Multi-agent Second Order Average Consensus with Prescribed Transient Behavior

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***Abstract*—The problem of consensus reaching with prescribed transient behavior for a group of double-integrator agents is addressed. The information exchange of the multi-agent system is described by a static communication network. We initially set time-dependent constraints on the transient response of the relative positions between neighboring agents and we propose a distributed control law consisting of a proportional term of the transformed error and an additional damping term based on absolute velocities measurements. We also design an agreement protocol that can additionally achieve prescribed performance for a combined error of positions and velocities. Under a sufficient condition for the damping gains, the proposed nonlinear time-dependent distributed controllers guarantee that the predefined constraints are not violated and that consensus is achieved with a convergence rate independent of the underlying communication graph. Furthermore, connectivity maintenance can be ensured by appropriately designing the performance bounds. Theoretical results are supported by simulations.**

I. INTRODUCTION

Distributed control of multi-agent systems is a relatively recent research area that is of great interest due to the large variety of applications. When dealing with a group of agents the control objectives can vary from agreement on a measured value, in the case of sensor networks, to rendez-vous, flocking or platooning, in the case of multiple vehicles. The problem of ensuring convergence to the same value is commonly known as consensus. For many linear consensus protocols both conditions for convergence and the rate of convergence depend on the eigenvalues of the Laplacian matrix [1], [2]. In [3] double-integrator systems are investigated, revealing that the maximum speed of convergence is determined by the largest and the smallest nonzero eigenvalues of the Laplacian matrix of the undirected connected graph. Moreover the consensus speed can be modified by choosing suitable feedback gains.

In order to achieve agreement, connectedness of the underlying communication graph is an essential requirement. Various solutions have been proposed to address connectivity maintenance [4]–[6]. For example, in [4] constraints in the edge's space are imposed, introducing potential functions which values go to infinity when approaching the critical distance between two agents. It is also shown that if the initial conditions are confined within a certain region, consensus is achieved while preserving connectedness. In [5], instead, convex state constraints are considered and the consensus protocol has been enriched by an auxiliary variable utilizing a logarithmic barrier function to form a convex potential.

In general, constraints in the evolution of the state or the output of both linear and nonlinear systems have been handled with a variety of control techniques, e.g. logarithmic barrier Lyapunov functions [7] and prescribed performance control [8]. A prescribed performance controller ensures that the error at steady state converges to an arbitrarily small residual set, with a rate not less than a given value and with an overshoot less than a specified value. Recently these controllers have been successfully applied to robot manipulators [9], [10], platooning [11] and formation control with connectivity maintenance for multi-agent systems [12].

Transient constraints are important for multi-agent systems but have not been tackled systematically up to now. Prescribed performance control is a promising tool in that direction. This paper builds on this observation and applies the framework of prescribed performance control to the consensus problem for multi-agent systems, first introduced in [13]. A set of time-dependent constraints are introduced on the edge's space, forcing the errors between communicating agents to evolve within predefined bounds. Two different nonlinear time-dependent agreement protocols are proposed: the first distributed controller is designed in order to ensure that the position errors between communicating agents evolve within some predefined bounds whereas the second also guarantees that additional time-dependent constraints on a combined error of relative positions and velocities are satisfied. The control input includes a prescribed performance term with an additional damping term depending on the absolute velocity of the agent. The use of the absolute velocities takes into account the cases in which the relative velocities are not available or are based on measurements that are not reliable. Both cases are very common in realistic multi-robot applications.

Prescribed performance control guarantees the existence of the solution that satisfies the constraints compared to classical techniques such as optimal control approaches, that only allow to impose indirect bounds, or constrained optimization problems resulting to LMIs where feasibility and solvability of the constrained problems are not guaranteed. Since the controller can guarantee that the multi-agent system response evolves with predefined time-dependent bounds, the convergence rate of the system can be made arbitrarily fast by appropriately setting the performance function parameters. Such behavior cannot be achieved by linear control laws: in [3] the case of second-order systems is investigated, finding that the tuning of the speed of convergence requires centralized information regarding the graph topology. By incorporating bounds on a state transformation in second order integrator multi-agent systems, prescribed performance control also allows to additionally introduce bounds on the time evolution of the velocities of the agents that can further improve the control input. The problem of constraining the evolution of the velocities of the agents has not been tackled before in distributed multi-agent control, but prescribed performance control has been successfully applied to bound joint-velocity in robot manipulators [10]. Furthermore, when connectivity maintenance depends on the maximum relative distance between two communicating agents such as in [4], [6], the controller can guarantee that the agents remain connected, given appropriately defined bounds.

Under certain assumptions, all the characteristics of the first-order protocol are preserved, including convergence to the invariant centroid. Lyapunov-like methods are used to investigate stability of the closed loop system and conditions that guarantee convergence are derived. The controllers are simulated considering different topologies for the underlying graph for comparison and demonstration purpose.

The paper is organized as follows: in Section II we introduce the model and the theoretical background; in Section III the first proposed

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controller is presented and stability and convergence properties are studied; in Section IV the second proposed controller is presented; Section V provides for simulation examples while Section VI summarizes the results and discusses future developments.

II. Preliminaries

A. Model and definitions

In this work we will consider a group of N agents each one described by a double-integrator model

$$\begin{aligned} \dot{x}_i &= v_i \\ \dot{v}_i &= u_i \end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}$ is the position and $v_i \in \mathbb{R}$ is the velocity of the i-th agent, for $i = 1, \ldots, N$. For simplicity, but without loss of generality, we take into account only one dimension for the theoretical analysis. Let $x = [x_1, x_2, \ldots, x_N]^\top$ and $v = \dot{x} = [v_1, v_2, \ldots, v_N]^\top$ be the stack vector of absolute positions and velocities, respectively. Let also $\bar{x} = [\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_m]^\top$ denote the stack vector of the *position errors* (or relative positions) between two communicating agents i and j, defined as $\bar{x}_k \triangleq x_{ij} = x_i - x_j$, with $k = 1, 2, \ldots, m$, being m the number of links. We also denote with $\bar{v}_k \triangleq v_{ij} = v_i - v_j$ the *velocity errors* (or relative velocities) with $\bar{v} = [\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_m]^\top$ the corresponding stack vector.

B. Graph theory

The communication graph $\mathcal{G}$ is characterized by two finite sets: $\mathcal{V} = \{1, 2, \ldots, N\}$ is the set of vertices, indexed by the agents, and $\mathcal{E} = \{(i,j) \in \mathcal{V} \times \mathcal{V} \mid j \in \mathcal{N}_i\}$ is the set of edges, containing pair of vertices that can exchange information. Let $m = |\mathcal{E}|$ be the number of edges. Each agent i can communicate only with agents belonging to its *neighborhood* $\mathcal{N}_i$. A *path* is a sequence of edges connecting two distinct vertices. If the starting and ending vertices coincide, then we have a *cycle*. A graph is *connected* if there exists a path between any pair of vertices. Additionally, a connected graph without cycles is referred to as a *tree*.

By assigning and orientation to each edge of $\mathcal{G}$ we can define the *incidence* matrix, a $N \times m$ matrix denoted by $B = B(\mathcal{G}) = [b_{ij}]$. The rows of B are indexed by the vertices and the columns are indexed by the edges. In particular $b_{ij} = 1$ if the vertex i is the head of the edge j, $b_{ij} = -1$ if i is the tail of edge j and $b_{ij} = 0$ otherwise. An important property of the incidence matrix is that the null space of its transpose, $\mathrm{Ker}(B^\top)$, is spanned by the vector $\mathbb{1} = [1, 1, \ldots, 1]^\top$. We can then obtain the *Laplacian* matrix of $\mathcal{G}$ as $L = BB^\top$. For an undirected graph L is a rank deficient, symmetric, positive semi-definite matrix. If the graph is connected, L has a single zero eigenvalue with $\mathbb{1}$ corresponding eigenvector [14]. Finally, it can be easily verified that $Lx = B\bar{x}$ and $\bar{x} = B^\top x$; moreover if $\bar{x} = 0$ we have that $Lx = 0$. The same identities are valid for v and $\bar{v}$.

C. Prescribed performance control

In this section we will introduce the theoretical background for prescribed performance control presented in [8], applied to the problem at hand. The objective is to prescribe the evolution of the relative positions x_{ij} between neighboring agents within the following bounds, described in Figure 1,

$$-M_{ij}\rho_{ij}(t) < x_{ij}(t) < \rho_{ij}(t) \qquad \text{if} \quad x_{ij}(0) > 0 \tag{2a}$$

$$-\rho_{ij}(t) < x_{ij}(t) < M_{ij}\rho_{ij}(t) \qquad \text{if} \quad x_{ij}(0) < 0 \tag{2b}$$

defined by the positive, smooth and decreasing *performance functions* $\rho_{ij}(t) : \mathbb{R}_+ \longrightarrow \mathbb{R}_+ \backslash \{0\}$ and indices M_{ij} representing the maximum allowed overshoot. Note that $\lim_{t\to\infty} \rho_{ij}(t) = \rho_\infty > 0$, where ρ_∞ represents the maximum allowable steady state error for $x_{ij}(t)$, while the rate of descent of $\rho_{ij}(t)$ is a lower bound on the speed of convergence. By normalizing $x_{ij}(t)$ with respect to the corresponding performance function we get the *modulated* (or *modified*) error

$$\hat{x}_{ij}(t) \triangleq \frac{x_{ij}(t)}{\rho_{ij}(t)} \tag{3}$$

and we can define the corresponding sets

$$D_{\hat{x}_{ij}} \triangleq \{\hat{x}_{ij}(t) : \hat{x}_{ij}(t) \in (-M_{ij}, 1)\} \quad \text{if} \quad \hat{x}_{ij}(0) > 0 \tag{4a}$$

$$D_{\hat{x}_{ij}} \triangleq \{\hat{x}_{ij}(t) : \hat{x}_{ij}(t) \in (-1, M_{ij})\} \quad \text{if} \quad \hat{x}_{ij}(0) < 0 \tag{4b}$$

that are equivalent to (2). Note that the choice of the set depends only on the sign of the initial value. Both sets can be used in case of zero initial value.

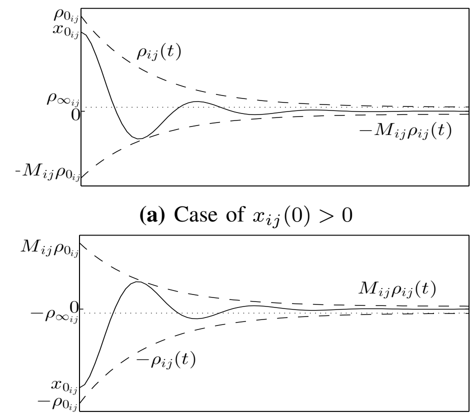
The modified errors are transformed through *transformation functions* that define smooth and strictly increasing mappings $T_{ij} : D_{\hat{x}_{ij}} \longrightarrow \mathbb{R}$, represented in Figure 2. We denote the elements of the *transformed error* $\varepsilon(\hat{\bar{x}}) \in \mathbb{R}^m$ with

$$\varepsilon_{ij}(\hat{x}_{ij}) = T_{ij}(\hat{x}_{ij}) \tag{5}$$

where we dropped the time argument t for notation convenience. Figure 2 shows that if the transformed error $\varepsilon_{ij}(\hat{x}_{ij})$ is bounded, then the modified error $\hat{x}_{ij}$ is confined within the regions (4). This implies that the relative positions x_{ij} evolve within the performance bounds (2), as shown in Figure 1.

Differentiating (5) with respect to time, we obtain

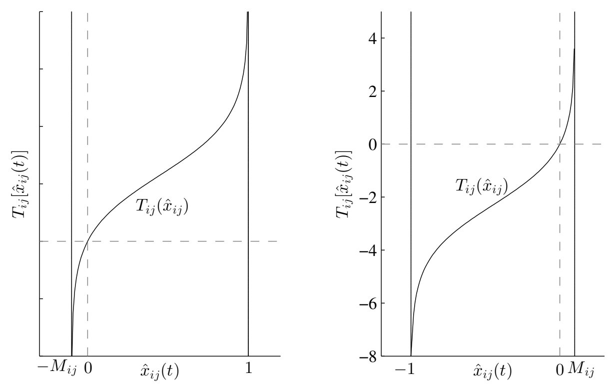
$$\dot{\varepsilon}_{ij}(\hat{x}_{ij}) = \mathcal{J}_{T_{ij}}(\hat{x}_{ij}, t)\left[\dot{x}_{ij} + \alpha_{ij}(t)x_{ij}\right] \tag{6}$$



(a) Case of $x_{ij}(0) > 0$

(b) Case of $x_{ij}(0) < 0$

Figure 1: Performance bounds (dashed lines)



(a) Case of $x_{ij}(0) \geq 0$

(b) Case of $x_{ij}(0) < 0$

Figure 2: Error transformation

where

$$\mathcal{J}_{T_{ij}}(\hat{x}_{ij},t) \triangleq \frac{\partial T_{ij}(\hat{x}_{ij})}{\partial \hat{x}_{ij}} \frac{1}{\rho_{ij}(t)} > 0 \tag{7}$$

$$\alpha_{ij}(t) \triangleq -\frac{\dot{\rho}_{ij}(t)}{\rho_{ij}(t)} \tag{8}$$

are the *normalized Jacobian* of the transformation T_{ij} and the normalized derivative of the performance function, respectively. The functions $\alpha_{ij}(t)$ are positive and converge to zero. By further requiring $T_{ij}(0) = 0$ we can derive the following inequality (cfr. [9], Sec. 2.2)

$$\hat{x}_{ij} \frac{\partial \varepsilon_{ij}(\hat{x}_{ij})}{\partial \hat{x}_{ij}} \varepsilon_{ij}(\hat{x}_{ij}) \geq \mu_{ij} \varepsilon_{ij}^2(\hat{x}_{ij}) \tag{9}$$

for some positive constant μ_{ij}, that is useful for the stability analysis.

III. Prescribed transient behavior for the position errors

A. Proposed controller

For each agent, the proposed controller is the composition of a term based on prescribed performance of the relative positions of the neighboring agents and a second term which is proportional to the absolute velocity of the agent:

$$u_i = -\sum_{j\in\mathcal{N}_i} g_{ij} \mathcal{J}_{T_{ij}}(\hat{x}_{ij},t) \varepsilon_{ij}(\hat{x}_{ij}) - \gamma v_i \quad i \in \mathcal{V} \tag{10}$$

with $g_{ij} = g_{ji}$ and γ being positive constants, $\forall (i,j) \in \mathcal{E}$. The terms $\mathcal{J}_{T_{ij}}(\hat{x}_{ij},t)$ and $\hat{x}_{ij}$ have been defined in Section II. The controller (10) will be referred to as PPC1. Independence of the evolution of the *centroid* (i.e. the average of the positions) of the prescribed performance term can be ensured by assuming that the graph $\mathcal{G}$ describing the communication topology of the multi-agent system is connected and that communicating agents share information about their performance functions, overshoot indices and transformation functions, i.e. by requiring $\rho_{ij}(t) = \rho_{ji}(t)$, $M_{ij} = M_{ji}$ and $T_{ji}(\hat{x}_{ji}) = -T_{ij}(-\hat{x}_{ij})$. This choice implies $\mathcal{J}_{T_{ij}}(\hat{x}_{ij},t)\varepsilon_{ij}(\hat{x}_{ij}) = -\mathcal{J}_{T_{ji}}(\hat{x}_{ji},t)\varepsilon_{ji}(\hat{x}_{ji})$ (cfr. [13], Sec. III). This also implies that the communication between the agents is bidirectional, i.e. the graph $\mathcal{G}$ is undirected.

The system (1) with the control input (10) can be written in vector form as follows:

$$\ddot{x} = -B\mathcal{J}_T(\hat{\bar{x}},t)G\varepsilon(\hat{\bar{x}}) - \gamma v \tag{11}$$

where $\hat{\bar{x}}$ is the stack vector of all modified errors $\hat{x}_{ij}$, $G \in \mathbb{R}^{m\times m}$ is a positive definite diagonal gain matrix with entries g_{ij} and $\mathcal{J}_T(\hat{\bar{x}},t)$ is a time varying diagonal matrix with diagonal entries $\mathcal{J}_{T_{ij}}(\hat{x}_{ij},t)$. The matrix B is the incidence matrix of the graph $\mathcal{G}$ describing the communication topology of the multi-agent system.

B. Stability analysis

The controller (10) is nonlinear and introduces a dependence on time, hence the closed loop system (11) is nonlinear and time-varying. We will use Lyapunov-like tools in order to prove that the equilibrium point is asymptotically stable and at the same time consensus and prescribed performance are guaranteed.

Theorem 1. *Consider the prescribed performance agreement protocol* (10) *applied to the double integrator dynamics* (1)*, with performance functions chosen s.t* $\dot{\rho}_{ij}(t) < \infty$ *and transformation functions s.t.* $T_{ij}(0) = 0\ \forall\ (i,j) \in \mathcal{E}$. *If the condition*

$$\gamma > \max_{t\geq 0} \alpha_{ij}(t) \tag{12}$$

holds and the initial conditions $x_{ij}(0)$ *are inside the performance bounds* (2) $\forall\ (i,j) \in \mathcal{E}$*, then i) the relative errors* $x_{ij}(t)$ *will evolve within the prescribed performance bounds and asymptotically converge to 0 and ii) the absolute velocities* $v_i(t)$ *will converge to zero for* $i = 1,\ldots,N$.

Proof. Let $\xi = \begin{bmatrix} x \\ v \end{bmatrix}$ be the state vector for the closed loop system (11) and let I_N be the $N\times N$ identity matrix. Let us also denote $\varepsilon(\hat{\bar{x}})$ with $\varepsilon_{\hat{\bar{x}}}$ and $\mathcal{J}_T(\hat{\bar{x}},t)$ with $\mathcal{J}_{T_{\hat{\bar{x}}}}$ for notation convenience. Consider a positive constant θ, with $\theta < \gamma$, and the following positive definite function

$$V(\xi,\hat{\bar{x}}) = \frac{1}{2}\xi^\top Q\xi + \frac{1}{2}\varepsilon_{\hat{\bar{x}}} G \varepsilon_{\hat{\bar{x}}} \tag{13}$$

with $Q = \begin{bmatrix} \theta\gamma I_N & \theta I_N \\ \theta I_N & I_N \end{bmatrix}$. Differentiating (13) along the trajectories of (11) and taking into account (6) we obtain

$$\dot{V}(\xi,\hat{\bar{x}}) = -\left[\theta I_m - A(t)\right] \bar{x}^\top \mathcal{J}_{T_{\hat{\bar{x}}}} G \varepsilon_{\hat{\bar{x}}} - (\gamma-\theta) v^\top v \tag{14}$$

which is negative semi-definite for $\theta < \gamma$ and $[\theta I_m - A(t)] > 0$, where $A(t)$ is the $m\times m$ diagonal matrix with diagonal entries $\alpha_{ij}(t)$. The last two conditions together yield (12). Note that since $\dot{\rho}_{ij}(t)$ is bounded $\max_{t\geq 0}\alpha_{ij}(t) < \infty$. Hence if (12) is satisfied, $\dot{V}$ is negative semi-definite, that in turn implies $V(\xi,\hat{\bar{x}}) \leq V(\xi(0),\hat{\bar{x}}(0))$. Therefore if $\hat{\bar{x}}(0)$ is chosen within the regions (4) then $V(\xi(0),\hat{\bar{x}}(0))$ is finite, implying that $\xi, \varepsilon_{\hat{\bar{x}}} \in \mathcal{L}_\infty$, where $\mathcal{L}_\infty$ is the space of all essentially bounded measurable functions. As explained in Section II-C the boundedness of the transformed error $\varepsilon_{\hat{\bar{x}}}$ implies that the position error $\bar{x}(t)$ evolves within the prescribed performance bounds $\forall t$ and we have proved the first part of i). Since $\varepsilon_{\hat{\bar{x}}}$ and $\dot{\varepsilon}_{\hat{\bar{x}}}$ are bounded, we can conclude that the second derivative of $V(\xi,\hat{\bar{x}})$ is bounded and subsequently $\dot{V}(\xi,\hat{\bar{x}})$ is uniformly continuous. Therefore, by applying Barbalat's Lemma, $\dot{V}(\xi,\hat{\bar{x}}) \to 0$ as $t\to\infty$. This means that the trajectories of (11) will converge to the set $E = \{(v,\hat{\bar{x}}) | v = 0, \hat{\bar{x}} = 0\}$ in which $\dot{V}$ is equal to zero. If the transformation functions T_{ij} are chosen such that $T_{ij}(0) = 0$, then $\dot{V}(\xi,\hat{\bar{x}}) \to 0$ implies $\bar{x} \to 0$ and $v \to 0$ as $t\to\infty$. This completes the proof of i) and also proves ii). □

Theorem 1 states that under certain assumptions the position errors of agents connected by an edge asymptotically converge to zero. It does not specify the state of the multi-agent system at the equilibrium. In that sense the Corollary below fills this gap:

Corollary 1. *The prescribed performance agreement protocol* (10) *applied to the multi-agent system* (1)*, under the assumptions of Theorem 1, ensures that the agents' positions converge to the centroid* $c(t) = \frac{1}{N}\sum_{i=1}^N x_i(t)$ *if the communication graph is connected.*

Proof. The centroid of the multi-agent system (11) evolves according to

$$c(t) = \frac{1}{N}\sum_{i=1}^N x_i(0) - \frac{1}{\gamma}\left(\frac{1}{N}\sum_{i=1}^N v_i(0)\right)\left(e^{-\gamma t} - 1\right) \tag{15}$$

Utilizing $\mathbb{1}^\top x(t) = \sum_{i=1}^N x_i(t)$ we can calculate $\beta \triangleq \lim_{t\to\infty} Nc(t)$ as follows:

$$\beta = \mathbb{1}^\top\left[x(0) + v(0)/\gamma\right] \tag{16}$$

On the other hand if the graph is connected Theorem 1 ensures that all the agents will asymptotically reach the same absolute position, i.e.

$$x_\infty \triangleq \lim_{t\to\infty} x(t) = \mathbb{1}\vartheta \tag{17}$$

By combining (16) and (17) we obtain the linear system

$$\begin{bmatrix} I_N & -\mathbb{1} \\ \mathbb{1}^\top & 0 \end{bmatrix}\begin{bmatrix} x_\infty \\ \vartheta \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \beta \end{bmatrix} \tag{18}$$

with $\mathbf{0} = [0, 0, \dots, 0]^\top$, which clearly implies $\vartheta = \frac{1}{N}\beta$. □

Remark 1. *Connectedness of the graph means that there exists a path connecting each pair of agents. Theorem 1 ensures that each pair of agents connected by an edge converges to the same position. Thus, connectedness is fundamental for* (17) *to hold. In case of disconnected graph, Corollary 1 still ensures average consensus of the agents forming the different connected components.*

Note that when $\mathbb{1}^\top v(0) = 0$, the centroid is time-invariant and $\vartheta = \frac{1}{N}\sum_{i=1}^{N} x_i(0)$, therefore average consensus is reached. Note also that assuming initial zero velocities is reasonable in this type of applications.

IV. Prescribed transient behavior for the position errors and the combined errors

A. Combined error

Beside the bounds (2) on the relative positions x_{ij}, we now impose additional bounds to the evolution of a linear combination of the velocity and the position error (the *combined error*). Weighted sums of position and velocity errors have been extensively used for generating sliding surfaces in order to transform the second order to a first order stabilization problem [15]. Let us define

$$\bar{q}_k \triangleq q_{ij} = v_{ij} + \gamma x_{ij} \qquad i \in \mathcal{V} \quad \text{and} \quad j \in \mathcal{N}_i \tag{19}$$

with γ positive constant and $k = 1, 2, \dots, m$. We want to constrain these quantities within the following bounds:

$$-M_{ij,\bar{q}}\rho_{ij,\bar{q}}(t) < q_{ij}(t) < \rho_{ij,\bar{q}}(t) \quad \text{if} \quad q_{ij}(0) > 0 \tag{20a}$$

$$-\rho_{ij,\bar{q}}(t) < q_{ij}(t) < M_{ij,\bar{q}}\rho_{ij,\bar{q}}(t) \quad \text{if} \quad q_{ij}(0) < 0 \tag{20b}$$

with $M_{ij,\bar{q}}$ overshoot index and $\rho_{ij,\bar{q}}(t)$ performance functions, $\forall$ $(i,j) \in \mathcal{E}$. By denoting with $\hat{q}_{ij}(t) = \frac{q_{ij}(t)}{\rho_{ij}(t)}$ the modified combined errors we can define the corresponding prescribed performance regions, $\forall$ $(i,j) \in \mathcal{E}$:

$$D_{\hat{q}_{ij}} \triangleq \{\hat{q}_{ij}(t) : \hat{q}_{ij}(t) \in (-M_{ij,\bar{q}}, 1)\} \text{ if } q_{ij}(0) > 0 \tag{21a}$$

$$D_{\hat{q}_{ij}} \triangleq \{\hat{q}_{ij}(t) : \hat{q}_{ij}(t) \in (-1, M_{ij,\bar{q}})\} \text{ if } q_{ij}(0) < 0 \tag{21b}$$

Let us also define $\hat{\bar{q}}$ as the stack vector of all combined errors $\hat{q}_{ij}$ and $\varepsilon_{\hat{\bar{q}}}(\hat{\bar{q}})$ as the stack vector of all transformed combined errors $\varepsilon_{ij,\hat{\bar{q}}}(\hat{q}_{ij})$.

Remark 2. *The bounds (20a) or (20b) on the combined error (19) imply a first order perturbed differential equation for the edge-space position error:*

$$\dot{x}_{ij} = -\gamma x_{ij} + z(t) \tag{22}$$

with $z(t)$ being a disturbance input satisfying (20a) *or* (20b). *Furthermore, equation* (22) *implies indirect bounds on the velocity error that depend on the bounds on the position and the combined errors and cannot be chosen arbitrarily. See [10] for more details.*

B. Proposed controller

Let $T_{ij,\hat{\bar{q}}}$ and $T_{ij,\hat{\bar{x}}}$ denote the transformation functions applied to the modified errors $\hat{q}_{ij}$ and $\hat{x}_{ij}$, respectively. In order to achieve the new control objective, the controller that we propose has an additional prescribed performance term depending on the transformed combined error:

$$\begin{aligned} u_i = &- \sum_{j\in\mathcal{N}_i} \mathcal{J}_{T_{ij},\hat{\bar{q}}}(\hat{q}_{ij}, t)\varepsilon_{ij,\hat{\bar{q}}}(\hat{q}_{ij}) \\ &- \sum_{j\in\mathcal{N}_i} \left[g_{ij}\mathcal{J}_{T_{ij},\hat{\bar{x}}}(\hat{x}_{ij}, t)\varepsilon_{ij,\hat{\bar{x}}}(\hat{x}_{ij})\right] - \gamma v_i \end{aligned} \tag{23}$$

for $i \in \mathcal{V}$, where $g_{ij} = g_{ji}$ are positive constant gains, $\forall (i,j) \in \mathcal{E}$, and all the other terms have already been defined. The controller (23) will be addressed as PPC2. Let $G \in \mathbb{R}^{m\times m}$ be a diagonal, positive definite, gain matrix with diagonal elements g_{ij} and let B be the incidence matrix of the graph $\mathcal{G}$ describing the communication topology of the multi-agent system. The closed loop system can be written as

$$\ddot{x} = -B\mathcal{J}_{T_{\hat{\bar{q}}}}(\hat{\bar{q}}, t)\varepsilon_{\hat{\bar{q}}}(\hat{\bar{q}}) - B\mathcal{J}_{T_{\hat{\bar{x}}}}(\hat{\bar{x}}, t)G\varepsilon_{\hat{\bar{x}}}(\hat{\bar{x}}) - \gamma v \tag{24}$$

If we define $\Gamma = \gamma I_N$ and $q = v + \Gamma x$, we can write (24) as a first order system:

$$\dot{q} = -B\mathcal{J}_{T_{\hat{\bar{q}}}}(\hat{\bar{q}}, t)\varepsilon_{\hat{\bar{q}}}(\hat{\bar{q}}) - B\mathcal{J}_{T_{\hat{\bar{x}}}}(\hat{\bar{x}}, t)G\varepsilon_{\hat{\bar{x}}}(\hat{\bar{x}}) \tag{25}$$

Such a form will be useful when studying the stability properties.

C. Stability analysis

By denoting with $\rho_{ij,\bar{x}}$ the performance function for x_{ij} and with $\alpha_{ij,\bar{x}}$ the corresponding normalized derivative, the conditions that ensure stability of (24) are given in the following theorem:

Theorem 2. *Consider the prescribed performance agreement protocol* (23) *applied to the double integrator dynamics* (1) *with performance functions $\rho_{ij,\bar{x}}(t)$ having bounded derivative $\forall t \geq 0$ and transformation functions s.t. $T_{ij,\hat{\bar{x}}}(0) = 0$ and $T_{ij,\hat{\bar{q}}}(0) = 0$ $\forall$ $(i,j) \in \mathcal{E}$. Assume also that $x_{ij}(0)$ is within the performance bounds* (2) *and that γ is chosen such that*

$$\gamma > \max_{t\geq 0}\ \alpha_{ij,\bar{x}}(t) \tag{26}$$

and $q_{ij}(0) = v_{ij}(0) + \gamma x_{ij}(0)$ is within the performance bounds (20) *$\forall$ $(i,j) \in \mathcal{E}$. Then the position error $\bar{x}(t)$ and the combined error $\bar{q}(t)$ evolve within the corresponding performance bounds and asymptotically converge to zero.*

Proof. In order to prove the theorem, we omit the arguments $\hat{\bar{q}}$, $\hat{\bar{x}}$ and t from $\varepsilon_{\hat{\bar{q}}}(\hat{\bar{q}})$, $\varepsilon_{\hat{\bar{x}}}(\hat{\bar{x}})$, $\mathcal{J}_{T_{\hat{\bar{q}}}}(\hat{\bar{q}}, t)$ and $\mathcal{J}_{T_{\hat{\bar{x}}}}(\hat{\bar{x}}, t)$ for compactness of notation. The proof consists of three parts: a) proof of the boundedness of the term $\varepsilon_{\hat{\bar{x}}}$, b) proof of the boundedness of the term $\varepsilon_{\hat{\bar{q}}}$, c) proof of the asymptotic stability of the equilibrium.

a) Consider the positive function

$$V_1(q, \hat{\bar{x}}) = \frac{1}{2}q^\top q + \frac{1}{2}\varepsilon_{\hat{\bar{x}}}^\top G\varepsilon_{\hat{\bar{x}}} \tag{27}$$

and its derivative along the trajectories of (24)

$$\dot{V}_1(\bar{q}, \hat{\bar{x}}) = -\bar{q}^\top\mathcal{J}_{T_{\hat{\bar{q}}}}\varepsilon_{\hat{\bar{q}}} - [\gamma I_m - A(t)]\,\bar{x}^\top\mathcal{J}_{T_{\hat{\bar{x}}}}G\varepsilon_{\hat{\bar{x}}} \tag{28}$$

Similar to the proof of Theorem 1, $\dot{V}_1$ is negative semi-definite if (26) holds $\forall$ $(i,j) \in \mathcal{E}$. Hence $q, \varepsilon_{\hat{\bar{x}}} \in \mathcal{L}_\infty$, which implies that $\bar{x}(t)$ evolves within the performance bounds (2). Furthermore the boundedness of $\varepsilon_{\hat{\bar{x}}} \in \mathcal{L}_\infty$ also implies that $\mathcal{J}_{T_{\hat{\bar{x}}}}$ is bounded.

b) Based on a), we can define $\mathrm{d}(t) = -B\mathcal{J}_{T_{\hat{\bar{x}}}}\varepsilon_{\hat{\bar{x}}} \in \mathcal{L}_\infty$ and write the system (25) in the form

$$\dot{q} + B\mathcal{J}_{T_{\hat{\bar{q}}}}\varepsilon_{\hat{\bar{q}}} = \mathrm{d}(t) \tag{29}$$

as a first order system evolving under the effect of a bounded disturbance. For this part of the proof, let $\theta > 0$ be an arbitrary constant and consider the following positive function

$$V_2(\varepsilon_{\hat{\bar{q}}}, q) = \frac{1}{2}\varepsilon_{\hat{\bar{q}}}^\top\varepsilon_{\hat{\bar{q}}} + \frac{\theta}{2}q^\top q \tag{30}$$

Taking into account (6) and (29) we get

$$\dot{\varepsilon}_{\hat{\bar{q}}} = -\mathcal{J}_{T_{\hat{\bar{q}}}}B^\top B\mathcal{J}_{T_{\hat{\bar{q}}}}\varepsilon_{\hat{\bar{q}}} + \mathcal{J}_{T_{\hat{\bar{q}}}}B^\top d(t) + \mathcal{J}_{T_{\hat{\bar{q}}}}A(t)\bar{q} \tag{31}$$

By differentiating (30) along the system's trajectories (25) and taking into account (31), we obtain

$$\begin{aligned}\dot{V}_2(\varepsilon_{\hat{\bar{q}}}, q) = &- \varepsilon_{\hat{\bar{q}}}^\top \mathcal{J}_{T_{\hat{\bar{q}}}} B^\top B \mathcal{J}_{T_{\hat{\bar{q}}}} \varepsilon_{\hat{\bar{q}}} - \varepsilon_{\hat{\bar{q}}}^\top \mathcal{J}_{T_{\hat{\bar{q}}}} \dot{P}(t) \hat{\bar{q}} \\ &- \theta \bar{q}^\top \mathcal{J}_{T_{\hat{\bar{q}}}} \varepsilon_{\hat{\bar{q}}} + \varepsilon_{\hat{\bar{q}}}^\top \mathcal{J}_{T_{\hat{\bar{q}}}} B^\top \mathrm{d}(t) + \theta q^\top \mathrm{d}(t) \end{aligned} \tag{32}$$

Let us recall the definition of $\mathcal{J}_{T_{\hat{\bar{q}}}}$ and $\alpha_{ij}(t)$ given in (7) and (8) respectively. Substituting $\hat{\bar{q}} = P(t)^{-1} B^\top q$, with $P(t)$ a $m \times m$ diagonal matrix with entries $\rho_{ij}(t)$ the following equality stands:

$$-\varepsilon_{\hat{\bar{q}}}^\top \mathcal{J}_{T_{\hat{\bar{q}}}} \dot{P}(t) \hat{\bar{q}} - \theta \bar{q}^\top \mathcal{J}_{T_{\hat{\bar{q}}}} \varepsilon_{\hat{\bar{q}}} = -\hat{\bar{q}}^\top \left[\theta I_m - A(t)\right] \frac{\partial \varepsilon_{\hat{\bar{q}}}}{\partial \hat{\bar{q}}} \varepsilon_{\hat{\bar{q}}} \tag{33}$$

The matrix $A(t)$ can be simply bounded from above, i.e. $\sup_t(\max_{(i,j)\in\mathcal{E}} |\alpha_{ij}(t)|) < \bar{\alpha}$, with some constant $\bar{\alpha}$. By setting $\bar{\mu} := \theta - \bar{\alpha}$ we can bound from above the term on the right of (33) with

$$-\hat{\bar{q}}^\top \left[\theta I_m - A(t)\right] \frac{\partial \varepsilon_{\hat{\bar{q}}}}{\partial \hat{\bar{q}}} \varepsilon_{\hat{\bar{q}}} \le -\bar{\mu} \hat{\bar{q}}^\top \frac{\partial \varepsilon_{\hat{\bar{q}}}}{\partial \hat{\bar{q}}} \varepsilon_{\hat{\bar{q}}} \tag{34}$$

Consider now the remaining terms on the right side of (32). We have the following inequalities:

$$-\varepsilon_{\hat{\bar{q}}}^\top \mathcal{J}_{T_{\hat{\bar{q}}}} B^\top \mathrm{d}(t) \le \zeta_1 \|B \mathcal{J}_{T_{\hat{\bar{q}}}} \varepsilon_{\hat{\bar{q}}}\|^2 + \frac{1}{4\zeta_1} \|\mathrm{d}(t)\|^2 \tag{35}$$

$$q^\top \mathrm{d}(t) \le \zeta_2 \|q\|^2 + \frac{1}{4\zeta_2} \|\mathrm{d}(t)\|^2 \tag{36}$$

for some appropriately chosen positive constants $\zeta_1 < 1$, and ζ_2. Considering inequalities (34), (35) and (36), we can bound $\dot{V}_2(\hat{\bar{q}}, q)$ with

$$\begin{aligned}\dot{V}_2(\hat{\bar{q}}, q) \le &- (1-\zeta_1) \|B \mathcal{J}_{T_{\hat{\bar{q}}}} \varepsilon_{\hat{\bar{q}}}\|^2 - \bar{\mu} \hat{\bar{q}}^\top \frac{\partial \varepsilon_{\hat{\bar{q}}}}{\partial \hat{\bar{q}}} \varepsilon_{\hat{\bar{q}}} \\ &+ \zeta_2 \|q\|^2 + \left(\frac{1}{4\zeta_1} + \frac{1}{4\zeta_2}\right) \|\mathrm{d}(t)\|^2 \end{aligned} \tag{37}$$

Being $(1-\zeta_1)\|B \mathcal{J}_{T_{\hat{\bar{q}}}} \varepsilon_{\hat{\bar{q}}}\|^2 \ge 0$, inequality (37) becomes

$$\dot{V}_2(\hat{\bar{q}}, q) \le -\bar{\mu} \hat{\bar{q}}^\top \frac{\partial \varepsilon_{\hat{\bar{q}}}}{\partial \hat{\bar{q}}} \varepsilon_{\hat{\bar{q}}} + \zeta_2 \|q\|^2 + \left(\frac{1}{4\zeta_1} + \frac{1}{4\zeta_2}\right) \|\mathrm{d}(t)\|^2 \tag{38}$$

Adding and subtracting the quantity $\mu_2 \|q\|^2$, with μ_2 being an appropriately chosen positive constant, (38) can be written as

$$\dot{V}_2(\hat{\bar{q}}, q) \le -\bar{\mu} \hat{\bar{q}}^\top \frac{\partial \varepsilon_{\hat{\bar{q}}}}{\partial \hat{\bar{q}}} \varepsilon_{\hat{\bar{q}}} - \mu_2 \|q\|^2 + \varphi(t) \tag{39}$$

where $\varphi(t) = (\mu_2 + \zeta_2)\|q\|^2 + \left(\frac{1}{4\zeta_1} + \frac{1}{4\zeta_2}\right)\|\mathrm{d}(t)\|^2$ is a bounded term given the boundedness of q and $\mathrm{d}(t)$.

Furthermore (9) yields

$$-\bar{\mu} \hat{\bar{q}}^\top \frac{\partial \varepsilon_{\hat{\bar{q}}}}{\partial \hat{\bar{q}}} \varepsilon_{\hat{\bar{q}}} \le -\bar{\mu} \mu_1 \|\varepsilon_{\hat{\bar{q}}}\|^2$$

where μ_1 is defined as $\mu_1 \triangleq \min_{(i,j)\in\mathcal{E}} \mu_{ij}$. Hence, by imposing $\lambda = 2\mu_2 = 2\bar{\mu}\mu_1$ (39) becomes

$$\dot{V}_2(\hat{\bar{q}}, q) \le -\lambda V_2(\varepsilon_{\hat{\bar{q}}}, q) + \varphi(t) \tag{40}$$

with

$$\varphi(t) = (\bar{\mu}\mu_1 + \zeta_2)\|q\|^2 + \left(\frac{1}{4\zeta_1} + \frac{1}{4\zeta_2}\right)\|\mathrm{d}(t)\|^2$$

bounded term, based on a). By applying Theorem 4.18 of [16], we can conclude that $\varepsilon_{\hat{\bar{q}}}$ and, consequently, $\mathcal{J}_{T_{\hat{\bar{q}}}}$ are bounded, thus proving that the combined error evolves within the bounds (20).

c) Let us recall the positive function (27) that we used in part a) and its first derivative (28). As showed in he proof of Theorem 1, based on the boundedness of $\dot{V}_1(\bar{q}, \hat{\bar{x}})$ we can apply Barbalat's Lemma to conclude that $\bar{q}(t) \to 0$ and $\bar{x}(t) \to 0$, thus completing the proof. □

Even in this case we can find an analytical solution for the time evolution of the centroid which is the same as (15). Therefore, along the lines of Corollary 1, we can state the following complementary result about the final consensus value:

Corollary 2. *Consider the prescribed performance agreement protocol* (23) *applied to the double integrator dynamics* (1) *with all the assumptions of Theorem 2. The agreement protocol ensures convergence of the agents' absolute positions to the centroid if the communication graph is connected.*

Proof. See proof of Corollary 1. □

Note that once again, we obtain average consensus if the sum of initial velocities is equal to zero.

V. Simulations

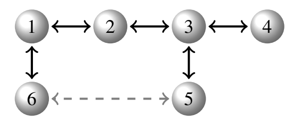


Figure 3: Communication graph of the multi-agent system used in the simulations: spanning tree $\mathcal{G}_1$ in solid lines and connected graph with one cycle $\mathcal{G}_2$ in solid and dashed lines.

In this section simulation results are presented in order to validate the theoretical findings of the previous sections. We consider $N = 6$ agents moving on a planar surface. Let $p_i = \begin{bmatrix}\mathrm{x}_i & \mathrm{y}_i\end{bmatrix}^\top$, $i \in \{1, 2, \ldots, N\}$, be the position of each agent and let d denote the sum of the distances between the agents and the centroid $d \triangleq \sum_{i=1}^6 \|p_i - p_c\|$ with $p_c \triangleq \frac{1}{6}\sum_{i=1}^6 p_i$ and u_{av} the average control input $u_{\mathrm{av}} \triangleq \frac{1}{6}\sum_{i=1}^6 \|\ddot{p}_i\|$.

The position errors are modulated by an exponentially decreasing performance function

$$\rho_{ij}(t) = (\rho_0 - \rho_\infty)\, e^{-\tau t} + \rho_\infty \tag{41}$$

which is the same for all the pairs $(i,j) \in \mathcal{E}$ of connected agents, and then transformed by the logarithmic function

$$T(\hat{y}) = \begin{cases} \ln\left(\frac{M + \hat{y}}{M(1-\hat{y})}\right) & \text{if} \quad \hat{y}(0) > 0 \\ \ln\left(\frac{M(1+\hat{y})}{M - \hat{y}}\right) & \text{if} \quad \hat{y}(0) < 0 \end{cases} \tag{42}$$

where $\hat{y} = y/\rho_{ij}(t)$ with $y \in \{\mathrm{x}_{ij}, \mathrm{y}_{ij}\}$ and $M = M_{ij} = 0.1$ $\forall\ (i,j) \in \mathcal{E}$. Note that the prescribed performance functions (41) are strictly decreasing, meaning that the relative distances between communicating agents will not be bigger than $\rho_{ij}(0)$. Hence if the connectivity range is at least equal to $\rho_{ij}(0)$, connectivity is maintained. With the performance and transformation functions chosen as in (41) and (42), the condition (12) on the gain, sufficient for the convergence of the trajectories, becomes $\gamma > \tau\left(\frac{\rho_0 - \rho_\infty}{\rho_0}\right)$. For the first controller (PPC1) we utilize (41) with $\rho_0 = 8$, $\rho_\infty = 10^{-2}$, and $\tau = 2$: hence the condition on the damping becomes $\gamma > 1.9975$. We also consider three different topologies for the connections between the agents (Figure 3): a spanning tree ($\mathcal{G}_1$), a connected graph with a cycle ($\mathcal{G}_2$) and also a complete graph ($\mathcal{G}_3$). The initial positions of the agents are: $p_{01} = \begin{bmatrix}-0.5 & -1\end{bmatrix}^\top$, $p_{02} = \begin{bmatrix}1 & -1.5\end{bmatrix}^\top$, $p_{03} = \begin{bmatrix}2 & 2\end{bmatrix}^\top$, $p_{04} = \begin{bmatrix}2.5 & 4.5\end{bmatrix}^\top$, $p_{05} = \begin{bmatrix}0 & 4\end{bmatrix}^\top$, $p_{06} = \begin{bmatrix}-1 & 1.5\end{bmatrix}^\top$. Furthermore their initial velocities are equal to 0. Given this set of initial conditions we have that $\mathrm{x}_{ij}(0)$ and $\mathrm{y}_{ij}(0)$ are within the bounds $\forall\ (i,j) \in \mathcal{E}$ and for each configuration of the communication graph. For the second controller (PPC2), we choose the same performance function for the relative positions, whereas the bounds for the combined errors are described

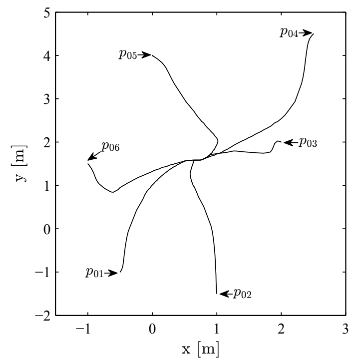


Figure 4: Trajectories on the $\mathrm{x} - \mathrm{y}$ plane for PPC1 in case of $\mathcal{G}_1$.

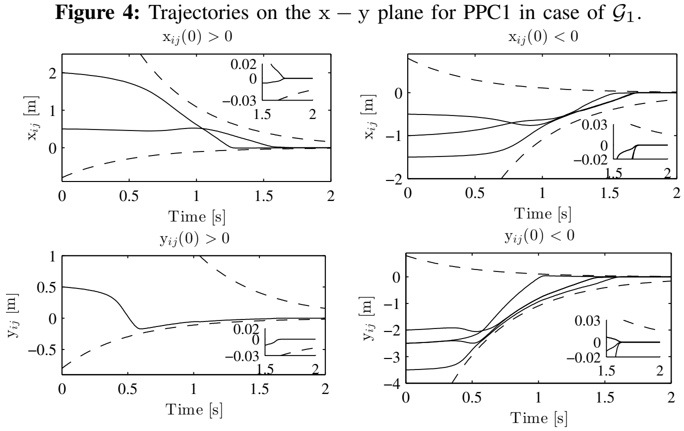


Figure 5: Relative postitions (solid lines) and performance bounds (dashed lines) for PPC1 in case of $\mathcal{G}_1$.

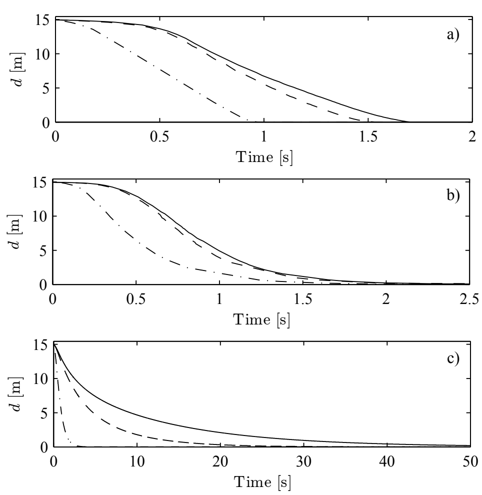


Figure 6: Total distance from the centroid depending on the graph topology: $\mathcal{G}_1$ (solid line), $\mathcal{G}_2$ (dashed line) and $\mathcal{G}_3$ $(- \cdot -)$. a) PPC1, b) PPC2, c) linear.

by $\rho_{ij,\bar{q}}(t) = \left(20 - 10^{-2}\right) e^{-3t} + 10^{-2}$ and the overshoot index is set to $M_{ij,\bar{q}} = 0.1 \; \forall \; (i,j) \in \mathcal{E}$.

PPC1 has been tuned with the following values for the parameters: $\gamma = 250$, $G = 36.5 I_m$. With this settings we have average consensus (Figure 4) and prescribed transient evolution (Figure 5). Given the choice of the performance functions $\rho_{ij}(t)$ each agent can know a priori the minimum rate of convergence of the system by setting the value of τ. However Figure 6a shows that when the graph contains cycles it is also possible to reach consensus faster.

Since the performance function for the quantities x_{ij} does not change, the same behavior is obtained with PPC2 by setting the parameters to $\gamma = 3$, $G = 2I_m$ (see Figure 6b). By comparing Figures 7a and 7b we can also notice that with the second controller the minimum rate of convergence imposed by $\rho_{ij}(t)$ can be achieved generating a smoother control input.

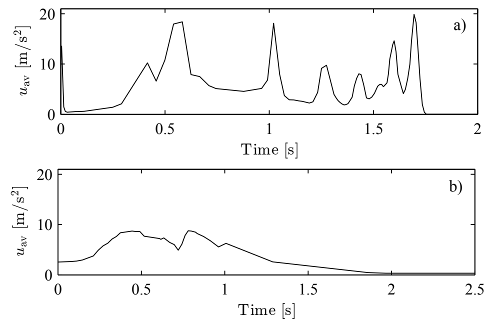


Figure 7: Average control input for a) PPC1 and b) PPC2 with $\mathcal{G}_1$.

We also compared the nonlinear protocol with the linear protocol $\dot{\xi} = \Gamma_L \xi$, where $\Gamma_L = \begin{bmatrix} \mathbf{0} & I_N \\ -L & -\gamma_L I_N \end{bmatrix}$, which is a particular case of the protocol investigated in [3]. The value of the parameter γ_L has been chosen in order to guarantee the minimum rate of convergence possible while ensuring that the spectrum of Γ_L is real, in order to avoid oscillations and obtain a behavior that is similar to our protocol. Figure 6, where the time scales of the three plots are different, illustrates the advantages of the nonlinear protocol with respect to the linear one. In fact, by decoupling the speed of convergence from the graph topology and by appropriately choosing the prescribed performance function, with $\mathcal{G}_1$ and $\mathcal{G}_2$ PPC1 and PPC2 ensure consensus in less than 2.5s, whereas the linear protocol takes 50s and 30s in case of $\mathcal{G}_1$ and $\mathcal{G}_2$, respectively.

VI. Conclusions and future work

In this paper we proposed two nonlinear distributed controllers able to reach consensus while guaranteeing predefined transient specifications such as maximum overshoot and minimum rate of convergence in the edge's space. We considered a group of agents described by a double-integrator model. By applying prescribed performance control we were able to constrain the evolution of the relative positions of neighboring agents within a priori imposed time-variant bounds, obtaining a rate of convergence which is independent of the topology of the network. This overcomes a typical problem of the linear consensus algorithms, in which the convergence is governed by the algebraic connectivity of the communication graph. In the second part of the paper, prescribed performance control was also applied to bound a linear combination of relative velocities and relative positions.

In both cases the stability analysis yielded a condition for the damping gain that guarantees convergence of the agents to the equilibrium, which coincides with the agents' centroid when the graph is connected. Note that the aforementioned condition does not depend on the graph topology but only on the agent's prescribed performance function. Moreover we proved that when the sum of initial velocities is equal to zero, time-invariance of the centroid is guaranteed provided that the neighboring agents share the same prescribed performance function and overshoot index and that the transformation functions have certain symmetry properties. Simulations validated the theoretical findings while demonstrating that by appropriately designing the performance function, the nonlinear protocol can achieve a faster convergence compared to the linear one. We also showed the advantages of the second controller in terms of control input.

Future work includes extension of the results to multi-agent systems with more complex dynamics, such as nonholonomic robots.

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