# Event-Triggered Predictor-Based Control of Networked Systems with Input Delays

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***Abstract*— Modern wireless systems can accurately estimate network latencies; this information could help design better controllers for systems prone to large delays. In this paper, we consider the problem of event-triggered control of linear networked systems with known input delays and address it via hybrid system approach. Specifically, we design a predictor-based state observer that incorporates input delay and propagated outputs to estimate the *future* state of the system. Subsequently, this estimated state is utilized to generate control inputs for transmissions over the network. In order to reduce the number of transmissions, we design a dynamic event-triggering mechanism (ETM) which makes decisions on whether or not to transmit the control inputs at pre-defined instants. The ETM, by its design, is devoid of Zeno behavior.**

***Index Terms*— Predictor-based control, Time-delay systems, Event-triggered control, Networked systems**

## I. Introduction

LATENCY related imperfections for both linear and nonlinear networked control systems (NCSs), over the years, have been handled through time-delay or impulsive/hybrid modeling methods which rely on the robustness of NCSs (largely) with respect to small, often unknown, delays, see [1]–[3]. However, it is widely known that modern communication systems can often accurately estimate the latencies and reliabilities associated with the network through resource allocation. This information, from NCSs' standpoint, can be exploited by the controllers to mitigate the effect of large delays as discussed in survey paper [4]. This notion is actively being pursued under the umbrella of predictor-based control for NCSs, see [4]. In this context, studies typically focus on two approaches, namely, discrete-time methods, see [5], and time-delay methods, see [6], [7]. Furthermore, most of the existing results that employ predictor-based controllers in NCSs, with some exceptions, are based on time-triggered/sampled-data approaches, see [5], [8]–[10].

On the other hand, scheduling in NCSs can benefit from adopting event-triggered methodologies, namely, transmissions are allowed only upon occurrence of specific events, see [11]–[13]. These event-triggered approaches for NCSs have been shown to significantly reduce the number of controller transmissions/updates that are needed to stabilize the system, see [12]. Among these approaches too there are two major categories: i) continuous-time event-triggering mechanisms (ETMs) where events are monitored continuously in time, see [11], [12], and ii) periodic ETMs, where events are monitored periodically (or intermittently) at pre-defined instants, see [14]. The later, namely, periodic ETMs, are often a preferred choice for digital implementation of NCSs. Moreover, it is noted that dynamic ETMs, first introduced in [13], could potentially outperform static ETMs, i.e., the former could lead to fewer number of events, see [15].

To counteract network latency and to simultaneously ensure the best use of network resources, the problem of designing predictor-based event-triggering controllers has gained a lot of attention in recent times, see [6], [7], [16]–[18] for linear NCSs and [19] for nonlinear NCSs. This problem was studied employing discrete-time methods in [16], [17], wherein, [16] considered a continuous-time plant with a (strictly) periodic output sampler and, subsequently, designed a discrete-time controller whereas [17] addressed the problem entirely in a discrete-time framework. Studies in [6], [7], [19] employed time-delay methods for modeling both periodic/aperiodic sampling effects and network latencies. While [19] studied continuous-time ETMs, the controllers in [6], [7], [18] were equipped with periodic ETMs.

*Statement of Contributions.* In this work, we design a predictor-based output feedback controller with intermittent ETM for continuous-time linear time-invariant plants. We model the NCS as a hybrid dynamical system and exploit the hybrid framework to neatly distinguish the aspects of sampling (or transmissions) from that of input delay; in this lies the main contribution of our work. Specifically, i) we adopt the methodologies in [3], [20], [21] to accommodate for aperiodic sampling and event-triggered transmissions, and ii) employ time-delay methods used in [2], solely, to address latency related concerns. In the context of event-triggered predictor-based control, the approach pursued here is in stark contrast to the studies in [6], [7] that completely rely on time-delay methods and the one in [18] that utilizes small-gain arguments. Furthermore, contrary to the studies in [6], [7], [16]–[18] that employ static ETMs, in this work, we design a hybrid dynamic ETM whose event-triggering condition is monitored intermittently at pre-defined instants. It is also worth noting that the methodology developed can accommodate other network-imperfections such as scheduling protocols and quantized transmissions.

## II. Preliminaries

In this section, we introduce notations used throughout this paper and refer the interested reader to [22] for preliminaries on hybrid systems.

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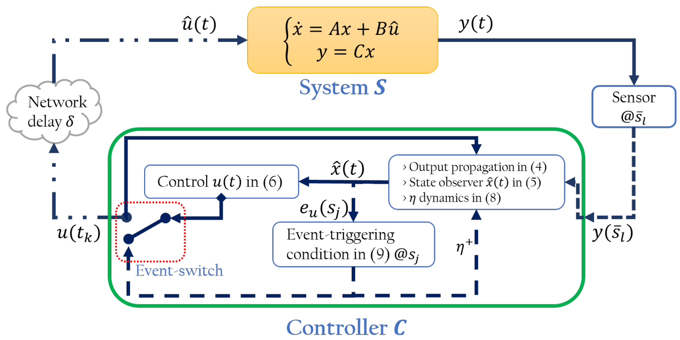


Fig. 1. Networked system $\mathcal{S}$ with controller $\mathcal{C}$

Denote $\mathbb{R}$ to be the set of real numbers, $\mathbb{Z}$ to be the set of integers; then $\mathbb{R}_{\geq 0} = [0, \infty)$ and $\mathbb{Z}_{\geq 0} = \{0, 1, 2, \dots\}$. Let $|x|$ be the Euclidean norm of an $n$-dimensional vector $x \in \mathbb{R}^n$ and, for a matrix $A \in \mathbb{R}^{m \times n}$, let $|A|$ denote its matrix norm. Denote $\mathbb{I}_n$ to be the identity matrix of dimension $n$. Let $\wedge$ and $\vee$ denote the 'and' and the 'or' operators, respectively. If $s_k, \forall k \in \mathbb{Z}_{\geq 0}$, be a sequence of numbers, then this sequence is also expressed as $\{s_k\}$ for brevity.

In this work, the following result from [23] (in slightly modified form) is used to compute the bound on sampling interval.

*Lemma 1 ( [23]):* Let $\theta : \mathbb{R}_{\geq 0} \to \mathbb{R}$ be the solution to

$$\dot{\theta}(s) = \begin{cases} -2\mathsf{L}\theta(s) - \gamma(\theta^2(s) + 1), & s \in [0, T_0]; \\ 0, & s > T_0, \end{cases}$$

with $\theta(0) = \theta^{\max}$. Then $\theta(s)$ is monotonically decreasing and $\theta(s) = \theta^{\min}$ for $s \geq T_0$.

Here, we define the bound $T_0$ as a quantity dependent on the parameters $(\gamma, \mathsf{L}, \theta^{\max}, \theta^{\min})$ and satisfying the conditions in Lemma 1. Note that the formulae to compute $T_0$ are closely related to those presented in [23] and are, therefore, left out of discussion here owing to space limitations.

## III. The Problem

Consider the following networked system:

$$\mathcal{S} : \begin{cases} \dot{x} = Ax + B\hat{u}, \\ y = Cx, \end{cases} \quad t \in [r_k, r_{k+1}] \tag{1}$$

depicted in Fig. 1, where $x(t) \in \mathbb{R}^n$, $\hat{u}(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^p$ are the system state, transmitted control input, and system output, respectively, and $A, B, C$ are system matrices of appropriate dimensions. In (1), $r_k, k \in \mathbb{Z}_{\geq 0}$, denote the arrival instants of the transmitted control input $\hat{u}$, i.e.,

$$\hat{u}(t) = u(t_k), \forall t \in [r_k, r_{k+1}], \tag{2}$$

where $t_k, \forall k \in \mathbb{Z}_{\geq 0}$, denote the transmission instants at the controller $\mathcal{C}$. The control input $u(t) \in \mathbb{R}^m$ and the transmission instant $t_k$, governed by ETMs, are described in Section IV. Furthermore, we make the following mild assumption on system $\mathcal{S}$:

*Assumption 1:* The system $\mathcal{S}$ in (1) is both controllable and observable.

In this work, we consider that the controller $\mathcal{C}$ assumes the knowledge of predicted delay, denoted by $\delta \in \mathbb{R}_{\geq 0}$, associated with the transmission of control inputs $u(t_k), \forall k \in \mathbb{Z}_{\geq 0}$, is constant and is known *apriori*; in other words, $r_k = t_k + \delta$ in (2). Moreover, the controller $\mathcal{C}$ has access to the sampled outputs $y(\bar{s}_l)$ at sampling instants $\bar{s}_l, \forall l \in \mathbb{Z}_{\geq 0}$ (defined shortly). Note that the transmission instants of the control inputs and the sampling instants of the system outputs may not be the same.

Briefly, the objective of this paper is to exploit the knowledge of input delay $\delta \geq 0$ to design an (intermittent) event-triggered output-feedback controller and, simultaneously, establish bounds on sampling intervals for system outputs.

## IV. Formulation and Modeling

In this section, we present the details of a observer-based controller that is employed to stabilize the system in (1). The controller $\mathcal{C}$ itself is composed of the following three components: i) output propagator, ii) predictor-based state observer, and iii) an event-triggering mechanism. Subsequently, we express the closed-loop system, incorporating the dynamics of system $\mathcal{S}$ and controller $\mathcal{C}$, as a hybrid system.

### A. Output propagation

Let $\bar{s}_l, \forall l \in \mathbb{Z}_{\geq 0}$, denote the sampling instants at which the output is sampled; these instants are such that

$$0 < \varepsilon_y \leq \bar{s}_{l+1} - \bar{s}_l \leq T_y, \ \forall l \in \mathbb{Z}_{\geq 0}, \tag{3}$$

where $\varepsilon_y$ is an arbitrarily small positive constant and $T_y$ is the upper bound on sampling interval that is to be designed. Note that $\varepsilon_y > 0$ ensures that Zeno solutions (see Chapter 2 in [22]) for the hybrid system, defined shortly in (14), are avoided.

Denote $\hat{y}(t) \in \mathbb{R}^p$ to be the propagated output. At $\bar{s}_l$, controller $\mathcal{C}$ collects the sampled output $y(\bar{s}_l) \in \mathbb{R}^p$ and propagates this via the following model:

$$\dot{\hat{y}}(t) = CA\hat{x}(t - \delta) + CB\hat{u}(t), \quad t \in [\bar{s}_l, \bar{s}_{l+1}] \tag{4a}$$

$$\hat{y}(\bar{s}_l^+) = y(\bar{s}_l), \quad t \in \{\bar{s}_l\} \tag{4b}$$

where $\hat{x}(t - \delta)$ denotes the observer state formally defined in Section IV-B.

### B. Predictor-based state observer & control

In this subsection, we design a state observer that employs the propagated output $\hat{y}(t)$ and known delay $\delta$ to estimate the system state at a future instant $t + \delta$, namely, $x(t + \delta)$. Let $\hat{x}(t) \in \mathbb{R}^n$ denote the observer state. The predictor-based observer dynamics is given by:

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + B\hat{u}(t + \delta) + L\Big(\hat{y}(t) - C\hat{x}(t) \\ &\quad + \int_t^{t+\delta} \big(CA\hat{x}(s - \delta) + CB\hat{u}(s)\big) ds\Big), \\ \hat{x}(t^+) &= \hat{x}(t) \end{aligned} \tag{5}$$

where $L \in \mathbb{R}^{n \times p}$ denotes the observer gain such that $A - LC$ is Hurwitz (from Assumption 1). We note that the integral term in (5), together with $\hat{y}(t)$, propagates $\hat{y}$ further into the future to time $t+\delta$ along the same dynamics from (4a); this term accounts for latency-based output prediction. Furthermore, $\hat{u}(t+\delta) = u(t_k), \forall t \in [t_k, t_{k+1}]$, i.e., as soon as $u(t_k)$ is transmitted, it is used in the observer dynamics in (5).

Lastly, the control input $u(t) \in \mathbb{R}^m$, given by

$$u(t) = K\hat{x}(t), \tag{6}$$

employs the observer state $\hat{x}(t)$ to account for the delay in transmission. Here, the control gain $K \in \mathbb{R}^{m\times n}$ is such that $A+BK$ is Hurwitz (from Assumption 1).

### C. Event-triggering mechanism (ETM)

Let $s_j, \forall j \in \mathbb{Z}_{\geq 0}$, denote the sampling instants at which the event-triggering condition (defined in (9)) is evaluated. The sampling instants $s_j$ are such that

$$0 < \varepsilon_u \leq s_{j+1} - s_j \leq T_u,\ \forall j \in \mathbb{Z}_{\geq 0}, \tag{7}$$

where $\varepsilon_u$ is a positive constant and $T_u$ is the to-be-designed upper bound on the sampling interval. Note that in the context of event-triggering control, existence of arbitrarily small $\varepsilon_u > 0$ (guaranteed by the existence of $T_u$) ensures the absence of Zeno behavior. In practice, this could be associated with the max. sampling frequency of the device.

Let $\eta(t) \in \mathbb{R}_{\geq 0}$ denote an auxiliary threshold variable which governs the transmission of control inputs from the controller $\mathcal{C}$ to the system $\mathcal{S}$. The hybrid dynamics of $\eta(t)$ is given by:

$$\begin{cases} \dot{\eta}(t) = f_\eta(\eta(t), \hat{x}(t)), & t \in [s_j, s_{j+1}] \\ \eta(t^+) = g_s(\eta(t), \hat{x}(t)), & t \in \{s_j\} \setminus \{t_k\} \\ \eta(t^+) = g_t(\eta(t), \hat{x}(t)), & t \in \{t_k\} \end{cases} \tag{8}$$

where recall that $\{t_k\}$ is the sequence of transmission instants at the controller in (2). Here, it is assumed that the functions $f_\eta(\cdot) : \mathbb{R}_{\geq 0} \times \mathbb{R}^n \to \mathbb{R}$, $g_s(\cdot) : \mathbb{R}_{\geq 0} \times \mathbb{R}^n \to \mathbb{R}$ and $g_t(\cdot) : \mathbb{R}_{\geq 0} \times \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ are continuous, have access to the observer state $\hat{x}$, and $f_\eta(\eta = 0, \cdot) \geq 0$; furthermore, the initial condition $\eta(0) \geq 0$ is a design parameter. The event-triggering condition provides the update law for the transmission instants and is defined as follows:

$$t_{k+1} = \{t > t_k | t \in \{s_j\}_{j=0}^{\infty},\ g_s(\cdot) \leq 0\}. \tag{9}$$

The event-triggering condition in (9) and the dynamics of $\eta$ in (8) are collectively referred to as the ETM.

### D. Hybrid system modeling

In this subsection, we model the overall networked system using the framework of hybrid systems. To facilitate this, we define the following errors: i) observer error $\tilde{x}(t) = \hat{x}(t) - x(t+\delta) \in \mathbb{R}^n$, since $\hat{x}(t)$ is estimating the future true state $x(t+\delta)$, ii) output error $e_y(t) = \hat{y}(t) - y(t) \in \mathbb{R}^p$, owing to output sampling, and iii) input error $e_u(t) = \hat{u}(t+\delta) - u(t) \in \mathbb{R}^m$, because the control law in (6) accounts for the delay in transmission.

From (1) and (5), the hybrid dynamics of the observer error $\tilde{x}$ is expressed as follows:

$$\begin{cases} \dot{\tilde{x}}(t) = A\hat{x}(t) + B\hat{u}(t+\delta) + L\big(\hat{y}(t) + \int_t^{t+\delta} \big(CA\hat{x}(s-\delta) \\ \qquad + CB\hat{u}(s)\big)ds - C\hat{x}(t)\big) - Ax(t+\delta) - B\hat{u}(t+\delta) \\ \quad = A\tilde{x}(t) + L\big(\hat{y}(t) + \int_t^{t+\delta} \big(CA\hat{x}(s-\delta) \\ \qquad + CB\hat{u}(s)\big)ds - C\hat{x}(t) + Cx(t+\delta) \\ \qquad - \int_t^{t+\delta} \big(CAx(s) + CB\hat{u}(s)\big)ds - y(t)\big) \\ \quad = (A - LC)\tilde{x}(t) + L(e_y(t) + CAv(t)) \\ \tilde{x}(t^+) = \tilde{x}(t) \end{cases} \tag{10}$$

where $v(t) = \int_{t-\delta}^{t} \tilde{x}(s)ds$ denotes the distributed delay term. From (10), we infer that the decay rate of $\tilde{x}(t)$ is governed by the tussle between frequent sampling of outputs (i.e., often resetting $e_y(t)$ to origin or, equivalently, having smaller sampling bound $T_y$) and the allowable length of input delay $\delta \geq 0$ (i.e., the contribution of $v(t)$).

Next, the hybrid dynamics of the output error $e_y(t)$ and the input error $e_u(t)$ (see Remark 1) are given by:

$$\begin{cases} \dot{e}_y(t) = CA\tilde{x}(t-\delta), & t \in [\bar{s}_l, \bar{s}_{l+1}], \\ e_y(t^+) = 0, & t \in \{\bar{s}_l\}, \end{cases} \tag{11}$$

and

$$\begin{cases} \dot{e}_u(t) = -K(\dot{\tilde{x}}(t) + \dot{x}(t+\delta)), \quad t \in [s_j, s_{j+1}], \\ \qquad = -K\Big((A+BK)x(t+\delta) + (A + BK \\ \qquad\quad - LC)\tilde{x}(t) + Be_u(t) + L(e_y(t) + CAv(t))\Big), \\ e_u(t^+) = \begin{cases} e_u(t), & t \in \{s_j\}, \\ 0, & t \in \{t_k\}, \end{cases} \end{cases} \tag{12}$$

respectively. Here, $\tilde{x}(t-\delta)$ in (11) is referred to as the discrete delay term in literature.

Finally, to help with time-keeping, we denote $\tau_u$ and $\tau_y$ as the timer variables that track the time elapsed since the last sampling instant i) at the ETM via $\{s_j\}$ and ii) at the output sampler via $\{\bar{s}_l\}$, respectively. Their hybrid dynamics are as follows:

$$\begin{cases} \dot{\tau}_u = 1, & \tau_u \in [0, T_u], \\ \tau_u^+ = 0, & \tau_u \in [\varepsilon_u, T_u], \end{cases} \text{ and } \begin{cases} \dot{\tau}_y = 1, & \tau_y \in [0, T_y], \\ \tau_y^+ = 0, & \tau_y \in [\varepsilon_y, T_y]. \end{cases} \tag{13}$$

Note that in (13), the jumps in timer variables $\tau_u$ and $\tau_y$ are pre-defined by the choice of sampling instants governed by (7) and (3), respectively.

We model the closed-loop networked system using the hybrid systems framework by combining the system dynamics in (1), the observer error dynamics in (10), the output error in (11), the input error dynamics in (12), and the event-triggering mechanism in (8). This model is as follows:

$$\begin{cases} \dot{\xi} = F(\xi), & \xi \in C, \\ \xi^+ \in G(\xi), & \xi \in D, \end{cases} \tag{14}$$

where $\xi := (x_\delta, \tilde{x}, e_y, e_u, \eta, \tau_u, \tau_y)$, $\xi^+$ is shorthand notation depicting jumps in $\xi$ and $x_\delta = x(t+\delta)$. The flow set $C$ and jump set $D$ are defined as:

$$C = \mathbb{R}^{2n+p+m} \times \mathbb{R}_{\geq 0} \times [0, T_u] \times [0, T_y], \text{ and}$$
$$D = \mathbb{R}^{2n+p+m} \times \mathbb{R}_{\geq 0} \times [\varepsilon_u, T_u] \times [\varepsilon_y, T_y],$$

respectively. The flow map $F(\xi)$:

$$F(\xi) = \begin{bmatrix} (A+BK)x_\delta + BK\tilde{x} + Be_u \\ (A-LC)\tilde{x} + Le_y + LCAv \\ CA\tilde{x}(t-\delta) \\ f_{e_u}(x_\delta, \tilde{x}, e_u, e_y) \\ f_\eta(\eta, x_\delta + \tilde{x}) \\ 1 \\ 1 \end{bmatrix}$$

where $f_{e_u}(\cdot) = -K\big((A+BK)x_\delta + (A+BK-LC)\tilde{x} + Be_u + L(e_y + CAv)\big)$. From the modeling detailed in this section, we can infer that the jumps in the hybrid system in (14) occur only when $t \in \{s_j\} \vee t \in \{\bar{s}_l\}$. Correspondingly, the jump map $G(\xi)$ is such that:

- If $g_s(\cdot) > 0 \wedge \tau_y^+ = 0$, then $G(\xi) = G_s(\xi) = [x_\delta^\top \;\; \tilde{x}^\top \;\; 0^\top \;\; e_u^\top \;\; g_s \;\; 0 \;\; 0]^\top$, and if $g_s(\cdot) > 0 \wedge \tau_y^+ \neq 0$, then $G(\xi) = G_s'(\xi) = [x_\delta^\top \;\; \tilde{x}^\top \;\; e_y^\top \;\; e_u^\top \;\; g_s \;\; 0 \;\; \tau_y]^\top$.
- If $g_s(\cdot) < 0 \wedge \tau_y^+ = 0$, then $G(\xi) = G_t(\xi) = [x_\delta^\top \;\; \tilde{x}^\top \;\; 0^\top \;\; 0^\top \;\; g_t \;\; 0 \;\; 0]^\top$, and if $g_s(\cdot) < 0 \wedge \tau_y^+ \neq 0$, then $G(\xi) = G_t'(\xi) = [x_\delta^\top \;\; \tilde{x}^\top \;\; e_y^\top \;\; 0^\top \;\; g_t \;\; 0 \;\; \tau_y]^\top$.
- If $g_s(\cdot) = 0 \wedge \tau_y^+ = 0$, then $G(\xi) \in \{G_s(\xi), G_t(\xi)\}$, and if $g_s(\cdot) = 0 \wedge \tau_y^+ \neq 0$, then $G(\xi) \in \{G_s'(\xi), G_t'(\xi)\}$.
- Finally, if only $\tau_y^+ = 0$, $G(\xi) = [x_\delta^\top \; \tilde{x}^\top \; 0^\top \; e_u^\top \; \eta \; \tau_u \; 0]^\top$.

It is worth noting that the flow map $F(\xi)$ and the jump map $G(\xi)$ are constructed such that the hybrid system in (14) is nominally well-posed[1].

*Remark 1:* (On scheduling protocols) Often in NCSs, transmissions over the network also involve adhering to scheduling protocols such as RR or TOD, see [3]. It is worth noting that these scheduling protocols (and other network-related imperfections such as quantization) can also be readily incorporated into the methods pursued in this paper. Specifically, the transmitted control $\hat{u}$ in (2), with the appropriate protocol $h_u(\cdot) \in \mathbb{R}^m$ at transmission instants $t_k$, results in $\hat{u}(t) = u(t_k) + h_u(t_k, e_u(t_k)), \forall t \in [r_k, r_{k+1}]$. These protocols, on the basis of $t_k$ and $e_u(t_k)$, determine which component/node of $u(t_k)$ needs to be transmitted over the network. Under hybrid system modeling, this inclusion of protocols would alter the jump dynamics of $e_u$ in (12) as follows:

$$e_u(t^+) = \begin{cases} e_u, & t \in \{s_j\} \\ h_u, & t \in \{t_k\} \end{cases}.$$

## V. Main Results

In this section, we present the dynamics of the ETM and establish bounds $T_u$ and $T_y$ such that the closed-loop system is exponentially stable through the following result.

*Theorem 1:* Let the dynamics of the threshold $\eta$ in (8) be:

$$\begin{aligned} f_\eta &= -\beta_1 \eta + \beta_2 |e_u|^2, \\ g_s &= \eta + \gamma_u(\theta_u^{\min} - \theta_u^{\max})|e_u|^2, \\ g_t &= \eta + \gamma_u \theta_u^{\min} |e_u|^2, \end{aligned} \tag{15}$$

where $\beta_1 > 0$, $\beta_2 \geq 0$, $\theta_u^{\max} > \theta_u^{\min} > 0$. Let the upper bounds on sampling instants $\{s_k\}$ and $\{\bar{s}_l\}$ be $T_u(\gamma_u, |KB|, \theta_u^{\max}, \theta_u^{\min})$ and $T_y(\gamma_y, 0, \theta_y^{\max}, \theta_y^{\min})$, respectively, (as per Lemma 1) where $\theta_y^{\max} > \theta_y^{\min} > 0$. Under Assumption 1, there exist $\delta \geq 0$, $P \succ 0$, $\tilde{P} \succ 0$, $\tilde{R} \succeq 0$, $\tilde{S} \succeq 0$, error gains $\gamma_u, \gamma_y \geq 0$, and scalar $\theta_u^{\max} > 0$, such that the following conditions hold:

$$-\Phi \succ 0, \quad \tilde{S} - |CA|^2 \mathbb{I}_n \succ 0, \tag{16}$$

where $\Phi = [\Phi_{ij}]$ is a symmetric matrix and its elements are such that:

$$\begin{aligned} &\Phi_{11} = P(A+BK) + (A+BK)^\top P + |K(A+BK)|^2 \mathbb{I}_n, \\ &\Phi_{12} = PBK, \quad \Phi_{14} = PB, \\ &\Phi_{22} = \tilde{P}(A-LC) + (A-LC)^\top \tilde{P} + \delta^2 \tilde{R} + \tilde{S} \\ &\qquad + |K(A+BK-LC)|^2 \mathbb{I}_n, \quad \Phi_{23} = \tilde{P}LCA, \\ &\Phi_{24} = \tilde{P}L, \Phi_{33} = -\tilde{R}, \quad \Phi_{34} = -\gamma_u \theta_u^{\max}(KLCA)^\top, \\ &\Phi_{44} = -\gamma_u^2 + \beta_2, \quad \Phi_{45} = -\gamma_u \theta_u^{\max} KL, \quad \Phi_{55} = -\gamma_y^2. \end{aligned}$$

Then, for the closed-loop system in (14) with the event-triggering condition in (9), the set $\{\xi \mid x_\delta = 0, \tilde{x} = 0, e_y = 0, e_u = 0\}$ is exponentially stable.

*Proof:* See Appendix for the proof.

*Remark 2:* (On design of dynamic ETMs) Note that the non-negative second term in $f_\eta(\cdot)$ of (15) aids in slowing down the decay of the threshold $\eta(t)$. Specifically, $f_\eta(\eta, \hat{x})$ (or, equivalently, $f_\eta(\eta, e_u)$) is designed such that the ETM can exploit the continuous availability of $e_u(t) = \hat{u}(t+\delta) - u(t) = \hat{u}(t+\delta) - K\hat{x}(t)$ at the controller $\mathcal{C}$. It is worth mentioning that using $e_u(t)$ in $f_\eta(\cdot)$ can be avoided by: i) setting $\beta_2 = 0$, or ii) by employing a sampled version of $e_u(t)$, namely, $e_u(s_k)$; both these approaches may increase conservatism.

*Remark 3:* (On the feasibility of (16)) For a given $\delta$, we note that, the condition $-\Phi \succ 0$ in (16) is not strictly an LMI owing to the coupled term $\gamma_u \theta_u^{\max}$ in $\Phi_{34}$, $\Phi_{45}$ and the quadratic term $\gamma_u^2$ in $\Phi_{44}$ (whereas $\gamma_y^2$ is standalone variable). This can easily be circumvented by treating $(\gamma_u \theta_u^{\max})$ and $(\gamma_u^2)$ as variables and, subsequently, obtaining $\theta_u^{\max}$ from the solution of the resultant LMI. Alternatively, the coupled terms in $\Phi$ could be disintegrated, using Young's inequality; however, this introduces further conservatism. It was also observed that, for a given $\delta$, the 'smallest' $\gamma_u^2$ or $\gamma_y^2$ could be iteratively obtained to ensure that $-\Phi \succ 0$.

*Remark 4:* (On comparison with [7], [16], [18]) In [7], the authors designed continuously-evaluated ETMs for networked systems with large delays. The effect of sampling, delays, and events were all analyzed through time-delay methods that result in less conservative delay bounds. More recently, studies in [16] and [18] designed ETMs which are evaluated *strictly* periodically wherein [16] employed discrete Lyapunov functions and [18] employed small-gain arguments. Contrary to the aforementioned studies, in this work, we adopt the hybrid system framework and design a dynamic ETM that is evaluated intermittently (i.e., allowing for drift in local device clocks). To enable this, the framework allows us to differentiate the aspects of periodic/aperiodic sampling from that of input delay; arguably, the approach pursued here may

[1]Interested readers can refer to Chapter 6 in [22] for the definition.

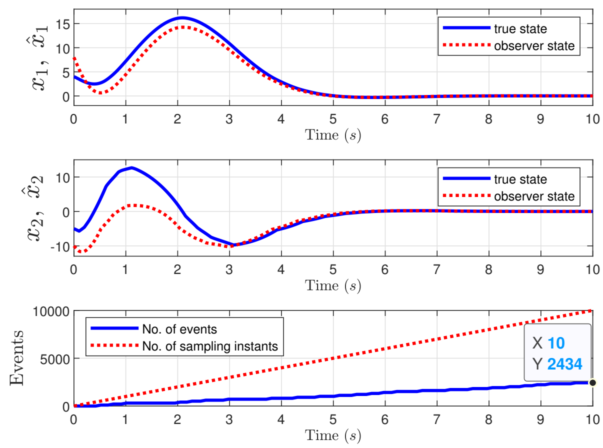


Fig. 2. Trajectories of state $\boldsymbol{x(t)}$, observer state $\boldsymbol{\hat{x}(t)}$, and comparison between the number of event instants $\boldsymbol{\{t_k\}}$ and sampling instants $\boldsymbol{\{s_j\}}$.

offer conservative bounds on admissible delays (compared to those in [7]). However, like [3], it can readily accommodate other sampling-related notions involving scheduling protocols and quantized transmissions.

## VI. Illustrative Example

Consider the following system:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -0.5 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \hat{u}, \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} x \tag{17}$$

where $x = [x_1 \quad x_2]^{\mathsf{T}}$. Let the control gain and observer gain be $K = -[1.5 \quad 3]$ and $L = [4 \quad 5.5]^{\mathsf{T}}$, respectively. For a known large delay of $\delta = 100$ ms (i.e., $\delta > T_u, T_y$), the matrix inequalities in (16) for the system in (17) yield error gains $\gamma_u = 137.6$ and $\gamma_y = 81.8$. We compute the bound on sampling periods using Lemma 1 as follows: $T_u(\gamma_u, |KB|, 1.08, 0.10) = 1.0$ ms and $T_y(\gamma_y, 0, 100, 0.1) = 19.1$ ms. The threshold $\eta$ hybrid dynamics, described by the functions in (15), are given by: $f_\eta = -\eta + 40.2|e_u|^2, g_s = \eta - 3.48|e_u|^2, g_t = \eta + 0.38|e_u|^2$. Furthermore, the integral term in observer dynamics in (5) is computed by sampling $\hat{x}(t)$ at $1/h$ Hz over $\delta$ interval where $h = 0.1$ ms is the time-step used for flow computations. Fig. 2 depicts the trajectories of the state $x(t)$, the state observer $\hat{x}(t)$ and the observer error $\tilde{x}(t)$ when $s_{j+1} - s_j = 1$ ms, $\forall j \in \mathbb{Z}_{\geq 0}$, and $\bar{s}_{l+1} - \bar{s}_l = 10$ ms, $\forall l \in \mathbb{Z}_{\geq 0}$. The dynamic ETM above offers significantly reduced control input transmissions, i.e., about 24% of the overall sampling instants $\{s_j\}$ as shown in the 3rd sub-figure of Fig. 2.

## VII. Conclusion

In this work, we addressed the problem of predictor-based output-feedback control of linear NCSs with event-triggered transmissions by exploiting hybrid system framework. The sampled output is propagated via a model and, subsequently, this propagated output along with the latency-based predictor term is incorporated into the design of the state observer. We employed time-delay methods to address the effect of delayed inputs transmissions and exploited network-induced errors (and their growth dynamics) to design event-triggered scheduling of the inputs. Furthermore, the ETM designed is dynamic and the event-triggering condition is intermittently evaluated ensuring it is Zeno-free by design. In the future, we intend to extend the approach for predictor-based control of NCSs with model uncertainties and multi-agent systems.

## Appendix

In this section, we present the proof to Theorem 1.

*Proof:* The Lyapunov candidate for the overall system is composed of four components, namely, one for each of the subsystems $x_\delta$, $\tilde{x}$, $e_y$, and $e_u$. We present each of these components individually and, subsequently, combine them for flow and jump analysis of the overall system.

- First, for state dynamics we consider $V_x = x_\delta^{\mathsf{T}} P x_\delta$; then

$$\dot{V}_x = x_\delta^{\mathsf{T}} \Big[ P(A+BK) + (A+BK)^{\mathsf{T}} P \Big] x_\delta + 2x_\delta^{\mathsf{T}} PBK\tilde{x}$$
$$2x_\delta^{\mathsf{T}} PBe_u$$

- Second, for the observer error dynamics we consider:

$$V_{\tilde{x}} = \tilde{x}^{\mathsf{T}} \tilde{P} \tilde{x} + \delta \int_{-\delta}^{0} \int_{t+\phi}^{t} \tilde{x}^{\mathsf{T}}(\tau) \tilde{R} \tilde{x}(\tau) d\tau d\phi + \int_{t-\delta}^{t} \tilde{x}^{\mathsf{T}}(\tau) \tilde{S} \tilde{x}(\tau) d\tau$$

based on Lyapunov functionals for discrete and distributed delays (see [2]). Then, $\dot{V}_{\tilde{x}}$ is upper bounded as follows:

$$\begin{aligned} \dot{V}_{\tilde{x}} \leq\ & \tilde{x}^{\mathsf{T}} \Big[ \tilde{P}(A-LC) + (A-LC)^{\mathsf{T}} \tilde{P} + \delta^2 \tilde{R} \Big] \tilde{x} - v^{\mathsf{T}} \tilde{R} v \\ & + 2\tilde{x}^{\mathsf{T}} \Big[ \tilde{P} LCA \Big] v + 2\tilde{x}^{\mathsf{T}} \tilde{P} L e_y + \tilde{x}^T \tilde{S} \tilde{x} \\ & - \tilde{x}(t-\delta)^T \tilde{S} \tilde{x}(t-\delta) \end{aligned}$$

where recall that $v = \int_{t-\delta}^{t} \tilde{x}(s) ds$.

- Third, for output error dynamics, we consider $V_y = \gamma_y \theta_y |e_y|^2$ where $\theta_y$ satisfies conditions in Lemma 1 with $\mathsf{L} = 0$, $\gamma = \gamma_y$. Then,

$$\begin{aligned} \dot{V}_y &= \gamma_y \dot{\theta}_y |e_y|^2 + 2\gamma_y \theta_y e_y^{\mathsf{T}} \dot{e}_y \\ &\leq \gamma_y \big( -\gamma_y - \gamma_y \theta_y^2 \big) |e_y|^2 + 2\gamma_y \theta_y |e_y| |CA| |\tilde{x}(t-\delta)| \\ &\leq -\gamma_y^2 |e_y|^2 + |CA|^2 |\tilde{x}(t-\delta)|^2 \end{aligned}$$

- Finally, for the input error dynamics, we consider $V_u = \gamma_u \theta_u |e_u|^2$ where $\theta_u$ satisfies conditions in Lemma 1 with $\mathsf{L} = \mathsf{L}_u = |KB|$ and $\gamma = \gamma_u$. Then,

$$\begin{aligned} \dot{V}_u \leq\ & \gamma_u \dot{\theta}_u |e_u|^2 + 2\gamma_u \theta_u \Big( \mathsf{L}_u |e_u|^2 + |e_u| H_u(x_\delta, \tilde{x}) \\ & - e_u^{\mathsf{T}} K L e_y - e_u^{\mathsf{T}} K L C A v \Big) \\ \leq\ & -\gamma_u^2 |e_u|^2 - \gamma_u^2 \theta_u^2 |e_u|^2 + 2\gamma_u \theta_u |e_u| H_u \\ & + 2\gamma_u \theta_u \Big( - e_u^{\mathsf{T}} K L e_y - e_u^{\mathsf{T}} K L C A v \Big) \\ \leq\ & -\gamma_u^2 |e_u|^2 - 2\gamma_u \theta_u \Big( e_u^{\mathsf{T}} K L e_y + e_u^{\mathsf{T}} K L C A v \Big) \\ & + 2|K(A+BK)|^2 |x_\delta|^2 + 2|K(A+BK-LC)|^2 |\tilde{x}|^2, \end{aligned}$$

where $H_u(x, \tilde{x}) = |K(A+BK)x + K(A+BK-LC)\tilde{x}|$.

► *Flow analysis of the overall system:*
Consider Lyapunov function $V = V_x + V_{\tilde{x}} + V_u + V_y + \eta$ for the closed-loop hybrid system in (14). Then,

$$\begin{aligned}\dot{V} \leq\ & x_\delta^{\mathrm{T}} Q x_\delta + 2x_\delta^{\mathrm{T}} PBK\tilde{x} + 2x_\delta^{\mathrm{T}} PBe_u - f_\eta - \tilde{x}^{\mathrm{T}}\Big[\tilde{Q} \\ & - \delta^2\tilde{R} - \tilde{S}\Big]\tilde{x} - v^{\mathrm{T}}\tilde{R}v + 2\tilde{x}^{\mathrm{T}}\big[\tilde{P}LCA\big]v + 2\tilde{x}^{\mathrm{T}}\tilde{P}Le_y \\ & - \tilde{x}(t-\delta)^T\tilde{S}\tilde{x}(t-\delta) - \gamma_y^2|e_y|^2 + |CA|^2|\tilde{x}(t-\delta)|^2 \\ & - \gamma_u^2|e_u|^2 - 2\gamma_u\theta_u\Big(e_u^{\mathrm{T}}KLe_y + e_u^{\mathrm{T}}KLCAv\Big) \\ & + |K(A+BK)|^2|x_\delta|^2 + |K(A+BK-LC)|^2|\tilde{x}|^2 \\ \leq\ & - x_\delta^{\mathrm{T}}\Big(Q - |K(A+BK)|^2\mathbb{I}\Big)x_\delta - \beta_1\eta - \tilde{x}^{\mathrm{T}}\Big[\tilde{Q} - \delta^2\tilde{R} \\ & - \tilde{S} - |K(A+BK-LC)|^2\mathbb{I}\Big]\tilde{x} - v^{\mathrm{T}}\tilde{R}v - \gamma_y^2|e_y|^2 \\ & - \big(\gamma_u^2 - \beta_2\big)|e_u|^2 - \tilde{x}(t-\delta)^T\big[\tilde{S} - |CA|^2\mathbb{I}\big]\tilde{x}(t-\delta) \\ & - 2\gamma_u\theta_u\Big(e_u^{\mathrm{T}}KLe_y + e_u^{\mathrm{T}}KLCAv\Big) + 2\tilde{x}^{\mathrm{T}}\tilde{P}Le_y \\ & + 2\tilde{x}^{\mathrm{T}}\big[\tilde{P}LCA\big]v + 2x_\delta^{\mathrm{T}}PBK\tilde{x} + 2x_\delta^{\mathrm{T}}PBe_u, \end{aligned} \tag{18}$$

where $-Q = P(A+BK) + (A+BK)^{\mathrm{T}}P$ and $-\tilde{Q} = \tilde{P}(A - LC) + (A-LC)^{\mathrm{T}}\tilde{P}$. Therefore, from (18) and conditions in (16), one can infer that there exists a positive constant $\zeta$ such that:

$$\dot{V}(\xi) \leq -\zeta V(\xi), \forall \xi \in C.$$

► *Jump analysis of the overall system:*
Recall that the jumps in the hybrid system in (14) occur only at instants $t \in \{s_j\}_{j=0}^{\infty} \vee t \in \{\bar{s}_l\}_{l=0}^{\infty}$. Furthermore, $x_\delta^+$ and $\tilde{x}$ do not experience any jumps as noted in the jump map $G(\xi)$ in (14). This implies, that at any jump instant $t$, we have: $\Delta V(t) = V(t^+) - V(t) = V_u(t^+) - V_u(t) + V_y(t^+) - V_y(t) + \eta(t^+) - \eta(t)$.
On output sampling, we have:

- if $t \in \{\bar{s}_l\}_{l=0}^{\infty}$, $e_y(t^+) = 0$; this implies,

$$\Delta V_y(t) = V_y(t^+) - V_y(t) = 0 - \gamma_y\theta_y|e_y|^2 \leq 0,$$

- if $t \notin \{\bar{s}_l\}_{l=0}^{\infty}$, $\Delta V_y(t) = 0$ since $e_y(t^+) = e_y(t)$.

On event-triggered input transmissions, using (15), we have:

- if no event occurs, i.e., if $t \in \{s_j\} \setminus \{t_k\}$, then

$$\Delta V = \gamma_u\theta_u^{\max}|e_u|^2 - \gamma_u\theta_u|e_u|^2 + g_s - \eta + \Delta V_y \leq 0,$$

- if an event occurs, i.e., if $t \in \{t_k\}$, then

$$\Delta V \leq -\gamma_u\theta_u|e_u|^2 + g_t - \eta + \Delta V_y \leq 0,$$

  since $e_u^+ = 0, \forall t \in \{t_k\}$.

Here, we note that the jumps associated with the output sampling have no instantaneous/immediate effect on event-triggered transmission of control inputs (and vice-versa). Therefore, at jump instants, we have:

$$V(G(\xi)) \leq V(\xi),\ \xi \in D.$$

for the hybrid system in (14). The remainder of this proof is completed following the arguments in the proof of Theorem 1 in [23]. ■

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