

Prescribed performance formation control for second-order multi-agent systems with connectivity and collision constraints

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Abstract

This paper studies the distributed formation control problem of second-order multi-agent systems (MASs) with limited communication ranges and collision avoidance constraints. A novel connectivity preservation and collision-free distributed control algorithm is proposed by combining prescribed performance control (PPC) and exponential zeroing control barrier Lyapunov functions (EZCBFs). In particular, we impose the time-varying performance constraints on the relative position and velocity errors between the neighboring agents, and then a PPC-based formation control algorithm is developed such that the connectivity of the communication graph can be preserved at all times, and the prescribed transient and steady performance on the relative position and velocity error can be achieved. Subsequently, by introducing the EZCBFs method, an inequality constraint condition on the control input is derived to guarantee the collision-free formation motion. By regarding the PPC-based formation controller as a nominal input, an actual formation control input is given by solving the quadratic programming (QP) problem such that each agent achieves collision-free formation motion while guaranteeing the connectivity and prescribed performance as much as possible. Finally, numerical simulation is carried out to validate the effectiveness of the proposed algorithm.

Key words: Distributed formation control, Prescribed performance control, Connectivity preservation, Collision avoidance, Control barrier function

1 Introduction

Distributed cooperative control of multi-agent systems has received considerable attention due to its wide applications including explorations, surveillance, monitoring and localization (Ren, Beard, Atkins, 2007). In general, specific problems of cooperative control have been considered including consensus, flocking, coverage, rendezvous and formation control (Dimarogonas, Kyriakopoulos, 2007; Oh, Park, Ahn, 2015). In particular, the formation control of MASs is

to derive a group of agents to form and maintain a prescribed formation configuration such that complex missions can be accomplished in a collaborative manner.

For the formation control problem of multi-agent system, most research results (e.g. Abdessameud, Tayebi (2011); Deghat, Anderson, Lin (2015); Lee, Ahn (2016); Li, Ge, He, et al. (2019); Lin, Wang, Han, Fu (2014)) assume that the communication graph is connected all the time and the communication ranges of all the agents are unlimited. Such an assumption, however, can not be satisfied in many practical applications since each agent has only limited communication ranges and the communication link of two agents may be lost if their distance is larger than the allowable communication ranges. This fact motivates the study of the connectivity preservation problem of MASs. In addition, consider that each agent has a geometrical size instead of being a mass point in practical applications and the collisions

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may occur when the distance between two agents is very small. Therefore, the collision avoidance problem is another basic problem of MASs.

The connectivity preservation or collision avoidance problem for multi-agent system has been widely studied in the related literature. For instance, a centralized control algorithm was first proposed by employing the Laplacian matrix such that the connectivity preservation can be guaranteed in Zavlanos, Pappas (2007). Note that the centralized method in Zavlanos, Pappas (2007) involves the global information that is generally unavailable for each agent. To deal with this issue, some distributed connectivity preservation algorithms have been proposed for first-order, second-order integrator and nonholonomic MASs (Ajlou, Aghdam, 2013; Dong, Su, Liu, Xu, 2018; Ji, Egerstedt, 2007). In addition, by considering collision avoidance and connectivity problems simultaneously, some collision-free and connectivity preservation distributed algorithms were proposed for various MASs (Feng, Hu, 2019; Meng, Lin, Ren, 2012; Poonawala, Satici, Eckert, Spong, 2014). Note that most results on connectivity preservation or collision avoidance are based on the potential function method. The main issue of the potential-based approach is the existence of local minima due to the conflicting objectives (Ge, Fua, 2005; Verginis, Nikou, Dimarogonas, 2019).

Different from the potential function methods, the prescribed performance control (PPC) method originally proposed in Bechlioulis, Rovithakis (2008) is also applied to solve the connectivity preservation or collision avoidance problem of formation control. In such a case, the collision or connectivity constraint can be transformed into the boundary constraints on the system states and these constraints can be satisfied all the times via the PPC method. In addition, a prominent advantage of the PPC method over potential function method is to guarantee the closed-loop system can achieve the prescribed transient and steady control performance, such as convergence speed, and maximum steady state error bounds. Inspired by these properties, some PPC-based distributed control algorithms were proposed to achieve formation control with connectivity preservation or collision avoidance. For instance, the longitudinal formation control problem with connectivity and collision constraints for large platoons of vehicles was considered in Verginis, Bechlioulis, Dimarogonas, et al (2017) while the PPC-based algorithm was proposed for the case of one dimension space. In Yoo, Park (2017), a distributed error transformation strategy as a special case of PPC method was proposed to solve the connectivity preserving problem of non-holonomic multi-robot system. In Verginis, Nikou, Dimarogonas (2019) and Mehdifar, Bechlioulis, Hashemzadeh, Baradarannia (2020), the formation control algorithms via PPC method were proposed to guarantee the satisfaction of the connectivity and

collision avoidance constraints. Note that the proposed algorithms in Mehdifar, Bechlioulis, Hashemzadeh, Baradarannia (2020); Verginis, Nikou, Dimarogonas (2019) are distance-based without involving global position measurements, but require the communication graph to be a tree structure or the minimally rigid condition of the formation graph, and only guarantee the collision avoidance among the agents that are initially connected rather than all the agents in the formation.

Motivated by above discussions, this paper focuses on the distributed formation control problem for second-order multi-agent system with limited communication ranges and collision constraints. Firstly, we propose a PPC-based distributed control algorithm to achieve the desired formation control with preserved connectivity and prescribed performance. Subsequently, the inequality constraint condition on the control input is derived by using the exponential zeroing control barrier Lyapunov functions (EZCBFs) method to guarantee the collision avoidance among the agents. By considering the PPC-based controller as a nominal one, we derive an actual control input by solving a quadratic programming (QP) problem with the collision avoidance constraint to achieve the collision-free, prescribed connectivity and prescribed performance formation control objective. Compared with the related results, the main contributions of this paper are three fold. (i) We develop a novel distributed formation control algorithm for second-order MASs by combining PPC and EZCBFs methods, which not only guarantees the connectivity preservation and collision avoidance, but also achieves the desired formation control with the prescribed transient and steady performance. (ii) We extend the formation algorithms in Verginis, Nikou, Dimarogonas (2019) and Mehdifar, Bechlioulis, Hashemzadeh, Baradarannia (2020) to a displacement-based PPC control one, which can be applied to the case of a general communication graph and achieving the collision avoidance problem among all the agents rather than only the initially connected ones. (iii) The proposed PPC-based formation control algorithm can be also applied when the input disturbance is considered. Compared with the related results in Dong, Su, Liu, Xu (2018); Feng, Hu (2019) where the steady-state error were shown in a conservative residual set relying on the control parameters and some unknown constants, the proposed formation algorithm guarantees the steady-state errors within the prespecified bounds only determined by the performance parameters.

The organization of the paper is as follows. In Section 2, the formation control problem is formulated. A PPC-based formation controller as a nominal one and collision-free formation controller are derived in Section 3. Numerical simulation is carried out in Section 4 and some conclusions are drawn in Section 5.

2 Problem statements and preliminaries

Notation: Let $\mathbb{R}^{n \times n}$ and \mathbb{R}^n be the sets of $n \times n$ real matrices and n -dimension real vectors. For any vector $x = [x_1, \dots, x_n]^T \in \mathbb{R}^n$, $\|x\|_\infty = \max(x_1, \dots, x_n)$ denotes its infinite norm, and $\|x\| = (\sum_{i=1}^n x_i^2)^{1/2}$ denotes its 2-norm. Let I_n be identity matrix in n dimensions, $\mathcal{K}_n = \{1, 2, \dots, n\}$ be the set of the natural numbers from 1 to n , $\text{diag}([x_i]_{i \in \mathcal{K}_n})$ denotes the diagonal matrix with diagonal entries being x_1 to x_n , and \otimes be the Kronecker product. For any square matrix A , let $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denotes the minimum and maximum eigenvalues, respectively.

In this section, we consider a group of N agents and their dynamics are described as

$$\dot{x}_i = v_i, \dot{v}_i = u_i, \quad (1)$$

where $x_i, v_i \in \mathbb{R}^n$ are the position and velocity vectors of the i th agent, respectively, $u_i \in \mathbb{R}^n$ is the control input to be designed later. Here, without loss of generality, $n = 1, 2, 3$ denotes the one, two and three dimensional space, respectively. The communication among N agents is described by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \dots, N\}$ is the node set and $\mathcal{E} = \{(i, j) \in \mathcal{V} \times \mathcal{V} | j \in \mathcal{N}_i\}$ is the edge set. We consider that each agent exchanges information with the other agents within its limited communication range. In particular, the neighbors of agent i are defined as $\mathcal{N}_i(t) = \{j \in \mathcal{V} | \|x_i(t) - x_j(t)\|_\infty \leq R_s\}$, where R_s is the communication range of the agents. The communication graph $\mathcal{G}(t)$ at the initial $t = 0$ is denoted by $\mathcal{G}_0 = (\mathcal{V}, \mathcal{E}_0)$ with $\mathcal{E}_0 = \{(i, j) | j \in \mathcal{N}_i(0)\}$. Let $m = |\mathcal{E}_0|$ be the number of the edges of the graph \mathcal{G}_0 . Since the graph \mathcal{G}_0 is undirected, we can arbitrarily assign the edge direction of the graph \mathcal{G}_0 and then define its incidence matrix $B = [b_{ij}] \in \mathbb{R}^{N \times m}$. The rows and columns of B are indexed by the vertices and edges, respectively. In particular, $b_{ij} = 1$ if the node i is the head of the edge j , $b_{ij} = -1$ if i is the tail of the edge j and $b_{ij} = 0$ otherwise. The Laplacian matrix of \mathcal{G}_0 can be defined as $L = BB^T$. It is easily obtained that L is symmetric and semi-definite matrix. If the graph \mathcal{G}_0 is connected, L has a simple zero eigenvalue such that $\lambda_N(L) \geq \dots \geq \lambda_2(L) > \lambda_1(L) = 0$, where $\lambda_i(\cdot)$ is the i th eigenvalue. Moreover, for any vector $x \in \mathbb{R}^N$ satisfying $1_N^T x = 0$, it follows that $x^T L x \geq \lambda_2 x^T x$.

Remark 2.1 We use the infinity norm $\|\cdot\|_\infty$ instead of the distance 2-norm $\|\cdot\|$ to define the neighbor set. The main reason is that the constraint on $\|x\|_\infty$ for any vector $x \in \mathbb{R}^n$ can be equivalently transformed into a constraint on its elements. This property is beneficial for designing PPC-based formation algorithm in the following part. Since $\|x\| \leq R_s$ implies that $\|x\|_\infty \leq R_s/\sqrt{n}$ holds, we know that $\|\cdot\|_\infty$ is a conservative way to define the communication ranges than $\|\cdot\|_2$ except $n = 1$.

In this paper, the formation control problem of second-

order MASs is considered. Based on the relative position of the agents, the desired formation configuration is given by $\mathcal{F}_t = \{x | x_i - x_j = \delta_{ij}^{des}, i, j \in \mathcal{V}\}$, where x is the stack vector of x_i , $\delta_{ij}^{des} = \delta_i^{des} - \delta_j^{des} \in \mathbb{R}^n$ is the desired position offset between agents i and j . The desired position offset δ_{ij}^{des} is properly defined such that the target formation configuration can be well-defined and is unique. In addition, the definitions of the connectivity preservation, collision avoidance and the allowable desired formation configuration \mathcal{F}_t are given as follows.

Definition 2.1 The desired formation configuration \mathcal{F}_t is feasible if the following conditions holds: (i) the communication topology of \mathcal{F}_t is connected. (ii) \mathcal{F}_t is collision-free, i.e., $\|\delta_{ij}^{des}\| > D_c$ for any $i, j \in \mathcal{V}$.

Definition 2.2 Agents $i, j \in \mathcal{V}$ are said to achieve collision avoidance if $\|x_i(t) - x_j(t)\| > D_c$ holds for $t \geq 0, i, j \in \mathcal{V}$, where $D_c > 0$ is the collision distance.

Definition 2.3 The connectivity of the initial graph \mathcal{G}_0 is preserved if $\|x_i(t) - x_j(t)\|_\infty \leq R_s$ holds for all $t \geq 0$ given that $\|x_i(0) - x_j(0)\|_\infty \leq R_s$.

Assumption 2.1 The desired formation configuration \mathcal{F}_t is feasible.

Assumption 2.2 The initial graph \mathcal{G}_0 is connected, and is congruent with the desired formation configuration \mathcal{F}_t , namely, if the edge $(i, j) \in \mathcal{E}_0$, then its desired offset δ_{ij}^{des} in \mathcal{F}_t is connected, i.e., $\|\delta_{ij}^{des}\|_\infty \leq R_s, \forall (i, j) \in \mathcal{E}_0$.

Remark 2.2 The connectivity of the initial graph is a standard condition in the coordinated control of MASs. In addition, the congruent requirement is necessary to preserve the connectivity of the initial graph \mathcal{G}_0 in the desired formation.

The control objective of this paper is to design a distributed control strategy for second-order MASs such that the following requirements are satisfied:

- (1) The connectivity of the initial graph \mathcal{G}_0 is preserved, i.e., if $\|x_i(0) - x_j(0)\|_\infty \leq R_s$ holds for any edge $(i, j) \in \mathcal{E}_0$, then $\|x_i(t) - x_j(t)\|_\infty \leq R_s$ for all $t \geq 0$;
- (2) The collision of all the agents is avoided, i.e., $\|x_i(0) - x_j(0)\| > D_c$, then $\|x_i(t) - x_j(t)\| > D_c$ for all $i, j \in \mathcal{V}$.
- (3) The desired formation configuration \mathcal{F}_t is achieved, i.e., $\lim_{t \rightarrow \infty} (x_i(t) - x_j(t)) = \delta_{ij}^{des}$ and $\lim_{t \rightarrow \infty} (v_i(t) - v_j(t)) = 0$ for all $i, j \in \mathcal{V}$.

3 Main results

In this section, we first propose PPC-based formation control algorithm to achieve the desired formation control with connectivity preservation and prescribed convergence. Secondly, the collision avoidance requirement is handled by using the EZCBFs method, and a necessary modification to the PPC-based formation controllers as the nominal ones is obtained by solving

QP problem in a distributed manner such that each agent achieves collision-free formation motion while guaranteeing the connectivity and prescribed performance as much as possible.

3.1 PPC-based formation control

For each edge $k \triangleq (i, j) \in \mathcal{E}_0$ of the initial graph \mathcal{G}_0 , we define the relative position error

$$e_k \triangleq e_{ij} = x_i - x_j - \delta_{ij}^{des}, \quad (2)$$

with $k \in \mathcal{K}_m = \{1, 2, \dots, m\}$ and $m = |\mathcal{E}_0|$ being the number of initial edges. In this section, the prescribed performance control (PPC) method, originally proposed in Bechlioulis, Rovithakis (2008), is introduced to achieve two objectives: (i) the prescribed transient and steady performance with respect to the relative position error e_k ; (ii) the connectivity preservation of all initially connected edges $k \in \mathcal{E}_0$. Similar to the results in Bechlioulis, Rovithakis (2008), the mathematical expression of prescribed performance in term of relative position error e_k is given by

$$-\underline{M}_{ij}^\mu \rho_{ij}^\mu(t) < e_k^\mu(t) < \overline{M}_{ij}^\mu \rho_{ij}^\mu(t), k \triangleq (i, j) \in \mathcal{E}_0, \quad (3)$$

where e_k^μ denotes the μ th element of e_k , $\mu \in \mathcal{K}_n = \{1, \dots, n\}$. $\underline{M}_{ij}^\mu, \overline{M}_{ij}^\mu$ are positive parameters and will be appropriately selected in the following to guarantee the connectivity of the graph \mathcal{G}_0 . $\rho_{ij}^\mu(t) = (1 - \rho_{ij}^\mu / \max(\underline{M}_{ij}^\mu, \overline{M}_{ij}^\mu))e^{-\pi_{ij}t} + \rho_{ij, \infty}^\mu / \max(\underline{M}_{ij}^\mu, \overline{M}_{ij}^\mu)$ is a smooth, positive and monotonically decreasing performance function. Note that $\lim_{t \rightarrow \infty} \overline{M}_{ij}^\mu \rho_{ij}^\mu(t) = \frac{\overline{M}_{ij}^\mu}{\max(\underline{M}_{ij}^\mu, \overline{M}_{ij}^\mu)} \rho_{ij, \infty}^\mu \leq \rho_{ij, \infty}^\mu$, where $\rho_{ij, \infty}^\mu > 0$ represents the maximum allowable steady state error. In addition, the parameter π_{ij} denotes the lower bound on the convergence speed of e_k^μ . By specifying the parameters $\pi_{ij}, \rho_{ij, \infty}^\mu$, the prescribed transient and steady performance can be obtained.

Next, the parameters $\underline{M}_{ij}^\mu, \overline{M}_{ij}^\mu$ are chosen to guarantee the connectivity of the initial graph \mathcal{G}_0 . Note that $\|x_i(0) - x_j(0)\|_\infty \leq R_s$ can be equivalently transformed element-wise for each dimension as

$$|x_i^\mu(0) - x_j^\mu(0)| \leq R_s, \forall (i, j) \in \mathcal{E}_0, \quad (4)$$

with x_i^μ being the μ th element of x_i , $\mu \in \mathcal{K}_n$. Then, in terms of the relative position error e_k , (4) is written as

$$-(R_s + \delta_{ij, \mu}^{des}) < e_k^\mu(0) < R_s - \delta_{ij, \mu}^{des}, \quad (5)$$

where $\delta_{ij, \mu}^{des}$ is the μ th element of δ_{ij}^{des} , $\mu \in \mathcal{K}_n$. In particular, we can select the parameters as $\underline{M}_{ij}^\mu = R_s - \delta_{ij, \mu}^{des}$, $\overline{M}_{ij}^\mu = R_s + \delta_{ij, \mu}^{des}$, and one can derive that $\underline{M}_{ij}^\mu > 0$ and $\overline{M}_{ij}^\mu > 0$ from Assumption 2.2. Then, it follows that $-\underline{M}_{ij}^\mu \rho_{ij}^\mu(0) < e_k^\mu(0) < \overline{M}_{ij}^\mu \rho_{ij}^\mu(0)$ holds. Based on the

decreasing property of $\rho_{ij}^\mu(t)$, it follows from (3) that $-\underline{M}_{ij}^\mu < e_k^\mu(t) < \overline{M}_{ij}^\mu$ for any $t \geq 0$, which implies $\|x_i(t) - x_j(t)\|_\infty \leq R_s$ holds for all $t > 0$. To this end, we have shown that the connectivity of the initial graph \mathcal{G}_0 is preserved if the inequality condition (3) is satisfied and $\underline{M}_{ij}^\mu, \overline{M}_{ij}^\mu$ are chosen according to (5).

We introduce a model transformation such that the control design problem with the state constraint (3) can be transformed into an unconstrained one. Define the modulated error $z_k^\mu = (\rho_{ij}^\mu)^{-1} e_k^\mu$ and obtain its domain $\mathcal{D}_{z_k^\mu} = \{z_k^\mu : z_k^\mu \in (-\underline{M}_{ij}^\mu, \overline{M}_{ij}^\mu)\}$. Then, a model transformation error with respect to z_k^μ is derived as

$$\varepsilon_k^\mu = T(z_k^\mu) = \ln\left(\frac{1 + (\underline{M}_{ij}^\mu)^{-1} z_k^\mu}{1 - (\overline{M}_{ij}^\mu)^{-1} z_k^\mu}\right), \quad (6)$$

and it is easily verified that $T(z_k^\mu)$ is a smooth and monotonically increasing function satisfying $T(0) = 0$, and its range is $(-\infty, \infty)$. Thus, we get that if the transformed error ε_k^μ is bounded, then z_k^μ is always constrained in the region $\mathcal{D}_{z_k^\mu}$. This also implies that the inequality constraint (3) is satisfied all the time. Differentiating (6) can yield

$$\dot{\varepsilon}_k = J_{z_k}(z_k, t)(\dot{e}_k + \alpha_k e_k), \quad (7)$$

where $\varepsilon_k, z_k \in \mathbb{R}^n$ are the stack vector of $\varepsilon_k^\mu, z_k^\mu, \mu \in \mathcal{K}_n$. $\alpha_k = \text{diag}([\alpha_{k, \mu}]_{\mu \in \mathcal{K}_n})$, $\alpha_{k, \mu} = -\frac{\rho_{ij}^\mu}{\rho_{ij}^\mu}$ and $J_{z_k}(z_k, t) = \text{diag}([J_{z_k^\mu}]_{\mu \in \mathcal{K}_n})$, $J_{z_k^\mu} = \frac{\partial T(z_k^\mu)}{\partial z_k^\mu} \frac{1}{\rho_{ij}^\mu}$. Based on the results in Eq. (16) of Karayiannidis, Papageorgiou, Doulgeri (2016), we get the following inequality $e_k^T J_{z_k}(z_k, t) \varepsilon_k \geq \varpi_{1, k} \varepsilon_k^T \varepsilon_k$ for some positive constants $\varpi_{1, k}$. These results are useful for the stability analysis.

Besides the prescribed bound constraints on the relative position error $e_k, k \in \mathcal{E}_0$, the performance constraints on the relative velocity are also considered. To this end, we introduce a linear combined error of relative position and velocity for each edge $k \in \mathcal{E}_0$ as

$$s_k \triangleq s_{ij} = v_{ij} + \lambda_s e_{ij}, \quad (8)$$

where $v_{ij} = v_i - v_j$ and λ_s is a positive constant. The prescribed performance constraint on the combined error $s_k = [s_k^1, s_k^2, \dots, s_k^n]^T$ is given as

$$-\underline{b}_k^\mu w_k^\mu(t) < s_k^\mu(t) < \overline{b}_k^\mu w_k^\mu(t), \quad (9)$$

where $\underline{b}_k^\mu, \overline{b}_k^\mu > 0, \mu \in \mathcal{K}_n$ and $w_k^\mu = (w_{k, 0}^\mu - w_{k, \infty}^\mu)e^{-c_k t} + w_{k, \infty}^\mu$ is the performance function with the parameters $w_{k, 0}^\mu, w_{k, \infty}^\mu > 0$ and $c_k > 0$. Define the modulated error $y_k^\mu = \frac{s_k^\mu}{w_k^\mu}$ and its domain is $\mathcal{D}_{y_k^\mu} = \{y_k^\mu : y_k^\mu \in (-\underline{b}_k^\mu, \overline{b}_k^\mu)\}$. Let y_k be the stack vector of y_k^μ . Similar to (6), we have

the transformation error $\zeta_k = T(y_k)$ and its derivative is

$$\dot{\zeta}_k = J_{y_k}(y_k, t)(\dot{s}_k + \beta_k s_k), \quad (10)$$

where $J_{y_k}(y_k, t)$ and β_k have the same formulations as J_{z_k} , α_k in (7). Similar to (7), we have that $s_k^T J_{y_k}(y_k, t) \zeta_k \geq \varpi_{2,k} \zeta_k^T \zeta_k$ with $\varpi_{2,k} > 0$.

To guarantee that the inequality constraints (3) and (9) holds for any $t \geq 0$, we develop a PPC-based formation control algorithm as

$$\begin{aligned} u_i = & -\gamma_1 \sum_{j \in \bar{\mathcal{N}}_i(t)} s_{ij} - k_1 \sum_{j \in \bar{\mathcal{N}}_i(t)} J_{z_{ij}}(z_{ij}, t) \varepsilon_{ij} \quad (11) \\ & - k_2 \sum_{j \in \bar{\mathcal{N}}_i(t)} J_{y_{ij}}(y_{ij}, t) \zeta_{ij}, \end{aligned}$$

where γ_1, k_1, k_2 are positive constants and $\bar{\mathcal{N}}_i(t) = \mathcal{N}_i(t) \cap \mathcal{N}_i(0)$ denotes the intersection of the agent i 's neighbor sets at the moments t and $t_0 = 0$. This indicates that only communication among the initially connected agents are considered for all $t \geq 0$. The controller (11) is composed of three parts. The first part is the consensus term of the combined error s_{ij} , and the second and third parts are based on the prescribed performance of the relative position error e_{ij} and s_{ij} , respectively. Note that $\rho_{ij}^\mu = \rho_{ji}^\mu, w_{ij}^\mu = w_{ji}^\mu, \forall (i, j) \in \mathcal{E}_0, \mu \in \mathcal{K}_n$. Also, we can derive that $T(z_{ij}) = -T(z_{ji})$ and $J_{z_{ij}}(z_{ij}, t) \varepsilon_{ij} = -J_{z_{ji}}(z_{ji}, t) \varepsilon_{ji}$. Similarly, $T(s_{ij}) = -T(s_{ji})$ and $J_{y_{ij}}(y_{ij}, t) \zeta_{ij} = -J_{y_{ji}}(y_{ji}, t) \zeta_{ji}$ can be obtained. Based on these results, the controller (11) can be written in a vector form as

$$\begin{aligned} u = & -\gamma_1 (B(t) \otimes I_n) s - k_1 (B(t) \otimes I_n) J_z(z, t) \varepsilon_e \\ & - k_2 (B(t) \otimes I_n) J_y(y, t) \zeta_s, \quad (12) \end{aligned}$$

where u is the stack vector of u_i , $B(t)$ is the incidence matrix of the subgraph of the graph $\mathcal{G}(t)$, $J_z(z, t)$ and $J_y(y, t)$ are time-varying diagonal matrix with the diagonal elements $J_{z_k}(z_k, t)$, $J_{y_k}(y_k, t)$ and $s, \varepsilon_e, \zeta_s$ are the stack vectors of $s_k, \varepsilon_k, \zeta_k, k \in \mathcal{K}_m$.

Theorem 3.1 Consider the second-order MASs (1). Under Assumptions 2.1-2.2 and the chosen parameters $\lambda_s > \pi_{ij}, \forall (i, j) \in \mathcal{E}_0, \gamma_1 > \lambda_s / \lambda_2(L), k_1 > 0, k_2 > 0$, the proposed distributed algorithm (11) guarantees that the prescribed performance constraints (3) and (9) hold for any $t \geq 0$. Moreover, $\lim_{t \rightarrow \infty} (x_i(t) - x_j(t)) = \delta_{ij}^{des}$ and $\lim_{t \rightarrow \infty} (v_i(t) - v_j(t)) = 0, \forall i, j \in \mathcal{V}$ with exponential convergence.

Proof: The proof of Theorem 3.1 includes three Steps.

Step 1: We first show the existence of a unique maximal solution for the variable $\Lambda_k^\mu(t) = [z_k^\mu(t), y_k^\mu(t)]^T$ over the set $\mathcal{D}_{z_k}^\mu \times \mathcal{D}_{y_k}^\mu$ for a time interval $[0, \tau_{\max})$. Differentiating $\Lambda_k^\mu(t) = [z_k^\mu(t), y_k^\mu(t)]^T$ yields

$$\dot{\Lambda}_k^\mu(t) = [\dot{z}_k^\mu(t), \dot{y}_k^\mu(t)]^T = [f_e(z_k^\mu, t), f_s(y_k^\mu, t)]^T, \quad (13)$$

where $f_e(z_k^\mu, t) = (\rho_{ij}^\mu)^{-1}(v_{ij}^\mu - \dot{\rho}_{ij}^\mu e_{ij}^\mu)$ and $f_s(y_k^\mu, t) = (w_k^\mu)^{-1}(\dot{v}_k^\mu + \lambda_k v_k^\mu - \dot{w}_k^\mu s_k^\mu)$. Selecting the parameters $\underline{M}_{ij}^\mu, \bar{M}_{ij}^\mu, b_k^\mu, \bar{b}_k^\mu$ in (3) and (10), we obtain that $\mathcal{D}_{z_k}^\mu \times \mathcal{D}_{y_k}^\mu$ is nonempty and open, and $\Lambda_k^\mu(0) \in \mathcal{D}_{z_k}^\mu \times \mathcal{D}_{y_k}^\mu$. Moreover, in the bounded set $\mathcal{D}_{z_k}^\mu \times \mathcal{D}_{y_k}^\mu$, we obtain that $f_e(z_k^\mu, t), f_s(y_k^\mu, t)$ are continuous and locally Lipschitz. Thus, the conditions of Theorem 54 in Sontag (1998) are satisfied, and then the existence and uniqueness of a maximal solution $\Lambda_k^\mu(t) = [z_k^\mu(t), y_k^\mu(t)]^T \in \mathcal{D}_{z_k}^\mu \times \mathcal{D}_{y_k}^\mu$ can be guaranteed for $\forall t \in [0, \tau_{\max})$.

Step 2: We next prove that algorithm (11) guarantees $\Lambda_k^\mu = [z_k^\mu, y_k^\mu]$ evolving in a bounded and compact subset of $\mathcal{D}_{z_k}^\mu \times \mathcal{D}_{y_k}^\mu$ for $\forall t \in [0, \tau_{\max})$, and τ_{\max} can be extended to ∞ by contradiction. This implies that the constraints (3) and (9) are satisfied all the time.

From Step 1, it follows that $z_k^\mu(t) \in \mathcal{D}_{z_k}^\mu, \forall t \in [0, \tau_{\max})$. This implies that any edge in the initial graph \mathcal{G}_0 is always connected for all $t \in [0, \tau_{\max})$. It then follows that $\bar{\mathcal{N}}_i(t) = \mathcal{N}_i(t) \cap \mathcal{N}_i(0) = \mathcal{N}_i(0)$ and $B(t) = B, \forall t \in [0, \tau_{\max})$, which implies that the graph $\mathcal{G}(t)$ is fixed for any $t \in [0, \tau_{\max})$. Denote the formation center as $x_c = \frac{1}{N} \sum_{i=1}^N x_i$ and its derivative is $v_c = \dot{x}_c = \frac{1}{N} \sum_{i=1}^N v_i$. The relative position and velocity errors with respect to (x_c, v_c) for each agent $i \in \mathcal{V}$ are defined as $\bar{x}_i = x_i - x_c - \delta_i^{des}$ and $\bar{v}_i = v_i - v_c$, respectively, where δ_i^{des} is the absolute desired position offset of agent $i \in \mathcal{V}$ with respect to x_c . It then follows that $\bar{x} = (Q \otimes I_n)(x - \delta^{des})$ and $\bar{v} = (Q \otimes I_n)v$, where $Q = I_N - \frac{1}{N} \mathbf{1}_N \mathbf{1}_N^T$ and $\bar{x}, \bar{v}, x, v, \delta^{des}$ are the stack vector of $\bar{x}_i, \bar{v}_i, x_i, v_i, \delta_i^{des}$, respectively. Based on $QB = B$ and (12), it follows that

$$\begin{aligned} \dot{\bar{v}} = & -\gamma_1 (B \otimes I_n) s - k_1 (B \otimes I_n) J_z(z, t) \varepsilon_e \quad (14) \\ & - k_2 (B \otimes I_n) J_y(y, t) \zeta_s. \end{aligned}$$

We consider the following Lyapunov function

$$\begin{aligned} V_e(\bar{x}, \bar{v}, \varepsilon_e) = & \gamma_1 \lambda_s \bar{x}^T (BB^T \otimes I_n) \bar{x} + \lambda_s \bar{x}^T \bar{v} \\ & + \frac{1}{2} \bar{v}^T \bar{v} + \frac{k_1}{2} \varepsilon_e^T \varepsilon_e. \end{aligned}$$

Note that $\lambda_2(L) \bar{x}^T \bar{x} \leq \bar{x}^T (BB^T \otimes I_n) \bar{x} = \bar{x}^T (L \otimes I_n) \bar{x} \leq \lambda_N(L) \bar{x}^T \bar{x}$, and then it follows that $V_e(\bar{x}, \bar{v}, \varepsilon_e) \geq \gamma_1 \lambda_s \lambda_2(L) \bar{x}^T \bar{x} + \lambda_s \bar{x}^T \bar{v} + \frac{1}{2} \bar{v}^T \bar{v} + \frac{k_1}{2} \varepsilon_e^T \varepsilon_e$ is positive definite if the inequality $2\gamma_1 \lambda_2(L) > \lambda_s$ is satisfied.

From the definitions of $e_k, s_k, k \in \mathcal{E}_0$, it follows that $e = (B^T \otimes I_n) \bar{x}$ and $s = (B^T \otimes I_n)(\bar{v} + \lambda_s \bar{x})$, where e is the stack of e_k . Based on (7) and (14), the derivative of $V_e(\bar{x}, \bar{v}, \varepsilon_e)$ is obtained as

$$\begin{aligned} \dot{V}_e(\bar{x}, \bar{v}, \varepsilon_e) = & 2\gamma_1 \lambda_s \bar{x}^T (BB^T \otimes I_n) \bar{v} + \lambda_s \bar{v}^T \bar{v} \\ & + (\bar{v} + \lambda_s \bar{x})^T \{-\gamma_1 (B \otimes I_n) s - k_1 (B \otimes I_n) J_z(z, t) \varepsilon_e \\ & - k_2 (B \otimes I_n) J_y(y, t) \zeta_s\} + k_1 \varepsilon_e^T J_z(z, t) (\dot{e} + \Xi e) \\ \leq & -\gamma_1 \lambda_s^2 \bar{x}^T (L \otimes I_n) \bar{x} - \bar{v}^T ((\gamma_1 L - \lambda_s I_N) \otimes I_n) \bar{v} \\ & - k_2 s^T J_y(y, t) \zeta_s - k_1 \varepsilon_e^T J_z(z, t) (\lambda_s I_{Nm} - \Xi) e, \end{aligned}$$

where $\Xi = \text{diag}([\alpha_k]_{k \in \mathcal{K}_m})$. According to (7), we have that $\alpha_{k,u} = -\frac{\rho_{ij}^\mu}{\rho_{ij}^\mu} = \pi_{ij}(\rho_{ij}^\mu - \rho_{ij,\infty}^\mu)/\rho_{ij}^\mu < \pi_{ij}$. Note that $-k_2 s^T J_y(y, t) \zeta_s \leq -\min(\varpi_{1,k}) k_2 \zeta_s^T \zeta_s \leq 0$ and $\bar{x}^T(L \otimes I_n) \bar{x} \geq \lambda_2(L) \bar{x}^T \bar{x}$. Then, if the parameters λ_s, γ_1 are chosen such that $\lambda_s > \pi_{ij}$ and $\gamma_1 \geq \frac{\lambda_s}{\lambda_2(L)}$, it follows that

$$\dot{V}_e \leq -\gamma_1 \lambda_2(L) \lambda_s^2 \bar{x}^T \bar{x} - l_1 \bar{v}^T \bar{v} - k_1 l_2 \varepsilon_e^T \varepsilon_e, \quad (15)$$

where $l_1 = \gamma_1 \lambda_2(L) - \lambda_s > 0$ and $l_2 = \min(\lambda_s - \pi_{ij}) > 0$. This infers that $V_e(t) \leq V_e(0), \forall t \in [0, t_{\max})$ and further derive that $\varepsilon_e^T \varepsilon_e \leq \frac{2}{k_1} V_e(0)$. Since $V_e(0)$ is bounded, we have that $\|\varepsilon_k\|$ is bounded and there exists some positive constant ϵ_0 such that $|\varepsilon_k^\mu| \leq \epsilon_0, \mu \in \mathcal{K}_n$ holds. Taking the inverse logarithmic function of (6), it follows that

$$-\underline{M}_k^\mu < -\frac{e^{-\epsilon_0} - 1}{e^{-\epsilon_0} + 1} \underline{M}_k^\mu < z_k^\mu(t) < \frac{e^{\epsilon_0} - 1}{e^{\epsilon_0} + 1} \overline{M}_k^\mu < \overline{M}_k^\mu,$$

for $\forall t \in [0, \tau_{\max})$. It is shown that algorithm (11) guarantees z_k^μ always evolving in a compact subset $\mathcal{D}'_{z_k^\mu} = [-\frac{e^{-\epsilon_0} - 1}{e^{-\epsilon_0} + 1} \underline{M}_k^\mu, \frac{e^{\epsilon_0} - 1}{e^{\epsilon_0} + 1} \overline{M}_k^\mu]$ of $\mathcal{D}_{z_k^\mu}$.

We next show that s_k^μ also evolves in a compact subset of \mathcal{D}_{s_k} . Differentiating $s = (B^T \otimes I_n)(\bar{v} + \lambda_s \bar{x})$ along the trajectory of (14) yields

$$\begin{aligned} \dot{s}(t) &= -\gamma_1 (B^T B \otimes I_n) s - k_1 (B^T B \otimes I_n) J_z(z, t) \varepsilon_e \\ &\quad - k_2 (B^T B \otimes I_n) J_y(y, t) \zeta_s + \lambda_s (B^T \otimes I_n) \bar{x}. \end{aligned}$$

Consider the Lyapunov function $V_s(\bar{x}, \bar{v}, \zeta_s) = \frac{1}{2} \zeta_s^T \zeta_s + \frac{\theta}{2} (\bar{v} + \lambda_s \bar{x})^T (\bar{v} + \lambda_s \bar{x})$ with a positive constant θ , and its derivative is simplified as

$$\begin{aligned} \dot{V}_s &\leq -\theta \gamma_1 s^T s - \zeta_s^T J_y(y, t) (\theta k_2 I_{Nm} - \beta) s \\ &\quad - \frac{k_2}{4} \zeta_s^T J_y(y, t) (B^T B \otimes I_n) J_y(y, t) \zeta_s + \Delta_s \\ &\leq -l_s \zeta_s^T \zeta_s - \theta \gamma_1 \lambda_2(L) (\bar{v} + \lambda_s \bar{x})^T (\bar{v} + \lambda_s \bar{x}) + \Delta_s \\ &\leq -\delta_s V_s + \Delta_s, \end{aligned} \quad (16)$$

where $\delta_s = \min(2l_s, 2\gamma_1 \lambda_2(L)) > 0, l_s = \min(\varpi_{1,k}(\theta k_2 - \beta_k)) > 0$ when the parameter θ is chosen such that $\theta > \max(\beta_k)/k_2$. In addition, $\Delta_s = \frac{k_1^2}{k_2} \varepsilon_e^T J_z(z, t) (B^T B \otimes I_n) J_z(z, t) \varepsilon_e + \frac{\gamma_1^2}{k_2} s^T (B^T B \otimes I_n) s + \frac{\lambda_s^2}{k_2} \bar{v}^T \bar{v} - \theta k_1 s^T J_z(z, t) \varepsilon_e + \theta \lambda_s (\bar{v} + \lambda_s \bar{x})^T \bar{v}$. Based on the results of Step 1, one can derive that $\bar{x}(t), \bar{v}(t), \varepsilon_e(t)$ are all bounded for $\forall t \in [0, \tau_{\max})$. In addition, note that $s^T s = (\bar{v} + \lambda_s \bar{x})^T (L \otimes I_n) (\bar{v} + \lambda_s \bar{x}) \leq \lambda_N(L) (\bar{v} + \lambda_s \bar{x})^T (\bar{v} + \lambda_s \bar{x})$. It follows that $s(t)$ are also bounded for $\forall t \in [0, \tau_{\max})$. Thus, we can deduce that Δ_s is bounded, i.e., $\|\Delta_s\| \leq \varrho_s$ for a bounded positive constant ϱ_s . From (16) and the comparison lemma (Khalil, 2002), we can obtain that $V_s(t) \leq e^{-\delta_s t} V_s(0) + (1 - e^{-\delta_s t}) \varrho_s / \delta_s$. It then follows that $V_s(\bar{x}(t), \bar{v}(t), \zeta_s(t))$ is bounded for $t \in [0, \tau_{\max})$, which implies that ζ_k^μ is bounded, i.e., $|\zeta_k^\mu| \leq \epsilon_s$ with a positive constant ϵ_s , and one can further

derive that

$$-b_k^\mu < -\frac{e^{-\epsilon_s} - 1}{e^{-\epsilon_s} + 1} b_k^\mu < y_k^\mu(t) < \frac{e^{\epsilon_s} - 1}{e^{\epsilon_s} + 1} \bar{b}_k^\mu < \bar{b}_k^\mu,$$

for $\forall t \in [0, \tau_{\max})$. It is shown that the solution y_k^μ always evolves in the compact subset $\mathcal{D}'_{y_k^\mu} = [-\frac{e^{-\epsilon_s} - 1}{e^{-\epsilon_s} + 1} b_k^\mu, \frac{e^{\epsilon_s} - 1}{e^{\epsilon_s} + 1} \bar{b}_k^\mu]$ of $\mathcal{D}_{y_k^\mu}$. To this end, we have proven that $\Lambda_k^\mu = [z_k^\mu, y_k^\mu]$ can always remain in the compact set $\mathcal{D}'_{z_k^\mu} \times \mathcal{D}'_{y_k^\mu} \subset \mathcal{D}_{z_k^\mu} \times \mathcal{D}_{y_k^\mu}$.

In addition, we further show that τ_{\max} can be extended to ∞ . Assume that $\tau_{\max} < \infty$ and since $\mathcal{D}'_{z_k^\mu} \times \mathcal{D}'_{y_k^\mu} \subset \mathcal{D}_{z_k^\mu} \times \mathcal{D}_{y_k^\mu}$, based on the results of Proposition 2.1, it is indicated that there exists a time instant $t' \in [0, \tau_{\max})$ such that $\Lambda_k^\mu \notin \mathcal{D}_{z_k^\mu} \times \mathcal{D}_{y_k^\mu}$. Based on the results obtained above, a contradiction occurs. Thereby, $\tau_{\max} = \infty$. This implies that the prescribed performance constraints (3) and (8) are satisfied for all $t \geq 0$.

Step 3: We finally prove that the desired formation configuration \mathcal{F} is achieved, i.e., $\lim_{t \rightarrow \infty} (x_i(t) - x_j(t)) = \delta_{ij}^{des}$ and $\lim_{t \rightarrow \infty} (v_i(t) - v_j(t)) = 0, \forall i, j \in \mathcal{V}$. Define the variable $\psi = [\bar{x}^T, \bar{v}^T, \varepsilon^T]^T$ and note that $V_e(\bar{x}, \bar{v}, \varepsilon_e) \leq \gamma_1 \lambda_s \lambda_N(L) \bar{x}^T \bar{x} + \lambda_s \bar{x}^T \bar{v} + \frac{1}{2} \bar{v}^T \bar{v} + \frac{k_1}{2} \varepsilon_e^T \varepsilon_e$. One can derive that $V_e(\psi) \leq \psi^T (\Sigma_m \otimes I_n) \psi$, where $\Sigma_m = [\gamma_1 \lambda_s \lambda_N(L) I_{Nn}, \frac{\lambda_s}{2} I_{Nn}, 0; \frac{\lambda_s}{2} I_{Nn}, \frac{1}{2} I_{Nn}, 0; 0, 0, \frac{k_1}{2} I_{Nn}]$ is positive definite if $\gamma_1 > \lambda_s / \lambda_N(L)$. It then follows from (15) that

$$V_e(\psi) \leq -\sigma_e \psi^T \psi \leq -\frac{\sigma_e}{\lambda_{\max}(\Sigma_m)} V_e(\psi) \quad (17)$$

where $\sigma_e = \min(\gamma_1 \lambda_2(L) \lambda_s^2, l_1, k_1 l_2) > 0$ and $\lambda_{\max}(\Sigma_m)$ is the largest eigenvalue of Σ_m . It follows from (17) that $\lim_{t \rightarrow \infty} \bar{x}(t) = 0$ and $\lim_{t \rightarrow \infty} \bar{v}(t) = 0$ exponentially. Since $\bar{x}_i - \bar{x}_j = x_i - x_j - \delta_{ij}^{des}$, it follows that $\lim_{t \rightarrow \infty} (x_i(t) - x_j(t)) = \delta_{ij}^{des}$. In addition, we can also get that $\lim_{t \rightarrow \infty} (v_i(t) - v_j(t)) = \lim_{t \rightarrow \infty} (\bar{v}_i(t) - \bar{v}_j(t)) = 0, \forall i, j \in \mathcal{V}$ with exponential convergence. ■

In fact, the results of Theorem 3.1 can easily be extended to the second-order MASs with input disturbance

$$\dot{x}_i = v_i, \dot{v}_i = u_i + d_i, \quad (18)$$

with $d_i \in \mathbb{R}^n$ being a bounded disturbance and satisfying $\|d_i\| \leq d_m$, where d_m is an unknown constant. Under the proposed PPC-based control algorithm (11), the following results can be obtained.

Theorem 3.2 Consider the second-order MASs (18). Under the same conditions of Theorem 3.1, the PPC-based formation control algorithm (11) guarantees (e_k, s_k) always evolving the performance constraints (3) and (9). Moreover, $\lim_{t \rightarrow \infty} |x_i^\mu(t) - x_j^\mu(t) - \delta_{ij}^\mu| \leq \rho_{ij,\infty}^\mu$ and $\lim_{t \rightarrow \infty} |v_i^\mu(t) - v_j^\mu(t)| \leq w_{ij,\infty}^\mu + \lambda_s \rho_{ij,\infty}^\mu$, where $\rho_{ij,\infty}^\mu, w_{ij,\infty}^\mu$ can be specified prior.

Proof: The proof of Theorem 3.2 includes three steps. Step 1 is almost same with that of Theorem 3.1 and is omitted here. In Step 2, we consider the Lyapunov function $W_e(\bar{x}, \bar{v}, \varepsilon_e) = \frac{1}{2}(\bar{v} + \lambda_s \bar{x})^T (\bar{v} + \lambda_s \bar{x}) + \frac{k_1}{2} \varepsilon_e^T \varepsilon_e$ and its derivative along system (18) is

$$\begin{aligned} \dot{W}_e &= (\bar{v} + \lambda_s \bar{x})^T (\dot{\bar{v}} + \lambda_s \dot{\bar{x}}) + k_1 \varepsilon_e^T J_z(z, t) (\dot{e} + \Xi e) \\ &\leq -\gamma_1 s^T s - k_1 \varepsilon_e^T J_z(z, t) (\lambda_s I_{Nm} - \Xi) e \\ &\quad - k_2 s^T J_y(y, t) \zeta_s + (\bar{v} + \lambda_s \bar{x})^T (\bar{v} + d), \end{aligned}$$

where d is the stack vector of d_i . As shown in Step 1 of Theorem 3.1, we know that $e_k(t), s_k(t), k \in \mathcal{E}_0$ are bounded for all $t \in [0, \tau_{\max})$. Noting that $\sum_{k=1}^m e_k^T e_k = \bar{x}^T (L \otimes I_n) \bar{x} \geq \lambda_2(L) \bar{x}^T \bar{x}$ and $\sum_{k=1}^m s_k^T s_k = s^T s = (\bar{x} + \lambda_s \bar{v})^T (L \otimes I_n) (\bar{x} + \lambda_s \bar{v}) \geq \lambda_2(L) (\bar{x} + \lambda_s \bar{v})^T (\bar{x} + \lambda_s \bar{v})$, we can get that $\bar{x}(t)$ and $\bar{v}(t)$ are also bounded for $\forall t \in [0, \tau_{\max})$. In addition, since $d^T d$ is also bounded, it follows that $\|(\bar{v} + \lambda_s \bar{x})^T (\bar{v} + d)\| \leq \|\bar{v} + \lambda_s \bar{x}\| \|\bar{v} + d\| \leq \Delta_m$ for $\forall t \in [0, \tau_{\max})$, where Δ_m is a positive constant. It then follows that

$$\begin{aligned} \dot{W}_e &\leq -\gamma_1 \lambda_2(L) (\bar{x} + \lambda_s \bar{v})^T (\bar{x} + \lambda_s \bar{v}) - k_1 l_2 \varepsilon_e^T \varepsilon_e + \Delta_m \\ &\leq -\delta_m W_e + \Delta_m, \end{aligned} \quad (19)$$

where $\delta_m = \min(2\gamma_1 \lambda_2(L), 2l_2) > 0$. Similar to the proof of Step 2 in Theorem 3.1, we have that $W_e(t)$ is bounded for $t \in [0, \tau_{\max})$, which implies that $e_k(t)$ always satisfies the performance bound constraint (3).

In addition, for the case of the combined error $s_k(t)$, consider the Lyapunov function $V_s(\bar{x}, \bar{v}, \zeta_s)$ given in Theorem 3.1 and its derivative is

$$\dot{V}_s \leq -\delta_s V_s + \tilde{\Delta}_s, \quad (20)$$

where $\delta_s = \min(2l_s, 2\gamma_1 \lambda_2(L)) > 0$ and $\tilde{\Delta}_s$ is bounded. Then, we also can obtain that $s_k(t)$ always remains in the performance bound (9). From (3) and (9), it follows that $\lim_{t \rightarrow \infty} |e_k^\mu(t)| = |x_i^\mu(t) - x_j^\mu(t) - \delta_{ij}^\mu| < \rho_{ij, \infty}^\mu$ and $\lim_{t \rightarrow \infty} |s_{ij}^\mu(t)| < w_{ij, \infty}^\mu$. Since $s_{ij} = v_{ij} + \lambda_s e_{ij}$, we can get that $\lim_{t \rightarrow \infty} |v_{ij}^\mu(t) - v_j^\mu(t)| = |v_{ij}^\mu(t)| = |s_{ij}^\mu(t) - \lambda_s e_{ij}(t)| < w_{ij, \infty}^\mu + \lambda_s \rho_{ij, \infty}^\mu$. ■

Remark 3.1 *As shown in Theorem 3.2, only the condition $\lambda_s > \pi_{ij}, \forall (i, j) \in \mathcal{E}_0$ needs to be satisfied and the conditions $\gamma_1 > \lambda_s / \lambda_2(L)$ as given in Theorem 3.1 that is involved with the global information are removed. Thus, the proposed PPC-based formation control algorithm (11) is easily implemented in practical applications. Since the unknown disturbance is considered, the zero-error formation control objective cannot be guaranteed while the final bounds of the relative errors can be specified prior to satisfy the practical control requirement.*

Remark 3.2 *The PPC-based formation algorithm (11) is partly motivated by the PPC-based average consensus results in Macellari, Karayiannidis, Dimarogonas (2016). Compared with the results therein, the main*

differences are two-fold. (i) The communication graph in our paper is a dynamic and time-varying network, in which some communication edges are allowed to be broken, whereas, the communication graph in Macellari, Karayiannidis, Dimarogonas (2016) is a static network. (ii) The proposed PPC-based formation algorithm can solve the connectivity preservation problem for multi-dimensional space thus extending Macellari, Karayiannidis, Dimarogonas (2016) which is only applied to a one-dimensional space. (iii) The proposed distributed algorithm (11) only involves the relative position and velocity information.

Note that the proposed formation algorithm (11) is displacement-based rather than distance-based as in Verginis, Nikou, Dimarogonas (2019) and Mehdifar, Bechlioulis, Hashemzadeh, Baradarannia (2020). This is a crucial reason why the proposed algorithm can be applied to the case of a general communication graph. For the cons, the proposed formation algorithm rely on a global or common coordinated system but the results Verginis, Nikou, Dimarogonas (2019) and Mehdifar, Bechlioulis, Hashemzadeh, Baradarannia (2020) do not. In addition, since the collision constraints can not be transformed effectively to the relative position constraints, the proposed PPC-based algorithm (11) cannot solve the collision avoidance problem for a general graph case. Motivated by the results in Ames, Xu, Grizzle, Tabuada (2016), the zeroing control barrier function (ZCBF) method can be used to satisfy the safety requirements, i.e., achieving collision avoidance. In what follows, we introduce the ZCBF method and a novel formation algorithm is proposed by combining PPC and ZCBF.

3.2 Collision avoidance

In this section, we employ the zeroing control barrier functions (ZCBFs) method to guarantee the collision-free formation maneuver and some definitions about ZCBFs can be seen in Ames, Xu, Grizzle, Tabuada (2016). Define the variable $q_i = [x_i^T, v_i^T]^T$ and $q \in \mathbb{R}^{2nN}$ is the stack vector of q_i , and we then introduce a ZCBF candidate $H_{0,ij}(q_i, q_j) = \|x_i - x_j\|^2 - D_c^2$. According to Definition 2.1, all the agents achieve collision-free motion if the following set

$$C_0 = \{q \in \mathbb{R}^{2nN} | H_{0,ij}(q_i, q_j) \geq 0, \forall i, j \in \mathcal{V}\}, \quad (21)$$

is forward invariant (The set C_0 is forward invariant, i.e., if $x(0) \in C_0$, then $x(t) \in C_0, \forall t \geq 0$).

Consider that the collision avoidance behavior for each agent occurs only when the distance between two agents is within a specified distance (less than the sensing or communication distance). Thus, beside the communication graph \mathcal{G} , we define another graph $\mathcal{G}_c = (\mathcal{V}, \mathcal{E}_c), \mathcal{E}_c = \{(i, j) \in \mathcal{V} | \|x_i - x_j\| \leq D_s, \forall i, j \in \mathcal{V}\}$, where $D_s > D_c$ is the distance within which the collision avoidance should be considered. Based on the graph \mathcal{G}_c ,

a neighbor set of agent $i \in \mathcal{V}$ for collision avoidance behavior is defined as $\mathcal{N}_{i,c} = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}_c\}$.

For system (1), we know that $H_{0,ij}(q_i, q_j)$ has a relative degree two and therefore the general ZCBF method in Ames, Xu, Grizzle, Tabuada (2016) cannot be employed. Motivated by the exponential CBF method given in Nguyen, Sreenath (2016), we define the constraint set

$$C_1 = \{q \in \mathbb{R}^{2nN} \mid H_{1,ij}(q_i, q_j) \geq 0, \forall i, j \in \mathcal{V}\}, \quad (22)$$

$$H_{1,ij}(q_i, q_j) = \dot{H}_{0,ij}(q_i, q_j) + c_1 H_{0,ij}(q_i, q_j),$$

for $c_1 > 0$ and one has that $H_{1,ij}(q_i, q_j)$ has a relative degree of 1.

Theorem 3.3 *Consider the second-order MASs in (1). Suppose that the initial state vector q_0 belongs to $C_0 \cap C_1$. Then, any Lipschitz continuous controller u_i satisfying the following inequality*

$$(x_j - x_i)^T (u_i - u_j) \leq \|v_i - v_j\|^2 \quad (23)$$

$$+ c_3 (x_i - x_j)^T (v_i - v_j) + \frac{1}{2} c_1 c_2 H_{0,ij}, j \in \mathcal{N}_{i,c},$$

with $c_3 = c_1 + c_2$ and $c_2 > 0$ guarantees that C_0 is forward invariant, i.e., achieving the collision-free formation maneuver.

Proof: The proof for Theorem 3.3 includes two steps.

Step 1: We will show that C_1 is forward invariant. Based on Proposition 1 given in Ames, Xu, Grizzle, Tabuada (2016), we have that C_1 is forward invariant if the following ZCBF condition

$$\dot{H}_{1,ij}(q_i, q_j) + c_2 H_{1,ij}(q_i, q_j) \geq 0 \quad (24)$$

is satisfied for $\forall i, j \in \mathcal{V}$. Based on (21) and (22), the ZCBF condition (24) is transformed as

$$2(x_j - x_i)^T (u_i - u_j) \leq 2\|v_i - v_j\|^2 \quad (25)$$

$$+ 2c_3 (x_i - x_j)^T (v_i - v_j) + c_1 c_2 H_{0,ij}, \forall j \in \mathcal{N}_{i,c},$$

Here, we only consider the neighbors $j \in \mathcal{N}_{i,c}$ since $H_{0,ij}(q_i, q_j) \geq 0$ for any agent $j \in \mathcal{V} \setminus \mathcal{N}_{i,c}$ with the fact $D_s > D_c$. Thus, we conclude that the set C_1 is forward invariant if the inequality (23) is satisfied.

Step 2: We next show that C_0 is forward invariant. Based on the results in Step 1, it follows that $H_{1,ij}(q_i(t), q_j(t)) \geq 0$ holds for $t \geq 0$. This implies that $\dot{H}_{0,ij}(q_i(t), q_j(t)) + c_1 H_{0,ij}(q_i(t), q_j(t)) \geq 0$. Based on the comparison Lemma (Khalil, 2002), it follows that $H_{0,ij}(q_i(t), q_j(t)) \geq H_{0,ij}(q_i(0), q_j(0))e^{-c_1 t}$. Since $q(0) \in C_0$, we have that $H_{0,ij}(q_i(0), q_j(0)) \geq 0$, and one can further derive that $H_{0,ij}(q_i(t), q_j(t)) \geq 0$ for any $t \geq 0$. It is shown that C_0 is forward invariant. Thus, we conclude that the collision avoidance can be achieved if the inequality condition (23) can be satisfied. ■

Note that the collision avoidance conditions (23) is not fully distributed since the neighbors' control input $u_j, j \in \mathcal{N}_i$ that consists of the agent j 's neighbors' state information is involved. This implies that two-hop neighbors' state information is required to establish the collision avoidance condition. This will result in more complex communication among the agents. To solve this problem, we replace the inequality (23) with the following collision avoidance conditions

$$2(x_j - x_i)^T u_i \leq \|v_i - v_j\|^2 \quad (26)$$

$$+ c_3 (x_i - x_j)^T (v_i - v_j) + \frac{1}{2} c_1 c_2 H_{0,ij}, j \in \mathcal{N}_{i,c},$$

Note that (26) is fully distributed with only involving one-hop neighbors' state information. However, (26) is relatively conservative with respect to (23) since the former implies the latter one but the converse is not true.

3.3 Collision-free formation control

In this section, we combine the above proposed PPC-based algorithm (11) and the collision-free condition (23) such that the connectivity preservation, prescribed performance and collision avoidance are achieved simultaneously. The main idea is to retain the PPC-based algorithm (11) if the collision-free condition is satisfied and modify the PPC controller (11) as little as possible otherwise. It can be formulated as the following optimization problem

$$u_i^* = \arg \min_{u_i \in \mathbb{R}^n} \|u_i - u_{\text{nom},i}\|^2, \text{ s.t. (26)}, \quad (27)$$

where $u_{\text{nom},i}$ denotes the nominal control input given in (11). The solution of optimization problem (27) is to guarantee that each agent achieves connectivity preservation and prescribed performance formation control as much as possible in the sense of minimizing $\|u_i - u_{\text{nom},i}\|^2$ while guarantees the collision-free formation motion by satisfying (26). In addition, note that the constraint condition (26) is linear with respect to u_i and then the optimization problem (27) can be easily solved by Quadratic Programs (QP) method. Moreover, the optimization problem (27) can be solved in a distributed manner since the nominal controller $u_{\text{nom},i}$ and the condition (26) only are involved with local information from its neighbors.

The feasibility analysis of QP problem: As formulated in QP problem (27), we prioritize the satisfaction of the collision avoidance (26) regarded as a *hard* constraint. If no collision avoidance occurs, the actual control input u_i equals to the nominal control input $u_{\text{nom},i}$, which guarantees the connectivity and prescribed performance. On the other hand, if collision avoidance occurs, the connectivity performance constraint (3) is not violated since the communication range is generally much larger than the collision distance. Thus, the nominal controller $u_{\text{nom},i}$ is bounded and the QP

problem (32) is still valid. To prioritize the satisfaction of the collision avoidance, we could temporarily relax the connectivity performance (determined by the nominal controller $u_{\text{nom},i}$) by solving the problem (32). In fact, the actual control input u_i that is obtained by solving the QP problem (32) can mimic the nominal controller $u_{\text{nom},i}$ completely when no collision avoidance occurs, and only modify the nominal controller $u_{\text{nom},i}$ as little as possible when the collision avoidance constraint is invoked. This implies that the solution of (32) guarantees that each agent achieves connectivity preservation as much as possible while guaranteeing collision-free formation motion. Finally, we admit that the feasibility analysis above is not a formal proof but can be still regarded as a heuristic one that usually works in practice. This is a shortcoming of our work, and we will consider it in the future work.

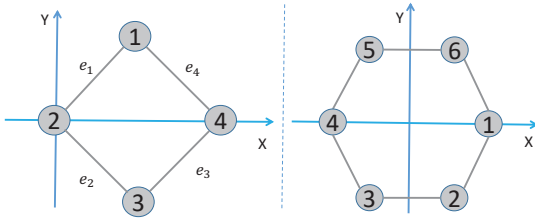


Fig. 1. The initial communication graph \mathcal{G}_0

4 Simulation results

In this section, numerical simulation results are given to verify the effectiveness of the theoretical results. We first consider a group of $N = 4$ agents moving on a planar space, which is shown in the left plot of Fig. 1. The desired formation configuration \mathcal{F}_t is a square with the relative position offsets $\delta_{12}^{des} = [0, 5]^T$, $\delta_{23}^{des} = [-5, 0]^T$, $\delta_{34}^{des} = [0, -5]^T$, $\delta_{41}^{des} = [5, 0]^T$. The initial positions and velocities of all the agents are $x_1(0) = [4, 4]^T$ m, $v_1(0) = [1, -1]^T$ m/s, $x_2(0) = [0, 0]^T$ m, $v_2(0) = [-1, 1]^T$ m/s, $x_3(0) = [4, -4]^T$ m, $v_3(0) = [2, 0]^T$ m/s, $x_4(0) = [8, 0]^T$ m, $v_4(0) = [0, -2]^T$ m/s. The communication range of each agent is $R_s = 6$ m. From the initial positions of all the agents, we get that the initial graph \mathcal{G}_0 shown in Fig. 1 is connected and is congruent with the desired formation configuration \mathcal{F}_t . There are four edges in \mathcal{G}_0 and the relative position errors associated with these edges are $e_1 = x_1 - x_2 - \delta_{12}^{des}$, $e_2 = x_2 - x_3 - \delta_{23}^{des}$, $e_3 = x_3 - x_4 - \delta_{34}^{des}$, $e_4 = x_4 - x_1 - \delta_{41}^{des}$. Based on the parameters R_s and δ_{ij}^{des} , we can obtain the values of the parameters $\underline{M}_{ij}^\mu = R_s - \delta_{ij,m}^{des}$, $\overline{M}_{ij}^\mu = R_s + \delta_{ij,m}^{des}$, $(i, j) \in \mathcal{E}_0$. The parameters of performance function ρ_{ij}^μ is set to be $\pi_{ij} = 0.12$, $\rho_\infty = 0.01$. In addition, the collision distance D_c is set to be $D_c = 3$, and then we get that the collision does not occur in the initial time.

The parameters of the PPC-based formation controller (11) are chosen as $\lambda_s = 0.5$, $\gamma_1 = 0.3$, $k_1 = 0.5$, $k_2 = 0.5$ such that $\lambda > \pi_{ij}$ is satisfied. By solving the QP problem

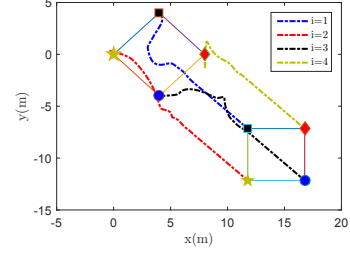


Fig. 2. The motion trajectories of the whole formation

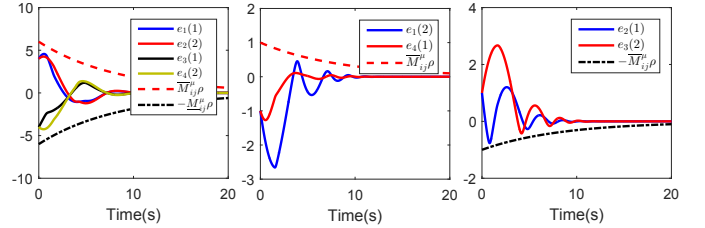


Fig. 3. Relative position error e_k , $k \in \mathcal{E}_0$ and their performance bounds

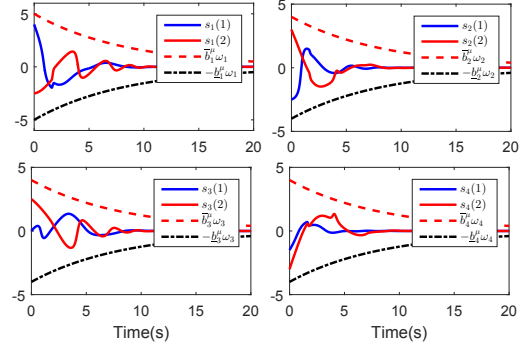


Fig. 4. The combined error s_k , $k \in \mathcal{E}_0$ and their performance bounds

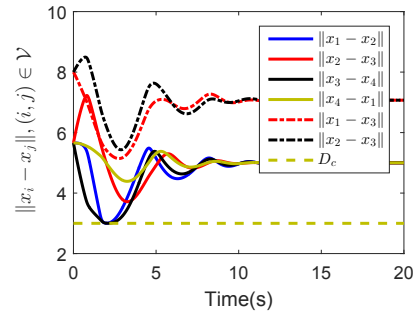


Fig. 5. The distances $\|x_i - x_j\|$, $i, j \in \mathcal{V}$

via Matlab tools, the actual control input u_i^* for each agent i is obtained.

Fig. 2 depicts that the trajectories of the whole formation motion on the planar space and the snapshots of the formation configuration at 0s, 20s are depicted. It is shown that the desired square formation pattern is achieved with a fast convergence speed. Figs. 3-4 describe the relative position error e_k , the combined

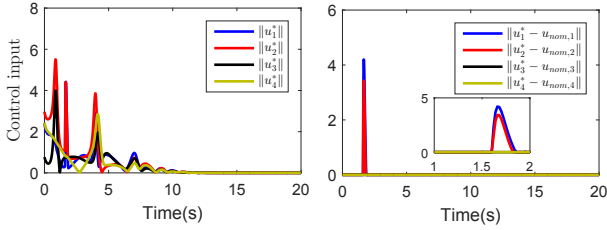


Fig. 6. The control input u_i^* and the difference $\|u_i^* - u_{nom,i}\|$

error s_k , $k = 1, 2, 3, 4$ and their performance bounds. As shown in Figs. 3-4, the relative position error e_k and the combined error s_k always evolve in the prescribed performance regions and eventually can converge to zero. The distances $\|x_i - x_j\|, i, j \in \mathcal{V}$ of all the agents are shown in Fig. 5. Based on the chosen collision distance $D_c = 3$, we observe from Fig. 5 that the distance between any two agents is not less than D_c . Fig. 6 describes the control inputs u_i^* obtained from QP and the differences with the nominal controller $u_{nom,i}$ formulated by (11). It can be seen from Fig. 6 that the actual control input u_i^* is equal to $u_{nom,i}$ except a small time interval where the collision constraint (23) is not satisfied.

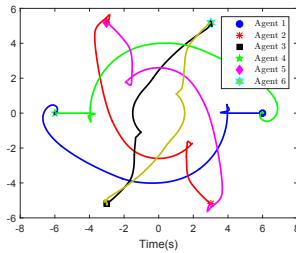


Fig. 7. The simulation results of swapping position task

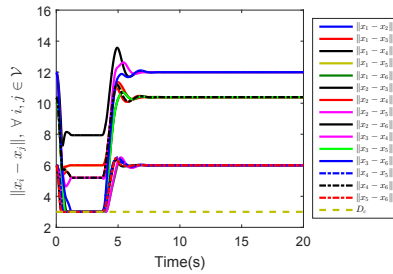


Fig. 8. The distance $\|x_i - x_j\|, i, j \in \mathcal{V}$

To further verify the effectiveness of the proposed algorithm, we consider a group of 6 agents consisting a regular hexagon shown in the right plot of Fig. 1. The initial positions of all the agents are set to be $x_1(0) = [6, 0]^T, x_2(0) = [3, -3\sqrt{3}]^T, x_3(0) = [-3, -3\sqrt{3}]^T, x_4(0) = [-6, 0]^T, x_5(0) = [-3, 3\sqrt{3}]^T, x_6(0) = [3, 3\sqrt{3}]^T$, and their velocities are zero. The formation task is to make all the agents swap their positions with the agents on the opposite angle in this regular hexagon. The communication range R_s and collision distance D_c are set to be $R_s = 8, D_c = 3$, and

the other control gains are the same as the above four-agents formation scene. Fig. 7 shows the trajectories of the whole formation motion, and Fig. 8 describes the distance $\|x_i - x_j\|, i, j \in \mathcal{V}$ of all the agents. From Figs. 7-8, we can observe that all the agents achieve their positions swapping on the opposite angle of the regular hexagon without collision.

5 Conclusion

This paper proposes a novel distributed control algorithm by combining PPC and EZCBFs for second-order MASs such that the collision-free formation control with prescribed performance and connectivity preservation is achieved. Firstly, a PPC-based distributed algorithm is proposed to achieve connectivity preservation and prescribed performance formation control. Subsequently, a control input condition for collision avoidance is derived by using the EZCBFs. Regarding the PPC-based control algorithm as a nominal one, each agent can obtain the actual control input by solving QP problem in a distributed manner to achieve connectivity preservation and collision-free formation control.

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