B^T P\right)$, $0 < \varepsilon_1 < 1$, P, R, Q are defined in (12) and p is the deadband factor. Without loss of generality, we assume $T_0 = 0$.

Theorem 2: Consider the closed loop system given by (7) with the event-triggered controller (21). Suppose Assumptions 1-2 hold and that the deadband condition (9) holds with $\alpha < \min\{(1 - \varepsilon_1)\hat{\gamma}_1 / (2\lambda_{\max}(P)), -\max_i \mathrm{Re}(\lambda_i(A - FC))\}$. Then, the origin of the closed loop system is globally asymptotically stable under the event condition (24). Moreover, Zeno behavior is excluded.

Remark 2: The PD principle can be seen as a trigger for communication and the event condition (24) can be seen as a trigger for controller update. They work asynchronously. From the definition of measurement error and event condition, one can see that the event condition is detected only at the communication time instants $\hat{t}_{\hat{\sigma}}$. That is to say, $\{T_k\} \subseteq \{\hat{t}_{\hat{\sigma}}\}$.

B. Framework II

In Framework II, the sensor and the controller are not colocated. Then, the closed loop system is described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \\ \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + F(y(t_\sigma) - C\hat{x}(\hat{t}_{\hat{\sigma}})) \end{aligned} \tag{26}$$

and the controller under tactile communication is given by

$$u(t) = -K\hat{x}(\hat{t}_{\hat{\sigma}}), \tag{27}$$

where $\hat{t}_{\hat{\sigma}}$ is the communication time sequence of the observer, which is defined in (8). $t_\sigma, \sigma \in \mathscr{Z}$ is the communication time sequence of the sensor output, which is updated by

$$t_{\sigma+1} = \inf\{t > t_\sigma : f_2(y(t), y(t_\sigma)) \geq 0\}, \tag{28}$$

where

$$f_2(y(t), y(t_\sigma)) = \begin{cases} \|y(t_\sigma) - y(t)\| - p\|y(t_\sigma)\|, \|y(t_\sigma)\| \geq I_0 \\ \|y(t_\sigma) - y(t)\| - I_0 e^{-\alpha t}, \|y(t_\sigma)\| < I_0, \end{cases}$$

with constants I_0 and α defined in (9). Similarly, for the case $\|y(t_\sigma)\| \geq I_0$, one can define the **deadband** of the sensor output at time t_σ by a finite set $\mathscr{F}_2(t_\sigma) = \{y(t) \,|\, \|y(t_\sigma) - y(t)\| \leq p\|y(t_\sigma)\|, t \in [t_\sigma, t_{\sigma+1})\}$ and $\mathscr{F}_2 = \{\mathscr{F}_2(t_0), \mathscr{F}_2(t_1), \mathscr{F}_2(t_2), \ldots\}$.

Define the sensor tactile error as $e(t) = y(t_\sigma) - y(t)$, then the dynamics of the observer can be rewritten as

$$\dot{\hat{x}}(t) = A\hat{x}(t) - BK(\hat{x}(t) + \hat{e}(t)) + F(C\tilde{x}(t) + e(t) - C\hat{e}(t)). \tag{29}$$

and the dynamics of the observation error is

$$\dot{\tilde{x}}(t) = (A - FC)\tilde{x}(t) - F(e(t) - C\hat{e}(t)). \tag{30}$$

Let $\psi \triangleq \left[\hat{x}^T, \tilde{x}^T\right]^T$, then the close loop system can be rewritten into vector form as

$$\begin{aligned} \dot{\psi} &= \begin{pmatrix} A - BK & FC \\ 0_n & A - FC \end{pmatrix} \begin{pmatrix} \hat{x} \\ \tilde{x} \end{pmatrix} + \begin{pmatrix} F \\ -F \end{pmatrix} e \\ &\quad + \begin{pmatrix} -BK - FC \\ FC \end{pmatrix} \hat{e} \\ &\triangleq \bar{A}\psi + G_1 e + G_2 \hat{e}. \end{aligned} \tag{31}$$

Based on Assumption 1, there exist gain matrices K and F such that the matrices $\bar{A}$ is Hurwitz, and hence there exists a positive definite matrix $\bar{P} = \bar{P}^T$ that satisfies the Lyapunov condition $\bar{P}\bar{A} + \bar{A}^T\bar{P} = -\bar{Q}$ for any given $\bar{Q} = \bar{Q}^T \succ 0$.

Assumption 3: $\mathscr{F}_1$ and $\mathscr{F}_2$ are both $\theta-$cover for

$$\theta = \frac{\lambda_{\min}\left(\bar{Q}\right)}{2\sqrt{2}\,\|\bar{P}G_1\|\,\|C\| + 2\,\|\bar{P}G_2\| + \lambda_{\min}\left(\bar{Q}\right)} \tag{32}$$

where $\bar{Q} = \bar{Q}^T \succ 0$, and $\bar{P}$ is the solution of the Lyapunov condition.

Theorem 3: Consider the closed loop system given by (26) with the control input (27). Suppose Assumptions 1, 3 hold and the deadband conditions (9) (28) hold with $\alpha < \min\{-\max_i \mathrm{Re}\left(\lambda_i\left(A-BK\right)\right), -\max_i \mathrm{Re}\left(\lambda_i\left(A-FC\right)\right)\}$. Then, the origin of the closed loop system is globally asymptotically stable. Moreover, Zeno behavior is excluded under the deadband conditions (9) (28).

Proof. Consider the following Lyapunov function

$$V(t) = \psi^T(t)\bar{P}\psi(t). \tag{33}$$

Differentiating $V(t)$ along the trajectories of (31), one has

$$\dot{V}(t) \leq -\psi^T(t)\bar{Q}\psi(t) + 2\psi^T(t)\bar{P}G_1 e(t) + 2\psi^T(t)\bar{P}G_2\hat{e}(t). \tag{34}$$

Similar to (17), one can get $\|e(t)\| \leq p/(1-p)\,\|y(t)\| + I_0 e^{-\alpha t}, \forall t \geq 0$. Besides, one has $\|\psi(t)\| \geq \|x(t)\|/\sqrt{2} \geq \|y(t)\|/(\sqrt{2}\,\|C\|)$ and $\|\psi(t)\| \geq \|\hat{x}(t)\|$. Thus (34) can be rewritten as

$$\begin{aligned}
\dot{V}(t) \leq & -\lambda_{\min}\left(\bar{Q}\right)\left(\omega_1 + (1-\omega_1)\right)\|\psi(t)\|^2 \\
& + 2\,\|\bar{P}G_1\|\,\|\psi(t)\|\left(\frac{p}{1-p}\,\|y(t)\| + I_0 e^{-\alpha t}\right) \\
& + 2\,\|\bar{P}G_2\|\,\|\psi(t)\|\left(\frac{p}{1-p}\,\|\hat{x}(t)\| + I_0 e^{-\alpha t}\right) \\
\leq & -\left(\frac{\lambda_{\min}\left(\bar{Q}\right)\omega_1}{\sqrt{2}\,\|C\|} - 2\frac{p}{1-p}\,\|\bar{P}G_1\|\right)\|y(t)\|\,\|\psi(t)\| \\
& -\left(\lambda_{\min}\left(\bar{Q}\right)(1-\omega_1) - 2\frac{p}{1-p}\,\|\bar{P}G_2\|\right)\|\hat{x}(t)\|\,\|\psi(t)\| \\
& + 2I_0\left(\|\bar{P}G_1\| + \|\bar{P}G_2\|\right)\|\psi(t)\|\,e^{-\alpha t},
\end{aligned}$$

where $0 < \omega_1 < 1$. Let $\omega_1 = \sqrt{2}\,\|C\|\,\|\bar{P}G_1\|/(\sqrt{2}\,\|C\|\,\|\bar{P}G_1\| + \|\bar{P}G_2\|)$ and $\gamma_2 = \lambda_{\min}\left(\bar{Q}\right)/(\sqrt{2}\,\|\bar{P}G_1\|\,\|C\| + \|\bar{P}G_2\|) - 2p/(1-p)$, one can further has

$$\begin{aligned}
\dot{V}(t) \leq & -\gamma_2\left(\|\bar{P}G_1\|\,\|y(t)\| + \|\bar{P}G_2\|\,\|\hat{x}(t)\|\right)\|\psi(t)\| \\
& + 2I_0\left(\|\bar{P}G_1\| + \|\bar{P}G_2\|\right)\|\psi(t)\|\,e^{-\alpha t}.
\end{aligned}$$

From Assumption 3, we can get $\lambda_{\min}\left(\bar{Q}\right)/(\sqrt{2}\,\|\bar{P}G_1\|\,\|C\| + \|\bar{P}G_2\|) - 2p/(1-p) > 0$. Similar to the remainder of Theorem 1, one can conclude that the closed loop system (26) with controller (27) is globally asymptotically stable, and Zeno behavior is excluded. The details are omitted due to space constraints but the proof follows the exact same steps as the corresponding parts in Theorem 1. ■

Similarly, ETC can then be implemented to reduce the controller updates as well. For the closed loop system (26), the event-triggered time instants T_k for the event-triggered controller (21) are determined by:

$$T_{k+1} = \inf\{t > T_k : g_2\left(\delta(t), \hat{x}(\hat{t}_{\hat{\sigma}}), y(t_\sigma)\right) \geq 0\}, \tag{35}$$

where

$$\begin{aligned}
& g\left(\delta(t), \hat{x}(\hat{t}_{\hat{\sigma}}), y(t_\sigma)\right) \\
& = \|\delta(t)\| - \varepsilon_2\hat{\gamma}_2\left(\|\bar{P}G_2\|\,\|\hat{x}(\hat{t}_{\hat{\sigma}})\| + \|\bar{P}G_1\|\,y(t_\sigma)\right) \\
& \quad \hat{\gamma}_2 = (1-p)\,\gamma_2/(2\,\|\bar{P}H\|),
\end{aligned} \tag{36}$$

with constant $0 < \varepsilon_2 < 1$ and the matrix $H = [-BK, 0_n]^T$.

Theorem 4: Consider the closed loop system given by (26) with the event-triggered controller (21). Suppose Assumptions 1, 3 hold and that the deadband conditions (9) (28) hold with $\alpha < \min\{(1-\varepsilon_2)\gamma_2\,\|\bar{P}G_2\|/(2\lambda_{\max}(\bar{P})), -\max_i \mathrm{Re}\left(\lambda_i\left(A-FC\right)\right)\}$. Then, the origin of the closed loop system is globally asymptotically stable under the event condition (35). Moreover, Zeno behavior is excluded.

IV. SIMULATION RESULTS

In this section, a simulation example is given to verify the theoretical results. Due to space limitations, only the simulation results for Framework II are presented. The initial state of the plant and the observer are chosen as $x(0) = [-35, 40]$ and $\hat{x}(0) = [-20, 30]$, respectively. The system matrices are chosen as

$$A = \begin{bmatrix} 0.1 & 0.5 \\ 0 & -2.5 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, F = \begin{bmatrix} 2.5 \\ 0.5 \end{bmatrix}, C = [1 \;\; 0]$$

and parameters $\varepsilon_1 = \varepsilon_2 = 0.95$. One can verify that (A, B) is stablizable and (A, C) is detectable.

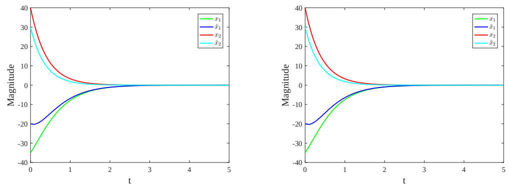
Given $\bar{Q} = I_4$, then solving the Lyapunov function $\bar{P}\bar{A} + \bar{A}^T\bar{P} = -\bar{Q}$, one can get $\theta = 0.1577$ from (32) and choose $p = 0.1$, (for tactile perception, i.e., force, limb position, and velocity, the JND is in the range from 5% to 15% [14]). The simulation results are shown in Figs. 1-3. Fig. 1(a) shows the evolution of state x and observer state $\hat{x}$ under tactile communication (TC), where x_1 and x_2 are the state components of the plant, $\hat{x}_1$ and $\hat{x}_2$ are the state components of the observer. Fig. 2 (b) shows the evolution of state x and observer state $\hat{x}$ with event-triggered controller under TC (ETC+TC). The communication time instants of the observer (CT1), the sensor output (CT2) and the controller update time instants (UT) for TC and ETC+TC are marked with blue $*$, green $\times$ and red o in Figs. 3 and 4, respectively. Without ETC, the controller is updated synchronously with the communication times of observer.

Table I summarises the simulation results for Framework II. The communication times of the observer, the communication times of sensor and the controller update times are given. Besides, the minimum inter-communication interval (MIC) of the observer, the minimum inter-communication interval (MIC) of the sensor and the minimum inter-event interval (MIE) of the controller are stated. It can be seen that the introduction of ETC reduces the frequency of controller update without increasing the communication times.

TABLE I

COMMUNICATION TIMES AND CONTROLLER UPDATE TIMES AND MINIMUM INTER-COMMUNICATION/EVENT INTERVALS

	CT1/MIC (TC)	CT2/MIC (TC)	UT/MIE (TC)	CT1/MIC (ETC+TC)	CT2/MIC (ETC+TC)	UT/MIE (ETC+TC)
Framework II	49/0.045	48/0.055	49/0.045	50/0.045	47/0.05	32/0.08



(a) TC (b) ETC+TC

Fig. 1. The evolution of state x_1, x_2 and observer state $\hat{x}_1, \hat{x}_2$.

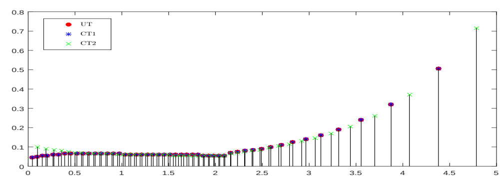


Fig. 2. The communication instants of the observer (CT1), the sensor (CT2) and the the controller update instants (UT) under TC.

V. CONCLUSION

In this paper, the event-triggered output feedback control of linear systems was investigated under tactile communication, and two different frameworks were considered. For each framework, firstly, an output feedback controller was proposed with tactile communications, and then ETC was further implemented for the output feedback controller. Under an assumption that the deadband factor was upper bounded with respect to the system model, it was proven that global asymptotic stability of the closed loop system was achieved. Furthermore, it was concluded that the integration of ETC and TC was capable of reducing both the frequency of communication and control update.

Single plant (agent) case was studied in this paper. In the future, the inclusion of coordination and communication between agents in the multiple agent case will be considered. Moreover, the analysis of communication constraints, such as quantization and disturbances are also parts of future work.

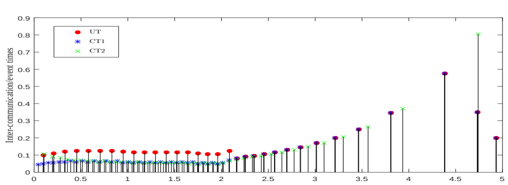


Fig. 3. The communication instants of the observer (CT1), the sensor (CT2) and the the controller update instants (UT) under ETC and TC.

REFERENCES

[1] P. Tabuada, Event-triggered real-time scheduling of stabilizing control tasks, *IEEE Trans. Autom. Control*, Vol. 52, 2007, pp 1680-1685.

[2] D.V. Dimarogonas, E. Frazzoli and K.H. Johansson, Distributed event-triggered control for multi-agent systems, *IEEE Trans. Autom. Control*, Vol. 57, 2012, pp 1291-1297.

[3] P. Tallapragada and N. Chopra, ”Event-triggered dynamic output feedback control for LTI systems”, *in Proc. Decision and Control (CDC)*, Maui, Hawaii, USA, Dec. 2012, pp 6597-6602.

[4] G.S. Seyboth, D.V. Dimarogonas, K.H. Johansson, Event-based broadcasting for multi-agent average consensus, *Automatica*, Vol. 49, 2013, pp 245-252.

[5] Y. Fan, G. Feng, Y. Wang Y and C. Song, Distributed event-triggered control of multi-agent systems with combinational measurements. *Automatica*, Vol. 49, 2013, pp 671-675.

[6] D. Yang, W. Ren, X. Liu and W. Chen, Decentralized event-triggered consensus for linear multi-agent systems under general directed graphs, *Automatica*, Vol. 69, 2016, pp 242-249.

[7] W. Heemels, M.C.F. Donkers and A.R. Teel, Periodic event-triggered control for linear systems, *IEEE Trans. Automat. Control*, Vol. 58, 2013, pp 847-861.

[8] A. Fuand M. Mazo, ”Periodic asynchronous event-triggered control”, *in Proc. Decision and Control (CDC)*, Las Vegas, USA, Dec. 2016, pp 1370-1375.

[9] G.P. Fettweis, The tactile internet: applications and challenges, *IEEE Veh. Technol. Mag.*, Vol. 9, 2014, pp 64-70.

[10] J.G. Andrews, S. Buzzi, W. Choi, S.V. Hanly, A. Lozano, A.C. Soong and J.C. Zhang, What will 5G be? *IEEE J. Sel. Areas Commun.*, Vol.32, 2014, pp 1065-1082.

[11] A. Ortega and Y. Liu, Lossy Compression of Haptic Data, *Englewood Cliffs, NJ: Prentice-Hall*, 2002, ch. 6, pp. 119-136.

[12] P.G. Otanez, J.R. Moyne, and D.M. Tilbury, ”Using deadbands to reduce communication in networked control systems”, *in Proc. Amer. Control Conf.*, Anchorage, AK, Nov. 2002, vol. 4, pp 3015-3020.

[13] D. Szab´, A. Gulyás, F. Fitzek and D.E.L Roetter, Towards the Tactile Internet: Decreasing Communication Latency with Network Coding and Software Defined Networking, *in European Wireless Conference 2015*, VDE Verlag GMBH, 2015, pp 428-433.

[14] P. Hinterseer, S. Hirche, S. Chaudhuri, E. Steinbach, and M. Buss, Perception-based data reduction and transmission of haptic data in telepresence and teleaction systems, *IEEE Trans. Signal Process.*, Vol. 56, 2008, pp 588-597.

[15] W.E. Heinrich, H.E. Ross and D.J. Murray, EH Weber on the tactile senses, *Psychology Press*, 1996.

[16] H.B. Barlow, Increment thresholds at low intensities considered as signal/noise discrimination, *J. Ptuisiol.*, Vol. 136, 1957, pp 469-488.

[17] S. Magnsson, K. Heal, C. Enyioha, N. Li, C. Fischione and V. Tarokh, ”Convergence of limited communications gradient methods”, *in American Control Conference (ACC)*, 2016, pp. 1421-1426.

[18] H.K. Khalil, Nonlinear Systems, *Prentice Hall*, 3rd edn. 2002.