Platoons Coordination Based on Decentralized Higher Order Barrier Certificates

Maryam Sharifi and Dimos V. Dimarogonas

Abstract— **This paper presents control strategies based on time-varying convergent higher order control barrier functions for the coordination of networks of platoons. This network could be modelled by a class of leader-follower multi-agent systems, where the leaders have knowledge on the associated tasks and control the performance of their platoon involved vehicles. The followers are not aware of the tasks, and do not have any control authority to reach them. They follow their platoon leader commands for the task satisfaction. Signal temporal logic (STL) tasks are defined for the platoons coordination. Robust solutions for the task satisfaction, based on the leader's accessibility to the follower vehicles' states are suggested. In addition, using the notion of higher order barrier functions, decentralized barrier certificates for each vehicle evolving in a formation dynamic structure are proposed. Our approach finds solutions to guarantee the satisfaction of STL tasks independent of the agents' initial conditions.**

Keywords: Network of platoons, leader-follower formation, control barrier functions, signal temporal logic.

I. Introduction

Vehicle platooning systems in which a group of vehicles maneuver cooperatively in order to fulfill transportation in an efficient way with respect to time and energy [1],[2] are one of the emergent applications of multi-agent systems. This research area has become popular as multi-vehicle coordination provides improved capabilities of handling task complexities and robustness to vehicle failures compared to single-vehicle performance. Among other works, fault-tolerant control problem for heterogeneous vehicular platoons with actuator faults and saturation, guaranteeing string stability [3], and control over ad hoc network with access-constrained fading channels [4] could be mentioned. However, the cooperative vehicular maneuvers contain complex tasks that can not be defined as stand-alone traditional control objectives and need employing more advanced approaches in order to define general specifications. Formal verification approaches that have been originally developed in computer science community allow to handle more complex tasks by defining them in temporal logic formulations which induce a sequence of control actions [5]. Signal temporal logic (STL) is one of these formulations which is more beneficial as is interpreted over continuous-time signals [6], allows for imposing tasks with strict deadlines and introduces quantitative robust semantics [7].

Another important property related to multi-vehicle coordination is scaleability. While assigning the same distributed control strategy to all vehicles may be suitable for simpler and more traditional control objectives, our aim here is to tackle high-level and more complex task specifications in the form of STL. We choose to consider a leader-follower approach to the problem due to the computationally free addition of followers. In addition, the leader-follower dynamical structure allows for robustness with respect to failures, and resource usability, since only a subset of the vehicular team, that act as the leaders, need to be actuated for the tasks fulfilment. There are a number of challenges regarding the best choice of the leader vehicles based on the dynamical structure and the graph topology. Leader selection to achieve the stabilization and tracking via the notion of manipulability is considered in [8]. However, majority of current approaches don't take into account complex tasks with space and time constraints prescribed by STL. A class of time-varying fixed-time convergent CBFs for coupled multi-agent systems under STL tasks has been introduced in [9]. In [10], some coordination tasks among platoons are expressed in STL and a CBF-based approach is employed to satisfy the STL constraints.

We have applied some theoretical results of our previous work [11] to network of platoons under STL tasks, where each platoon is represented by a leader-follower multi-agent system. In this framework, the leader and its followers are dynamically connected through a formation structure. We have formulated different platoon coordination dynamic structures and address the high relative degree constraints using *time-varying convergent higher order control barrier functions (TCHCBF)*. The platoons leader vehicles have the knowledge of the associated tasks and are responsible for their satisfaction. The follower vehicles are not aware of the prescribed tasks and don't have any control authority to meet them. They obey their leaders commands according to the formation dynamic structures of the platoon network.

Based on the partial knowledge of the platoon leaders from the followers' states and network topology, relaxed control barrier certificates are introduced. We assume the connectedness for the platoons network topology and guarantee the convergence and forward invariance of the desired sets built based on the specifications. Furthermore, in order to improve scalability of the network control solution and account for more general STL formulas, we have constructed individual barrier certificates for each vehicle. We take

This work was supported by the ERC CoG LEAFHOUND, the Swedish Research Council (VR), the Knut & Alice Wallenberg Foundation (KAW) and ABB Corporate Research, Sweden.

Maryam Sharifi is with ABB Corporate Research, Västerås, Sweden. Email: maryam.sharifi@se.abb.com. Dimos V. Dimarogonas is with Division of Decision and Control Systems, School of Electrical Engineering and Computer Science, KTH Royal Institute of Technology, Stockholm, Sweden. Email: dimos@kth.se.

care of maintaining the constraints by just using the leader vehicle's control input, utilizing the higher order barrier functions of the follower vehicles according to the formation dynamic structure of the network. In addition, we consider coordination tasks between platoons such as merging and splitting, where more than one leader is involved in the task. For these cases, we provide decentralized barrier certificates as functions of the leader vehicles' control signal. Then, the task satisfactions are guaranteed for a network of platoons using the decentralized barrier certificates.

The rest of the paper is organized as follows. Section II gives some preliminaries on STL, leader-follower platoon network dynamical structure, time-varying barrier functions, and relevant results for first order dynamic systems which are used as a foundation for the main results. In Section III, the main results for the platoon network dynamic systems and decentralized barrier certificates are provided. Simulation results for a network of platoons are provided in Section IV. Finally, some concluding points are presented in Sections V.

II. Preliminaries and problem formulation

A. Signal temporal logic (STL)

Signal temporal logic (STL) [6] is based on predicates ν which are obtained by evaluation of a continuously differentiable predicate function $h:\mathbb{R}^d \to \mathbb{R}$ as $\nu := \top$ (True) if $h(\mathbf{x}) \geq 0$ and $\nu := \bot$ (False) if $h(\mathbf{x}) < 0$ for $\mathbf{x} \in \mathbb{R}^d$. The STL syntax is then given by

$$\phi ::= \top | \nu | \neg\phi | \phi' \wedge \phi'' | \phi' U_{[a,b]} \phi'',$$

where $\neg$ and $\wedge$ denote negation and conjunction, respectively and ϕ', ϕ'' are STL formulas, and $U_{[a,b]}$ is the until operator with $a \leq b < \infty$. In addition, define $F_{[a,b]}\phi := \top U_{[a,b]}\phi$ (eventually operator) and $G_{[a,b]}\phi := \neg F_{[a,b]}\neg\phi$ (always operator). Note that $\neg\mu$ can be encoded in the STL syntax above by defining $\bar{\mu} := \neg\mu$ and $\bar{h}(\nu) := -h(\nu)$. Let $(\mathbf{x}, t) \models \phi$ denote the satisfaction relation, i.e., a formula ϕ is satisfiable if $\exists \mathbf{x}: \mathbb{R}_{\geq 0} \to \mathbb{R}^d$ such that $(\mathbf{x}, t) \models \phi$. We consider the STL fragment

$$\psi ::= \top | \nu | \psi' \wedge \psi'', \tag{1a}$$

$$\phi ::= G_{[a,b]}\psi | F_{[a,b]}\psi | \psi' U_{[a,b]} \psi'' | \phi' \wedge \phi'', \tag{1b}$$

where ψ', ψ'' are formulas of class ψ in (1a) and ϕ', ϕ'' are formulas of class ϕ in (1b). It is worth mentioning that these formulas can be extended to consider disjunctions ($\vee$) using automata based approaches [11].

B. Dynamical model of network of platoons

Consider a connected undirected graph $\mathcal{G} := (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} := \{1, \cdots, n\}$ indicates the set consisting of n vehicles and $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$ represents communication links between them. Each vehicle belongs to one platoon which consists of one leader and a number of followers. Without loss of generality, we suppose the first n_f vehicles as followers and the last n_l ones as leaders, with corresponding vertices sets $\mathcal{V}_f := \{1, \cdots, n_f\}$ and $\mathcal{V}_l := \{n_f+1, \cdots, n_f+n_l\}$, respectively, with $n_f + n_l = n$. We will have a time-varying graph topology which switches among different structures according to the coordination phases, e.g., merging, splitting, etc. The overall graph of the multi platoon network can be specified according to the following Laplacian matrix.

$$L_{pl} = \begin{bmatrix} L_{ff} & L_{fl} \\ L_{lf} & L_{ll} \end{bmatrix}, \tag{2}$$

where L_{ff} corresponds to the laplacian matrix of followers interconnections, L_{fl} and L_{lf} model the communications from the leaders to followers and vice versa, respectively, and L_{ll} demonstrates the communications among the leaders of platoons. We consider directed communication from the leaders to their followers. Hence $L_{fl} \neq L_{lf}^\top$ and $L_{lf} = \mathbf{0}$. In addition, the communication among the leaders of platoons is assumed to be undirected. In this manner, while each platoon is subject to its local tasks, there is no coordination between platoons, i.e., $L_{ll} = \mathbf{0}$. When the platoon coordinations such as merging or splitting are considered, the Laplacian matrix will change according to the new graph topology. Let $p_i \in \mathbb{R}$, $v_i \in \mathbb{R}$ and $u_i \in \mathbb{R}$ denote the position, velocity and control input of vehicle $i \in \mathcal{V}$, respectively. Moreover, $\mathcal{N}_i$ denotes the set of neighbors of vehicle i and $|\mathcal{N}_i|$ determines the cardinality of the set $\mathcal{N}_i$. In addition, $\mathfrak{f}_i : \mathbb{R}^{2+2|\mathcal{N}_i|} \to \mathbb{R}$, $\mathfrak{g}_i : \mathbb{R} \to \mathbb{R}$ are assumed to be locally Lipschitz continuous functions. We define the stacked vector of all elements in the set $\mathcal{X}$ with cardinality $|\mathcal{X}|$, as $[x_i]_{i\in\mathcal{X}} := [x_{i_1}^\top, \cdots, x_{i_{|\mathcal{X}|}}^\top]^\top$, $i_1, \cdots, i_{|\mathcal{X}|} \in \mathcal{X}$, and write the stacked dynamics for the network of platoons containing 2^{nd} order dynamics vehicles $i \in \mathcal{V}$, as

$$\dot{x} = \mathfrak{f}_{pl}(x) + \mathfrak{g}_{pl}(x)u, \tag{3}$$

where $x := [x_i]_{i\in\mathcal{V}} = [p_i; v_i]_{i\in\mathcal{V}} \in \mathcal{S} \subseteq \mathbb{R}^{2n}$, $\mathfrak{f}_{pl}(\cdot) = [\mathfrak{f}_i(\cdot)]_{i\in\mathcal{V}} \in \mathbb{R}^{2n}$, $\mathfrak{f}_{pl}(x) := \begin{bmatrix} 0_{6\times 6} & I_6 \\ -L_{pl} & -L_{pl} \end{bmatrix} x$. In addition, the local dynamic functions $\mathfrak{f}_{i,i}(x_i)$ correspond to the terms of $\mathfrak{f}_i(x)$ which are only dependent on x_i, and $\mathfrak{f}_{i,j}(x_i, x_j)$ contains the terms of $\mathfrak{f}_i(x)$ which are dependent on agent $j \in \mathcal{V}, j \neq i$ as well. For the case of one platoon (one leader), with follower and leader sets $\mathcal{V}_f := \{1, \cdots, n-1\}$ and $\mathcal{V}_l := \{n\}$, respectively, the input matrix and control input signal are defined as $\mathfrak{g}_{pl}(\cdot) := \begin{bmatrix} 0_{2n-1\times 1}^T, \mathfrak{g}_n(\cdot) \end{bmatrix}^T$, and $u := u_n \in \mathbb{R}$. In addition, for networks containing more than one platoon (multiple leaders), with $\mathcal{V}_f := \{1, \cdots, n_f\}$ and $\mathcal{V}_l := \{n_f+1, \cdots, n\}$, $n = n_f + n_l$, the associated matrices are denoted as $\mathfrak{g}_{pl}(\cdot) := \begin{bmatrix} 0_{n_e+n_f\times n_l}^T, \mathfrak{g}_{n_l}(\cdot) I_{n_l} \end{bmatrix}^T$, $\mathfrak{g}_{n_l}(\cdot) = [\mathfrak{g}_i(\cdot)]_{i\in\{n_f+1,\cdots,n\}}$, and $u := [u_i]_{i\in\{n_f+1,\cdots,n\}} \in \mathbb{R}^{n_l}$. Hence, the input matrix $\mathfrak{g}_{pl}(\cdot)$ is not full row rank. We also denote by m_i as the minimum of the length of paths between vehicle i and its corresponding platoon leader.

We assume that the dynamics of the vehicles are input-to-state stable (ISS). Then, it could be shown that the ISS property of the whole platoon network is guaranteed [11].

C. Time-varying barrier functions

In this subsection, we introduce time-varying barrier functions. For the sake of simplicity, we consider 1^{st} order dynamic systems and then extend the results to the considered

2^{nd} order dynamic platoons in Section III. Consider the following 1^{st} order dynamic network:

$$\dot{x}^s = \mathfrak{f}^s(x^s) + \mathfrak{g}^s(x^s)u, \tag{4}$$

(4), where $\mathfrak{f}^s(\cdot) = [\mathfrak{f}_i^s(\cdot)]_{i\in\mathcal{V}} \in \mathbb{R}^n$, $\mathfrak{g}^s(\cdot) = [\mathfrak{g}_i^s(\cdot)]_{i\in\mathcal{V}}$ and $u \in \mathbb{R}$, $\mathfrak{f}_i^s : \mathbb{R}^{1+|\mathcal{N}_i|} \to \mathbb{R}$, $\mathfrak{g}_i^s : \mathbb{R} \to \mathbb{R}$, corresponds to the dynamics, input function and the control input of the system, respectively. Consider formulas ϕ^s and ϕ^d of the form (1b), corresponding to the systems (4) and (3), respectively. These formulas consist of a number of temporal operators and their satisfaction depends on the behavior of the set of vehicles $\mathcal{V} = \{1, \cdots, n\}$.

Assumption 1. *Predicate functions in ϕ^s (resp. ϕ for the second order dynamical platoon networks) are concave.*

Following the procedure in [12], we construct the barrier function $\mathfrak{h}^s(x^s,t) : \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}$, for the conjunctions of a number of q^s single temporal operators, by using a smooth under-approximation of the min-operator. Then, the corresponding barrier function to ϕ^s could be constructed as

$$\mathfrak{h}^s(x^s,t) := -\frac{1}{\eta^s}\ln\Big(\sum_{j=1}^{q^s}\exp(-\eta^s\mathfrak{h}_j^s(x^s,\mathrm{t}))\Big), \tag{5}$$

where each $\mathfrak{h}_j^s(x^s,t)$ is related to an always or eventually operator specified for the time interval $[a_j, b_j]$, and parameter $\eta^s > 0$ is proportionally related to the accuracy of this approximation.. Whenever the jth temporal operator is satisfied, its corresponding barrier function $\mathfrak{h}_j^s(x^s,t)$ is deactivated and hence a switching occurs in $\mathfrak{h}^s(x^s,t)$. This time-varying strategy helps reducing the conservatism in the presence of large numbers of conjunctions [12]. Due to the knowledge of $[a_j, b_j]$, the switching instants can be known in advance.

Definition 1. *[13] (Forward Invariance) The set $\mathfrak{C}^s(t) := \{x^s \in \mathbb{R}^n | \mathfrak{h}^s(x^s,t) \geq 0\}$ is forward invariant with a given control law u for* (4)*, if for each initial condition $x_0^s \in \mathfrak{C}^s(t_0)$, there exists a unique solution $x^s : [t_0,t_1] \to \mathbb{R}^n$ with $x(t_0) = x_0^s$, such that $x^s(t) \in \mathfrak{C}^s(t)$ for all $t \in [t_0,t_1]$.*

Definition 2. *We denote the set $\mathfrak{C}^s(t)$ to be **fixed-time convergent** for* (4)*, if there exists a user-defined, independent of the initial condition, and finite time $T^s > t_0$, such that $\lim_{t\to T^s} x^s(t) \in \mathfrak{C}^s(t)$. Moreover, the set $\mathfrak{C}^s(t)$ is **robust fixed-time convergent** if $\lim_{t\to T^s} x^s(t) \in \mathfrak{C}_{rf}^s(t)$, where $\mathfrak{C}_{rf}^s(t) \supset \mathfrak{C}^s(t)$, and **robust convergent** for* (4)*, if $\lim_{t\to\infty} x^s(t) \in \mathfrak{C}_{rf}^s(t)$. The set $\mathfrak{C}_{rf}^s(t)$ is characterized as $\mathfrak{C}_{rf}^s(t) := \{x^s \in \mathbb{R}^n | \mathfrak{h}^s(x^s,t) \geq -\epsilon_{\max}^s\}$, where $\epsilon_{\max}^s$ is a bounded and positive value.*

The same properties hold for the barrier functions $\mathfrak{h}^d(x^d,t)$ and the set $\mathfrak{C}^d(t)$ corresponding to the 2^{nd} order dynamic platoon network (3) under the task ϕ^d.

Definition 3. *[11, Theorem 1] Consider a network of platoons subject to the dynamics* (4) *containing one leader, under STL task ϕ^s of the form* (1b) *satisfying Assumption 1. Let $\mathfrak{h}^s(x^s,t)$ be a time-varying barrier function associated with the task ϕ^s, specified in Section II-C. If for some constants $\mu^s > 1$, $k^s > 1$, $\gamma_1^s = 1-\frac{1}{\mu^s}$, $\gamma_2 = 1+\frac{1}{\mu^s}$, $\alpha^s > 0$, $\beta^s > 0$, for some open set $\mathcal{S}^s$ with $\mathcal{S}^s \supset \mathfrak{C}^s(t)$, $\forall t \geq 0$, and for all $(x^s,t) \in \mathcal{S}^s \times [\tau_l, \tau_{l+1})$, $l \in \{0, \cdots, p^s-1\}$, there exists a control law u_n such that*

$$\begin{aligned}&\textstyle\sum_{i\in\mathcal{N}_n} \frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_i^s}\mathfrak{f}_{i,i}^s(x_i^s) + \big(\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_n^s} + \frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_i^s}\big)\mathfrak{f}_{n,i}^s(x_n^s,x_i^s)\\ &+\frac{\partial\mathfrak{h}_e^s(x^s,t)}{\partial x_n^s}\mathfrak{f}_{n,n}^s(x_n^s) + \frac{\partial\mathfrak{h}^s(x^s,t)}{\partial t} + \frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_n^s}\mathfrak{g}_n^s(x_n^s)u_n\\ &\geq -\alpha^s\, sgn(\mathfrak{h}^s(x^s,t))|\mathfrak{h}^s(x^s,t)|^{\gamma_1}\\ &-\beta^s\, sgn(\mathfrak{h}^s(x^s,t))|\mathfrak{h}^s(x^s,t)|^{\gamma_2^s},\end{aligned} \tag{6}$$

with

$$\begin{aligned}T^s &\leq \begin{cases} \frac{\mu^s}{\alpha^s(c^s-b^s)}\log\big(\frac{|1+c^s|}{|1+b^s|}\big) & ;\delta^s > 2\sqrt{\alpha^s\beta^s}\\ \frac{\mu^s}{\sqrt{\alpha^s\beta^s}}\big(\frac{1}{k^s-1}\big) & ;\delta^s = 2\sqrt{\alpha^s\beta^s}\\ \frac{\mu^s}{\alpha^s k_1^s}\big(\frac{\pi}{2} - \tan^{-1}k_2^s\big) & ;0 \leq \delta^s < 2\sqrt{\alpha^s\beta^s}\end{cases}\\ &\leq \min_{l\in\{0,\cdots,p^s-1\}}\{\tau_{l+1}-\tau_l\},\end{aligned} \tag{7}$$

where b^s, c^s are the solutions of $\gamma^s(s) = \alpha^s s^2 - \delta^s s + \beta^s = 0$, $k_1^s = \sqrt{\frac{4\alpha^s\beta^s - \delta^{s2}}{4\alpha^{s2}}}$, $k_2^s = -\frac{\delta^s}{\sqrt{4\alpha^s\beta^s-\delta^{s2}}}$, and δ^s is introduced satisfies $\|\sum_{i\in\mathcal{N}_n, j\notin\mathcal{N}_n}\frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_j^s}\mathfrak{f}_j^s(x^s) + \frac{\partial\mathfrak{h}^s(x^s,t)}{\partial x_i^s}\mathfrak{f}_{i,j}^s(x_i^s,x_j^s)\| \leq \delta^s$, $\forall(x^s,t) \in \mathcal{S}^s \times [\tau_l,\tau_{l+1})$, $l \in \{0, \cdots, p^s-1\}$, then, the set $\mathfrak{C}_{rf}^s(t) \supset \mathfrak{C}^s(t)$ defined by

$$\mathfrak{C}_{rf}^s(t) := \{x^s \in \mathbb{R}^n | \mathfrak{h}^s(x^s,t) \geq -\epsilon_{\max}^s\}$$

with

$$\epsilon_{\max}^s = \begin{cases}\big(\frac{\delta^s + \sqrt{\delta^{s2} - 4\alpha^s\beta^s}}{2\alpha^s}\big)^{\mu^s} & ;\delta^s > 2\sqrt{\alpha^s\beta^s}\\ k^{s\mu^s}\big(\frac{\beta^s}{\alpha^s}\big)^{\frac{\mu^s}{2}} & ;\delta^s = 2\sqrt{\alpha^s\beta^s}\\ \frac{\delta^s}{2\sqrt{\alpha^s\beta^s}} & ;0 \leq \delta^s < 2\sqrt{\alpha^s\beta^s},\end{cases} \tag{8}$$

is forward invariant and fixed-time convergent within T^s time units, defined in (7).

III. Main Results

In this section, we consider second order dynamics platoon systems and in order to tackle their higher relative degree specifications, provide a class of higher order control barrier functions with the property of convergence to the desired sets and robustness with respect to uncertainties. We first provide definitions which are used to get the main results.

Definition 4. *Consider the autonomous system*

$$\dot{\mathbf{x}} = f(\mathbf{x}), \tag{9}$$

with $\mathbf{x} \in \mathbb{R}^n$ and locally Lipschitz continuous function $f : \mathbb{R}^n \to \mathbb{R}^n$. A class C^m function $\mathfrak{h}(\mathbf{x},t) : \mathbb{R}^n \times [t_0,\infty) \to \mathbb{R}$ is a time-varying convergent higher order barrier function (TCHBF) of degree m for the system (9)*, if there exist extended class $\mathcal{K}$ functions $\lambda_k(\cdot)$, $k = 1, \cdots, m-1$, constants $0 < \gamma_{1m} < 1, \gamma_{2m} > 1, \alpha_m > 0, \beta_m > 0$, and an open set $\mathfrak{D}$ with $\mathfrak{C} := \cap_{k=1}^m \mathfrak{C}_k \subset \mathfrak{D} \subset \mathbb{R}^n$ such that*

$$\psi_m(\mathbf{x},t) \geq 0,\ \forall(\mathbf{x},t) \in \mathfrak{D} \times \mathbb{R}_{\geq 0},$$

where the functions $\psi_k : \mathbb{R}^n \times [t_0, \infty) \to \mathbb{R}^n$, $0 \le k \le m$, *are given as*

$$\begin{aligned}\psi_0(\mathbf{x}, t) &:= \mathfrak{h}(\mathbf{x}, t),\\ \psi_k(\mathbf{x}, t) &:= \dot{\psi}_{k-1}(\mathbf{x}, t)\\ &\quad + \lambda_k(\psi_{k-1}(\mathbf{x}, t)),\ 1 \le k \le m-1,\\ \psi_m(\mathbf{x}, t) &:= \dot{\psi}_{m-1}(\mathbf{x}, t)\\ &\quad + \alpha_m \mathrm{sgn}(\psi_{m-1}(\mathbf{x}, t))|\psi_{m-1}(\mathbf{x}, t)|^{\gamma_{1m}}\\ &\quad + \beta_m \mathrm{sgn}(\psi_{m-1}(\mathbf{x}, t))|\psi_{m-1}(\mathbf{x}, t)|^{\gamma_{2m}}, \quad (10)\end{aligned}$$

and

$$\mathfrak{C}_k(t) := \{\mathbf{x} \in \mathbb{R}^n | \psi_{k-1}(\mathbf{x}, t) \ge 0\}. \quad (11)$$

Definition 5. *Consider the system*

$$\dot{\mathbf{x}} = f(\mathbf{x}) + g(\mathbf{x})\mathbf{u}(\mathbf{x}), \quad (12)$$

with locally Lipschitz continuous functions f and g. A class C^m function $\mathfrak{h}(\mathbf{x}, t) : \mathbb{R}^n \times [t_0, \infty) \to \mathbb{R}$, associated with the task ϕ of the form (1b)*, is called a time-varying convergent higher order control barrier function (TCHCBF) of degree m for this system under task ϕ of the form* (1b)*, if for some constants $0 < \gamma_{1m} < 1$, $\gamma_{2m} > 1$, $\alpha_m > 0$, $\beta_m > 0$, and an open set $\mathfrak{D}$ with $\mathfrak{C} := \cap_{k=1}^{m} \mathfrak{C}_k \subset \mathfrak{D} \subset \mathbb{R}^n$, $\mathfrak{C}_k, k = 1, \cdots, m$, defined as in* (11)*, there exists a control law $\mathbf{u}(\mathbf{x})$ such that*

$$\begin{aligned}&\frac{\partial \psi_{m-1}(\mathbf{x}, t)}{\partial \mathbf{x}} (f(\mathbf{x}) + g(\mathbf{x})\mathbf{u}(\mathbf{x})) + \frac{\partial \psi_{m-1}(\mathbf{x}, t)}{\partial t}\\ &\quad \ge -\alpha_m sgn(\psi_{m-1}(\mathbf{x}, t))|\psi_{m-1}(\mathbf{x}, t)|^{\gamma_{1m}}\\ &\quad - \beta_m sgn(\psi_{m-1}(\mathbf{x}, t))|\psi_{m-1}(\mathbf{x}, t)|^{\gamma_{2m}}, \quad (13)\end{aligned}$$

where $\psi_{m-1}(\mathbf{x}, t)$ is given by (10).

Next, we use the introduced TCHCBFs for 2^{nd} order platoons.

A. Second order dynamics platoon

Consider a group of n number of vehicles with 2^{nd} order dynamics as in (3), under the task ϕ. We will formulate a quadratic program that renders the set $\mathfrak{C}_{pl} := \cap_{k=1}^{2} \mathfrak{C}_k \subset \mathcal{S} \subset \mathbb{R}^{2n}$ corresponding to functions $\mathfrak{h}(x, t)$ and $\psi_1(x, t)$, defined by (11), *robust convergent*, under the following Assumption.

Assumption 2. *Consider one 2^{nd} order platoon* (3) *with the leader $i = n$. There exists a positive constant δ satisfying $\| \sum_{i \in \mathcal{N}_n, j \notin \mathcal{N}_n} \frac{\partial \psi_1(x,t)}{\partial x_j} \mathfrak{f}_j(x) + \frac{\partial \psi_1(x,t)}{\partial x_i} \mathfrak{f}_{i,j}(x_i, x_j) \| \le \delta$, $\forall (x, t) \in \mathcal{S} \times [\tau_l, \tau_{l+1})$, $l \in \{0, \cdots, p-1\}$).*

In the following, a control input u_n will be found such that for all initial conditions $x(t_0)$, and under Assumption 2, the trajectories of (3) converge to a set $\mathfrak{C}_{rf}(t) \supset \mathfrak{C}_{pl}(t)$ in a fixed-time $t \le T + t_0$, $T > 0$. We provide the statement of the Theorem based on a quadratic problem formulation based on our previous work [11, Theorem 2] and present here for the sake of completeness. We do not provide the proof as it follows the one in [11].

Theorem 1. *Consider a given TCHCBF $\mathfrak{h}(x, t)$ introduced in Definition 5 with the associated functions $\psi_k(x, t)$, $k \in \{1, 2\}$, as defined in* (10)*. Define $z = [u_n, \varepsilon]^T \in \mathbb{R}^2$, and consider the following optimization problem.*

$$\min_{u_n \in \mathbb{R}, \varepsilon \in \mathbb{R}_{\ge 0}} \frac{1}{2} z^T z$$

s.t.

$$\begin{aligned}&\textstyle\sum_{i \in \mathcal{N}_n} \{ \frac{\partial \psi_1(x,t)}{\partial x_i} \mathfrak{f}_{i,i}(x_i)\\ &+ (\frac{\partial \psi_1(x,t)}{\partial x_n} + \frac{\partial \psi_1(x,t)}{\partial x_i}) \mathfrak{f}_{n,i}(x_n, x_i) \}\\ &+ \frac{\partial \psi_1(x,t)}{\partial x_n} \mathfrak{g}_n(x_n) u_n + \frac{\partial \psi_1(x,t)}{\partial x_n} \mathfrak{f}_{n,n}(x_n)\\ &+ \frac{\partial \psi_1(x,t)}{\partial t} \ge -\alpha_2\ sgn(\psi_1(x, t))|\psi_1(x, t)|^{\gamma_{12}}\\ &\qquad - \beta_2\ sgn(\psi_1(x, t))|\psi_1(x, t)|^{\gamma_{22}} - \varepsilon,\end{aligned} \quad (14)$$

where $\alpha_2 > 0, \beta_2 > 0$, $0 < \gamma_{12} < 1$, $\gamma_{22} > 1$. Any control signal $u_n : \mathbb{R} \to \mathbb{R}$ which solves the quadratic program (14) renders the set $\mathfrak{C}_{pl}(t)$ robust convergent for the network (3), under Assumption 2.

Barrier certificates proposed in the previous sections provide one constraint, which relies on the leader vehicle control signal as a central coordination unit. This may cause limitations on the network scalability and robustness properties. Next, we define individual barrier certificates for each vehicle according to the tasks that it is involved in, and based on the formation structure of the network, in order to guarantee the task satisfaction. In particular, according to the length of the path between each follower and the leader vehicle, higher order barrier certificate for each follower vehicle is built.

Definition 6. *[11, Lemma 2]* ***Individual barrier certificates:*** *Consider one platoon network* (3) *containing one leader $i = n$, under the task ϕ of the form* (1b) *satisfying Assumption 2. Then, the individual barrier certificate for each follower vehicle $i \in \{1, \cdots, n-1\}$ can be given as*

$$\psi_{m_i, i}(x, t) \ge 0, \quad (15)$$

where

$$\begin{aligned}\psi_{0,i}(x, t) &:= \mathfrak{h}_i(x, t),\\ \psi_{k,i}(x, t) &:= \dot{\psi}_{k-1,i}(x, t)\\ &\quad + \lambda_{k,i}(\psi_{k-1,i}(x, t)),\ 1 \le k \le m_i - 1,\\ \psi_{m_i,i}(x, t) &:= \dot{\psi}_{m_i-1,i}(x, t)\\ &\quad + \alpha_i \mathrm{sgn}(\psi_{m_i-1,i}(x, t))|\psi_{m_i-1,i}(x, t)|^{\gamma_{1,i}}\\ &\quad + \beta_i \mathrm{sgn}(\psi_{m_i-1,i}(x, t))|\psi_{m_i-1,i}(x, t)|^{\gamma_{2,i}},\end{aligned} \quad (16)$$

with $\mathfrak{h}_i(x, t) := \frac{\partial \mathfrak{h}(x,t)}{\partial x_i} \mathfrak{f}_i(x)$, $i \in \{1, \cdots, n-1\}$, is the higher order barrier certificate, $\lambda_{k,i}(\cdot)$, $k = 1, \cdots, m_i - 1$, $(m_i - k)^{th}$-order differentiable extended class $\mathcal{K}$ functions, and m_i the length of path between the follower i and the leader, as mentioned in Section II-B.

Note that the individual barrier certificates in the network of platoons are constructed with respect to the closest leader according to the graph topology.

B. Decentralized barrier certificates for network of platoons

In the previous sections, one platoon of vehicles were considered, i.e., the network consists of one leader vehicle. Hence, the satisfaction of specifications had to be achieved in a centralized way by the only leader vehicle. For the case of a network of platoons, there exists a larger number of leader vehicles that are responsible to control the corresponding followers vehicle within their platoons to satisfy the specifications in a decentralized fashion. In other words, the leaders of the platoons communicate with each other according to the corresponding graph topology in each configuration, e,g, splitting, merging, etc. We next formulate the barrier certificates in a decentralized scheme for these scenarios.

Definition 7. *[11, Lemma 3]* ***Decentralized barrier certificates:*** *The decentralized barrier certificates for the platoon network* (3) *with the first n_f vehicles as followers and the last n_l ones as leaders and the higher order barrier certificate $\mathfrak{h}(x,t)$, can be constructed with respect to each leader vehicle, by the following n_l inequalities.*

$$\begin{aligned}
&\textstyle\sum_{i\in\mathcal{N}_{n_f+1}}\{\frac{\partial\mathfrak{h}(x,t)}{\partial x_i}\mathfrak{f}_{i,i}(x_i)\\
&+(\frac{\partial\mathfrak{h}(x,t)}{\partial x_{n_f+1}}+\frac{\partial\mathfrak{h}(x,t)}{\partial x_i})\mathfrak{f}_{n_f+1,i}(x_{n_f+1},x_i)\}\\
&+\frac{\partial\mathfrak{h}(x,t)}{\partial x_{n_f+1}}\mathfrak{f}_{n_f+1,n_f+1}(x_{n_f+1})+\frac{\partial\mathfrak{h}(x,t)}{\partial x_{n_f+1}}\mathfrak{g}_{n_f+1}u_{n_f+1}\\
&\geq-\frac{\tau_1}{\sum_{j\in\{1,\cdots,n_l\}}\tau_j}[\frac{\partial\mathfrak{h}(x,t)}{\partial t}+\alpha\, sgn(\mathfrak{h}(x,t))|\mathfrak{h}(x,t)|^{\gamma_1}\\
&\qquad+\beta\, sgn(\mathfrak{h}(x,t))|\mathfrak{h}(x,t)|^{\gamma_2}],\\
&\vdots\\
&\textstyle\sum_{i\in\mathcal{N}_{n_f+n_l}}\{\frac{\partial\mathfrak{h}(x,t)}{\partial x_i}\mathfrak{f}_{i,i}(x_i)\\
&+(\frac{\partial\mathfrak{h}(x,t)}{\partial x_{n_f+n_l}}+\frac{\partial\mathfrak{h}(x,t)}{\partial x_i})\mathfrak{f}_{n_f+n_l,i}(x_{n_f+n_l},x_i)\}\\
&+\frac{\partial\mathfrak{h}(x,t)}{\partial x_{n_f+n_l}}\mathfrak{f}_{n_f+n_l,n_f+n_l}(x_{n_f+n_l})+\frac{\partial\mathfrak{h}(x,t)}{\partial x_{n_f+n_l}}\mathfrak{g}_{n_f+n_l}u_{n_f+n_l}\\
&\geq-\frac{\tau_{n_l}}{\sum_{j\in\{1,\cdots,n_l\}}\tau_j}[\frac{\partial\mathfrak{h}(x,t)}{\partial t}+\alpha\, sgn(\mathfrak{h}(x,t))|\mathfrak{h}(x,t)|^{\gamma_1}\\
&\qquad+\beta\, sgn(\mathfrak{h}(x,t))|\mathfrak{h}(x,t)|^{\gamma_2}],
\end{aligned} \tag{17}$$

for positive constants τ_j, $j\in\{1,\cdots,n_l\}$.

IV. Simulation Results

We consider the problem of coordination among second order dynamics platoon networks using higher order CBFs. We consider merging and splitting scenarios for a number of platoons, specified through STL specifications. The STL tasks are specified on the position signals. Then, we need to employ higher order CBFs in order to guarantee the task satisfactions. Consider three number of platoons as in Figure 1. In this scenario, according to (2) the Laplacian matrix of the whole network can be specified by

$$L_{indvs}=\begin{bmatrix}1&0&0&-1&0&0\\0&1&0&0&-1&0\\0&0&1&0&0&-1\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix}.$$

Then, the overall dynamics of the platoon network is

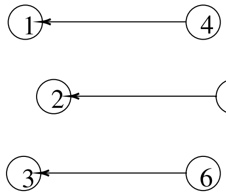

Fig. 1: 3 Platoons of vehicles

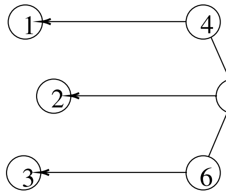

Fig. 2: Platoons merging

according to (3), where $\mathfrak{f}_{pl}(x):=\begin{bmatrix}0_{6\times6}&I_6\\-L_{indvs}&-L_{indvs}\end{bmatrix}x$ and $\mathfrak{g}_{pl}(x):=\begin{bmatrix}0_9^T,1_3^T\end{bmatrix}^T$. Assume the merging task of the second platoon (composed of agents 2 and 5), to the first platoon (composed of agents 1 and 4) and the third platoon (composed of agents 3 and 6) in the time interval $[a,b]$. This scenario could be specified using the *until operator* as a fragment of STL semantics as below.

$$\begin{aligned}\Phi_{merge}:=&(\|p_5-p_6\|\leq d_{com})\\&\wedge(p_4-p_6>d_{space})U_{[a,b]}(\|p_5-p_{road}\|\leq\epsilon)\\&\wedge(\|p_5-p_4\|<d_{com}),\end{aligned}$$

where p_i is the position of agent i, p_{road} is the lane of interest, d_{com} specifies the distance threshold for establishing the merging, d_{space} provides a collision avoidance safety distance, and ϵ maintains the lane keeping. In this phase, the Laplacian graph of the whole multi leader network corresponding to Figure 2 is specified as

$$L_{merge}=\begin{bmatrix}1&0&0&-1&0&0\\0&1&0&0&-1&0\\0&0&1&0&0&-1\\0&0&0&1&-1&0\\0&0&0&-1&2&-1\\0&0&0&0&-1&1\end{bmatrix}.$$

which substitutes L_{indivs} in the network dynamics. This formula is a function of the positions of the platoon leaders, which makes it of relative degree 2. Then, the corresponding barrier function to this task would be of order $m=2$. Furthermore, consider the splitting of the second platoon from the others in the time interval $[c,d]$, which could be specified using the eventually operator as

$$\Phi_{split}:=F_{[c,d]}(p_4-p_5>d_{split}),$$

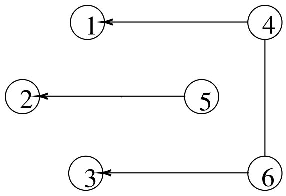

Fig. 3: Third Platoon splitting from the others

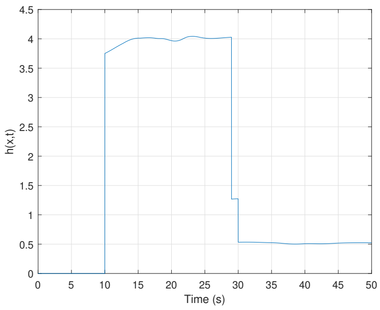

Fig. 4: The evolution of control barrier function.

with the corresponding Laplacian matrix

$$L_{split} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}$$

according to Figure 3. Similar to the merging scenario, this task is also of relative degree 2, which makes the corresponding barrier function of order $m = 2$.

For the sake of simulations, consider the merging and splitting time instants as $a = 10, b = c = 30, d = 50$. In addition $d_{com} = 10$, $d_{space} = 15$, $\epsilon = 2$, $d_{split} = 8.5$, $p_{road} = 12$. The evolution of the control barrier function, the relative position trajectories of the leader and follower vehicles of the platoons are presented in Figures 4, 5, and 6, respectively.

V. Conclusion

Based on a class of time-varying convergent higher order control barrier functions, we have presented feedback control strategies to find solutions for the platoon coordination problem consisting of a number of leader-follower platoons under STL tasks. Appropriate individual and decentralized barrier certificates are also introduced to maintain more general formulas in a simpler framework. Future work will extend these results to high level specifications including leader selection methods to find the optimal solution with respect to task specifications.

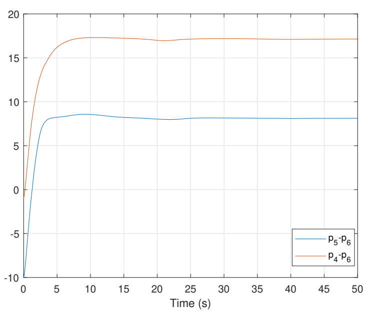

Fig. 5: Relative positions of the platoon leader vehicles.

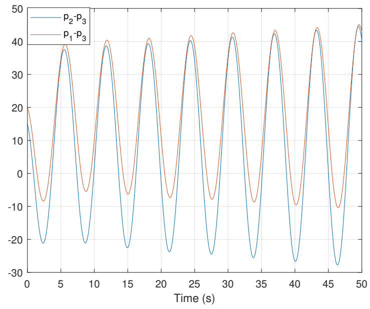

Fig. 6: Relative positions of the platoon follower vehicles.

References

[1] A. Johansson, E. Nekouei, K. H. Johansson, and J. Mårtensson, "Strategic hub-based platoon coordination under uncertain travel times," *IEEE Transactions on Intelligent Transportation Systems*, vol. 23, no. 7, pp. 8277–8287, 2021.

[2] V. Lesch, M. Breitbach, M. Segata, C. Becker, S. Kounev, and C. Krupitzer, "An overview on approaches for coordination of platoons," *IEEE Transactions on Intelligent Transportation Systems*, vol. 23, no. 8, pp. 10 049–10 065, 2021.

[3] G. Guo, P. Li, and L.-Y. Hao, "A new quadratic spacing policy and adaptive fault-tolerant platooning with actuator saturation," *IEEE Transactions on Intelligent Transportation Systems*, vol. 23, no. 2, pp. 1200–1212, 2020.

[4] G. Guo and L. Wang, "Control over medium-constrained vehicular networks with fading channels and random access protocol: A networked systems approach," *IEEE Transactions on Vehicular Technology*, vol. 64, no. 8, pp. 3347–3358, 2014.

[5] G. E. Fainekos, A. Girard, H. Kress-Gazit, and G. J. Pappas, "Temporal logic motion planning for dynamic robots," *Automatica*, vol. 45, no. 2, pp. 343–352, 2009.

[6] O. Maler and D. Nickovic, "Monitoring temporal properties of continuous signals," in *Formal Techniques, Modelling and Analysis of Timed and Fault-Tolerant Systems*. Springer, 2004, pp. 152–166.

[7] G. E. Fainekos and G. J. Pappas, "Robustness of temporal logic specifications for continuous-time signals," *Theoretical Computer Science*, vol. 410, no. 42, pp. 4262–4291, 2009.

[8] H. Kawashima and M. Egerstedt, "Leader selection via the manipulability of leader-follower networks," in *American Control Conference*. IEEE, 2012, pp. 6053–6058.

[9] M. Sharifi and D. V. Dimarogonas, "Fixed-time convergent control barrier functions for coupled multi-agent systems under STL tasks," in *European Control Conference*. IEEE, 2021.

[10] M. Charitidou and D. V. Dimarogonas, "Splitting and merging control of multiple platoons with signal temporal logic," in *2022 IEEE Conference on Control Technology and Applications (CCTA)*. IEEE, 2022, pp. 1031–1036.

[11] M. Sharifi and D. V. Dimarogonas, "Higher order barrier certificates for leader-follower multi-agent systems," *IEEE Transactions on Control of Network Systems*, 2022.

[12] L. Lindemann and D. V. Dimarogonas, "Barrier function-based collaborative control of multiple robots under signal temporal logic tasks," *IEEE Transactions on Control of Network Systems*, vol. 7, no. 4, pp. 1916–1928, 2020.

[13] F. Blanchini, "Set invariance in control," *Automatica*, vol. 35, no. 11, pp. 1747–1767, 1999.