Funnel Control for Fully Actuated Systems under a Fragment of Signal Temporal Logic Specifications ⋆

Lars Lindemann[a,*], Dimos V. Dimarogonas[a]

[a] *Division of Decision and Control Systems, School of Electrical Engineering and Computer Science, KTH Royal Institute of Technology, Malvinas väg 10, SE-10044, Stockholm, Sweden*

Abstract

Temporal logics have lately proven to be a valuable tool for various control applications by providing a rich specification language. Existing temporal logic-based control strategies discretize the underlying dynamical system in space and/or time. We will not use such an abstraction and consider continuous-time systems under a fragment of signal temporal logic specifications by using the associated robust semantics. In particular, this paper provides computationally-efficient funnel-based feedback control laws for a class of systems that are, in a sense, feedback equivalent to single integrator systems, but where the dynamics are partially unknown for the control design so that some degree of robustness is obtained. We first leverage the transient properties of a funnel-based feedback control strategy to maximize the robust semantics of some atomic temporal logic formulas. We then guarantee the satisfaction for specifications consisting of conjunctions of such atomic temporal logic formulas with overlapping time intervals by a suitable switched control system. The result is a framework that satisfies temporal logic specifications with a user-defined robustness when the specification is satisfiable. When the specification is not satisfiable, a least violating solution can be found. The theoretical findings are demonstrated in simulations of the nonlinear Lotka-Volterra equations for predator-prey models.

Keywords: Robust control; formal methods; signal temporal logic; autonomous systems; switched systems.

1. Introduction

Temporal logics allow to express temporal properties in a logical framework that may represent specifications imposed on a dynamical system. For-

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*Corresponding author

Email addresses: llindem@kth.se (Lars Lindemann), dimos@kth.se (Dimos V. Dimarogonas)

mal verification techniques, such as model checking [1], can then be used to check whether or not the system satisfies these specifications. Motivated by the trade-off between complexity and expressivity, linear temporal logic (LTL) has widely been used in formal verification. More expressive temporal logics are metric temporal logic (MTL) and signal temporal logic (STL) that allow to impose quantitative time constraints, as opposed to qualitative time constraints in LTL. Formal methods-based control has emerged due to the need of more complex system specifications in areas such as robotics and intelligent transportation systems. In contrast to formal verification, the task is to design the control input so that the dynamical system satisfies the given specifications. Recent advances in this area, with a focus on LTL, have been reported in [2] and [3]. Formal methods-based control strategies for multi-agent systems have also appeared in [4] and [5]. The multi-agent case is special in the sense that global and/or local, i.e., individual, specifications may be assigned to the agents, potentially requiring collaboration and communication among them. At the same time, additional agent couplings such as connectivity maintenance and/or collision avoidance need to be taken care of. All previously mentioned approaches rely on automata-based formal verification techniques. This implies that the temporal logic specification is translated into a language equivalent Büchi automaton, while the dynamical system is abstracted into a discrete transition system that needs to satisfy some equivalence properties. If the abstracted system is non-deterministic, e.g., modelled as a Markov decision process, then a Rabin automaton instead of a Büchi automaton is needed. A search algorithm is then run on their product automaton to find a specification-satisfying discrete path. These automata-based approaches, however, may be subject to the state-space explosion problem. The works in [6] and [7] are based on game theoretical results and consider a special fragment of LTL, called the generalized reactivity(1) fragment, which explicitly accounts for adversarial and dynamical environments, and hence establish reactive control synthesis frameworks. This fragment allows for computationally more efficient control synthesis methods similarly to [8] and [9] where sampling-based methods are introduced. Robustness of temporal logics has been introduced in [10] where MTL specifications have been interpreted over continuous-time signals. In particular, the robustness degree measures the distance of a signal to the set of signals where the boolean evaluation of the MTL specification changes. The authors in [10] also introduce robust semantics, which are easier to compute than the robustness degree and which are an under-approximation of the robustness degree. STL is a predicate logic interpreted over continuous-time signals [11] entailing space robustness [12], a special form of the robust semantics. Formal methods-based control of dynamical systems under STL specifications is a difficult task due to the nonlinear, nonconvex, noncausal, and nonsmooth semantics. This has been considered for discrete-time systems by means of model predictive control in [13] where space robustness is incorporated into a mixed integer linear program. Robust extensions of this approach have been reported in [14–16], while the extension to multi-agent systems with a special focus on communication is discussed in [17]. Optimization-based approaches to this problem are summarized in [18]. Reinforcement learning-based control strategies have been derived in [19]. Our recent work in [20] neither involves learning nor optimization and embeds a funnel-based feedback control law into a hybrid system framework to satisfy STL specifications. Despite losing optimality guarantees, our framework applies directly to continuous-time systems and is computationally tractable and robust. An extension to coupled multi-agent systems under local and possibly conflicting STL specifications can be found in [21]. A learning-based extension of these funnel-based feedback control laws to account for optimality can be found in [22]. Prescribed performance control (PPC) [23, 24] is a funnel-based feedback control strategy that explicitly takes the transient and steady-state behavior of a tracking error into account. A user-defined performance function prescribes a desired temporal behavior to this error that is subsequently achieved by a continuous feedback control law. In other words, the performance function defines a funnel and the task of the continuous feedback control law is to keep the error within this funnel. In this work, we leverage this funnel and replace, in a suitable way, the tracking error by the robust semantics of the STL specification at hand.

We consider nonlinear continuous-time systems that are, in a sense, feedback equivalent to single integrator systems with, however, partially unknown dynamics. For a fragment of STL specifications, we cast the control problem into a PPC control problem. We first leverage the transient properties of the performance function to derive a continuous feedback control law that maximizes the robust semantics of some *atomic temporal logic formulas*. Subsequently, a switched control system is presented to satisfy specifications consisting of conjunctions of such atomic temporal logic formulas with overlapping time intervals. For the case of an unsatisfiable specification, a least violating solution can directly be found due to the use of the robust semantics. To the best of the authors' knowledge, the approach presented in this paper is the first approach deriving a continuous-time feedback control law for temporal logic specifications, while discretizing neither the system dynamics nor the environment in space or time and without resorting to automata representations of the temporal logic specification. The advantages of our approach are low computational complexity and inherent robustness properties of the feedback control laws. Compared with [13] and its robust extensions, the approach presented in this paper considers continuous-time systems and also admits nonlinear predicates. This paper extends [20] by allowing specifications with overlapping time intervals, providing all technical proofs, and presenting an extension to the case when specifications are not satisfiable.

Section 2 states preliminaries, while Section 3 illustrates the underlying main idea. Section 4 proposes a feedback control law that satisfies atomic temporal logic formulas, while Section 5 proposes a switched control system for a set of atomic temporal logic formulas. Section 6 presents simulations of a predator-prey system using the Lotka-Volterra equations, followed by conclusions in Section 7.

2. Notation and Preliminaries

True and false are denoted by $\top$ and $\bot$ with $\mathbb{B} := \{\top, \bot\}$; $\mathbb{R}^n$ is the n-dimensional vector space over the real numbers $\mathbb{R}$ and $\mathbf{0}_n \in \mathbb{R}^n$ consists of n zeros. The non-negative and positive real numbers are $\mathbb{R}_{\geq 0}$ and $\mathbb{R}_{>0}$, respectively. Let $\|\boldsymbol{x}\|$ denote the Euclidean norm of $\boldsymbol{x} \in \mathbb{R}^n$. All technical proofs of Lemmas and Theorems, derived in this paper, are provided in the Appendix.

At time t, let $\boldsymbol{x}(t) \in \mathbb{R}^n$, $\boldsymbol{u}(t) \in \mathbb{R}^m$, and $\boldsymbol{w}(t) \in \mathcal{W} \subset \mathbb{R}^n$ be the state, input, and additive noise of the system

$$\dot{\boldsymbol{x}}(t) = f(\boldsymbol{x}(t)) + g(\boldsymbol{x}(t))\boldsymbol{u}(t) + \boldsymbol{w}(t),\ \boldsymbol{x}(0) := \boldsymbol{x}_0 \tag{1}$$

where $\mathcal{W}$ is a bounded set, $\boldsymbol{x}_0$ is the initial state, and $f(\boldsymbol{x})$ is unknown apart from a regularity assumption. Similarly and in the case when $m = n$, $g(\boldsymbol{x})$ is unknown apart from a regularity and a controllability assumption.

Assumption 1. *The functions $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ are locally Lipschitz continuous and $g(\boldsymbol{x})g(\boldsymbol{x})^T$ is positive definite for all $\boldsymbol{x} \in \mathbb{R}^n$. The function $\boldsymbol{w} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ is piecewise continuous.*

Remark 1. *We emphasize that $f(\boldsymbol{x})$ is unknown so that* (1) *is* not *feedback equivalent to $\dot{\boldsymbol{x}}(t) = \boldsymbol{u}(t) + \boldsymbol{w}(t)$. Knowledge of the function $g(\boldsymbol{x})$ is, however, assumed if $m > n$. If $m = n$ and $g(\boldsymbol{x})$ is positive definite for all $\boldsymbol{x} \in \mathbb{R}^n$, then it is possible to derive a model-free approach, i.e., no exact knowledge of $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$ are needed. Furthermore, we remark that $g(\boldsymbol{x})g(\boldsymbol{x})^T$ is positive definite if and only if $g(\boldsymbol{x})$ has full row rank, which implies that $m \geq n$.*

2.1. Signal Temporal Logic (STL)

Signal temporal logic (STL) is based on continuous-time signals. Let hence $\boldsymbol{x} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ be a continuous-time signal that may possibly be the solution to (1). STL consists of predicates μ that are obtained after evaluation of a continuously differentiable predicate function $h : \mathbb{R}^n \to \mathbb{R}$. For $\boldsymbol{\zeta} \in \mathbb{R}^n$, let $\mu := \top$ if $h(\boldsymbol{\zeta}) \geq 0$ and $\mu := \bot$ if $h(\boldsymbol{\zeta}) < 0$. Hence, h maps from $\mathbb{R}^n$ to $\mathbb{R}$, while μ maps from $\mathbb{R}^n$ to $\mathbb{B}$. The STL syntax is given by

$$\phi \ ::= \ \top \mid \mu \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid \phi_1 \, U_{[a,b]} \, \phi_2$$

where ϕ_1, ϕ_2 are STL formulas and $U_{[a,b]}$ encodes the until operator with $a \leq b < \infty$. Define the eventually and always operators as $F_{[a,b]}\phi := \top \, U_{[a,b]} \, \phi$ and $G_{[a,b]}\phi := \neg F_{[a,b]}\neg\phi$. The satisfaction relation $(\boldsymbol{x}, t) \models \phi$ denotes that the signal $\boldsymbol{x} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ satisfies ϕ at time t. The STL semantics [11, Definition 1] are recursively given by $(\boldsymbol{x}, t) \models \mu$ if and only if $h(\boldsymbol{x}(t)) \geq 0$, $(\boldsymbol{x}, t) \models \neg\phi$ if and only if $\neg((\boldsymbol{x}, t) \models \phi)$, $(\boldsymbol{x}, t) \models \phi_1 \wedge \phi_2$ if and only if $(\boldsymbol{x}, t) \models \phi_1 \wedge (\boldsymbol{x}, t) \models \phi_2$, and $(\boldsymbol{x}, t) \models \phi_1 \, U_{[a,b]} \, \phi_2$ if and only if $\exists t_1 \in [t+a, t+b]$ s.t. $(\boldsymbol{x}, t_1) \models \phi_2 \wedge \forall t_2 \in [t, t_1]$,$(\boldsymbol{x}, t_2) \models \phi_1$. A formula ϕ is satisfiable if $\exists \boldsymbol{x} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ such that $(\boldsymbol{x}, 0) \models \phi$. Robust semantics for STL have been presented in [12], referred to as space robustness.

Definition 1 (STL Robust Semantics). *The semantics of space robustness [12, Definition 3] are recursively given by:*

$$
\begin{aligned}
\bar{\rho}^{\mu}(\boldsymbol{x},t) &:= h(\boldsymbol{x}(t))\\
\bar{\rho}^{\neg\phi}(\boldsymbol{x},t) &:= -\bar{\rho}^{\phi}(\boldsymbol{x},t)\\
\bar{\rho}^{\phi_1\wedge\phi_2}(\boldsymbol{x},t) &:= \min\big(\bar{\rho}^{\phi_1}(\boldsymbol{x},t),\bar{\rho}^{\phi_2}(\boldsymbol{x},t)\big)\\
\bar{\rho}^{\phi_1\, U_{[a,b]}\,\phi_2}(\boldsymbol{x},t) &:= \max_{t_1\in[t+a,t+b]} \min\big(\bar{\rho}^{\phi_2}(\boldsymbol{x},t_1), \min_{t_2\in[t,t_1]}\bar{\rho}^{\phi_1}(\boldsymbol{x},t_2)\big),\\
\bar{\rho}^{F_{[a,b]}\phi}(\boldsymbol{x},t) &:= \max_{t_1\in[t+a,t+b]}\bar{\rho}^{\phi}(\boldsymbol{x},t_1)\\
\bar{\rho}^{G_{[a,b]}\phi}(\boldsymbol{x},t) &:= \min_{t_1\in[t+a,t+b]}\bar{\rho}^{\phi}(\boldsymbol{x},t_1).
\end{aligned}
$$

It holds that $(\boldsymbol{x},t)\models\phi$ if $\bar{\rho}^{\phi}(\boldsymbol{x},t)>0$ [10, Proposition 16]. Space robustness determines how robustly a signal $\boldsymbol{x}$ satisfies the formula ϕ at time t, i.e., for a formula ϕ and two signals $\boldsymbol{x}_1,\boldsymbol{x}_2:\mathbb{R}_{\geq 0}\to\mathbb{R}^n$ with $\bar{\rho}^{\phi}(\boldsymbol{x}_1,t)>\bar{\rho}^{\phi}(\boldsymbol{x}_2,t)>0$ it holds that $\boldsymbol{x}_1$ satisfies ϕ more robustly at time t than $\boldsymbol{x}_2$ does.

2.2. Prescribed Performance Control (PPC)

Prescribed performance control (PPC) [23, 24] is a funnel-based feedback control strategy that constrains a generic error $\boldsymbol{e}:\mathbb{R}_{\geq 0}\to\mathbb{R}^n$ to a user-designed, time-varying funnel. For instance, consider $\boldsymbol{e}(t):=\begin{bmatrix}e_1(t) & \dots e_n(t)\end{bmatrix}^T:=\boldsymbol{x}(t)-\boldsymbol{x}_d(t)$, where $\boldsymbol{x}_d$ is a desired trajectory. In order to prescribe transient and steady-state behavior to this error, define the performance function $\gamma:\mathbb{R}_{\geq 0}\to\mathbb{R}_{>0}$, which is a continuously differentiable, bounded, positive, and non-increasing function given by

$$\gamma(t):=(\gamma_0-\gamma_\infty)\exp(-lt)+\gamma_\infty$$

where $\gamma_0,\gamma_\infty\in\mathbb{R}_{>0}$ with $\gamma_0\geq\gamma_\infty$ and $l\in\mathbb{R}_{\geq 0}$. Also define the transformation function $S:(-1,M)\to\mathbb{R}$ with $M\in[0,1]$, which is a smooth and strictly increasing function. In particular, let $S(\xi):=\ln\Big(-\frac{\xi+1}{\xi-M}\Big)$. Now assume that γ_i is a performance function. The task is to synthesize a continuous feedback control law such that

$$-\gamma_i(t)<e_i(t)<M\gamma_i(t)\quad\forall t\in\mathbb{R}_{\geq 0},\forall i\in\{1,\dots,n\}\tag{2}$$

given that $-\gamma_i(0)<e_i(0)<M\gamma_i(0)$ for all $i\in\{1,\dots,n\}$; γ_i is a design parameter by which transient and steady-state behavior of e_i can be prescribed. As with M in the right inequality of (2), another constant could be added to the left inequality. This will, however, not be considered here; (2) is a constrained control problem with n constraints subject to the dynamics in (1). By defining the normalized error $\xi_i(t):=\frac{e_i(t)}{\gamma_i(t)}$, dividing (2) by $\gamma_i(t)$, and applying the transformation function S, an unconstrained control problem $-\infty<S\big(\xi_i(t)\big)<\infty$

is obtained where $\epsilon_i(t) := S\big(\xi_i(t)\big)$ is the transformed error. If now, for each $i \in \{1, \dots, n\}$, $\epsilon_i(t)$ is bounded for all $t \geq \mathbb{R}_{\geq 0}$, then (2) is satisfied.

3. Main Idea and Problem Formulation

A fragment of STL is considered in this paper. Considering the predicate μ, we first define the STL formulas

$$\psi ::= \top \mid \mu \mid \neg\mu \mid \psi_1 \wedge \psi_2 \tag{3a}$$

$$\phi ::= F_{[a,b]}\psi \mid G_{[a,b]}\psi \mid F_{[\underline{a},\underline{b}]}G_{[\bar{a},\bar{b}]}\psi \tag{3b}$$

where ψ in (3b) and ψ_1, ψ_2 in (3a) are formulas of class ψ given in (3a) and where $a, b, \underline{a}, \underline{b}, \bar{a}, \bar{b} \in \mathbb{R}_{\geq 0}$ with $a \leq b$, $\underline{a} \leq \underline{b}$ and $\bar{a} \leq \bar{b}$. Disjunctions are not considered here as they can, in general, not be handled by continuous feedback control laws unless the formula is trivially simplified as illustrated next. Consider the system $\dot{\boldsymbol{x}}(t) = \boldsymbol{u}(t)$ and the formula $\phi := F_{[a,b]}(\mu_1 \vee \mu_2)$ where μ_1 and μ_2 are associated with $h_1(\boldsymbol{x}) := 0.5 - |\boldsymbol{x} + 1|$ and $h_2(\boldsymbol{x}) := 0.5 - |\boldsymbol{x} - 1|$, respectively. For an initial condition of $\boldsymbol{x}(0) := -\epsilon$ (or $\boldsymbol{x}(0) := \epsilon$) for a small $\epsilon > 0$, the control law $\boldsymbol{u}_1(\boldsymbol{x}) := -(\boldsymbol{x} + 1)$ (or $\boldsymbol{u}_2(\boldsymbol{x}) := -(\boldsymbol{x} - 1)$) drives $\boldsymbol{x}(t)$ towards μ_1 (or μ_2). Note in particular that there exists no combination of $\boldsymbol{u}_1(\boldsymbol{x})$ and $\boldsymbol{u}_2(\boldsymbol{x})$ that is continuous at $\boldsymbol{x} = 0$ and can satisfy $\phi := F_{[a,b]}(\mu_1 \vee \mu_2)$ unless one decides to only use either $\boldsymbol{u}_1(\boldsymbol{x})$ or $\boldsymbol{u}_2(\boldsymbol{x})$ which technically reduces ϕ to $F_{[a,b]}\mu_1$ or $F_{[a,b]}\mu_2$, respectively. In this sense, there exists no continuous feedback control law that satisfies $\phi := F_{[a,b]}(\mu_1 \vee \mu_2)$ without trivially simplifying the formula. To avoid such simplifications, higher level decision making, for instance using switched systems, resulting in discontinuous control laws can be used (see [25, Chapter 4] for similar arguments).

We refer to ψ given in (3a) as *non-temporal formulas*, i.e., boolean formulas, and to ϕ given in (3b) as *atomic temporal formulas* due to the use of eventually and always operators that makes t appear explicitly in $\bar{\rho}^{\phi}(\boldsymbol{x}, t)$. Let $\mathcal{T}$ be the set of all possible signals with time domain $\mathbb{R}_{\geq 0}$, and note that $\bar{\rho}^{\psi}(\boldsymbol{x}, t)$ and $\bar{\rho}^{\phi}(\boldsymbol{x}, t)$ map from $\mathcal{T} \times \mathbb{R}_{\geq 0}$ to $\mathbb{R}$. For non-temporal formulas ψ, however, we can equivalently use $\bar{\rho}^{\psi}(\boldsymbol{x}(t))$ by a slight change of notation where now $\bar{\rho}^{\psi} : \mathbb{R}^n \to \mathbb{R}$ and with $\bar{\rho}^{\psi}(\boldsymbol{x}(t)) = \bar{\rho}^{\psi}(\boldsymbol{x}, t)$. The notation of $\bar{\rho}^{\psi}(\boldsymbol{x}(t))$ is introduced to highlight that t is only contained in $\bar{\rho}^{\psi}$ through the composition of $\bar{\rho}^{\psi}$ with $\boldsymbol{x}$, which becomes important later. In Section 4 we derive a control law $\boldsymbol{u}(\boldsymbol{x}, t)$ to satisfy ϕ as in (3b). In Section 5, we then introduce a switched control system to deal with specifications consisting of a conjunction of formulas as in (3b) with possibly overlapping time intervals. We first derive a control strategy for formulas of the form $\psi' \, U_{[a,b]} \, \psi''$ where ψ' and ψ'' are as in (3a) and then consider, for each $k \in \{1, \dots, K\}$,

$$\phi_k ::= F_{[a_k,b_k]}\psi'_k \mid G_{[a_k,b_k]}\psi'_k \mid F_{[\underline{a}_k,\underline{b}_k]}G_{[\bar{a}_k,\bar{b}_k]}\psi'_k \mid \psi'_k \, U_{[a_k,b_k]} \, \psi''_k$$

with ψ'_k and ψ''_k being formulas of class ψ given in (3a). Finally, the STL fragment under consideration is

$$\theta ::= \bigwedge_{k=1}^{K} \phi_k. \tag{4}$$

Formulas of class θ given in (4) are referred to as *temporal formulas*, including atomic temporal formulas ϕ formulas in (3b) as a subcase.

To derive, in Section 4, a continuous-time feedback control law $\boldsymbol{u}(\boldsymbol{x}, t)$ for atomic temporal formulas ϕ, the non-smooth robust semantics of $\psi_1 \wedge \psi_2$ are replaced by a smooth approximation similarly to [19] and [26]. The new robust semantics are denoted by $\rho^{\phi}(\boldsymbol{x}, t)$ instead of $\bar{\rho}^{\phi}(\boldsymbol{x}, t)$. We emphasize that a smooth approximation is only needed and used for conjunctions of non-temporal formulas ψ. For the set of formulas given in (3), we explicitly define

$$\begin{aligned}
\rho^{\mu}(\boldsymbol{x}(t)) &:= h(\boldsymbol{x}(t)) \\
\rho^{\neg\mu}(\boldsymbol{x}(t)) &:= -\rho^{\mu}(\boldsymbol{x}(t)) \\
\rho^{\psi_1 \wedge \psi_2}(\boldsymbol{x}(t)) &:= -\frac{1}{\eta} \ln\Big(\exp\big(-\eta \rho^{\psi_1}(\boldsymbol{x}(t))\big) + \exp\big(-\eta \rho^{\psi_2}(\boldsymbol{x}(t))\big)\Big) \\
\rho^{F_{[a,b]}\psi}(\boldsymbol{x}, t) &:= \max_{t_1 \in [t+a, t+b]} \rho^{\psi}(\boldsymbol{x}(t_1)) \\
\rho^{G_{[a,b]}\psi}(\boldsymbol{x}, t) &:= \min_{t_1 \in [t+a, t+b]} \rho^{\psi}(\boldsymbol{x}(t_1)) \\
\rho^{F_{[\underline{a},\underline{b}]} G_{[\bar{a},\bar{b}]}\psi}(\boldsymbol{x}, t) &:= \max_{t_1 \in [t+\underline{a}, t+\underline{b}]} \min_{t_2 \in [t_1+\bar{a}, t_1+\bar{b}]} \rho^{\psi}(\boldsymbol{x}(t_2)),
\end{aligned}$$

where $\eta > 0$ determines the accuracy by which $\rho^{\psi_1 \wedge \psi_2}(\boldsymbol{x}(t))$ approximates $\bar{\rho}^{\psi_1 \wedge \psi_2}(\boldsymbol{x}(t))$. In fact, it can be proven that $\rho^{\psi_1 \wedge \psi_2}(\boldsymbol{x}(t)) = \bar{\rho}^{\psi_1 \wedge \psi_2}(\boldsymbol{x}(t))$ as $\eta \to \infty$. Regardless of the value of η, it holds that $\rho^{\psi_1 \wedge \psi_2}(\boldsymbol{x}(t)) \leq \bar{\rho}^{\psi_1 \wedge \psi_2}(\boldsymbol{x}(t))$.

Lemma 1. *Consider a conjunction of* q non-temporal formulas ψ_i *as* $\psi := \wedge_{i=1}^{q} \psi_i$ *where each* ψ_i *does not contain any further conjunctions itself. Then, it holds that*

$$\rho^{\psi}(\boldsymbol{x}(t)) \leq \bar{\rho}^{\psi}(\boldsymbol{x}(t)) \leq \rho^{\psi}(\boldsymbol{x}(t)) + \frac{\ln(q)}{\eta}.$$

Note that each formula ψ in (3a) can be written as $\psi := \wedge_{i=1}^{q} \psi_i$ where ψ_i does not contain any further conjunctions itself. As stated before, it holds that $(\boldsymbol{x}, t) \models \psi$ if $\bar{\rho}^{\psi}(\boldsymbol{x}(t)) > 0$ and $(\boldsymbol{x}, t) \not\models \psi$ if $\bar{\rho}^{\psi}(\boldsymbol{x}(t)) < 0$ due to [10, Proposition 16]. These two inferences establish a direct relation between semantics and robust semantics. For $\rho^{\psi}(\boldsymbol{x}(t))$ and due to Lemma 1 it holds that $(\boldsymbol{x}, t) \models \psi$ if $\rho^{\psi}(\boldsymbol{x}(t)) > 0$. However, $\rho^{\psi}(\boldsymbol{x}(t)) < 0$ can imply both $(\boldsymbol{x}, t) \models \psi$ and $(\boldsymbol{x}, t) \not\models \psi$. It holds that $(\boldsymbol{x}, t) \not\models \psi$ if $\rho^{\psi}(\boldsymbol{x}(t)) + \frac{\ln(q)}{\eta} < 0$. We aim for a control law $\boldsymbol{u}(\boldsymbol{x}, t)$ that achieves $r \leq \rho^{\phi}(\boldsymbol{x}, 0)$ where $r \in \mathbb{R}$ is a robustness measure and $\boldsymbol{x} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ is the closed-loop solution of (1) with $\boldsymbol{x}(0) := \boldsymbol{x}_0$. Note that $(\boldsymbol{x}, 0) \models \phi$ if $r > 0$. For technical reasons, we introduce $\rho_{\max} \in \mathbb{R}$ with $r < \rho_{\max}$

and also aim for $\rho^{\phi}(\boldsymbol{x},0) \leq \rho_{\max}$. As will be shown in Section 4, this will not induce any conservatism. For a given ϕ as in (3b), let ψ be the non-temporal formula appearing in ϕ, then we achieve

$$r \leq \rho^{\phi}(\boldsymbol{x},0) \leq \rho_{\max}$$

by prescribing a temporal behavior to $\rho^{\psi}(\boldsymbol{x}(t))$ through the design parameters γ and $\rho_{\max}$ and the funnel

$$-\gamma(t) + \rho_{\max} < \rho^{\psi}(\boldsymbol{x}(t)) < \rho_{\max}. \tag{5}$$

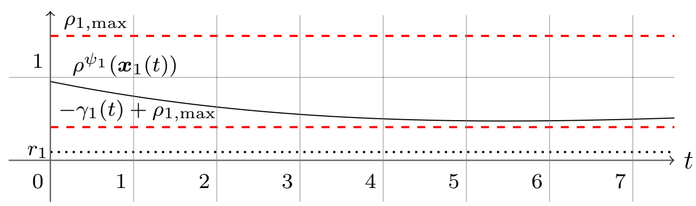
Note the use of $\rho^{\psi}(\boldsymbol{x}(t))$ and not $\rho^{\phi}(\boldsymbol{x},0)$ itself. The connection between the non-temporal $\rho^{\psi}(\boldsymbol{x}(t))$ and the temporal $\rho^{\phi}(\boldsymbol{x},0)$ is made by the choice of the performance function γ. In fact, γ prescribes temporal behavior that, in combination with $\rho^{\psi}(\boldsymbol{x}(t))$, mimics $\rho^{\phi}(\boldsymbol{x},0)$. Section 4 will explain in detail how to select γ and $\rho_{\max}$ so that satisfaction of (5) for all $t \geq 0$ implies $r \leq \rho^{\phi}(\boldsymbol{x},0) \leq \rho_{\max}$, while some intuition is given in the following example.

Example 1. *Consider $\phi_1 := G_{[0,7]}\psi_1$ and $\phi_2 := F_{[0,7]}\psi_2$ with $\psi_1 := \mu_1$ and $\psi_2 := \mu_2 \wedge \mu_3$ where, for $\boldsymbol{x} := \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T \in \mathbb{R}^n$, $h_1(\boldsymbol{x}) := 1 - \|\boldsymbol{x}\|$, $h_2(\boldsymbol{x}) := x_1$, and $h_3(\boldsymbol{x}) := x_2 + 0.5$ are associated with μ_1, μ_2, and μ_3, respectively. Figs. 1b and 1a show the funnel in* (5) *prescribing a desired temporal behavior to satisfy ϕ_1 and ϕ_2, respectively. If now the signals $\boldsymbol{x}_1, \boldsymbol{x}_2 : \mathbb{R}_{\geq 0} \to \mathbb{R}^2$, possibly solutions to* (1), *are such that $\rho^{\psi_1}(\boldsymbol{x}_1(t))$ and $\rho^{\psi_2}(\boldsymbol{x}_2(t))$ are contained within these funnels, it holds that $(\boldsymbol{x}_1, 0) \models \phi_1$ and $(\boldsymbol{x}_2, 0) \models \phi_2$. In particular, this follows by the choice of γ_1, γ_2, $\rho_{1,max}$, $\rho_{2,max}$ and since $\rho^{\psi_1}(\boldsymbol{x}_1(t)) \in (-\gamma_1(t) + \rho_{1,max}, \rho_{1,max})$ and $\rho^{\psi_2}(\boldsymbol{x}_2(t)) \in (-\gamma_2(t) + \rho_{2,max}, \rho_{2,max})$ for all $t \in \mathbb{R}_{\geq 0}$, i.e.,* (5) *is satisfied for all $t \in \mathbb{R}_{\geq 0}$. Note also that, for instance, in Fig. 1b the lower funnel $-\gamma_2(t) + \rho_{2,max}$ enforces $\rho^{\psi_2}(\boldsymbol{x}_2(t)) \geq r_2 := 0.1$ for all $t \geq 6$. Thus, ϕ_2 is robustly satisfied with $\rho^{\phi_2}(\boldsymbol{x}_2, 0) \geq r_2$.*

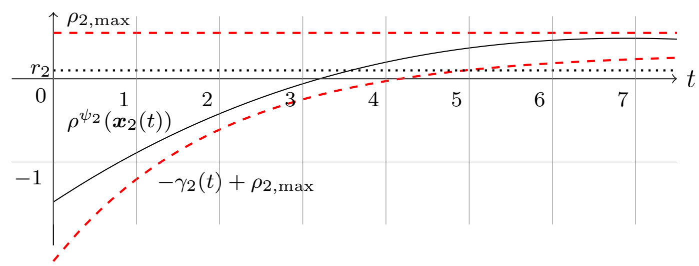
In the remainder, and for non-temporal formulas of class ψ given in (3a), we use $\rho^{\psi}(\boldsymbol{x}(t))$ when we talk about the solution $\boldsymbol{x} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ to (1) and $\rho^{\psi}(\boldsymbol{x})$ when we refer to the function $\rho^{\psi} : \mathbb{R}^n \to \mathbb{R}$ evaluated at $\boldsymbol{x} \in \mathbb{R}^n$. Furthermore, two assumptions on the function $\rho^{\psi}(\boldsymbol{x})$ for ψ formulas that are contained in (3b) and (4) are posed.

Assumption 2. *Each formula of class ψ given in* (3a) *that is contained in* (3b) *and* (4) *is: 1) such that $\rho^{\psi} : \mathbb{R}^n \to \mathbb{R}$ is concave and 2) well-posed in the sense that for all $\chi \in \mathbb{R}$ there exists $\bar{C} \geq 0$ such that, for all $\boldsymbol{x} \in \mathbb{R}^n$ with $\rho^{\psi}(\boldsymbol{x}) \geq \chi$, it holds that $\|\boldsymbol{x}\| \leq \bar{C} < \infty$.*

Remark 2. *Recall the syntax of class ψ formulas in* (3a) *and consider $\psi := \psi_1 \wedge \neg\psi_2$, which is build of all possible components in* (3a). *Part 1) of Assumption 2 is satisfied, i.e. $\rho^{\psi}(\boldsymbol{x})$ is concave, if $\rho^{\psi_1}(\boldsymbol{x})$ and $\rho^{\neg\psi_2}(\boldsymbol{x})$ are concave. This can be seen as follows: due to [27, Section 3.5] it holds that* $\exp(-\eta\rho^{\psi_1}(\boldsymbol{x}))$ *and* $\exp(-\eta\rho^{\psi_2}(\boldsymbol{x}))$ *are log-convex. It also holds that a sum of log-convex functions*



(a) Funnel $(-\gamma_1(t) + \rho_{1,\max}, \rho_{1,\max})$ (dashed lines) for $\phi_1 = G_{[0,7]}\psi_1$ s.t. $\rho^{\phi_1}(\boldsymbol{x}_1, 0) \geq 0.1 =: r_1$ (dotted line) and $\rho^{\psi_1}(\boldsymbol{x}_1(t))$ (solid line).



(b) Funnel $(-\gamma_2(t) + \rho_{2,\max}, \rho_{2,\max})$ (dashed lines) for $\phi_2 = F_{[0,7]}\psi_2$ s.t. $\rho^{\phi_2}(\boldsymbol{x}_2, 0) \geq 0.1 =: r_2$ (dotted line) and $\rho^{\psi_2}(\boldsymbol{x}_2(t))$ (solid line).

Figure 1: Illustration of the connection between $\rho^{\psi}\big(\boldsymbol{x}(t)\big)$ and $\rho^{\phi}(\boldsymbol{x}, 0)$ in Example 1.

is log-convex, i.e., $\exp(-\eta\rho^{\psi_1}(\boldsymbol{x}))+\exp(-\eta\rho^{\psi_2}(\boldsymbol{x}))$ *is log-convex. Consequently,* $\rho^{\psi}(\boldsymbol{x})$ *is concave. Part 1) of Assumption 2 hence imposes a restriction on the predicates allowed in* (3)*. A predicate function* $h(\boldsymbol{x})$ *needs to be concave when the associated predicate* μ *is considered in* (3a)*, while* $h(\boldsymbol{x})$ *needs to be convex when the associated predicate in negated form, i.e.,* $\neg\mu$*, is considered in* (3a)*. Note that affine predicate functions* $h(\boldsymbol{x})$ *can hence be considered for both* μ *and* $\neg\mu$*. Part 1) of Assumption 2 is needed since the controller that will be presented in Section 4 uses gradient information of* $\frac{\partial\rho^{\psi}(\boldsymbol{x})}{\partial\boldsymbol{x}}$*. Local extrema or saddle points may hence lead the system to get stuck. For instance, specifications such as* eventually reach a region while avoiding another region *induce a nonconvex and nonconcave* $\rho^{\psi}(\boldsymbol{x})$*. For these problems, there exist, in general, no continuous control law that solves the problem globally [25, Ch. 4].*

Remark 3. *Part 2) of Assumption 2 will ensure, in the proof of Theorem 1, bounded solutions* $\boldsymbol{x}(t)$ *to* (1) *so that it can be shown that* $\boldsymbol{x}(t)$ *is defined for all* $t \geq 0$*. This assumption is merely a technical assumption and not restrictive in practice since* $\mu_{Ass.2} := (\|\boldsymbol{x}\| \leq \bar{C})$ *can be combined with the desired* ψ *for a sufficiently large* $\bar{C}$ *so that* $\psi \wedge \mu_{Ass.2}$ *is well-posed.*

In Section 4, the smooth approximation $\rho^{\psi}(\boldsymbol{x})$ will be used for control instead of the original non-smooth semantics $\bar{\rho}^{\psi}(\boldsymbol{x})$. As shown in Lemma 1, a large enough η provides a sufficiently accurate approximation. We next explain how conservatism, potentially arising by using this approximation, is avoided. Let the optimum of $\bar{\rho}^{\psi}(\boldsymbol{x})$ and $\rho^{\psi}(\boldsymbol{x})$ (for a given η) be

$$\bar{\rho}^{\psi}_{\text{opt}} := \sup_{\boldsymbol{x}\in\mathbb{R}^n} \bar{\rho}^{\psi}(\boldsymbol{x}) \text{ and } \rho^{\psi}_{\text{opt}} := \sup_{\boldsymbol{x}\in\mathbb{R}^n} \rho^{\psi}(\boldsymbol{x}) \tag{6}$$

where ρ^{ψ}_{opt} is computationally easy to calculate since $\rho^{\psi}(\boldsymbol{x})$ is concave. Note also that ρ^{ψ}_{opt} is finite due to Assumption 2. There always exists an η such that $\rho^{\psi}_{\text{opt}} > 0$ if $\bar{\rho}^{\psi}_{\text{opt}} > 0$ since $\rho^{\psi}(\boldsymbol{x}) = \bar{\rho}^{\psi}(\boldsymbol{x})$ as $\eta \to \infty$. We then find η by solving a convex feasibility problem [27], i.e., by selecting η such that $\rho^{\psi}(\boldsymbol{x}) > 0$ for some $\boldsymbol{x} \in \mathbb{R}^n$ which consequently implies that $\rho^{\psi}_{\text{opt}} > 0$; $\rho^{\psi}_{\text{opt}} > 0$ means that ψ and hence also ϕ (recall (3b)) are satisfiable. Otherwise, i.e., if $\bar{\rho}^{\psi}_{\text{opt}} \leq 0$, there exist no η such that $\rho^{\psi}_{\text{opt}} > 0$ and we select $\eta := 1$ (in fact, any $\eta > 0$ can be selected). It then holds that ϕ is not satisfiable if $\rho^{\psi}_{\text{opt}} + \frac{\ln(q)}{\eta} < 0$ with q as in Lemma 1. If $\rho^{\psi}_{\text{opt}} \leq 0$, let $\rho_{\text{gap}} > 0$ be a parameter indicating how much ϕ may be violated. If $\boldsymbol{x} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ is such that $r \leq \rho^{\phi}(\boldsymbol{x}, 0)$ for r with $\rho^{\psi}_{\text{opt}} - \rho_{\text{gap}} \leq r < \rho^{\psi}_{\text{opt}}$, we say that $\boldsymbol{x}$ is a *least violating solution* with a given gap of ρ_{gap}. To distinguish between these two cases, we pose the next assumption.

Assumption 3. *The optimum of $\rho^{\psi}(\boldsymbol{x})$ is s.t. $\rho^{\psi}_{opt} > 0$.*

To derive, in Section 5, a switched control system to satisfy temporal formulas θ, we define $\rho^{\theta}(\boldsymbol{x}, t)$ without introducing another smooth approximation. Recall also that the smooth approximation introduced before only applies for conjunctions of non-temporal formulas. For formulas θ given in (4), $\rho^{\theta}(\boldsymbol{x}, t)$ is defined as

$$\rho^{\theta}(\boldsymbol{x}, t) := \min_{k\in\{1,\ldots,K\}} \rho^{\phi_k}(\boldsymbol{x}, t).$$

Let $\rho^{\theta}_{\text{opt}} := \sup_{\boldsymbol{x}\in\mathbb{R}^n} \min_{k\in\{1,\ldots,K\}} \rho^{\psi_k}(\boldsymbol{x})$ where η for each $\rho^{\psi_k}(\boldsymbol{x})$ has been obtained as described previously. Let $\hat{r}$ be such that $0 < \hat{r} < \rho^{\theta}_{\text{opt}}$ if $\rho^{\theta}_{\text{opt}} > 0$ and $\rho^{\theta}_{\text{opt}} - \rho_{\text{gap}} \leq \hat{r} < \rho^{\theta}_{\text{opt}}$ otherwise where $\rho_{\text{gap}} > 0$ is again a parameter indicating how much θ may be violated so that, in the latter case, again a least violating solution with a given gap of ρ_{gap} can be found. For these formulas, we construct a performance function γ_k for each ϕ_k and combine and extend the previously derived funnel-based control laws. Finally, let $\hat{\rho}_{\max} \in (\max_{k\in\{1,\ldots,K\}} \rho^{\psi_k}_{\text{opt}}, \infty)$.

Problem 1. *Consider the system in* (1) *and a temporal formula θ as in* (4). *Let η be selected as instructed above and $\rho_{gap} > 0$ be given, design a control law $\boldsymbol{u}(\boldsymbol{x}, t)$ such that $\hat{r} \leq \rho^{\theta}(\boldsymbol{x}, 0) \leq \hat{\rho}_{max}$.*

We remark that $\hat{r}$, as defined above, may be conservatively small. In our proposed problem solution, the lower bound $\hat{r}$ may be less conservative allowing for much larger values (see Theorem 4).

4. A Funnel-based Feedback Control Law for Atomic Temporal Formulas

Define first the one-dimensional error, the normalized error, and the transformed error as

$$e(\boldsymbol{x}) := \rho^{\psi}(\boldsymbol{x}) - \rho_{\max} \tag{7}$$

$$\xi(\boldsymbol{x},t) := \frac{e(\boldsymbol{x})}{\gamma(t)} \tag{8}$$

$$\epsilon(\boldsymbol{x},t) := S\big(\xi(\boldsymbol{x},t)\big) = \ln\Big(-\frac{\xi(\boldsymbol{x},t)+1}{\xi(\boldsymbol{x},t)}\Big), \tag{9}$$

respectively. When referring to the solution $\boldsymbol{x} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ to (1), we use the notation $e(t) := e(\boldsymbol{x}(t))$, $\xi(t) := \xi(\boldsymbol{x}(t),t)$, and $\epsilon(t) := \epsilon(\boldsymbol{x}(t),t)$, while we use $e(\boldsymbol{x})$, $\xi(\boldsymbol{x},t)$, and $\epsilon(\boldsymbol{x},t)$ when we want to emphasize dependence on the state $\boldsymbol{x} \in \mathbb{R}^n$. Equation (5) can now be written as $-\gamma(t) < e(t) < 0$, which resembles (2) by setting $M := 0$ and can further be written as $-1 < \xi(t) < 0$. Applying the function S results in $-\infty < \epsilon(t) < \infty$. If now $\epsilon(t)$ is bounded for all $t \geq 0$, then (5) holds for all for all $t \geq 0$. We remark that $\xi\big(\boldsymbol{x}(0),0\big) \in \Omega_\xi := (-1,0)$ needs to hold initally. Assume now that there exists a control law such that (5) holds for all $t \in \mathbb{R}_{\geq 0}$. Then the design parameters γ and $\rho_{\max}$ need to be selected in a particular way that will be discussed at the end of this section. This choice will ensure the desired transient behavior as illustrated in Example 1, i.e., the connection between $\rho^{\psi}(\boldsymbol{x}(t))$ and $\rho^{\phi}(\boldsymbol{x},0)$, and hence imply that $r \leq \rho^{\phi}(\boldsymbol{x},0) \leq \rho_{\max}$. Hence, the first objective of this section is to derive a continuous feedback control law $\boldsymbol{u}(\boldsymbol{x},t)$ such that (5) holds for all $t \in \mathbb{R}_{\geq 0}$.

Theorem 1. *Consider the system* (1) *and an atomic temporal formula* ϕ *as in* (3b) *with the corresponding* ψ*. If* $-\gamma_0 + \rho_{max} < \rho^{\psi}(\boldsymbol{x}_0)$ *and* $-\gamma_\infty + \rho_{max} < \rho^{\psi}_{opt} < \rho_{max}$*, and Assumptions 1-2 are satisfied, then the control law*

$$\boldsymbol{u}(\boldsymbol{x},t) := -\epsilon(\boldsymbol{x},t) g(\boldsymbol{x})^T \frac{\partial \rho^{\psi}(\boldsymbol{x})}{\partial \boldsymbol{x}} \tag{10}$$

guarantees that (5) *is satisfied for all* $t \in \mathbb{R}_{\geq 0}$ *with all closed-loop signals being continuous and bounded.*

Remark 4. *If* $m = n$*, it can be shown that the control law* (10) *in Theorem 1 can be replaced by* $\boldsymbol{u}(\boldsymbol{x},t) := -\epsilon(\boldsymbol{x},t)\frac{\partial \rho^{\psi}(\boldsymbol{x})}{\partial \boldsymbol{x}}$ *so that a model-free approach is obtained.*

Remark 5. *Recall that* $\boldsymbol{w}(t) \in \mathcal{W}$ *where* $\mathcal{W}$ *is an arbitrary yet bounded set. Naturally, allowing* $\|\boldsymbol{w}(t)\|$ *to be large implies potentially larger control inputs as indicated by the constant* k_1 *in the proof of Theorem 1.*

The second step is to show that the control law (10) in Theorem 1 results in $r \leq \rho^{\phi}(\boldsymbol{x},0) \leq \rho_{\max}$ if γ is properly designed. Select $\rho_{\max} \in (\rho^{\psi}_{\text{opt}}, \infty)$ according to Theorem 1. Next, define the variable

$$t_* \in \begin{cases} \{a\} & \text{if } \phi = G_{[a,b]}\psi \\ [a,b] & \text{if } \phi = F_{[a,b]}\psi \\ [\underline{a}+\bar{a}, \underline{b}+\bar{a}] & \text{if } \phi = F_{[\underline{a},\underline{b}]}G_{[\bar{a},\bar{b}]}\psi. \end{cases}$$

Our goal is to enforce $r \leq \rho^{\psi}(\boldsymbol{x}(t)) \leq \rho_{\max}$ for all $t \geq t_*$ by the choice of γ and the use of the funnel-based feedback control law (10). This will consequently lead to $r \leq \rho^{\phi}(\boldsymbol{x}, 0) \leq \rho_{\max}$ by the choice of t_*. We select $r \in (0, \rho^{\psi}_{\text{opt}})$ if Assumption 3 holds, i.e., $\rho^{\psi}_{\text{opt}} > 0$ implies that $r > 0$ is a feasible choice. It holds that $0 < r < \rho^{\psi}_{\text{opt}} < \rho_{\max}$. We will now introduce a feasibility notion of the funnel-based feedback control law and ψ with respect to the initial condition $\boldsymbol{x}_0$, the desired robustness r, and the choice of t_*.

Definition 2 (Feasibility). *The formula ψ is feasible with respect to r, $\boldsymbol{x}_0$, and t_*, if $t_* > 0$ or if $\rho^{\psi}(\boldsymbol{x}_0) > r$.*

If a formula is not feasible w.r.t r, $\boldsymbol{x}_0$, and t_*, it may be possible to adjust r and t_* accordingly. Otherwise, the formula can be relaxed as discussed in [28] by modifying ϕ. If a formula ψ is feasible w.r.t. r, $\boldsymbol{x}_0$, and t_*, it means that it is possible to achieve $r \leq \rho^{\psi}(\boldsymbol{x}(t)) \leq \rho_{\max}$ for all $t \geq t_*$ by the choice of γ and by means of the funnel-based feedback control law in (10). Note again that $r \leq \rho^{\psi}(\boldsymbol{x}(t)) \leq \rho_{\max}$ for all $t \geq t_*$ leads to $r \leq \rho^{\phi}(\boldsymbol{x}, 0) \leq \rho_{\max}$ by the choice of t_*. For $t_* > 0$ it is always possible to design γ such that $-\gamma_0 + \rho_{\max} < \rho^{\psi}(\boldsymbol{x}_0)$ as required by Theorem 1 and such that $r \leq \gamma(t) + \rho_{\max}$ for all $t \geq t_*$. The control law (10) can then be applied to achieve $r \leq \gamma(t) + \rho_{\max} < \rho^{\psi}(\boldsymbol{x}(t)) < \rho_{\max}$ for all $t \geq t_*$. For $t_* = 0$, however, it needs to hold that $\rho^{\psi}(\boldsymbol{x}_0) > r$ so that γ can similarly be designed such that $r \leq -\gamma_0 + \rho_{\max} < \rho^{\psi}(\boldsymbol{x}_0)$ and $r \leq \gamma(t) + \rho_{\max}$ for all $t \geq t_*$. If, however, $t_* = 0$ and $\rho^{\psi}(\boldsymbol{x}_0) \leq r$, the initial condition $r \leq -\gamma_0 + \rho_{\max} < \rho^{\psi}(\boldsymbol{x}_0)$ is not satisfied since $\rho^{\psi}(\boldsymbol{x}_0) \leq r$. Hence, the formula ψ is not feasible w.r.t. r, $\boldsymbol{x}_0$, and t_*.

For the design of γ, assume that ψ is feasible w.r.t. r, $\boldsymbol{x}_0$, and t_* and recall that $\gamma(t) := (\gamma_0 - \gamma_\infty)\exp(-lt) + \gamma_\infty$. The crucial part of Theorem 1 are the assumptions that $-\gamma_0 + \rho_{\max} < \rho^{\psi}(\boldsymbol{x}_0)$ and $\rho^{\psi}_{\text{opt}} < \rho_{\max}$, i.e., $\xi(\boldsymbol{x}_0, 0) \in \Omega_\xi$. It is possible to choose γ_0 such that $\xi(\boldsymbol{x}_0, 0) \in \Omega_\xi$, which is equivalent to $-1 < \frac{\rho^{\psi}(\boldsymbol{x}_0) - \rho_{\max}}{\gamma(0)} < 0$. It should also hold that $r \leq -\gamma_0 + \rho_{\max}$ if $t_* = 0$ due to (5) and since we want $r \leq \rho^{\psi}(\boldsymbol{x}(t))$ for all $t \geq t_*$ as illustrated in Fig. 1a where $t_* = 0$ and $r := 0.1$ and where it should therefore hold that the lower funnel satisfies $r := 0.1 \leq -\gamma_0 + \rho_{\max}$. Finally, γ_0 is chosen as

$$\gamma_0 \in \begin{cases} (\rho_{\max} - \rho^{\psi}(\boldsymbol{x}_0), \infty) & \text{if } t_* > 0 \\ (\rho_{\max} - \rho^{\psi}(\boldsymbol{x}_0), \rho_{\max} - r] & \text{if } t_* = 0. \end{cases} \tag{11}$$

At $t = \infty$, it is required that $\max(-\gamma_0 + \rho_{\max}, r) \leq -\gamma_\infty + \rho_{\max} < \rho^{\psi}_{\text{opt}}$ where the first inequality enforces that $-\gamma + \rho_{\max}$ is a non-decreasing function, which in turn leads to γ being non-increasing, and the second inequality is required by Theorem 1. The smaller γ_∞ is selected, the tighter the funnel in (5) will become as $t \to \infty$. The parameter γ_∞ is hence selected as

$$\gamma_\infty \in \Big(\rho_{\max} - \rho^{\psi}_{\text{opt}}, \min\big(\gamma_0, \rho_{\max} - r\big)\Big). \tag{12}$$

For the calculation of l, three cases need to be distinguished: 1) $\rho^{\psi}(\boldsymbol{x}_0) > r$, 2) $\rho^{\psi}(\boldsymbol{x}_0) \leq r$ and $t_* > 0$, and 3) $\rho^{\psi}(\boldsymbol{x}_0) \leq r$ and $t_* = 0$. Case 3) can be excluded since ψ is feasible w.r.t. r, $\boldsymbol{x}_0$, and t_*. Next, select l as

$$l \in \begin{cases} 0 & \text{if } -\gamma_0 + \rho_{\max} \geq r \\ -\frac{\ln\left(\frac{r+\gamma_\infty-\rho_{\max}}{-(\gamma_0-\gamma_\infty)}\right)}{t_*} & \text{if } -\gamma_0 + \rho_{\max} < r \end{cases} \tag{13}$$

which ensures that $-\gamma(t_*) + \rho_{\max} = r$ if $-\gamma_0 + \rho_{\max} < r$ and $-\gamma(t_*) + \rho_{\max} \geq r$ otherwise.

Theorem 2. *Consider the system* (1) *and an* atomic temporal formula ϕ *as in* (3b). *If Assumptions 1-3 hold,* $\rho_{max} \in (\rho^{\psi}_{opt}, \infty)$, $r \in (0, \rho^{\psi}_{opt})$, *the control law in* (10) *is used, and* ψ *is feasible w.r.t.* r, $\boldsymbol{x}_0$, *and* t_*, *then choosing* γ_0, γ_∞, *and* l *as in* (11), (12), *and* (13), *respectively, ensures that* $0 < r \leq \rho^{\phi}(\boldsymbol{x}, 0) \leq \rho_{max}$, *i.e.,* $(\boldsymbol{x}, 0) \models \phi$.

Example 2. *Consider* $\phi := G_{[0,7]}\psi$ *with* $\psi := \mu$ *that is associated with* $h(\boldsymbol{x}) := 1 - \|\boldsymbol{x}\|$ *(this is the same formula as* ϕ_1 *in Example 1) for which* $\rho^{\psi}_{opt} = 1$ *(note that* ρ^{ψ}_{opt} *is independent of* η *in this case) and let* $\boldsymbol{x}_0 := \begin{bmatrix}-0.05 & 0\end{bmatrix}^T$. *We hence select, as instructed,* $\rho_{max} := 1.5 \in (\rho^{\psi}_{opt}, \infty)$, $r := 0.1 \in (0, \rho^{\psi}_{opt})$, *and* $t_* := 0$ *as well as* $\gamma_0 := 1.1 \in (\rho_{max} - \rho^{\psi}(\boldsymbol{x}_0), \rho_{max} - r] = (0.55, 1.4]$, $\gamma_\infty := 1 \in (\rho_{max} - \rho^{\psi}_{opt}, \min(\gamma_0, \rho_{max} - r)) = (0.5, 1.1)$, *and* $l := 0$. *Note that exactly these parameters are plotted in Fig. 1a. Now applying* (10) *results, according to Theorem 2, in* $0 < r \leq \rho^{\phi}(\boldsymbol{x}, 0) \leq \rho_{max}$.

Corollary 1. *Let all the conditions of Theorem 2 hold except of Assumption 3 and let* r *instead be chosen as* $r \in [\rho^{\psi}_{opt} - \rho_{gap}, \rho^{\psi}_{opt})$, *then* $r \leq \rho^{\phi}(\boldsymbol{x}, 0) \leq \rho_{max}$ *and a least violating solution with a given gap of* ρ_{gap} *is found.*

Remark 6. *Theorem 2 and Corollary 1 provide a theoretical guarantee that* $r < \rho^{\phi}(\boldsymbol{x}, 0) < \rho_{max}$ *holds. However, a steep performance function* γ *may result in a large, although bounded, control input* $\|\boldsymbol{u}\|$ *that may conflict with input saturations. Therefore, it may be advisable to select* t_* *as big as possible and* γ_0 *and* l *as small as possible. In simulations, it also turns out that inserting a sufficiently large gain* $\kappa > 0$ *into* (10) *as* $\boldsymbol{u}(\boldsymbol{x}, t) := -\kappa\epsilon(\boldsymbol{x}, t)g(\boldsymbol{x})^T \frac{\partial \rho^{\psi}(\boldsymbol{x})}{\partial \boldsymbol{x}}$ *can compensate for inaccuracies resulting from the digital implementation of the continuous-time feedback control law.*

5. A Switched Control System for Temporal Formulas with Overlapping Time Intervals

Let us first consider $\theta := \psi' \, U_{[a,b]} \, \psi''$ where ψ' and ψ'' are as in (3a). The main idea is to combine two funnel-based control laws as derived in the previous section. Hence, let us decompose $\psi' \, U_{[a,b]} \, \psi''$ into $G_{[0,t_*]}\psi'$ and $F_{[t_*,t_*]}\psi''$ where $t_* \in [a,b]$. Let $\rho_{\max} \in (\max(\rho_{\text{opt}}^{\psi'}, \rho_{\text{opt}}^{\psi''}), \infty)$ and let $\gamma'(t) := (\gamma_0' - \gamma_\infty') \exp(-l't) + \gamma_\infty'$ and $\gamma''(t) := (\gamma_0'' - \gamma_\infty'') \exp(-l''t) + \gamma_\infty''$ be performance functions for $\rho^{\psi'}(\boldsymbol{x})$ and $\rho^{\psi''}(\boldsymbol{x})$, respectively. Our goal is to achieve

$$-\gamma'(t) + \rho_{\max} < \rho^{\psi'}(\boldsymbol{x}(t)) < \rho_{\max} \tag{14a}$$

$$-\gamma''(t) + \rho_{\max} < \rho^{\psi''}(\boldsymbol{x}(t)) < \rho_{\max} \tag{14b}$$

for all $t \in \mathcal{I}$ where $\mathcal{I} := [0,b]$. Let us define $\hat{\rho}^{\psi''}(\boldsymbol{x},t) := \rho^{\psi''}(\boldsymbol{x}) - \gamma'(t) + \gamma''(t)$ and introduce another approximation of the min operator as

$$\hat{\rho}(\boldsymbol{x},t) := -\frac{1}{\hat{\eta}} \ln\big(\exp\big(-\hat{\eta}\rho^{\psi'}(\boldsymbol{x})\big) + \exp\big(-\hat{\eta}\hat{\rho}^{\psi''}(\boldsymbol{x},t)\big)\big)$$

where $\hat{\eta} > 0$. Note again $\hat{\rho}(\boldsymbol{x},t) \leq \min(\rho^{\psi'}(\boldsymbol{x}), \hat{\rho}^{\psi''}(\boldsymbol{x},t))$ with $\hat{\rho}(\boldsymbol{x},t) = \min(\rho^{\psi'}(\boldsymbol{x}), \hat{\rho}^{\psi''}(\boldsymbol{x},t))$ as $\hat{\eta} \to \infty$. If now

$$-\gamma'(t) + \rho_{\max} < \hat{\rho}(\boldsymbol{x}(t),t) < \rho_{\max} \tag{15}$$

for all $t \in \mathcal{I}$, then (14) holds for all $t \in \mathcal{I}$. Let $\hat{e}(\boldsymbol{x},t) := \hat{\rho}(\boldsymbol{x},t) - \rho_{\max}$, $\hat{\xi}(\boldsymbol{x},t) := \frac{\hat{e}(\boldsymbol{x},t)}{\gamma'(t)}$, $\hat{\epsilon}(\boldsymbol{x},t) := S(\hat{\xi}(\boldsymbol{x},t)) = \ln\big(-\frac{\hat{\xi}(\boldsymbol{x},t)+1}{\hat{\xi}(\boldsymbol{x},t)}\big)$, and $\hat{\rho}_{\text{opt}}(t) := \sup_{\boldsymbol{x}\in\mathbb{R}^n} \hat{\rho}(\boldsymbol{x},t)$.

Theorem 3. *Consider the system* (1) *and the formula* $\theta := \psi' \, U_{[a,b]} \, \psi''$ *where* ψ' *and* ψ'' *are as in* (3a). *If* $-\gamma_0' + \rho_{max} < \hat{\rho}(\boldsymbol{x}_0, 0)$ *and* $-\gamma'(t) + \rho_{max} < \hat{\rho}_{opt}(t)$ *for all* $t \in \mathcal{I}$, *and Assumptions 1-2 are satisfied, then the control law*

$$\boldsymbol{u}(\boldsymbol{x},t) := -\hat{\epsilon}(\boldsymbol{x},t) g(\boldsymbol{x})^T \frac{\partial \hat{\rho}(\boldsymbol{x},t)}{\partial \boldsymbol{x}} \tag{16}$$

guarantees that (14) *is satisfied for all* $t \in \mathcal{I}$ *with all closed-loop signals being continuous and bounded.*

Recall that $\rho_{\text{opt}}^{\theta} := \sup_{\boldsymbol{x}\in\mathbb{R}^n} \min\big(\rho^{\psi'}(\boldsymbol{x}), \rho^{\psi''}(\boldsymbol{x})\big)$ and select $r \in (0, \rho_{\text{opt}}^{\theta})$ if $\rho_{\text{opt}}^{\theta} > 0$ and $r \in [\rho_{\text{opt}}^{\theta} - \rho_{\text{gap}}, \rho_{\text{opt}}^{\theta})$ otherwise. Satisfaction of $\psi' \, U_{[a,b]} \, \psi''$ in the sense of Theorem 2 and Corollary 1 follows under the conditions of Theorem 3.

Corollary 2. *Consider the system* (1) *and the formula* $\theta := \psi' \, U_{[a,b]} \, \psi''$ *where* ψ' *and* ψ'' *are as in* (3a). *Let Assumptions 1-2 hold and* $\gamma'(t)$ *and* $\gamma''(t)$ *be as in* (11), (12), *and* (13) *for* $G_{[0,t_*]}\psi'$ *and* $F_{[t_*,t_*]}\psi''$, *respectively, where* $t_* \in [a,b]$. *If* ψ' *and* ψ'' *are feasible w.r.t.* r, $\boldsymbol{x}_0$, *and* t_* *and* $-\gamma_0' + \rho_{max} < \hat{\rho}(\boldsymbol{x}_0, 0)$ *as well as* $-\gamma'(t) + \rho_{max} < \hat{\rho}_{opt}(t)$ *for all* $t \in \mathcal{I}$, *then* (16) *ensures that* $r \leq \rho^{\theta}(\boldsymbol{x}, 0) \leq \rho_{max}$.

Example 3. *Consider* $\theta := \psi' \, U_{[0,7]} \, \psi''$ *with* $\psi' := \mu_1$ *and* $\psi'' := \mu_2 \wedge \mu_3 \wedge \mu_{Ass.2}$ *where* μ_1*,* μ_2*, and* μ_3 *are as in Example 1,* $\mu_{Ass.2} := \mu_1$ *is to satisfy Assumption 2, and where again* $\boldsymbol{x}_0 := \begin{bmatrix}-0.05 & 0\end{bmatrix}^T$ *as in Example 2 so that* $(\boldsymbol{x}_0, 0) \models \psi'$ *but* $(\boldsymbol{x}_0, 0) \not\models \psi''$*. For* $\eta := 5$*, we have* $\rho_{opt}^{\psi''} = 0.31$ *and* $\rho^{\psi''}(\boldsymbol{x}_0) = -0.06$ *so that we select* $\rho_{max} := 1.5 \in (\max(\rho_{opt}^{\psi'}, \rho_{opt}^{\psi'}), \infty) = (1, \infty)$*. Furthemore,* $\rho_{opt}^{\theta} := \sup_{\boldsymbol{x} \in \mathbb{R}^n} \min\big(\rho^{\psi'}(\boldsymbol{x}), \rho^{\psi''}(\boldsymbol{x})\big) = \rho_{opt}^{\psi''} = 0.31$ *so that, as instructed, we select* $r := 0.1 \in (0, \rho_{opt}^{\theta})$*. Let* $\gamma'(t) := \gamma_0' := 1.1$ *be as derived in Example 2 so that it remains to define* $\gamma''(t)$*. We select* $t_* := 7 \in [0, 7]$ *as well as* $\gamma_0'' := 3 \in (\rho_{max} - \rho^{\psi''}(\boldsymbol{x}_0), \infty) = (1.56, \infty)$*,* $\gamma_\infty'' := 1.25 \in (\rho_{max} - \rho_{opt}^{\psi''}, \min(\gamma_0'', \rho_{max} - r)) = (1.19, 1.4)$*, and* $l'' := -\ln\big(\frac{r + \gamma_\infty'' - \rho_{max}}{-(\gamma_0'' - \gamma_\infty'')}\big)/t_* = 0.51$*. Let* $\hat{\eta} := 5$ *so that* $-\gamma_0' + \rho_{max} < \hat{\rho}(\boldsymbol{x}_0, 0)$ *is satisfied and further, for all* $t \in [0, 7]$*, it holds that* $-\gamma'(t) + \rho_{max} < \hat{\rho}_{opt}(t)$ *so that all assumptions of Corollary 2 are satisfied and* (16) *results in* $r \leq \rho^{\theta}(\boldsymbol{x}, 0) \leq \rho_{max}$*.*

Remark 7. *The funnel-based control law for formulas of class* ϕ *(Theorem 2 and Corollary 1) allows some freedom in the choices of* ρ_{max}*,* t_**, and* $\gamma(t)$ *that naturally affect the performance of the control system. The proposed solution is, however, complete. For formulas* $\theta := \psi' \, U_{[a,b]} \, \psi''$ *(Corollary 2), the choices of* ρ_{max}*,* t_**,* $\gamma'(t)$*, and* $\gamma''(t)$ *do affect the completeness of our proposed solution. In fact, it additionally needs to hold that* $-\gamma_0' + \rho_{max} < \hat{\rho}(\boldsymbol{x}_0, 0)$ *and* $-\gamma'(t) + \rho_{max} < \hat{\rho}_{opt}(t)$ *for all* $t \in \mathcal{I}$ *as illustrated in Example 3. The functions* $\gamma'(t)$ *and* $\gamma''(t)$ *are still selected according to* (11)*,* (12)*, and* (13)*. These equations can be seen as templates for an atomic temporal formula in* θ *that, along with the choices of* t_**,* r*,* ρ_{max}*, and* $\hat{\eta}$*, provide some freedom to satisfy these sufficient conditions.*

Let us now look at the general class of θ formulas given in (4). The basic idea is to develop a switching strategy to apply and combine the previously introduced funnel-based control laws. For the sake of readability, we treat each formula ϕ_k in (4) either as an eventually or as an always formula. Note in particular that $F_{[\underline{a}_k, \underline{b}_k]} G_{[\bar{a}_k, \bar{b}_k]} \psi_k'$ can be, according to Section 4, treated as $G_{[t_*, t_* + \bar{b}_k]} \psi_k'$ where $t_* \in [\underline{a}_k + \bar{a}_k, \underline{b}_k + \bar{a}_k]$, while $\psi_k' \, U_{[a_k, b_k]} \, \psi_k''$ can be, as discussed in the beginning of this section, treated as $G_{[0, t_*]} \psi_k' \wedge F_{[t_*, t_*]} \psi_k''$ where $t_* \in [a_k, b_k]$. Let us hence select $t_* \in [\underline{a}_k + \bar{a}_k, \underline{b}_k + \bar{a}_k]$ and $t_* \in [a_k, b_k]$ for each of these operators. Note again that this will introduce conservatism. We here, however, want to focus on the underlying scheduling and extend the idea of using several funnels simultaneously as presented in Theorem 3 and Corollary 2. Consequently, assume that each ϕ_k is either $\phi_k := F_{[a_k, b_k]} \psi_k$ or $\phi_k := G_{[a_k, b_k]} \psi_k$ and that each ϕ_k is associated with $t_{k,*}$; $t_{k,*}$ for ϕ_k resembles t_* for ϕ as in Section 4, but now equipped with the subscript k.

In Algorithm 1, we construct the switching sequence, i.e., the times at which the funnel-based control law changes, and determine the $t_{k,*}$ that are used to calculate $\gamma_k(t)$. We denote this sequence by $\mathcal{S} := \{s_0 := 0, s_1, \ldots, s_q\}$ and remark that q will be finite since θ consists of a finite number of conjunctions and atomic temporal formulas. Define $\mathfrak{i}_k := 0$ if $\phi_k := F_{[a_k, b_k]} \psi_k'$ and $\mathfrak{i}_k = 1$ if $\phi_k := G_{[a_k, b_k]} \psi_k'$. Similarly, let $\mathfrak{i}_k[a_k, b_k] := \emptyset$ if $\mathfrak{i}_k = 0$ and $\mathfrak{i}_k[a_k, b_k] :=$

$[a_k, b_k]$ if $\mathfrak{i}_k = 1$. We also separate the ϕ_k into independent subsets. Consider therefore the undirected graph $\mathcal{G} := (\mathcal{V}, \mathcal{E})$ with $\mathcal{V} := \{1, \ldots, K\}$. There exists an edge $(v_i, v_j) \in \mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ if and only if the predicate functions contained in ψ_i share at least one element of $\boldsymbol{x}$ with the predicate functions contained in ψ_j; $\Xi \subseteq \mathcal{V}$ is a dependency cluster if and only if $\forall v_i, v_j \in \Xi$ there is a path from v_i to v_j in $\mathcal{G}$ and $\nexists v_k \in \mathcal{V} \setminus \Xi$ such that there is a path from v_i to v_k in $\mathcal{G}$. For simplicity and without loss of generality, assume that Ξ_1, Ξ_2, ... correspond to $\{\phi_1, \ldots, \phi_{|\Xi_1|}\}$, $\{\phi_{|\Xi_1|+1}, \ldots, \phi_{|\Xi_1|+|\Xi_2|}\}$, ..., respectively, and assume that L dependency clusters exist in total. Algorithm 1 iterates through each dependency cluster Ξ_l with $l \in \{1, \ldots, L\}$ (lines 2-16) to determine, for each ϕ_k for which $v_k \in \Xi_l$, candidate $t_{k,*}$ for $\phi_k = F_{[a_k,b_k]}\psi'_k$ (lines 5-12), while $t_{k,*} := a_k$ if $\phi_k = G_{[a_k,b_k]}\psi'_k$ (line 15). The idea is, for the eventually operators, to find a $t_{k,*}$ that does not intersect with other time intervals of temporal operators in this cluster (lines 6-8). In particular, $\mathcal{D}_1$ denotes the union of all time intervals of always operators in Ξ_l, while $\mathcal{D}_2$ denotes the union of all $t_{k,*}$ already determined for eventually operators in Ξ_l. We remark here that Algorithm 1, and the way the $t_{k,*}$ are found, is not complete. The switching sequence $\mathcal{S}$ is constructed in lines 1, 13, and 16.

Algorithm 1 Calculation of $\mathcal{S} := \{s_0 := 0, s_1, \ldots, s_q\}$

1: $\mathcal{S} = \{0\}$
2: **for** $l := 1$, $l := l + 1$, while $l \leq L$ **do**
3: $\quad N := \sum_{j=1}^{l-1} |\Xi_j|$
4: $\quad$ **for** $k := N + 1$, $k := k + 1$, while $k \leq N + |\Xi_l|$ **do**
5: $\quad\quad$ **if** $\mathfrak{i}_k = 0$ **then**
6: $\quad\quad\quad \mathcal{D}_1 := \bigcup_{j \in \Xi_l \setminus \{k\}} \mathfrak{i}_j [a_j, b_j]$
7: $\quad\quad\quad \mathcal{D}_2 := \bigcup_{j=N+1}^{k-1} (1 - \mathfrak{i}_j) t_{j,*}$
8: $\quad\quad\quad \mathcal{K} := [a_k, b_k] \setminus \{\mathcal{D}_1 \cup \mathcal{D}_2\}$
9: $\quad\quad\quad$ **if** $\mathcal{K} \neq \emptyset$ **then**
10: $\quad\quad\quad\quad t_{k,*} \in \mathcal{K}$
11: $\quad\quad\quad$ **else**
12: $\quad\quad\quad\quad t_{k,*} \in [a_k, b_k]$
13: $\quad\quad\quad \mathcal{S} := \mathcal{S} \cup \{t_{k,*}\}$
14: $\quad\quad$ **else**
15: $\quad\quad\quad t_{k,*} := a_k$
16: $\quad\quad\quad \mathcal{S} := \mathcal{S} \cup \{b_k\}$

Based on $\mathcal{S}$, we propose a switched control law $\boldsymbol{u}(\boldsymbol{x}, t)$, i.e., $\boldsymbol{u}(\boldsymbol{x}, t)$ is discontinuous at $t = s_j$ for each $j \in \{0, 1, \ldots, q\}$. At s_j, we consider, for the calculation of $\boldsymbol{u}(\boldsymbol{x}, t)$, all atomic temporal formulas that are active during $(s_j, s_{j+1}]$. Let $\mathcal{A}_j$ denote the set of indices corresponding to active atomic temporal formulas during $[s_j, s_{j+1}]$. In particular, $k \in \mathcal{A}_j$ if $[a_k, b_k] \cap (s_j, s_{j+1}] \neq \emptyset$ in case that $\phi_k := G_{[a_k,b_k]}\psi'_k$ or if $t_{k,*} \cap (s_j, s_{j+1}] \neq \emptyset$ in case that $\phi_k := F_{[a_k,b_k]}\psi'_k$.

Example 4. *Consider* $\boldsymbol{x} := \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T \in \mathbb{R}^4$ *and* $\phi_1 := G_{[0,5]}\mu_1$, $\phi_2 := F_{[2,6]}\mu_2$, $\phi_3 := F_{[1,7]}\mu_3$, *and* $\phi_4 := G_{[6,7]}\mu_4$ *with associated predicate functions* $h_1(\boldsymbol{x}) := x_1 - 2$, $h_2(\boldsymbol{x}) := x_1 - x_2$, $h_3(\boldsymbol{x}) := x_3 - x_4$, *and* $h_4(\boldsymbol{x}) := 2 - x_4$. *The dependency clusters are hence* $\Xi_1 := \{v_1, v_2\}$ *and* $\Xi_2 := \{v_3, v_4\}$. *Running Algorithm 1, we obtain, for instance,* $t_{1,*} := 0$, $t_{2,*} := 5.5$, $t_{3,*} := 2$, *and* $t_{4,*} := 6$ *and correspondingly* $\mathcal{S} := \{0, 2, 5, 5.5, 7\}$. *Furthermore,* $\mathcal{A}_0 = \{1, 3\}$, $\mathcal{A}_1 = \{1\}$, $\mathcal{A}_2 = \{2\}$, *and* $\mathcal{A}_3 = \{4\}$.

At each $t = s_j$, let $\rho^j_{\max} \in (\max_{k\in\mathcal{A}_j} \rho^{\psi_k}_{\text{opt}}, \infty)$, $\rho^j_{\text{opt}} := \sup_{\boldsymbol{x}\in\mathbb{R}^n} \min_{k\in\mathcal{A}_j} \rho^{\psi_k}(\boldsymbol{x})$, and $r^j \in (\max(0, \hat{r}), \rho^j_{\text{opt}})$ if $\rho^j_{\text{opt}} > 0$ or $r^j \in [\max(\rho^j_{\text{opt}} - \rho_{\text{gap}}, \hat{r}), \rho^j_{\text{opt}})$ otherwise. Note here in particular that $\rho^j_{\text{opt}} \geq \rho^\theta_{\text{opt}}$. Also, construct a performance function $\gamma^j_k(t) := (\gamma^j_{k,0} - \gamma^j_{k,\infty})\exp(-l^j_k t) + \gamma^j_{k,\infty}$ for each $k \in \mathcal{A}_j$ with

$$\gamma^j_{k,0} \in \begin{cases} (\rho^j_{\max} - \rho^{\psi_k}(\boldsymbol{x}(s_j)), \infty) & \text{if } t_{k,*} > s_j \\ (\rho^j_{\max} - \rho^{\psi_k}(\boldsymbol{x}(s_j)), \rho^j_{\max} - r^j] & \text{if } t_{k,*} = s_j. \end{cases}$$

$$\gamma^j_{k,\infty} \in \left(\rho^j_{\max} - \rho^j_{\text{opt}}, \min\left(\gamma^j_{k,0}, \rho^j_{\max} - r^j\right)\right)$$

$$l^j \in \begin{cases} 0 & \text{if } -\gamma^j_{k,0} + \rho^j_{\max} \geq r^j \\ -\frac{\ln\left(\frac{r^j + \gamma^j_{k,\infty} - \rho^j_{\max}}{-(\gamma^j_{k,0} - \gamma^j_{k,\infty})}\right)}{t_{k,*} - s_j} & \text{if } -\gamma^j_{k,0} + \rho^j_{\max} < r^j. \end{cases}$$

The goal is, similarly to (14), to achieve, for each $k \in \mathcal{A}_j$, $-\gamma^j_k(t) + \rho^j_{\max} < \rho^{\psi_k}(\boldsymbol{x}(t)) < \rho^j_{\max}$ for all $t \in [s_j, s_{j+1}) =: \mathcal{I}^j$. Pick $k' \in \mathcal{A}_j$ and define, for each $k \in \mathcal{A}_j$, $\hat{\rho}^{\psi_k}(\boldsymbol{x}, t) := \rho^{\psi_k}(\boldsymbol{x}) - \gamma^j_{k'}(t) + \gamma^j_k(t)$ for $t \in \mathcal{I}^j$. Let again $\hat{\rho}(\boldsymbol{x}, t) := -\frac{1}{\hat{\eta}} \ln\big(\exp \sum_{k\in\mathcal{A}_j} \big(-\hat{\eta}\hat{\rho}^{\psi_k}(\boldsymbol{x}, t)\big)$ and aim at, similarly to (15), satisfying $-\gamma^j_{k'}(t) + \rho^j_{\max} < \hat{\rho}(\boldsymbol{x}(t), t) < \rho^j_{\max}$ for all $t \in \mathcal{I}^j$. Now, let $\boldsymbol{u}(\boldsymbol{x}, t) := -\hat{\epsilon}(\boldsymbol{x}, t - s_j) g(\boldsymbol{x})^T \frac{\partial\hat{\rho}(\boldsymbol{x},t)}{\partial\boldsymbol{x}}$ for $s_j \leq t < s_{j+1}$ where $\hat{\epsilon}(\boldsymbol{x}, t)$ is as defined previously. Also recall that $\hat{\rho}_{\text{opt}}(t) := \sup_{\boldsymbol{x}\in\mathbb{R}^n} \hat{\rho}(\boldsymbol{x}, t)$.

Theorem 4. *Consider the system* (1) *and the formula* θ *as in* (4). *Under Assumptions 1-2 and if 1)* ψ_k *is, for each* $k \in \mathcal{A}_0$, *feasible w.r.t.* r^0, $\boldsymbol{x}_0$, *and* $t_{k,*}$; *2)* $-\gamma^j_{k',0} + \rho^j_{max} < \hat{\rho}(\boldsymbol{x}(s_j), s_j)$ *at each* $s_j \in \mathcal{S}$ *as well as* $-\gamma^j_{k'}(t) + \rho^j_{max} < \hat{\rho}_{opt}(t)$ *for all* $t \in \mathcal{I}^j$, *then* $\boldsymbol{u}(\boldsymbol{x}, t)$ *ensures that* $\hat{r} \leq \min_j r^j \leq \rho^\theta(\boldsymbol{x}, 0) \leq \max_j \rho^j_{max} \leq \hat{\rho}_{max}$.

6. Simulations

A modified version of the Lotka-Volterra equations for predator-prey models is considered. In particular, consider three species x_1, x_2, and x_3 where x_3 is a predator hunting x_1 and x_2, while x_2 is a predator hunting x_1. The dynamics of this system are given by

$$\dot{x}_1 = \beta_1 x_1 - \hat{\beta}_{1,2} x_1 x_2 - \hat{\beta}_{1,3} x_1 x_3 + u_1$$

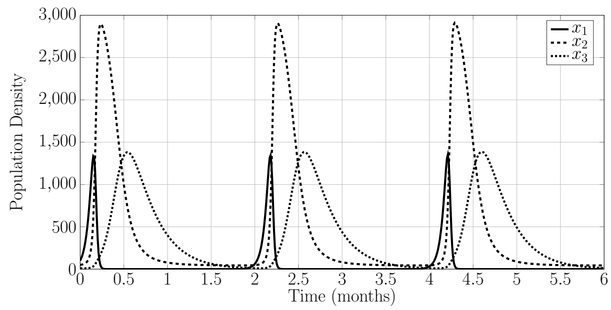


Figure 2: Population densities when no control is used.

$$\dot{x}_2 = \beta_{2,1}x_1x_2 - \hat{\beta}_{2,3}x_2x_3 + u_2$$
$$\dot{x}_3 = \beta_{3,1}x_1x_3 + \beta_{3,2}x_2x_3 - \hat{\beta}_3x_3 + u_3$$

where β_1 is the growth rate of x_1, while $\beta_{2,1}x_1$ and $\beta_{3,1}x_1 + \beta_{3,2}x_2$ are the growth rates of x_2 and x_3, respectively. The terms $\hat{\beta}_{1,2}x_1x_2$ and $\hat{\beta}_{1,3}x_1x_3$ measure the impact of predation by x_2 and x_3 on the population of x_1, while $\hat{\beta}_{2,3}x_2x_3$ measures the impact of predation by x_3 on the population of x_2. The parameter $\hat{\beta}_3$ indicates the death rate of x_3. In compact form, this system can be written as in (1) with $\boldsymbol{x} := \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$, $\boldsymbol{u} := \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^T$, $f(\boldsymbol{x}) := \begin{bmatrix} \beta_1x_1 - \hat{\beta}_{1,2}x_1x_2 - \hat{\beta}_{1,3}x_1x_3 \\ \beta_{2,1}x_1x_2 - \hat{\beta}_{2,3}x_2x_3 \\ \beta_{3,1}x_1x_3 + \beta_{3,2}x_2x_3 - \hat{\beta}_3x_3 \end{bmatrix}^T$, and $g(\boldsymbol{x}) := I_3$, where I_3 is the 3×3-identity matrix. To have a more specific example, let x_1, x_2, and x_3 represent fish population densities within an aquarium that needs to exhibit certain population densities for the visitors of the aquarium throughout the year (x_1: non-predatory fish, x_2: trout, and x_3: pike). The inputs u_1, u_2, and u_3, which are additionally inserted into the original Lotka-Volterra model, represent the actions of inserting or removing fish. From now on, let the initial state be $\boldsymbol{x}_0 := \begin{bmatrix} 100 & 50 & 10 \end{bmatrix}^T$. The parameters β_1, $\hat{\beta}_{1,2}$, $\hat{\beta}_{1,3}$, $\beta_{2,1}$, $\hat{\beta}_{2,3}$, $\beta_{3,1}$, $\beta_{3,2}$, and $\hat{\beta}_3$ have been selected so that the resulting trajectory is a periodic orbit when no control is used, i.e., $\boldsymbol{u} := \boldsymbol{0}_3$. Therefore, $\beta_1 := 21.8$, $\hat{\beta}_{1,2} := \beta_{2,1} := 0.03$, $\hat{\beta}_{1,3} := \beta_{3,1} := 0.02$, $\hat{\beta}_{2,3} := \beta_{3,2} := 0.0055$, and $\hat{\beta}_3 := 4$. The resulting trajectory is shown in Fig. 2. Note that this system exhibits different equilibria at $\{\boldsymbol{x} \in \mathbb{R}^3 | x_1 = 0, x_3 = 0\}$ and $\{\boldsymbol{x} \in \mathbb{R}^3 | x_1 = 200, x_2 = 0, x_3 = 1090\}$, and that different initial conditions may not lead to a periodic orbit.

Section 6.1 considers an unsatisfiable STL specification, while Section 6.2 considers a satisfiable STL specification consisting of temporal formulas with overlapping time intervals. All simulations have been performed in real-time on a four-core 1,4 GHz CPU with 8 GB of RAM.

6.1. Scenario 1

This scenario will, in particular, show how least violating solutions with a given gap of $\rho_{\text{gap}} := 30$ can be found. Assume now that the aquarium owner wants to establish certain fish populations between the first six months of the year, but does so, as will be verified, by imposing an unsatisfiable STL specification. During all six months, there should not be too many or too few fish in the lake. During the first two months, the aquarium owner wants to eventually establish a roughly equal population density of all fish. Between months two and three, it should eventually hold that $x_2 \geq 1100$, while between months three and four it should hold that $x_2 \geq 1100$ and $x_3 \geq 1000$. For the last two months, it should eventually hold that $x_1 \geq 300$, $x_2 \leq 200$, and $x_3 \leq 900$. The given high-level STL specification is $\theta := \phi_1 \wedge \phi_2 \wedge \phi_3 \wedge \phi_4$ with the sub-formulas as follows: $\phi_1 := F_{[0,2]}\psi_1'$, $\phi_2 := F_{[2,3]}\psi_2'$, $\phi_3 := G_{[3,4]}\psi_3'$, and $\phi_4 := F_{[4,6]}\psi_4'$ where $\psi_1' := (\|\boldsymbol{x} - \begin{bmatrix}500 & 500 & 500\end{bmatrix}^T\| \leq 500)$, $\psi_2' := (\|\boldsymbol{x} - \begin{bmatrix}500 & 500 & 500\end{bmatrix}^T\| \leq 500) \wedge (x_2 \geq 1100)$, $\psi_3' := (\|\boldsymbol{x} - \begin{bmatrix}500 & 500 & 500\end{bmatrix}^T\| \leq 500) \wedge (x_2 \geq 1100) \wedge (x_3 \geq 1000)$, and $\psi_4' := (\|\boldsymbol{x} - \begin{bmatrix}500 & 500 & 500\end{bmatrix}^T\| \leq 500) \wedge (x_1 \geq 300) \wedge (x_2 \leq 200) \wedge (x_3 \leq 900)$. Note that the sub-formula $\|\boldsymbol{x} - \begin{bmatrix}500 & 500 & 500\end{bmatrix}^T\| \leq 500$ that is contained in ψ_1', ψ_2', ψ_3', and ψ_4' ensures that part 2) in Assumption 2 is satisfied. For $\eta = 1$, it holds that $\rho_{\text{opt}}^{\psi_1'} = 500$, $\rho_{\text{opt}}^{\psi_2'} = -50.69$, $\rho_{\text{opt}}^{\psi_3'} = -117.84$, and $\rho_{\text{opt}}^{\psi_4'} = -1.08$ (note that $\bar{\rho}_{\text{opt}}^{\psi_4} = 0$). The time intervals are, in this example, not overlapping and Algorithm 1 trivially gives, for instance, $\mathcal{S} := \{s_0 := 0, s_1 := 1.7, s_2 := 2.7, s_3 := 4, s_4 := 6\}$ with $t_{1,*} := 1.7$, $t_{2,*} := 2.7$, $t_{3,*} := 3$, and $t_{4,*} := 6$ and $\mathcal{A}_0 = \{1\}$, $\mathcal{A}_1 = \{2\}$, $\mathcal{A}_2 = \{3\}$, and $\mathcal{A}_3 = \{4\}$. Since $\rho_{\text{gap}} := 30$, we select $r^0 := 450$, $\rho_{\max}^0 := 600$, $r^1 := -80$, $\rho_{\max}^1 := -20$, $r^2 := -145$, $\rho_{\max}^2 := -50$, $r^3 := -30$, and $\rho_{\max}^3 := 5$. Without control, it is obvious from Fig. 2 that no least violating solution with a given gap of $\rho_{\text{gap}} := 30$ can be found and the fish demand can not be served. The simulation results are shown in Fig. 3. From Fig. 3a it follows that θ is not satisfied, which comes at no surprise since the formula is unsatisfiable, i.e., $\nexists \boldsymbol{x} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ s.t. $(\boldsymbol{x}, 0) \models \theta$. However, we found a least violating solution with a gap of $\rho_{\text{gap}} = 30$ since the third subformula is the most violating subformula with $\rho_{\text{opt}}^{\psi_3'} = -117.84$ and since we selected $r^2 := -145$. It can be concluded that $\hat{r} := -145 \leq \rho^{\theta}(\boldsymbol{x}, 0)$, which is also illustrated in Fig. 3b. Note in Fig. 3b that $\hat{\rho}(\boldsymbol{x}(t), t) := \rho^{\psi_i'}(\boldsymbol{x}(t))$ for $t \in \mathcal{I}^j$ if $|\mathcal{A}_j| = 1$ and $\mathcal{A}_j = \{i\}$. Fig. 3c shows the bounded and piecewise continuous control inputs. Note again that no knowledge of $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$, except of those stated in Assumption 1, are needed. We remark that the couplings in $f(\boldsymbol{x})$ are nonlinear and quite strong, especially when the STL formula requires to increase x_1. Increasing x_1 automatically leads to an increase of x_2 and x_3 (predators) that in turn decrease x_1 (prey).

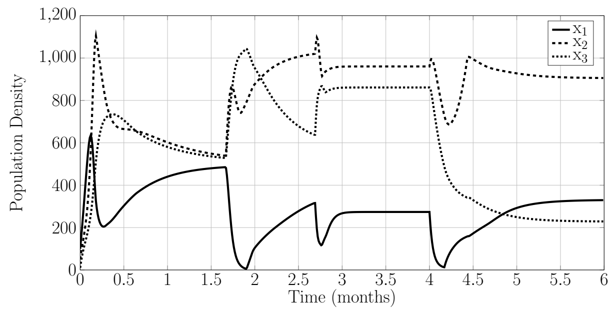
Remark 8. *The computation of $\boldsymbol{u}(\boldsymbol{x}, t)$ took, on average,* 0.1 *ms since $\boldsymbol{u}(\boldsymbol{x}, t)$ is given by a closed-form expression. For a comparison of computation times, we used the toolbox in [29] for the mixed inter linear programming (MILP) method* *originally presented in [13] for a discretized form of the system with a sampling time of* 0.1 *s. All nonlinear predicates functions have been converted to linear ones by using the infinity vector norm instead of the Euclidean norm. Creating the parametric MILP took* 177 *seconds, while solving this parametric MILP (which provides an open-loop control law) took* 8.5 *seconds. Learning-based approaches, such as in [19], can also address the problem for discrete-time systems. Note, however, that such an approach, which uses Q-learning, does not guarantee formula satisfaction during learning, hence not guaranteeing safe learning. Furthermore, these approaches do not provide any formal guarantees, even after completed learning. Since the learning process is in general quite expensive, we used, in our recent work [22], funnel-based control laws as presented in this paper to accelerate the learning process. Our proposed method can hence be seen as a complement to existing appraoches that are based on Q-learning.*

6.2. Scenario 2

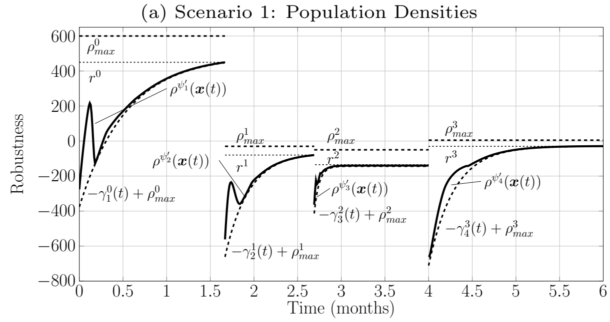
Consider next a satisfiable STL specification $\theta := \phi_1 \wedge \phi_2 \wedge \phi_3 \wedge \phi_4 \wedge \phi_5$ where ϕ_1, ϕ_2, ϕ_3, ϕ_4, and ϕ_5 have overlapping time intervals. In particular, let $\phi_1 := G_{[2,5]}\psi_1'$, $\phi_2 := G_{[2.5,3.5]}\psi_2'$, $\phi_3 := F_{[3.5,3.5]}\psi_3'$, $\phi_4 := F_{[0,6]}\psi_4'$, and $\phi_5 := F_{[3,6]}\psi_5'$ where $\psi_1' := (x_1 \leq 500) \wedge (x_2 \leq 300) \wedge (x_3 \leq 200)$, $\psi_2' := (0 \leq x_2) \wedge (x_2 \leq 200) \wedge (0 \leq x_3) \wedge (x_3 \leq 200)$, $\psi_3' := (x_1 \geq 300)$, $\psi_4' := (\|\boldsymbol{x} - \begin{bmatrix}100 & 100 & 100\end{bmatrix}^T\| \leq 100)$, and $\psi_5' := (\|\boldsymbol{x} - \begin{bmatrix}400 & 400 & 400\end{bmatrix}^T\| \leq 100)$. The time intervals are now overlapping and Algorithm 1 returns, for instance, $\mathcal{S} := \{s_0 := 0, s_1 := 1.3, s_2 := 3.5, s_3 := 5, s_4 := 6\}$ with $t_{1,*} := 2$, $t_{2,*} := 2.5$, $t_{3,*} := 3.5$, $t_{4,*} := 1.3$, and $t_{5,*} := 6$. Note also that $\mathcal{A}_0 = \{4\}$, $\mathcal{A}_1 = \{1, 2, 3\}$, $\mathcal{A}_2 = \{1\}$, and $\mathcal{A}_3 = \{5\}$ so that three funnel-based constraints are active between $[s_1, s_2]$ (illustrated in particular in Fig. 4c). To satisfy Assumption 2, each ϕ_k has been modified as stated in Remark 3 and we set $\rho^j_{\text{max}} := 600$ for each $j \in \{0, 1, 2, 3\}$, while $\rho^j_{\text{opt}} = 100$ and $r^j := 50$. The results are shown in Fig. 4 and it can be verified, both in Figs. 4a and 4b, that $\hat{r} := 50 \leq \rho^\theta(\boldsymbol{x}, 0) \leq 600$.

7. Conclusion

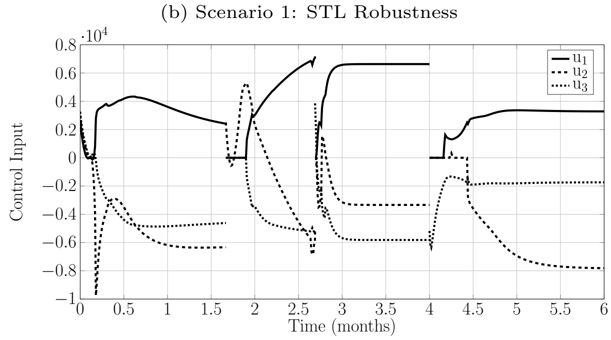
We considered the control of fully actuacted systems under a fragment of signal temporal logic specifications without discretizing neither the system dynamics nor the environment in space or time. A continuous feedback control law for *atomic temporal logic formulas* was derived by considering a funnel-based feedback control strategy where the transient behavior of the funnel was exploited. This control law is robust in two ways. First, bounded additive noise does not affect the satisfaction of the signal temporal logic specification. Second, the robust semantics of signal temporal logic, stating how robustly a specification is satisfied, have been used for the control synthesis. Hence, a user-defined robustness can be imposed, i.e., giving the user the choice of how robustly the specification should be satisfied. A switched control systems allows to use several funnel constraints at the same time to control systems under



(a) Scenario 1: Population Densities

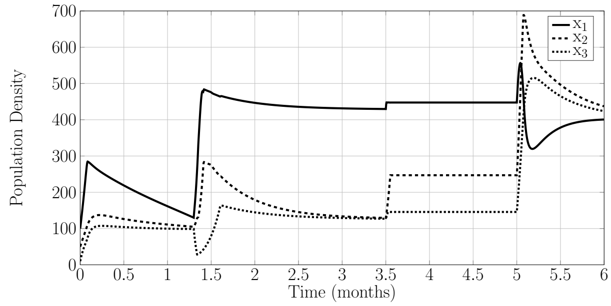


(b) Scenario 1: STL Robustness

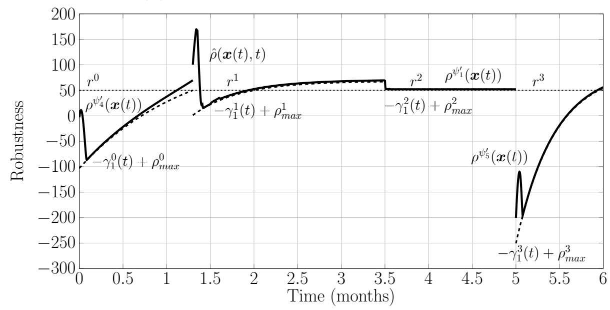


(c) Scenario 1: Control Inputs

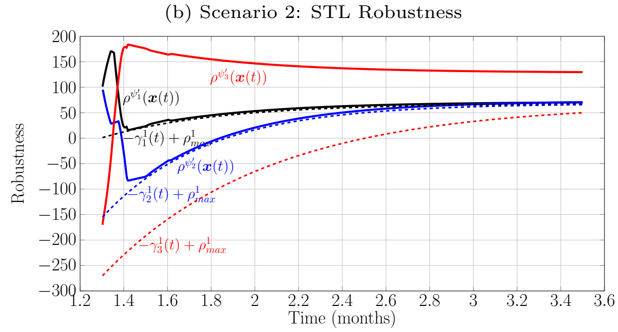
Figure 3: Simulation results for Scenario 1.



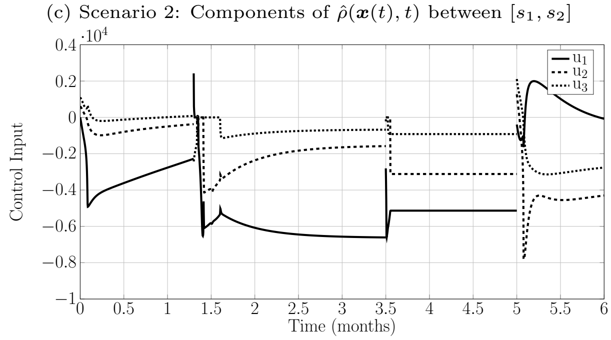
(a) Scenario 2: Population Densities



(b) Scenario 2: STL Robustness



(c) Scenario 2: Components of $\hat{\rho}(\boldsymbol{x}(t), t)$ between $[s_1, s_2]$



(d) Scenario 2: Control Inputs

Figure 4: Simulation results for Scenario 2.

more nested signal temporal logic specifications. The framework can find least violating solutions when the specification is not satisfiable. The control law is given by a closed-form expression and hence computationally tractable.

Appendix

Proof of Lemma 1:

For $v_1, \ldots, v_q \in \mathbb{R}$, we know from [27, p.72] that

$$\max_{i\in\{1,\ldots,q\}} v_i \leq \ln\Big(\sum_{i=1}^{q} \exp(v_i)\Big) \leq \max_{i\in\{1,\ldots,q\}} v_i + \ln(q).$$

Set $\max_{i\in\{1,\ldots,q\}} v_i = -\min_{i\in\{1,\ldots,q\}}\{-v_i\}$, substitute $v_i := -\eta\rho^{\psi_i}(\boldsymbol{x}(t))$, and note that $\rho^{\psi_i}(\boldsymbol{x}(t)) = \bar{\rho}^{\psi_i}(\boldsymbol{x}(t))$ since ψ_i contains no further conjunctions. Multiply the inequalities on each side by $\frac{1}{\eta}$ and the result follows. ■

Proof of Theorem 1:

We proceed as follows: in Step A, we use [30, Theorem 54] and show that there exists a unique and maximal solution $\boldsymbol{x} : \mathcal{J} \to \mathbb{R}^n$ of (1) such that $\xi(t) \in \Omega_\xi := (-1, 0)$ for all $t \in \mathcal{J}$ where $\mathcal{J} := [0, \tau_{\max}) \subseteq \mathbb{R}_{\geq 0}$ with $\tau_{\max} > 0$. Step B consists of using [30, Proposition C.3.6] to show that $\boldsymbol{x}$ is also complete, i.e., $\tau_{\max} = \infty$. With (7), (8), and (9), the dynamics of ϵ are given by

$$\dot{\epsilon} = \frac{\partial\epsilon}{\partial\xi}\dot{\xi} = -\frac{1}{\gamma\xi(1+\xi)}\Big(\frac{\partial\rho^{\psi}(\boldsymbol{x})}{\partial\boldsymbol{x}}^T\dot{\boldsymbol{x}} - \xi\dot{\gamma}\Big),$$

which can be derived since $\frac{\partial\epsilon}{\partial\xi} = -\frac{1}{\xi(1+\xi)}$ and $\dot{\xi} = \frac{1}{\gamma}(\dot{e} - \xi\dot{\gamma})$. Note also that $\dot{e} = \frac{\partial e(\boldsymbol{x})}{\partial\boldsymbol{x}}^T\dot{\boldsymbol{x}}$ with $\frac{\partial e(\boldsymbol{x})}{\partial\boldsymbol{x}} = \frac{\partial\rho^{\psi}(\boldsymbol{x})}{\partial\boldsymbol{x}}$.

Step A: First, define the stacked vector $\boldsymbol{y} := \begin{bmatrix}\boldsymbol{x}^T & \xi\end{bmatrix}^T$. Consider the closed-loop system that is obtained by inserting (10) into (1) resulting in $\dot{\boldsymbol{x}} = H_1(\boldsymbol{x}, \xi, t)$ with

$$H_1(\boldsymbol{x}, \xi, t) := f(\boldsymbol{x}) - \ln\Big(-\frac{\xi+1}{\xi}\Big)g(\boldsymbol{x})g(\boldsymbol{x})^T\frac{\partial\rho^{\psi}(\boldsymbol{x})}{\partial\boldsymbol{x}} + \boldsymbol{w}(t).$$

We also obtain $\dot{\xi} = H_2(\boldsymbol{x}, \xi, t)$ with

$$H_2(\boldsymbol{x}, \xi, t) := \frac{1}{\gamma(t)}\Big(\frac{\partial\rho^{\psi}(\boldsymbol{x})}{\partial\boldsymbol{x}}^T H_1(\boldsymbol{x}, \xi, t) - \xi\dot{\gamma}(t)\Big),$$

which finally results in $\dot{\boldsymbol{y}} = H(\boldsymbol{y}, t)$ with $H(\boldsymbol{y}, t) := \Big[H_1(\boldsymbol{x}, \xi, t)^T \quad H_2(\boldsymbol{x}, \xi, t)\Big]^T$. Since $\rho^{\psi}_{\text{opt}} < \rho_{\max}$ and $-\gamma_0 + \rho_{\max} < \rho^{\psi}(\boldsymbol{x}_0)$, $\boldsymbol{x}_0$ is such that $\xi(\boldsymbol{x}_0, 0) \in \Omega_\xi :=$

$(-1,0)$ so that $\boldsymbol{x}_0 \in \Omega_{\boldsymbol{x}}(0)$ where $\Omega_{\boldsymbol{x}}(t)$ is defined as

$$\Omega_{\boldsymbol{x}}(t) := \Big\{\boldsymbol{x} \in \mathbb{R}^n | -1 < \xi(\boldsymbol{x},t) = \frac{\rho^{\psi}(\boldsymbol{x}) - \rho_{\max}}{\gamma(t)} < 0\Big\}.$$

The time-varying set $\Omega_{\boldsymbol{x}}(t)$ has the property that for $t_1 < t_2$ it holds that $\Omega_{\boldsymbol{x}}(t_1) \supseteq \Omega_{\boldsymbol{x}}(t_2)$ since γ is non-increasing. Note also that $\Omega_{\boldsymbol{x}}(t)$ is non-empty for each $t \geq 0$ since $-\gamma_\infty + \rho_{\max} < \rho^{\psi}_{\text{opt}} < \rho_{\max}$, and that $\Omega_{\boldsymbol{x}}(t)$ is bounded due to part 2) in Assumption 2. The former is important to ensure a non-empty funnel, i.e., (5) is always well-defined (used later to ensure that, for $\boldsymbol{x}(t')$ with $\xi(\boldsymbol{x}(t'),t') \in \Omega_\xi$ it holds that $\boldsymbol{x}(t') \in \Omega_{\boldsymbol{x}}(t')$, which guarantees the existence of a solution $\boldsymbol{y} : [t',\tau_{\max}) \to \mathbb{R}^{n+1}$ with $\tau_{\max} > 0$ to $\dot{\boldsymbol{y}} = H(\boldsymbol{y},t)$). Define also $\Omega_{\boldsymbol{x}_0} := \Omega_{\boldsymbol{x}}(0)$. Due to [31, Proposition 1.4.4], the inverse image of an open (closed) set under a continuous function is open (closed). With $\xi_0(\boldsymbol{x}) := \xi(\boldsymbol{x},0)$, it holds that the inverse image ${\xi_0}^{-1}(\Omega_\xi) = \Omega_{\boldsymbol{x}}(0)$ is open. Finally, define the open, bounded, and non-empty set $\Omega_{\boldsymbol{y}} := \Omega_{\boldsymbol{x}_0} \times \Omega_\xi$ for which it holds that $\boldsymbol{y}_0 := \begin{bmatrix}{\boldsymbol{x}_0}^T & \xi(\boldsymbol{x}_0,0)\end{bmatrix}^T \in \Omega_{\boldsymbol{y}}$. Next, the conditions of [30, Theorem 54] for the initial value problem $\dot{\boldsymbol{y}} = H(\boldsymbol{y},t)$ with $\boldsymbol{y}_0 \in \Omega_{\boldsymbol{y}}$ and $H(\boldsymbol{y},t) : \Omega_{\boldsymbol{y}} \times \mathbb{R}_{\geq 0} \to \mathbb{R}^{n+1}$ need to be checked: 1) $H(\boldsymbol{y},t)$ is locally Lipschitz continuous on $\boldsymbol{y}$ since $f(\boldsymbol{x})$, $g(\boldsymbol{x})$, $\frac{\partial \rho^{\psi}(\boldsymbol{x})}{\partial \boldsymbol{x}}$, and $\epsilon = \ln\big(-\frac{\xi+1}{\xi}\big)$ are locally Lipschitz continuous on $\boldsymbol{y}$ for each $t \in \mathbb{R}_{\geq 0}$; 2) $H(\boldsymbol{y},t)$ is continuous on t for each fixed $\boldsymbol{y} \in \Omega_{\boldsymbol{y}}$ due to continuity of $\gamma(t)$ and $\dot{\gamma}(t)$. Since $\Omega_{\boldsymbol{y}}$ is non-empty and open, there exists a unique and maximal solution $\boldsymbol{y} : \mathcal{J} \to \Omega_{\boldsymbol{y}}$ with $\mathcal{J} := [0,\tau_{\max})$ and $\tau_{\max} > 0$ so that $\xi(t) \in \Omega_\xi$ and $\boldsymbol{x}(t) \in \Omega_{\boldsymbol{x}_0}$ for all $t \in \mathcal{J}$.

Step B: From Step A), it is known that $\boldsymbol{y}(t) \in \Omega_{\boldsymbol{y}}$ for all $t \in \mathcal{J}$. Next, we show that $\boldsymbol{y}$ is complete, i.e., $\tau_{\max} = \infty$, by contradiction of [30, Proposition C.3.6]. Therefore, assume $\tau_{\max} < \infty$ and consider first $\boldsymbol{x} \in \Omega_{\boldsymbol{x}_0}$ such that $\frac{\partial \rho^{\psi}(\boldsymbol{x})}{\partial \boldsymbol{x}} = \boldsymbol{0}_n$. It then holds that $\rho^{\psi}(\boldsymbol{x}) = \rho^{\psi}_{\text{opt}}$ since $\rho^{\psi}(\boldsymbol{x})$ is concave. Recall that $-\gamma_\infty + \rho_{\max} < \rho^{\psi}_{\text{opt}} < \rho_{\max}$. There exist, for all $t \in \mathcal{J}$, some $\zeta > 0$ with $\frac{\zeta}{\gamma_0} \in (0,1)$ and a neighborhood $\mathcal{U}$ around $\boldsymbol{x}$ such that $-\gamma(t) + \rho_{\max} \leq \rho^{\psi}(\boldsymbol{x}') - \zeta$ and $\rho^{\psi}(\boldsymbol{x}') + \zeta \leq \rho_{\max}$ for each $\boldsymbol{x}' \in \mathcal{U}$. Consequently, it can be concluded that $|\epsilon(\boldsymbol{x}',t)| \leq \max\big(|S(-1+\frac{\zeta}{\gamma_0})|, |S(-\frac{\zeta}{\gamma_0})|\big) =: \bar{\epsilon}_1$. Consider next $\boldsymbol{x} \in \Omega_{\boldsymbol{x}_0} \setminus \mathcal{U}$ and let $V(\epsilon) = \frac{1}{2}\epsilon^2$ with

$$\begin{aligned}\dot{V} = \epsilon\dot{\epsilon} &= \epsilon\Big(-\frac{1}{\gamma\xi(1+\xi)}\big(\frac{\partial \rho^{\psi}(\boldsymbol{x})}{\partial \boldsymbol{x}}^T \dot{\boldsymbol{x}} - \xi\dot{\gamma}\big)\Big) \\ &= \epsilon\alpha\Big(\frac{\partial \rho^{\psi}(\boldsymbol{x})}{\partial \boldsymbol{x}}^T \big(f(\boldsymbol{x}) + g(\boldsymbol{x})\boldsymbol{u} + \boldsymbol{w}\big) - \xi\dot{\gamma}\Big)\end{aligned}$$

where $\alpha := -\frac{1}{\gamma\xi(1+\xi)}$ satisfies $\alpha(t) \in [\frac{4}{\gamma_0},\infty) \in \mathbb{R}_{>0}$ for all $t \in \mathcal{J}$. This follows since $\frac{4}{\gamma_0} \leq -\frac{1}{\gamma_0 \xi(1+\xi)} \leq -\frac{1}{\gamma \xi(1+\xi)} \leq -\frac{1}{\gamma_\infty \xi(1+\xi)} < \infty$ for $\xi \in \Omega_\xi$. Consequently,

$$\begin{aligned}\dot{V} &\leq |\epsilon|\alpha\big(\|\frac{\partial \rho^\psi(\boldsymbol{x})}{\partial \boldsymbol{x}}\|\|f(\boldsymbol{x}) + \boldsymbol{w}\| + |\xi\dot{\gamma}|\big) + \epsilon\alpha\frac{\partial \rho^\psi(\boldsymbol{x})}{\partial \boldsymbol{x}}^T g(\boldsymbol{x})\boldsymbol{u} \\ &\leq |\epsilon|\alpha k_1 + \epsilon\alpha\frac{\partial \rho^\psi(\boldsymbol{x})}{\partial \boldsymbol{x}}^T g(\boldsymbol{x})\boldsymbol{u}, \end{aligned} \tag{17}$$

where the positive constant k_1 is derived as follows; $\|f(\boldsymbol{x})\|$ and $\|\frac{\partial \rho^\psi(\boldsymbol{x})}{\partial \boldsymbol{x}}\|$ are upper bounded due to continuity of $f(\boldsymbol{x})$ and $\frac{\partial \rho^\psi(\boldsymbol{x})}{\partial \boldsymbol{x}}$ on $\mathrm{cl}(\Omega_{\boldsymbol{x}_0})$, where cl denotes the closure operation, and since $\mathrm{cl}(\Omega_{\boldsymbol{x}_0})$ is a compact set. Note that $\boldsymbol{x}(t) \in \Omega_{\boldsymbol{x}}(t) \subseteq \Omega_{\boldsymbol{x}_0}$ for all $t \in \mathcal{J}$. Furthermore, $\boldsymbol{w} \in \mathcal{W}$, $\xi \in \Omega_\xi$, and $\dot{\gamma}$ are bounded and thus the upper bound k_1 follows. Next, insert the control law (10) into (17), which results in

$$\dot{V} \leq |\epsilon|\alpha k_1 - \epsilon^2\alpha\frac{\partial \rho^\psi(\boldsymbol{x})}{\partial \boldsymbol{x}}^T g(\boldsymbol{x})g(\boldsymbol{x})^T\frac{\partial \rho^\psi(\boldsymbol{x})}{\partial \boldsymbol{x}} \leq |\epsilon|\alpha\big(k_1 - |\epsilon|\lambda_{\min}\|\frac{\partial \rho^\psi(\boldsymbol{x})}{\partial \boldsymbol{x}}\|^2\big),$$

where $\lambda_{\min} > 0$ is the minimum eigenvalue of $g(\boldsymbol{x})g^T(\boldsymbol{x})$, which is positive according to Assumption 1. It further holds that $0 < k_2 \leq \|\frac{\partial \rho^\psi(\boldsymbol{x})}{\partial \boldsymbol{x}}\|^2$ for a positive constant k_2 since we consider, at this point, only $\boldsymbol{x} \in \Omega_{\boldsymbol{x}_0} \setminus \mathcal{U}$. Hence,

$$\dot{V} \leq |\epsilon|\alpha\big(k_1 - |\epsilon|\lambda_{\min}k_2\big)$$

so that $\dot{V} \leq 0$ if $\bar{\epsilon}_2 := \frac{k_1}{\lambda_{\min}k_2} \leq |\epsilon|$. Now, again considering $\boldsymbol{x} \in \Omega_{\boldsymbol{x}_0}$, i.e., both of the above cases simultaneously, $\dot{V} \leq 0$ if $\max(\bar{\epsilon}_1, \bar{\epsilon}_2) \leq |\epsilon|$ and it can be concluded that $|\epsilon(t)|$ will be upper bounded due to the level sets of $V(\epsilon)$ as $|\epsilon(t)| \leq \max\left(|\epsilon(0)|, \bar{\epsilon}_1, \bar{\epsilon}_2\right)$, which leads to the conclusion that $\epsilon(t)$ is upper and lower bounded by some constants ϵ_u and ϵ_l, respectively. In other words, it holds that $\epsilon_l \leq \epsilon(t) \leq \epsilon_u$ for all $t \in \mathcal{J}$. By using the inverse of S, the normalized error $\xi(t)$ can be bounded as $-1 < \xi_l := -\frac{1}{\exp(\epsilon_l+1)} \leq \xi(t) \leq \xi_u := -\frac{1}{\exp(\epsilon_u+1)} < 0$ for all $t \in \mathcal{J}$, which means that

$$\xi(t) \in [\xi_l, \xi_u] =: \Omega'_\xi \subset \Omega_\xi \text{ for all } t \in \mathcal{J}$$

and consequently $\boldsymbol{x}(t) \in \Omega'_{\boldsymbol{x}}(t)$ for all $t \in \mathcal{J}$ where

$$\Omega'_{\boldsymbol{x}}(t) := \Big\{\boldsymbol{x} \in \Omega_{\boldsymbol{x}} | \xi_l \leq \xi(\boldsymbol{x}, t) = \frac{\rho^\psi(\boldsymbol{x}) - \rho_{\max}}{\gamma(t)} \leq \xi_u\Big\}.$$

Note in particular that $\Omega'_{\boldsymbol{x}}(t) \subset \Omega_{\boldsymbol{x}}(t) \subseteq \Omega_{\boldsymbol{x}_0}$ and that $\Omega'_{\boldsymbol{x}}(t)$ is non-empty for each $t \geq 0$ since $\Omega_{\boldsymbol{x}}(t)$ is non-empty for each $t \geq 0$ and $\xi(\boldsymbol{x}, t)$ is continuous. Also note that it can be shown that $\Omega'_{\boldsymbol{x}}(t)$ is compact by again using [31, Proposition 1.4.4]. By defining $\Omega'_{\boldsymbol{x}_0} := \Omega'_{\boldsymbol{x}}(0)$, it can be concluded that

$$\boldsymbol{x}(t) \in \Omega'_{\boldsymbol{x}_0} \subset \Omega_{\boldsymbol{x}_0} \text{ for all } t \in \mathcal{J}.$$

Define the compact set $\Omega'_{\boldsymbol{y}} := \Omega'_{\boldsymbol{x}_0} \times \Omega'_{\xi}$ and notice that $\Omega'_{\boldsymbol{y}} \subset \Omega_{\boldsymbol{y}}$ by which it follows that there is no $t \in \mathcal{J} := [0, \tau_{\max})$ such that $\boldsymbol{y} \notin \Omega'_{\boldsymbol{y}}$. By contradiction of [30, Proposition C.3.6] it follows that $\tau_{\max} = \infty$, i.e., $\mathcal{J} = \mathbb{R}_{\geq 0}$. The control law $\boldsymbol{u}(\boldsymbol{x}, t)$ is locally Lipschitz continuous and bounded, because $\rho^{\psi}(\boldsymbol{x})$, $\epsilon(\boldsymbol{x}, t)$, $g(\boldsymbol{x})$, and $\gamma(t)$ are locally Lipschitz continuous and bounded on $\Omega'_{\boldsymbol{x}}$ and $\mathcal{J}$. ■

Proof of Theorem 2:

Note that $\rho_{\max} \in (\rho^{\psi}_{\text{opt}}, \infty)$ ensures $\rho^{\psi}_{\text{opt}} < \rho_{\max}$, while γ_0 as in (11) ensures $-\gamma_0 + \rho_{\max} < \rho^{\psi}(\boldsymbol{x}_0)$. Additionally choosing γ_∞ as in (12) ensures $-\gamma_\infty + \rho_{\max} < \rho^{\psi}_{\text{opt}}$ so that the assumptions of Theorem 1 are satisfied. Note also that l as in (13) then ensures $r \leq \rho^{\psi}(\boldsymbol{x}(t)) \leq \rho_{\max}$ for all $t \geq t_*$ if (10) is applied due to Theorem 1, which leads to $r \leq \rho^{\phi}(\boldsymbol{x}, 0) \leq \rho_{\max}$ by the choice of t_*. This follows since γ is such that $r \leq -\gamma(t) + \rho_{\max}$ for all $t \geq t_*$. Hence, it holds that $0 < r \leq \rho^{\phi}(\boldsymbol{x}, 0) \leq \rho_{\max}$, which in turn results in $(\boldsymbol{x}, 0) \models \phi$. ■

Proof of Theorem 3:

The dynamics of $\hat{\epsilon}$ are now given by

$$\dot{\hat{\epsilon}} = \frac{\partial \hat{\epsilon}}{\partial \hat{\xi}} \dot{\hat{\xi}} = -\frac{1}{\gamma' \hat{\xi}(1+\hat{\xi})} \Big(\frac{\partial \hat{\rho}(\boldsymbol{x}, t)}{\partial \boldsymbol{x}}^T \dot{\boldsymbol{x}} + \frac{\partial \hat{\rho}(\boldsymbol{x}, t)}{\partial t} - \hat{\xi} \dot{\gamma}' \Big),$$

which can be derived since, again, $\frac{\partial \hat{\epsilon}}{\partial \hat{\xi}} = -\frac{1}{\hat{\xi}(1+\hat{\xi})}$ and $\dot{\hat{\xi}} = \frac{1}{\gamma'}(\dot{\hat{e}} - \hat{\xi}\dot{\gamma}')$, but now with $\dot{\hat{e}} = \frac{\partial \hat{e}(\boldsymbol{x}, t)}{\partial \boldsymbol{x}}^T \dot{\boldsymbol{x}} + \frac{\partial \hat{e}(\boldsymbol{x}, t)}{\partial t}$ where $\frac{\partial \hat{e}(\boldsymbol{x}, t)}{\partial \boldsymbol{x}} = \frac{\partial \hat{\rho}(\boldsymbol{x}, t)}{\partial \boldsymbol{x}}$ and $\frac{\partial \hat{e}(\boldsymbol{x}, t)}{\partial t} = \frac{\partial \hat{\rho}(\boldsymbol{x}, t)}{\partial t}$.

Note first that $\hat{\rho}_{\text{opt}}(t) < \rho_{\max}$ for all $t \in \mathcal{I}$ due to the choice of $\rho_{\max}$ and since $\hat{\rho}(\boldsymbol{x}, t) \leq \min(\rho^{\psi'}(\boldsymbol{x}), \hat{\rho}^{\psi''}(\boldsymbol{x}, t))$. By assumption, $-\gamma'(t) + \rho_{\max} < \hat{\rho}_{\text{opt}}(t)$ for all $t \in \mathcal{I}$ which ensures that (15) is well defined for all $t \in \mathcal{I}$, i.e., $\hat{\Omega}_{\boldsymbol{x}}(t) := \big\{\boldsymbol{x} \in \mathbb{R}^n | -1 < \hat{\xi}(\boldsymbol{x}, t) = \frac{\hat{\rho}(\boldsymbol{x}, t) - \rho_{\max}}{\gamma'(t)} < 0\big\}$ is non-empty for each $t \in \mathcal{I}$. Additionally, the assumption $-\gamma'_0 + \rho_{\max} < \hat{\rho}(\boldsymbol{x}_0, 0)$ ensures that $\boldsymbol{x}_0 \in \hat{\Omega}_{\boldsymbol{x}}(0) =: \hat{\Omega}_{\boldsymbol{x}_0}$. Similarly to Step A in the proof of Theorem 1, we can define $\boldsymbol{y} := \begin{bmatrix}\boldsymbol{x}^T & \hat{\xi}\end{bmatrix}^T$ and conclude that there exists a unique and maximal solution $\boldsymbol{y} : \mathcal{J} \to \Omega_{\boldsymbol{y}}$ with $\mathcal{J} := [0, \tau_{\max}) \subseteq \mathcal{I}$ and $\tau_{\max} > 0$ so that $\hat{\xi}(t) \in \Omega_{\xi}$ and $\boldsymbol{x}(t) \in \hat{\Omega}_{\boldsymbol{x}_0}$ for all $t \in \mathcal{J}$. Similarly to Step B in the proof of Theorem 1 and by defining $V(\hat{\epsilon}) = \frac{1}{2}\hat{\epsilon}^2$ we have

$$\begin{aligned}
\dot{V} = \hat{\epsilon}\dot{\hat{\epsilon}} &= \hat{\epsilon}\Big(\frac{-1}{\gamma' \hat{\xi}(1+\hat{\xi})} \Big(\frac{\partial \hat{\rho}(\boldsymbol{x}, t)}{\partial \boldsymbol{x}}^T \dot{\boldsymbol{x}} + \frac{\partial \hat{\rho}(\boldsymbol{x}, t)}{\partial t} - \hat{\xi}\dot{\gamma}' \Big)\Big) \\
&= \hat{\epsilon}\hat{\alpha}\Big(\frac{\partial \hat{\rho}(\boldsymbol{x}, t)}{\partial \boldsymbol{x}}^T \big(f(\boldsymbol{x}) + g(\boldsymbol{x})\boldsymbol{u} + \boldsymbol{w} \big) + \frac{\partial \hat{\rho}(\boldsymbol{x}, t)}{\partial t} - \xi\dot{\gamma} \Big)
\end{aligned}$$

where $\hat{\alpha} := -\frac{1}{\gamma' \hat{\xi}(1+\hat{\xi})}$ satisfies $\hat{\alpha}(t) \in [\frac{4}{\gamma'_0}, \infty) \in \mathbb{R}_{>0}$ for all $t \in \mathcal{J}$. It can again be shown that $\dot{V} \leq |\hat{\epsilon}|\hat{\alpha}k_1 + \hat{\epsilon}\hat{\alpha}\frac{\partial \hat{\rho}(\boldsymbol{x}, t)}{\partial \boldsymbol{x}}^T g(\boldsymbol{x})\boldsymbol{u}$ where k_1 now, compared to (17), also depends on an upper bound of $\big|\frac{\partial \hat{\rho}(\boldsymbol{x}, t)}{\partial t}\big|$, which exists since $\frac{\partial \hat{\rho}(\boldsymbol{x}, t)}{\partial t}$ is continuous and $\mathcal{I}$ is compact. By again noting that (15) implies (14), the rest of the proof is the same as in the proof of Theorem 1. ■

Proof of Theorem 4:

Condition 1) only needs to be ensured at s_0, but not at s_j for $j \in \{1, \dots, q\}$. This holds since the set $\mathcal{A}_j$ considers, to determine the active atomic temporal formulas, the time interval $(s_j, s_{j+1}]$ (including s_{j+1}), so that only initial feasibility in the sense of Definition 2 is needed. The set $\mathcal{S}$ is, in Algorithm 1, constructed such that only dependent atomic temporal formulas are grouped into one cluster. The parameter $t_{k,*}$ as well as $\gamma_k^j(t)$ are constructed to account for ϕ_k. By the same arguments as in Theorem 3 and since condition 2) holds for each time interval $\mathcal{I}^j$, we know that $-\gamma_{k'}^j(t) + \rho_{\max}^j < \hat{\rho}(\boldsymbol{x}(t), t) < \rho_{\max}^j$ for all $t \in \mathcal{I}^j$, by which also $-\gamma_k^j(t) + \rho_{\max}^j < \rho^{\psi_k}(\boldsymbol{x}(t)) < \rho_{\max}^j$ holds for all $t \in \mathcal{I}^j$ and for each $k \in \mathcal{A}_j$. Hence, $\min_{j \in \{0,\dots,q\}} r^j \leq \rho^\theta(\boldsymbol{x}, 0) \leq \max_{j \in \{0,\dots,q\}} \rho_{\max}^j$ and consequently $\hat{r} \leq \rho^\theta(\boldsymbol{x}, 0) \leq \hat{\rho}_{\max}$. ■

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