Feedback Control Strategies for Multi-Agent Systems
under a Fragment of Signal Temporal Logic Tasks

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Abstract

Multi-agent systems under temporal logic tasks have great potential due to their ability to deal with complex tasks. The control of these systems, however, poses many challenges and the majority of existing approaches result in large computational burdens. We instead propose computationally-efficient and robust feedback control strategies for a class of systems that are, in a sense, feedback equivalent to single integrator systems, but where the dynamics are partially unknown for the control design. A bottom-up scenario is considered in which each agent is subject to a local task from a limited signal temporal logic fragment. Notably, the satisfaction of a local task may also depend on the behavior of other agents. We provide local continuous-time feedback control laws that, under some sufficient conditions, guarantee satisfaction of the local tasks. Otherwise, a local detection & repair scheme is proposed in combination with the previously derived feedback control laws to deal with infeasibilities, such as when local tasks are conflicting. The efficacy of the proposed method is demonstrated in simulations.

Key words: Multi-agent systems; formal methods-based control; signal temporal logic; robust control; autonomous systems; hybrid systems.

1 Introduction

Control of multi-agent systems is a promising research area where scholars have addressed multi-agent navigation (Dimarogonas et al., 2006), consensus (Ren and Beard, 2005), formation control (Tanner et al., 2003), and connectivity maintenance problems (Zavlanos and Pappas, 2008), see Mesbahi and Egerstedt (2010) for an overview. More recently, ideas from model checking (Baier and Katoen, 2008) have been used where the task, which is imposed on the system, is a complex temporal logic formula. Control of single-agent systems under linear temporal logic (LTL) tasks has been considered in Bela et al. (2007); Fainekos et al. (2009); Kress-Gazit et al. (2009) while the multi-agent case has been addressed in Filippidis et al. (2012); Guo and Dimarogonas (2013, 2017); Kloetzer and Bela (2010); Tumova and Dimarogonas (2016). The aforementioned works rely on an abstraction of the workspace and the agent dynamics. This abstraction process and the subsequent plan synthesis are limited, especially for multi-agent systems, due to their high computational complexity. For single-agent systems, probabilistically optimal and complete sampling-based methods have been proposed in Vasile and Bela (2013) and Karaman and Frazzoli (2012) to avoid these computational burdens by incrementally building and model-checking a transition system. For the multi-agent setup, two paradigms exist: one where all agents are subject to a global task (top-down) and one where each agent is subject to a local task (bottom-up). These local tasks can be obtained in two ways. Either a global task is decomposed into local ones as in Schillinger et al. (2016), which in some sense is a mix of top-down and bottom-up, or each agent is assigned a local task regardless of what other agents are assigned. Especially in the latter case it may happen that the satisfaction of a local task also depends on the behavior of other agents. By behavior of an agent we mean the corresponding agent trajectory. A challenge is hence that local tasks may be in conflict, i.e., satisfiability of each local task does not imply satisfiability of the conjunction of all local tasks. The authors in Guo and Dimarogonas (2013) find least violating solutions in these conflicting situations. Opposed to the aforemen-

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tioned works using LTL, signal temporal logic (STL) (Maler and Nickovic, 2004) is based on continuous-time signals and suited to impose continuous-time tasks. STL entails space robustness (Donzé and Maler, 2010), a form of the robust semantics (Fainekos and Pappas, 2009), which states how robustly a signal satisfies a task. These robust semantics are mainly used for control under STL tasks, which is, however, a difficult problem due to the nonlinear, nonconvex, noncausal, and nonsmooth semantics. Control of discrete-time single-agent systems under STL tasks has been considered in Raman et al. (2014) and Lindemann and Dimarogonas (2019); Raman et al. (2014) obtain a computationally expensive mixed integer linear program for which robust extensions have been presented in Raman et al. (2015) and Sadraddini and Belta (2015), while an extension to multi-agent systems has been reported in Liu et al. (2017). The authors in Pant et al. (2018) use smooth approximations of the robust semantics within a non-convex optimization problem for which robust extensions have been presented in Raman et al. (2015) and Sadraddini and Belta (2015), while providing conservative continuous-time guarantees for the corresponding continuous-time system.

We consider continuous-time multi-agent systems in a bottom-up fashion. Each local task stems from a limited STL fragment and may depend on the behavior of other agents, while agents may also be dynamically coupled. The agent dynamics are described by nonlinear control-affine systems where the corresponding driftless system is feedback equivalent to a single integrator system, but where the drift term is unknown. The limited STL fragment allows to encode concave temporal tasks such as eventually within 5 sec reach a region and stay there for the next 10 sec while staying close to other agents. Specifications such as eventually within 5 sec reach a region while always avoiding another region induce a mix of convex and concave temporal tasks and are not permitted here. These assumptions are necessary to achieve finite time stability results under arbitrarily short deadlines and to obtain closed-form and continuous feedback control laws. We derive a robust continuous-time feedback control law that, under some sufficient conditions, guarantees satisfaction of all local tasks. When these conditions do not hold, we propose a local detection & repair scheme, expressed as a hybrid control system (Goebel et al., 2009), that detects critical events to repair them in a three-stage procedure. We discretize neither the environment nor the agent dynamics in space or time. To the best of our knowledge, this is the first approach not making use of discretizations in space or time and directly providing continuous-time satisfaction guarantees. Our main contribution is a robust, abstraction-free, and computationally-efficient control strategy for coupled multi-agent systems that finds least violating solutions for conflicting local specifications. This paper is an extension of Lindemann and Dimarogonas (2018).

Section 2 presents preliminaries and the problem definition. Section 3 states our proposed problem solution, while simulations in Section 4 demonstrate the efficacy of our method. Conclusions are given in Section 5.

2 Preliminaries

The set of integers, natural, non-negative, and positive real numbers are $\mathbb{Z}$, $\mathbb{N}$, $\mathbb{R}_{\geq 0}$, and $\mathbb{R}_+$, respectively. True and false are $\top$ and $\bot$, and $\mathbb{0}$ is the $n$-dimensional zero vector. For the column vectors $\mathbf{c}_1$ and $\mathbf{c}_2$ and instead of $[\mathbf{c}_1^T \mathbf{c}_2^T]^T$, we let $[\mathbf{c}_1 \mathbf{c}_2]$ be a column vector, while partial derivatives are assumed to be row vectors.

2.1 Signal Temporal Logic (STL)

Signal temporal logic (STL) (Maler and Nickovic, 2004) consists of predicates $\mu$ that are obtained by evaluating a continuously differentiable predicate function $h: \mathbb{R}^d \to \mathbb{R}$ as $\mu := \top$ if $h(\zeta) \geq 0$ and $\mu := \bot$ if $h(\zeta) < 0$ for $\zeta' \in \mathbb{R}^d$. The STL syntax is then defined as

\[
\varphi :::= \top | \mu | \neg \varphi \lor \varphi'' | F_{[a,b]} \varphi \lor G_{[a,b]} \varphi,
\]

where $\varphi''$ and $\varphi''$ are STL formulas and where $\neg$, $\land$, $F_{[a,b]}$, and $G_{[a,b]}$ are the negation, conjunction, eventually and always operators, respectively, with $a, b \in \mathbb{R}_{\geq 0}$. For a continuous-time signal $\zeta: \mathbb{R}_{\geq 0} \to \mathbb{R}^d$, the satisfaction relation $(\zeta, t) \models \varphi$ indicates if $\zeta$ satisfies $\varphi$ at time $t$. These STL semantics are formally defined in Maler and Nickovic (2004). Robust semantics have been introduced in Fainekos and Pappas (2009) as a robustness measure for temporal logics. Space robustness (Donzé and Maler, 2010) are robust semantics for STL and defined as

\[
\bar{\rho}^h(\zeta, t) := h(\zeta(t))
\]
\[
\bar{\rho}^{\varphi'}(\zeta, t) := \bar{\rho}^{\varphi''}(\zeta, t)
\]
\[
\bar{\rho}^{F_{[a,b]} \varphi}(\zeta, t) := \min_{t_1 \in [t+a, t+b]} \bar{\rho}^{\varphi'}(\zeta, t_1)
\]
\[
\bar{\rho}^{G_{[a,b]} \varphi}(\zeta, t) := \min_{t_1 \in [t+a, t+b]} \bar{\rho}^{\varphi'}(\zeta, t_1).
\]

The function $\bar{\rho}$ determines how robustly $\zeta$ satisfies $\varphi$ at time $t$ and it holds that $(\zeta, t) \models \varphi$ if $\bar{\rho}(\zeta, t) > 0$. For a robust STL satisfaction of the STL introduced above. Let

\[
\psi :::= \top | \mu | \neg \mu | \psi(1) \land \psi(2)
\]
\[
\phi :::= F_{[a,b]} \psi \lor G_{[a,b]} \psi \lor F_{[a,b]} G_{[a,b]} \psi
\]

where $\psi$ in (1b) and $\psi(1), \psi(2)$ in (1a) are formulas of class $\psi$ given in (1a) and where $a, b, a, b \in \mathbb{R}_{\geq 0}$ and $b, b, b \in \mathbb{R}_{\geq 0}$ $\cup \infty \in \mathbb{R}_{\geq 0}$ $\cup \infty$ with $a \leq b, a \leq b$, and $a \leq b$. We refer to $\psi$ as non-temporal (boolean) formulas and to $\phi$ as temporal formulas. Let $S$ be the set of all possible signals with time domain $\mathbb{R}_{\geq 0}$. Note that $\bar{\rho}(\zeta, t)$ and $\bar{\rho}(\zeta, t)$ map from $S \times \mathbb{R}_{\geq 0}$ to $\mathbb{R}$. For non-temporal formulas $\psi$, we can
equivalently use \( \bar{\rho}^\psi(\zeta(t)) \) by a change of notation where now \( \bar{\rho}^\psi : \mathbb{R}^n \to \mathbb{R} \). The notation of \( \bar{\rho}^\psi(\zeta(t)) \) is used to highlight that \( t \) is only contained in \( \bar{\rho}^\psi \) through the composition of \( \bar{\rho}^\psi \) with \( \zeta \). The non-smooth robust semantics of \( \psi_i(\bar{x}) \land \psi_j(\bar{x}) \) are replaced by a smooth approximation, denoted by \( \rho^\psi(\zeta,t) \). For the formulas in (1), we define

\[
\begin{align*}
\rho^\psi(\zeta(t)) &:= h(\zeta(t)) \\
\rho^{\psi_1(\zeta) \land \psi_2(\zeta)}(\zeta(t)) &:= - \frac{1}{\eta} \ln \left( \sum_{j=1}^{2} \exp \left( - \eta \rho^{\psi_j(\zeta)}(\zeta(t)) \right) \right) \\
\rho^{\psi_i(\zeta)}(\zeta(t)) &:= \max_{t_i \in [t+a,t+b]} \rho^\psi(\zeta(t_i)) \\
\rho^{\psi_i(\zeta) \land \psi_2(\zeta)}(\zeta(t)) &:= \min_{t_i \in [t+a,t+b]} \min \rho^\psi(\zeta(t_2)) \\
\rho^{\psi_i(\zeta)}(\zeta(t)) &:= \max_{t_i \in [t+a,t+b]} \min \rho^\psi(\zeta(t_2)) \\
\rho^{\psi_i(\zeta)}(\zeta(t)) &:= \min_{t_i \in [t+a,t+b]} \min \rho^\psi(\zeta(t_2))
\end{align*}
\]

where \( \eta > 0 \) is a design parameter that determines the accuracy by which \( \rho^{\psi_1(\zeta) \land \psi_2(\zeta)}(\zeta(t)) \) approximates \( \rho^{\psi_1(\zeta) \land \psi_2(\zeta)}(\zeta(t)) \). In fact, it can be shown that

\[
\rho^{\psi(\zeta)}(\zeta(t)) = \rho^{\psi(\zeta)}(\zeta(t)) \quad \text{as} \quad \eta \to \infty.
\]

Remark 1 The fragment in (1) can be extended to include sequential formulas. For single-agent systems, this extension is explained in Lindemann et al. (2017) by employing a hybrid control strategy. For the multi-agent setup, this extension can be done in the same way and is omitted. Note that the fragment in (1) does not allow to pose reach-avoid specifications, which require at least discontinuous feedback control laws (Liberzon, 2003, Ch. 4), while we focus on continuous feedback control laws.

2.2 A Bottom-up Approach for Multi-Agent Systems

Consider \( M \) agents modeled by an undirected graph \( G := (V,E) \) (Mesbah and Egerstedt, 2010) where \( V := \{v_1, \ldots, v_M\} \) while \( E \subset V \times V \) indicates communication links. At time \( t \), let \( x_i(t) \in \mathbb{R}^{n_i}, u_i(t) \in \mathbb{R}^{m_i}, \) and \( w_i(t) \in \mathbb{W}_i \subset \mathbb{R}^{n_i} \) be the state, input, and additive noise of agent \( v_i \), with \( \mathbb{W}_i \subset \mathbb{R}^{n_i} \) being a bounded set and

\[
x_i(t) = f_i(x_i(t)) + f_i^*(x_i(t)) + g_i(x_i(t))u_i(t) + w_i(t) \tag{2}
\]

where \( x_i := [x_1(t), \ldots, x_M(t)] \in \mathbb{R}^n \) with \( n := n_1 + \cdots + n_M. \). Also define \( x_i^{\text{ext}}(t) := [x_{j_1}(t), \ldots, x_{j_M}(t)] \) such that \( v_{j_1}, \ldots, v_{j_M} \subset V \setminus \{v_i\} \). We also define the behavior of agent \( v_i \) to be agent \( v_i \)'s trajectory, i.e., the solution \( x_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^{n_i} \) to (2). The functions \( f_i \) and \( f_i^* \) are unknown apart from a regularity assumption.

Assumption 1 The functions \( f_i : \mathbb{R}^{n_i} \to \mathbb{R}^{n_i}, f_i^* : \mathbb{R}^{n_i} \to \mathbb{R}^{n_i}, \text{and } g_i : \mathbb{R}^{n_i} \to \mathbb{R}^{n_i \times m_i} \) are locally Lipschitz continuous, and \( g_i(x_i)g_i(x_i)^T \) is positive definite for all \( x_i \in \mathbb{R}^{n_i}, w_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^{n_i} \) is piecewise continuous.

Remark 2 We emphasize that \( f_i \) and \( f_i^* \) are unknown so that (2) is not feedback equivalent to \( x_i(t) = u_i(t) + w_i(t). \) Note that \( g_i(x_i)g_i(x_i)^T \) is positive definite if and only if \( g_i(x_i) \) has full row rank. This assumption captures, for instance, the dynamics of omnidirectional robots as in Section 4; \( f_i^* \) describes given dynamical couplings between agents, e.g., a physical connection when two or more agents collaboratively carry an object. Using \( u_i = g_i(x_i)^T [y_i(x_i)g_i(x_i)^T]^{-1}f_i(x_i) + v_i, \) the dynamics \( x_i = f_i(x_i) + f_i^*(x_i) + g_i(x_i)v_i + w_i \) resemble (2) if \( f_i^*(x_i) \) is locally Lipschitz continuous; \( f_i^*(x_i) \) can be used for other objectives such as collision avoidance, consensus, formation control, or connectivity maintenance.

Each agent \( v_i \in V \) is now subject to a local STL formula \( \phi_i \) of class \( \phi \) given in (1b), i.e., \( \phi_i \) is of the form

\[
\phi_i := F[\alpha, \beta]\psi_i | G[\alpha, \beta]\psi_i | F[\alpha, \beta]G[\alpha, \beta]\psi_i. \tag{3}
\]

where \( \psi_i \) is a non-temporal formula of the form (1a). Satisfaction of \( \phi_i \) depends on the behavior of \( v_i \) and may also depend on the behavior of agents in \( V \setminus \{v_i\}. \) If the satisfaction of \( \phi_i \) depends on the behavior of \( v_j \in V, \) we say that \( \phi_i \) depends on \( v_j. \) Equivalently, we then say that agent \( v_j \) is participating in \( \phi_i; \phi_i \) consequently depends on a set of agents denoted by \( P_i := \{v_{j_1}, \ldots, v_{j_{|P|}}\} \subset V \) where \( P_i \) indicates the total number of agents that participate in \( \phi_i. \) We call \( \phi_i \) a non-collaborative formula if \( P_i = 1. \) Otherwise, i.e., if \( P_i > 1, \) we call \( \phi_i \) a collaborative formula. Let \( p_i := n_{j_1} + \cdots + n_{j_{|P_i|}} \) and define

\[
\bar{x}_i(t) := [x_{j_1}(t), \ldots, x_{j_{|P_i|}}(t)] \in \mathbb{R}^{p_i}. \tag{4}
\]

Assumption 2 Each formula \( \psi_i \) in (3) is: 1) s.t. \( \rho^{\psi_i}(\bar{x}_i) \) is concave and 2) well-posed in the sense that \( \rho^{\psi_i}(\bar{x}_i) > 0 \) implies \( \|\bar{x}_i\| \leq C < \infty \) for some \( C \geq 0. \)

Remark 3 Recall the syntax of class \( \psi \) formulas in (1a) and consider \( \psi_i := \psi(\bar{x}_i) \land \psi(\bar{x}_i). \) Part 1 of Assumption 2 is satisfied if \( \rho^{\psi(\bar{x}_i)}(\bar{x}_i) \) and \( \rho^{\psi(\bar{x}_i)}(\bar{x}_i) \) are concave. This assumption is needed since the controller presented in this paper uses the gradient of \( \rho^{\psi(\bar{x}_i)}(\bar{x}_i). \) Local extrema may hence lead the system to get stuck. Part 2 of Assumption 2 will ensure bounded solutions and is not restrictive since \( \psi_i^{\text{Ass.2}} := (\|\bar{x}_i\| < C) \) can be combined with \( \psi_i \) for a sufficiently large \( C \) so that \( \psi_i \land \psi_i^{\text{Ass.2}} \) is well-posed.
The signal $\bar{x}_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ locally satisfies $\phi_i$ if $(\bar{x}_i, 0) \models \phi_i$ and $\phi_i$ is locally satisfiable if $\exists \bar{x}_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ globally satisfies $\{\phi_1, \ldots, \phi_M\}$ if $\bar{x}_i$, which is naturally contained in $\bar{x}_i$, locally satisfies $\phi_i$ for all agents $v_i \in \mathcal{V}$. The set of formulas $\{\phi_1, \ldots, \phi_M\}$ is globally satisfiable if $\exists \bar{x}_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ such that $\bar{x}_i$ globally satisfies $\{\phi_1, \ldots, \phi_M\}$. Consider also the undirected graph $G_d := (V, E_d)$ where there is an edge $(v_i, v_j) \in E_d \subseteq \mathcal{V} \times \mathcal{V}$ if $\phi_i$ depends on $v_j$; $\Xi \subseteq \mathcal{V}$ is a maximal dependency cluster if $\forall v_i, v_j \in \Xi$ there is a path from $v_i$ to $v_j$ in $G_d$ and $\mathbb{V}_i \in \mathbb{V}_k \in \mathcal{V} \setminus \Xi$ such that there is a path from $v_i$ to $v_k$. Here, a path is a sequence $v_i, v_k, \ldots, v_{kp}, v_j$ such that $(v_i, v_k), \ldots, (v_{kp}, v_j) \in E_d$. A multi-agent system under $\{\phi_1, \ldots, \phi_M\}$ hence induces $L \leq M$ maximal dependency clusters $\Xi := \{\Xi_1, \ldots, \Xi_L\}$.

**Assumption 3** For each $\Xi_l$ with $l \in \{1, \ldots, L\}$ it holds that $(v_i, v_j) \in \mathcal{E}$ for each $v_i, v_j \in \Xi_l$.

In Section 3, each agent $v_i \in \mathcal{V}$ is equipped with logical variables so that $v_i$ is associated with a hybrid state $z_i \in \mathcal{Z}_i \subseteq \mathbb{R}^{n_i}$ where $\mathcal{Z}_i$ is a hybrid domain of dimension $n_i$. Hybrid systems with internal and external inputs $u_i^{\text{int}} \in U_i^{\text{int}}$ and $u_i^{\text{ext}} \in U_i^{\text{ext}}$, respectively, have been presented in Sanfelice (2016) where $U_i^{\text{int}}$ and $U_i^{\text{ext}}$ are input domains. Let $\mathcal{H} := \mathcal{Z}_i \times U_i^{\text{int}} \times U_i^{\text{ext}}$. The value of the state $z_i$, after a jump, i.e., after an instantaneous change in $z_i$, is denoted by $\hat{z}_i$. A hybrid system is now a tuple $H := (C_i, F_i, D_i, G_i)$ where $C_i \subseteq \mathcal{H}_i, D_i \subseteq \mathcal{H}_i, F_i : \mathcal{H}_i \to \mathcal{H}_i$, and $G_i : \mathcal{H}_i \to \mathcal{H}_i$. A detailed review can be found in Goebel et al. (2009, 2012).

**2.3 Problem Statement**

Let the supremum of $\rho^\psi_i(\bar{x}_i)$ and $\rho^\psi_i(\hat{x}_i)$ be

$$\rho^\psi_i^{\text{opt}} := \sup_{\bar{x}_i \in \mathbb{R}^n} \rho^\psi_i(\bar{x}_i) \text{ and } \rho^\psi_i^{\text{opt}} := \sup_{\hat{x}_i \in \mathbb{R}^n} \rho^\psi_i(\hat{x}_i)$$

where $\rho^\psi_i^{\text{opt}}$ is in particular straightforward to compute since $\rho^\psi_i(\hat{x}_i)$ is continuously differentiable and concave.

**Assumption 4** $\rho^\psi_i^{\text{opt}}$ is such that $\rho^\psi_i^{\text{opt}} > 0$.

Assumption 4 implies that $\psi_i$ and hence $\phi_i$ are locally satisfiable. Note that always exists an $\eta$ such that $\rho^\psi_i^{\text{opt}} > 0$ if $\rho^\psi_i^{\text{opt}} > 0$ since $\rho^\psi_i(\bar{x}_i) = \rho^\psi_i(\hat{x}_i)$ as $\eta \to \infty$. We then find $\eta$ by solving a convex feasibility problem by selecting $\eta > 0$ such that $\rho^\psi_i(\bar{x}_i) > 0$ for some $\bar{x}_i \in \mathbb{R}^n$. If this feasibility problem, however, is not feasible, we set $\eta := 1$. It then holds that $\rho^\psi_i^{\text{opt}} \leq 0$ and $\phi_i$ is not satisfiable if $\rho^\psi_i^{\text{opt}} + \ln(q) < 0$ with $q$ as in Lemma 1. Let $\rho^\psi_i^{\text{opt}} > 0$ be a parameter indicating how much $\phi_i$ is violated. If $\bar{x}_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ is such that $r_i \leq \rho^\psi_i(\bar{x}_i, 0)$ for $r_i$ with $\rho^\psi_i^{\text{opt}} - \rho^\psi_i^{\text{opt}} \leq r_i < \rho^\psi_i^{\text{opt}}$, we say that $\bar{x}_i$ is a least violating solution with a given gap of $\rho^\psi_i^{\text{opt}}$. The goal is to derive $u_i(x_i, t)$ such that $r_i \leq \rho^\psi_i(\bar{x}_i, 0) \leq \rho^\psi_i^{\text{opt}}$ where $r_i \in \mathbb{R}$ is a robustness measure, while $\rho^\psi_i^{\text{opt}} \in \mathbb{R}$ with $r_i < \rho^\psi_i^{\text{opt}}$ is a robustness delimiter.

**Problem 1** Let $\eta$ be selected for each $\psi_i$ as instructed above. Given the parameters $\rho^\psi_i^{\text{opt}} > 0$ and $\delta_i > 0$, derive for each cluster $\Xi_l \in \Xi$ a control strategy as follows: Case A) Under the assumption that $\phi_i \neq \phi_i$ for each $v_i, v_j \in \Xi_l$, design $u_i(x_i, t)$ such that $r_i \leq \rho^\psi_i(\bar{x}_i, 0) \leq \rho^\psi_i^{\text{opt}}$ for each $v_i \in \Xi_l$ and where $r_i$ is chosen such that $0 < r_i < \rho^\psi_i^{\text{opt}}$ if $\rho^\psi_i^{\text{opt}} > 0$ and $0 < r_i \geq \rho^\psi_i^{\text{opt}} - \rho^\psi_i^{\text{opt}}$ if $\rho^\psi_i^{\text{opt}} \leq 0$. Case B) Otherwise, i.e., $\exists v_i, v_j \in \Xi_l$ such that $\phi_i \neq \phi_j$, assume nevertheless that each agent $v_i \in \Xi_l$ initially applies the derived control law $u_i(x_i, t)$ for Case A. Design a local online detection & repair scheme for each $v_i \in \Xi_l$ such that $r_i \leq \rho^\psi_i(\bar{x}_i, 0) \leq \rho^\psi_i^{\text{opt}}$ where either $r_i > 0$ or $r_i$ is maximized up to a precision of $\delta_i > 0$.

By a maximization up to a precision of $\delta_i$ we mean that $r_i$ is successively reduced by $\delta_i$ whenever it turns out that $u_i(x_i, t)$ can not achieve $r_i \leq \rho^\psi_i(\bar{x}_i, 0) \leq \rho^\psi_i^{\text{opt}}$.

**3 Proposed Problem Solution**

Following a funnel-based control strategy (Bechlioulis and Rovithakis, 2014), we define a performance function

$$\gamma_i(t) := (\gamma_i^0 - \gamma_i^\infty) \exp(-l_it) + \gamma_i^\infty$$

where $\gamma_i^0, \gamma_i^\infty \in \mathbb{R}_{\geq 0}$ with $\gamma_i^0 \geq \gamma_i^\infty$ and $l_i \in \mathbb{R}_{\geq 0}$. We achieve $r_i \leq \rho^\psi_i(\bar{x}_i, 0) \leq \rho^\psi_i^{\text{max}}$ by prescribing a temporal behavior to $\rho^\psi_i(\bar{x}_i(t))$ through $\gamma_i$ and $\rho^\psi_i^{\text{opt}}$ as

$$-\gamma_i(t) + \rho^\psi_i^{\text{max}} < \rho^\psi_i(\bar{x}_i(t)) < \rho^\psi_i^{\text{max}}$$

If $\gamma_i$ and $\rho^\psi_i^{\text{max}}$ are chosen properly (as shown later) and if (5) holds for all $t \geq 0$, then $r_i \leq \rho^\psi_i(\bar{x}_i, 0) \leq \rho^\psi_i^{\text{max}}$. Let $e_i(x_i) := \rho^\psi_i(\bar{x}_i) - \rho^\psi_i^{\text{max}}, \xi_i(x_i, t) := \frac{e_i(x_i)}{\gamma_i(t)}$, and $e_i(x_i, t) := S(\xi_i(x_i, t))$, where $S(\xi_i) := \ln\left(-\frac{\xi_i(t)}{\gamma_i(t)}\right)$, then (5) is equivalent to $-\gamma_i(t) < e_i(t) < 0$ and hence to $-1 < \xi_i(t) < 0$ where $e_i(t) := e_i(x_i(t), t)$ and $\xi_i(t) := \xi_i(x_i(t), t)$. Applying
$S(·) \to -1 < \xi_i(t) < 0$ gives $-\infty < \epsilon_i(t) < \infty$ with 
$$
\epsilon_i(t) := \epsilon_i(\bar{x}_i(t), t).
$$
If $\epsilon_i(t)$ is bounded for all $t \geq 0$, i.e., 
$$\xi_i(t) \in \Omega_{\xi} \supseteq \mathbb{R}_{\geq 0} \text{ and } \tau_{\max} > 0 \text{ and hence } \xi(t) \in \Omega_{\xi} \text{ and } x(t) \in \Omega_x \text{ for all } t \in J.$$

Step B: We show that $\tau_{\max} = \infty$ by contradiction of

\begin{equation}
\frac{d\epsilon_i}{dt} = -\frac{1}{\gamma_i(1+\xi_i)} \left( \frac{\partial \rho^{\psi_i}(x_i)}{\partial x} \dot{x} - \xi_i \dot{\gamma}_i \right) \tag{15}
\end{equation}

since $\frac{d\epsilon_i}{dt} = -\frac{1}{\gamma_i(1+\xi_i)} \frac{d\epsilon_i}{dt}$ and $\frac{d\epsilon_i}{dt} = \frac{1}{\gamma_i} (\frac{d\epsilon_i}{dt} - \xi_i \dot{\gamma}_i)$. It also holds that $\frac{d\epsilon_i}{dt} = \frac{d\epsilon_i}{dt}$ with $\frac{d\epsilon_i}{dt} = \frac{d\epsilon_i}{dt}$.

Step A: First, define $\xi := [\xi_1 \ldots \xi_M]$ and $y := [x]$. By inserting (14) into (2), we get $\dot{x} = H_{x_i}(x_i, \xi)$ with

$$H_{x_i}(x_i, \xi) := f_i(x_i) + f_i^*(x) - \left( -\frac{\xi_i + 1}{\gamma_i} g_i(x_i) g_i(x_i)^T \frac{\partial \rho^{\psi_i}(x_i)}{\partial x_i} \right).$$

Let $H_{x_i}(x_i, \xi) := [H_{x_{i_1}}(x_{i_1}, \xi) \ldots H_{x_{i_M}}(x_{i_M}, \xi)]$ so that $\dot{x} = H_{x}(x, \xi)$. We also obtain $\frac{d\epsilon_i}{dt} = H_{\epsilon_i}(x_i, \xi, t)$ with

$$H_{\epsilon_i}(x_i, \xi, t) := \frac{1}{\gamma_i(t)} \left( \frac{\partial \rho^{\psi_i}(x_i)}{\partial x} H_{x_i}(x_i, \xi) - \xi_i(t) \right).$$

Let now $H_{\epsilon_i}(x_i, \xi, t) := [H_{\epsilon_{i_1}}(x_{i_1}, \xi_1) \ldots H_{\epsilon_{i_M}}(x_{i_M}, \xi_M)]$ so that $\dot{\xi} = H_{\xi}(x, \xi, t)$, which results in $y = H(y, t)$ with $H(y, t) := [H_{x}(x, \xi) H_{\epsilon}(x, \xi)]$. Note that $x(0)$ is such that $\xi_i(x_i(0), 0) \in \Omega_{\xi} := (-1, 0)$ holds for all agents $i \in \mathcal{E}$ due to the choice of $\gamma_i^0$. Now define $\Omega_1(t) := \{x_i \in \mathbb{R}^p | -1 < \xi_i(x_i, t) < 0\}$ and note that $\Omega_1(t_2) \subseteq \Omega_1(t_1)$ for $t_2 < t_1$ since $\gamma_i$ is non-increasing in $t$. Note also that $\bar{x}_i(0) \in \Omega_1(0)$ and that $\Omega_1(t)$ is bounded due to Assumption 2 and since $\gamma_i$ is bounded. Due to Aubin and Frankowska (2009, Prop. 1.4.4) the inverse image of an open set under a continuous function is open. By defining $\xi_{i_0,0}(x_i) := \xi_i(x_i, 0)$, it holds that $\xi_{i_0,0}^{-1}(\Omega_i) = \Omega_i(0)$ is open. Next, select $v_i \in \mathcal{E}$ for each $t \in \{1, \ldots, L\}$ and define $\Xi_{\xi} := \Omega_{x_i} \times \ldots \times \Omega_{x_L} \subseteq \mathbb{R}^p$, $\Omega_{\xi} := \Omega_{x_i} \times \ldots \times \Omega_{x_L} \subseteq \mathbb{R}^{n+M}$, and $\Omega_{\gamma} := \Omega_{x_i} \times \Omega_{\gamma} \subseteq \mathbb{R}^{n+M}$, which are open, non-empty, and bounded sets so that $y(0) = [x(0) \xi_0(0)] \in \Omega_y$. Let us check the existence conditions of solutions for the initial value problem $y = H(y, t)$ with $y(0) \in \Omega_y$ and $H(y, t) := [H_{x}(x, \xi) H_{\epsilon}(x, \xi)]$. Note that $x(0)$ is such that $\xi_i(x_i(0), 0) \in \Omega_{\xi} := (-1, 0)$ holds for all agents $i \in \mathcal{E}$ due to the choice of $\gamma_i^0$. Now define $\Omega_1(t) := \{x_i \in \mathbb{R}^p | -1 < \xi_i(x_i, t) < 0\}$ and note that $\Omega_1(t_2) \subseteq \Omega_1(t_1)$ for $t_2 < t_1$ since $\gamma_i$ is non-increasing in $t$. Note also that $\bar{x}_i(0) \in \Omega_1(0)$ and that $\Omega_1(t)$ is bounded due to Assumption 2 and since $\gamma_i$ is bounded.

Step B: We show that $\tau_{\max} = \infty$ by contradiction of

3.1 Global and Local Task Satisfaction Guarantees

We first present global task satisfaction guarantees.

**Theorem 1** Let Assumptions 1-4 hold. Assume for each $\Xi_{\xi} \in \mathcal{E}$ that for all $v_i, v_j \in \Xi_{\xi}; 1) \phi_i = \phi_j$, and (2) $\xi_i^{max} = \xi_j^{max}$, $r_i = r_j$, and $\gamma_i = \gamma_j$ are chosen as in (6)-(11). If each agent $v_i \in V$ applies

\begin{equation}
u_i(x_i, t) := -\epsilon_i(x_i, t) g_i(x_i)^T \frac{\partial \rho^{\psi_i}(x_i)}{\partial x_i}, \tag{14}\end{equation}

then it holds that $0 < r_i \leq \rho^{\psi_i}(x_i) \leq \rho_{i_{\text{max}}}^{\psi_i}$ for all agents $v_i \in V$, i.e., $\{\phi_1, \ldots, \phi_M\}$ are globally satisfied. All closed-loop signals are continuous and bounded.

**PROOF.** In Step A, we apply Sontag (2013, Thm. 54) and show that for each $v_i \in V$ there exists a maximal solution $\xi_i(t)$ such that $\xi_i(t) := \xi_i(x_i(t), t) \in \Omega_{\xi} := (-1, 0)$ for all $t \in \mathcal{J} := [0, \tau_{\max}] \subseteq \mathbb{R}_{\geq 0}$ where $\tau_{\max} > 0$, which is the same as requiring that (5) holds for all $t \in J$.

Step B consists of using Sontag (2013, Prop. C.3.6) to show that $\tau_{\max} = \infty$. Note beforehand that

\begin{equation}
\frac{d\epsilon_i}{dt} = \frac{d\epsilon_i}{dt} \frac{d\epsilon_i}{dt} = \frac{1}{\gamma_i(1+\xi_i)} \frac{d\epsilon_i}{dt} = \frac{1}{\gamma_i} (\frac{d\epsilon_i}{dt} - \xi_i \dot{\gamma}_i). \tag{15}
\end{equation}

since $\frac{d\epsilon_i}{dt} = \frac{d\epsilon_i}{dt}$ and $\frac{d\epsilon_i}{dt} = \frac{1}{\gamma_i} \frac{d\epsilon_i}{dt}$. It also holds that $\frac{d\epsilon_i}{dt} = \frac{d\epsilon_i}{dt}$ with $\frac{d\epsilon_i}{dt} = \frac{d\epsilon_i}{dt}$.
Sontag (2013, Prop. C.3.6). Therefore, assume $\tau_{\text{max}} < \infty$. Note that $\xi_i(\bar{x}_i, t) = \xi_j(\bar{x}_j, t)$, $\epsilon_i(\bar{x}_i, t) = \epsilon_j(\bar{x}_j, t)$, and $\rho^\psi_i(\bar{x}_i) = \rho^\psi_j(\bar{x}_j)$ for all $v_i, v_j \in \Xi$ since $\bar{x}_i = \bar{x}_j$ and since $\rho^\psi_i = \rho^\psi_j$ and $\gamma_i = \gamma_j$. We first show that $\epsilon_i(t)$ is bounded for all $t \in \mathbb{R}^n$ and then it consequently follows that $\epsilon_j(t)$ is bounded for all $v_j \in \Xi \setminus \{v_i\}$. Since the clusters are maximal we can then deduce the same result for the other clusters. Consider the Lyapunov function candidate $V(\epsilon_i) := \frac{1}{2} \epsilon_i^T \epsilon_i$. By using (15), it follows
\[ V(\epsilon_i) = \frac{d\epsilon_i}{dt} = \epsilon_i \left( -\frac{1}{\gamma_i(1 + \xi_i)} \frac{\partial \rho^\psi_i(\bar{x}_i)}{\partial \bar{x}} \bar{x} - \xi_i \right) \leq \epsilon_i(\gamma_i + \xi_i) \frac{\partial \rho^\psi_i(\bar{x}_i)}{\partial \bar{x}} H_{\bar{x}}(\epsilon_i, \xi_i) \]
and since $\partial \rho^\psi_i(\bar{x}_i)/\partial \bar{x}$ and $\gamma_i$ are bounded as in (6)-(11). Assume further that $\epsilon_i(v) \in V(\phi)$ applies (14) and that Assumptions 2 and 4 become upper bounded as $\epsilon_i(t) \leq -\frac{1}{\epsilon_i(\gamma_i + 1)} \xi_i(t)$ is bounded by $-1 < \xi_i(t) \leq \xi_i^\text{u} < 0$, which translates to $\xi_i(t) \in \Omega_i^\text{b} := [\xi_i^\text{l}, \xi_i^\text{u}] \subset \Omega_i$ for all $t \in J$. Note that if $\xi_i(t)$ evolves in a compact set, then $\rho^\psi_i(\bar{x}_i(t))$ will evolve in a compact set $\Omega_i^\text{c} := [\rho^\psi_i^\text{l}, \rho^\psi_i^\text{u}]$ for some constants $\rho^\psi_i^\text{l}$ and $\rho^\psi_i^\text{u}$. Due to Aubin and Frankowska (2009, Prop. 1.4.4) it again holds that
\[ \Omega_i^\prime := \rho^\psi_i^{\text{u}^{-1}}(\Omega_i^\prime_0) = [\bar{x}_i \in \Omega_i(0)|\rho^\psi_i \leq \rho^\psi_i(\bar{x}_i) \leq \rho^\psi_i^\text{u}] \]
and since $\Omega_i^\prime \subset \Omega_i(0)$ for all $t \in J$ and all $v_i$, define the compact sets $\Omega_i^\prime_0 := \Omega_i^\prime \times \cdots \times \Omega_i^\prime \subset \mathbb{R}^n$, $\Omega_i^\prime_0 := \Omega_i^\prime \times \cdots \times \Omega_i^\prime \subset \mathbb{R}^M$, and $\Omega_i^\prime_0 := \Omega_i^\prime \times \Omega_i^\prime \subset \mathbb{R}^{n+M}$ for which it holds that $y(t) \in \Omega_i^\prime_0$ for all $t \in J$. It is also true that $\Omega_i^\prime_0 \subset \Omega_i$ by which it follows that there is no $e \in J := (0, \tau_{\text{max}})$ such that $y(t) \notin \Omega_i^\prime_0$. By contradiction of Sontag (2013, Prop. C.3.6) it holds that $\tau_{\text{max}} = \infty$, i.e., $J = \mathbb{R}^n$. This means that (5) holds for all $v_i \in V$ and all $t \in \mathbb{R}^n$. By the choice of $\rho^\psi_i^\text{u}$, $r_i$, and $\gamma_i$ as in (7)-(11), it then holds that $0 < r_i \leq \rho^\psi_i(\bar{x}_i, t) \leq \rho^\psi_i^\text{u}$ for each $v_i \in \Omega$. The control law $u_i(\bar{x}_i, t)$ is continuous and bounded because $\rho^\psi_i(\bar{x}_i)$, $\epsilon_i(\bar{x}_i, t)$, and $g_i(\bar{x}_i)$ are continuous. Furthermore, $\gamma_i$ is continuous with $0 < \gamma_i(t) < \infty$. Due to the compact domain $\Omega_\phi^\prime$, these functions are also bounded.

Remark 5 If $L = M$, i.e., $|V_i| = 1$ for each agent $v_i \in V$, Theorem 1 trivially applies. If $m_i = n_i$ and $g_i(\bar{x}_i)$ is positive definite for all $\bar{x}_i \in \mathbb{R}^n$, the control law (14) can be replaced by $u_i(\bar{x}_i, t) := -\epsilon_i(\bar{x}_i, t) \frac{\partial \rho^\psi_i(\bar{x}_i)}{\partial x_i}$, i.e., no knowledge of $f_i$, $g_i$, and $g_i$ needed.

For a cluster $\Xi \subset \Xi$ that satisfies Case A in Problem 1, we next guarantee local satisfaction of $\phi_i$ for each $v_i \in \Xi$, while one or more clusters $\Xi_k \subset \Xi$, do not satisfy the assumption of Case A. We first derive a more general result by disregarding clusters. Therefore, consider a formula $\phi$ as in (1b) and the set of agents $\mathcal{V}_i \subset V$ that participate in $\phi$. Satisfaction of $\phi$ will be guaranteed when the trajectories of agents in $V \setminus \mathcal{V}_i$ remain bounded.

Assumption 5 Given, for each $v_i \in \mathcal{V}_i$, a sufficiently regular control law $u_i$, there exists, for each $v_k \in \mathcal{V}_i \setminus \mathcal{V}_i$, a sufficiently regular control law $u_i^k$ such that, for each $v_i \in \mathcal{V}_i$, there is a solution $\bar{x}_i : [0, \tau_{\text{max}}) \rightarrow \mathbb{R}^n$ to (2) with $\tau_{\text{max}} > 0$ so that, for each $v_k \in \mathcal{V}_i \setminus \mathcal{V}_i$, $\bar{x}_i(t)$ stays in a compact set. By "sufficiently regular" we mean a controller so that the conditions in Sontag (2013, Thm. 54) are satisfied.

Theorem 2 Let each agent $v_i \in V$ satisfy Assumption 1. Consider $\phi$ as in (1b) and let each $v_i \in \mathcal{V}_i$ be subject to $\phi_i := \phi$. Assume that for all $v_i, v_j \in \mathcal{V}_i$ it holds that: 1) $(v_i, v_j) \in \mathcal{E}$ and 2) $t_i^* = t_j^*, \rho^\psi_i^\text{max} = \rho^\psi_j^\text{max} = r_i = r_j$, and $\gamma_i = \gamma_j$ are chosen as in (6)-(11). Assume further that each $v_i \in \mathcal{V}_i$ applies (14) and that Assumptions 2 and 4
hold. If Assumption 5 holds and all agents \( v_k \in V \setminus V_\varphi \) apply \( u'_k \), then it holds that \( 0 < r_i \leq \rho^\varphi(\bar{x}_i, 0) \leq \rho^\max_i \).

**Proof.** Define \( \xi := [\xi_1, \ldots, \xi_{|V \setminus V_\varphi|}] \) and \( y := [x \xi] \) where \( v_1, \ldots, v_{|V \setminus V_\varphi|} \in V_\varphi \). Due to \( u_i \) and \( u'_i \) being sufficiently regular and similarly as in the proof of Theorem 1, there exists a maximal solution \( y(t) \in \Omega_y \) for all \( t \in J := [0, \tau^\max] \) with \( \tau^\max > 0 \) and for an open, non-empty, and bounded set \( \Omega_y \subset \mathbb{R}^{n+|V_\varphi|} \). Since \( x_k(t) \) evolves in a compact set for all \( t \in J \), it can again be shown that \( y \in \Omega'_{y'} \subset \Omega_y \) for a compact set \( \Omega'_{y'} \) so that \( \tau^\max = \infty \). It follows that \( 0 < r_i \leq \rho^\varphi(\bar{x}_i, 0) \leq \rho^\max_i \) \( \blacksquare \).

If all \( v_i \in V_\varphi \) apply (14) under the conditions in Theorem 2, we say that the agents in \( V_\varphi \) use collaborative control to satisfy \( \varphi \). Theorem 2 solves Case A of Problem 1 with \( \rho^\opt_i > 0 \) even when Theorem 1 does not apply, i.e., some other clusters do not satisfy Case A. Note also that Theorem 2 is more general than that as exemplified next.

**Example 1** Consider \( V := \{v_1, v_2, v_3, v_4\} \) with the tasks \( \phi_1 := \phi_2 := F_{[0,5]}(||x_1 - x_2|| \leq 1), \phi_3 := F_{[0,5]}(||x_3 - x_2|| \leq 1 \wedge ||x_3 - x_4|| \leq 1), \phi_4 := F_{[0,5]}(||x_4|| \leq 1), \) hence inducing only one maximal dependency cluster \( \Xi := \Xi_1 \) to which Case B of Problem 1 applies. If agents \( v_1 \) and \( v_2 \) use collaborative control to satisfy \( \phi_1 = \phi_2 \), then Theorem 2 guarantees satisfaction of \( \phi_3 = \phi_4 \).

Extensions of Theorems 1 and 2 to solve Case A when \( \rho^\opt_i \leq 0 \) are stated in Corollary 1.

**Corollary 1** Assume that all assumptions of Theorem 1 (Theorem 2) hold except for Assumption 4 and the choice of \( \rho^\max_i \) and \( r_i \). If instead \( \rho^\max_i \) and \( r_i \) are as in (12) and (13), respectively, then it holds that \( r_i \leq \rho^\varphi(\bar{x}_i, 0) \leq \rho^\max_i (r_i < \rho^\varphi(\bar{x}_i, 0) \leq \rho^\max_i) \).

### 3.2 An Online Detection & Repair Scheme

Assume now that \( \Xi_1 \in \tilde{\Xi} \) may not satisfy the assumption of Case A in Problem 1 and Case B applies. We propose that each agent \( v_i \in \Xi_1 \) initially applies (14) with properly chosen initial parameters \( t_1^*, \rho^\max_i, r_i \), and \( \gamma_i \); (14) consists of two components, one determining the control strength and one the control direction. The closer \( \xi_i(\bar{x}_i, t) \) gets to \( \Omega_i := \{-1, 0\} \), the bigger will \( e_i(\bar{x}_i, t) \) and consequently \( \|u_i(\bar{x}_i, t)\| \) become, i.e., \( \|u_i(\bar{x}_i, t)\| \Rightarrow \infty \) as \( \xi_i(\bar{x}_i, t) \Rightarrow \Omega_i \). The control direction is given by \( -\frac{\partial e_i(\bar{x}_i)}{\partial \bar{x}_i} \). We reason that applying (14) is hence a good initial choice. However, the resulting trajectory \( \bar{x}_i \) may still lead to \( \xi_i(\bar{x}_i(t)) = \{-1, 0\} \), which is equivalent to a violation of (5), for some \( t \geq 0 \) and result in critical events. The next two examples exhibit such behavior.

**Example 2** Consider \( V := \{v_1, v_2, v_3, v_4\} \) with the collaborative formula \( \phi_1 := G_{[10,15]}(||x_1 - x_2|| \leq 25) \wedge (||x_1 - x_3|| \leq 25) \wedge (||x_1 - x_4|| \leq 25) \) and the non-collaborative formulas \( \phi_2 := G_{[10,15]}(||x_2 - [10 90]|| \leq 5), \phi_3 := G_{[10,15]}(||x_3 - [10 10]|| \leq 5), \) and \( \phi_4 := G_{[10,15]}(||x_4 - [45 20]|| \leq 5) \). The set of formulas \( \{\phi_1, \phi_2, \phi_3, \phi_4\} \) is not globally satisfiable, although each formula is locally satisfiable. Under (14), agents \( v_2, v_3, \) and \( v_4 \) move to \([10 90], [10 10], \) and \([45 20]\), respectively. Agent \( v_1 \) can hence not satisfy \( \phi_1 \) and will violate (5) for some \( t \geq 0 \). A solution is to decrease the robustness online so that \( r_1 < \rho^\varphi(\bar{x}_1, 0) \leq 0 \).

Even if the set \( \{\phi_1, \ldots, \phi_2\} \) is globally satisfiable, the resulting trajectory may violate (5) for some \( t \geq 0 \).

**Example 3** Consider \( V := \{v_5, v_6, v_7\} \) with the collaborative formula \( \phi_5 := F_{[5,10]}(||x_5 - x_6|| \leq 10) \wedge (||x_5 - x_7|| \leq 10) \wedge (||x_5 - [110 20]|| \leq 5) \) and the non-collaborative formulas \( \phi_6 := F_{[5,10]}(||x_5 - [50 20]|| \leq 5) \) and \( \phi_7 := F_{[5,10]}(||x_7 - [110 80]|| \leq 5) \). Under (14), \( v_5 \) and \( v_7 \) move to \([50 20]\) and \([110 80]\) by at latest 15 time units, respectively. However, \( v_5 \) is forced to move to \([110 20]\) and be close to \( v_5 \) and \( v_7 \) by at latest 10 time units. This may lead to critical events where \( v_5 \) violates (5) for some \( t \geq 0 \). If \( v_5 \) and \( v_7 \) collaborate, satisfaction of \( \phi_6 \) can be postponed and \( \phi_5 \) can be locally satisfied first, e.g., by using collaborative control for \( \phi_5 \).

We propose an online detection & repair scheme by using a low hybrid control system \( H_i := (G_i, F_i, D_i, G_i) \) for each \( v_i \in \Xi_2 \). We first detect critical events, i.e., when (5) is violated, by using \( D_i \). Then, a three-stage repair procedure is initiated as illustrated in Fig. 1 where repairs are according to \( G_i \). In the first repair stage, detected by \( D'_{i,1} \), the design parameters \( t_i^*, \rho^\max_i, r_i \), and \( \gamma_i \) are modified locally without communication among agents by the jump map \( G'_{i,1} \). If this is not successful, collaboration among agents will be considered in the second repair stage. Here, critical events of agent \( v_i \) are detected by \( D'_{i,2} \) and collaborative control is requested from agents \( v_j \in V \setminus \{v_i\} \) by \( D'_{j,2} \) to handle \( \varphi_i \) as in Example 3. If this second repair stage is not applicable, the third repair stage is detected by \( D'_{i,3} \) that successively decreases \( r_i \) by \( \delta_i > 0 \). First, define \( p'_i := [n, c_i] \).
where \( n_i \in \mathbb{N} \) indicates the number of repair attempts in the first repair stage, while \( c_i \) is used in the second repair stage; \( c_i \in \{1, \ldots, M\} \) indicates collaborative control for the formula \( \phi_i \). If \( c_i = 0 \), then \( v_i \) tries to locally satisfy \( \phi_i \) by itself and if \( c_i = -1 \), then \( v_i \) is free, i.e., not subject to a task. For convenience, we abbreviate \( p_i^f := \begin{bmatrix} \gamma_i^0 & \gamma_i^\infty \end{bmatrix} l_i \) and \( p_i^c := \begin{bmatrix} r_i & p_i^c \end{bmatrix} \)
where \( p_i^f \) defines the funnel (5). For agent \( v_i \), define \( z_i := [x_i; p_i^f; p_i^c] \in \mathcal{Z}_i \) where \( \mathcal{Z}_i := \mathbb{R}_n^i \times \mathbb{R}_{\geq 0}^i \times \mathbb{Z}^2 \).

The initial state is \( z_i(0) := [x_i(0); p_i^f(0); 0] \) with

\[
p_i^f(0) := \begin{cases} t_i^r \rho_{\text{opt}} \gamma_i^0 l_i & \text{if } \rho_{\text{opt}}^i > 0 \\ t_i^r \rho_{\text{max}} \gamma_i^0 l_i & \text{if } \rho_{\text{opt}}^i \leq 0 \end{cases}
\]

where \( t_i^r, \rho_{\text{max}}^i, r_i, \gamma_i^0, \gamma_i^\infty, l_i, \rho_{\text{opt}}^i \), and \( \tilde{r}_i \) are chosen according to (6)-(13), respectively, and where \( \rho_{\text{max}}^i > \tilde{r}_i \) is maximized up to a precision of \( \delta_i > 0 \). Additionally, \( p_i^c(0) = p_i^c(0) \) if Case A holds for all agents \( v_i, v_j \in \mathcal{E}_i \). The flow map is given by

\[
F_i(z_i, u_i^{\text{int}}, u_i^{\text{ext}}) := [f_i(x_i) + f_i^f(x_i) + g_i(x_i)u_i^{\text{int}} + w_i, 0]
\]

where the control law is given by

\[
u_i^{\text{int}} := \begin{cases} -c_i(x_i, t)g_i(x_i)^T \frac{\partial \rho^{\text{opt}}(x_i)}{\partial x_i} & \text{if } c_i = 0 \quad (18a) \\ -c_i\tilde{r}_i(x_i, t)g_i(x_i)^T \frac{\partial \rho^{\text{opt}}(x_i)}{\partial x_i} & \text{if } c_i > 0. \quad (18b)\end{cases}
\]

If \( c_i > 0 \), then collaborative control for \( \phi_i \) is used. External inputs contained in \( u_i^{\text{ext}} = w_i \) and \( x_i^{\text{ext}} \). By assuming \( v_i \in \mathcal{E}_i \), we define \( c_i^{\text{ext}} := [c_j, \ldots, c_{j|\mathcal{E}_i| - 1}] \) and \( p_i^{\text{ext}} := \begin{bmatrix} p_i^f, \ldots, p_{j|\mathcal{E}_i| - 1}^f \end{bmatrix} \) such that \( v_j, \ldots, v_{j|\mathcal{E}_i| - 1} \in \mathcal{E}_i \setminus \{v_i\} \). Note that \( c_i^{\text{ext}} \) and \( p_i^{\text{ext}} \) contain states of all agents in the same dependency cluster \( \mathcal{E}_i \). Ultimately, define the external input \( u_i^{\text{ext}} := [w_i, x_i^{\text{ext}}, c_i^{\text{ext}}, p_i^{\text{ext}}] \).

Detection: To detect a critical event, the jump set

\( \mathcal{D}_i^j := \{(z_i; u_i^{\text{int}}, u_i^{\text{ext}}) \in \mathcal{S}_i | \zeta_i(t) \in \{-1, 0\}, c_i = 0\} \)

is used. Throughout this section, we assume that agent \( v_i \) detects the critical event, while the agents with subscript \( j \) as \( v_j \in \mathcal{V}_i \setminus \{v_i\} \) are possibly asked to collaborate. We introduce the notation \( \{\tilde{z}_i \in \mathcal{Z}_i | \tilde{z}_i = z_i \text{; exception}\} \) denoting the set of \( \tilde{z}_i \) such that \( \tilde{z}_i = z_i \) after the jump except for the elements exception in \( \tilde{z}_i \) that change during a jump and that are explicitly mentioned after the semicolon. The set \( \mathcal{D}_i^j \) is split into disjoint sets indicating repairs of the first, second, and third repair stage.

Repair Stage 1: The first repair stage is indicated by

\( \mathcal{D}_{i,1} := \mathcal{D}_i^j \cap (\{z_i; u_i^{\text{int}}, u_i^{\text{ext}}\) \in \mathcal{S}_i | n_i < N_i\} \)

where \( N_i \in \mathbb{N} \) is a design parameter that represents the maximum number of repair attempts in the first stage. A good choice of \( N_i \) may in practice depend on the dynamics and tasks of the agents. If \( (z_i, u_i^{\text{int}}, u_i^{\text{ext}}) \in \mathcal{D}_{i,1} \), we first relax the parameters \( \rho_i, \rho_{\text{max}}^i, r_i \), and \( \gamma_i \).

Example 4 Consider \( \phi_i := F_{[4,6]} \psi_i \) with \( r_i := 0.4 \) (initial robustness), which is supposed to be achieved at \( t_i^* := 4.5 \). The initial funnel with \( \rho_{\text{max}}^i \) and \( -\gamma_i + \rho_{\text{max}}^i \) is shown in Fig. 2. Without detection of a critical event, it would hold that \( \rho^{\text{opt}}(x_i, 0) \geq r_i \) since \( \rho^{\text{opt}}(x_i(t_i^*)) \geq r_i \). However, at \( t_r = 2 \), where \( t_r \) indicates the time where a critical event is detected, the trajectory \( \rho^{\text{opt}}(x_i(t_r)) \) touches the lower funnel boundary and repair action is needed. This is done by setting \( \tilde{t}_i := 6 \) (time relaxation), \( \tilde{r}_i := 0.15 \) (robustness relaxation), \( \tilde{\rho}_{\text{max}}^i := 1.1 \) (upper funnel relaxation), and adjusting \( \gamma_i \) (lower funnel relaxation). The funnel is hence relaxed to \( \tilde{\rho}_{\text{max}}^i \) and \( -\gamma_i + \rho_{\text{max}}^i \) as in Fig. 2. At the time \( t_r \), the lower funnel is relaxed to \( -\gamma_i(t_r) + \rho_{\text{max}}^i \) where we denote \( \gamma_i(t_r) \) as in Fig. 2. Due to repair action, \( x_i \) locally satisfies \( \phi_i \) as shown in Fig. 2.

With Example 4 in mind, set

\[
\mathcal{G}_{i,1} := \{\tilde{z}_i \in \mathcal{Z}_i | \tilde{z}_i = z_i ; t_i^* = T_i, \tilde{\rho}_{\text{max}}^i = \rho_{\text{opt}}^i + \zeta_i^i, \tilde{r}_i = R_i, \tilde{p}_i = \tilde{p}_i^{\text{new}}, \tilde{n}_i = n_i + 1\}
\]

where, to achieve time relaxation, we define

\[
T_i := \begin{cases} \{b_i\} & \text{if } \phi_i = F_{[\tilde{b}_i, \tilde{b}_i]} \psi_i \\ \{a_i\} & \text{if } \phi_i = G_{[\tilde{a}_i, \tilde{a}_i]} \psi_i \end{cases}
\]

and \( \phi_i = F_{[\tilde{a}_i, \tilde{a}_i]} G_{[\tilde{a}_i, \tilde{a}_i]} \psi_i \).

The parameter \( r_i \) is decreased to \( \tilde{r}_i \in (0, r_i) \) if \( r_i > 0 \) and set to \( \tilde{r}_i := -\gamma_i - r_i \) otherwise (robustness relaxation), i.e.,

\[
\mathcal{R}_i := \{\tilde{r}_i \in \mathbb{R} | \tilde{r}_i \in \{0, r_i\} \text{ if } r_i > 0, \tilde{r}_i = -\gamma_i - r_i \text{ if } r_i \leq 0\}
\]

The variable \( \zeta_i^i \) relaxes the upper funnel and needs to be such that \( \tilde{\rho}_{\text{max}}^i := \rho_{\text{max}}^i + \zeta_i^i < \rho_{\text{opt}}^i \) (upper funnel relaxation) according to (7), i.e., let \( \zeta_i^i \in (0, \rho_{\text{opt}}^i - \rho_{\text{max}}^i) \).

![Fig. 2. Funnel repair in the first stage for \( \phi_i := F_{[4,6]} \psi_i \).](image)
At $t_r$, the detection time of a critical event, we set $\gamma^r_i := \hat{\gamma}_i(t_r) := \hat{\rho}_i^{\max} - \rho_i^\psi(\bar{x}_i(t_r)) + \zeta_i$ with
\[
\zeta_i := \begin{cases} 
\mathbb{R}_{>0} & \text{if } t_i^* > t_r \\
(0, \rho_i^\psi(\bar{x}_i(t_r)) - \hat{\rho}_i) & \text{otherwise,}
\end{cases}
\]
which resembles (9) (lower funnel relaxation). Let $\tilde{\rho}_{i}^{\max,\text{new}} := \left[ \gamma_i^{0,\text{new}}, \gamma_i^{\infty,\text{new},\text{new}} \right]$ and select
\[
\gamma_i^{\infty,\text{new}} \in \left(0, \min(\gamma_i^r - \hat{\rho}_i^{\max} - \hat{\rho}_i)\right)
\]
\[
\tilde{\rho}_{i}^{\text{max,\text{new}}} := \left\{ \begin{array}{ll}
0 & \text{if } \gamma_i^{0,\text{new}} \leq 0 \\
\exp(\ln(\frac{\gamma_{i}^{0,\text{new}}}{\gamma_{i}^{0,\text{new}} - \gamma_{i}^{\infty,\text{new}}}) \frac{t_i - t_r}{\hat{\rho}_i}) & \text{else}
\end{array} \right.
\]
similarly to (10) and (11). Finally, set $\gamma_i^{0,\text{new}} := (\gamma_i^r - \gamma_i^{\infty,\text{new}}) \exp(\ln(\frac{\gamma_{i}^{0,\text{new}}}{\gamma_{i}^{0,\text{new}} - \gamma_{i}^{\infty,\text{new}}}) \frac{t_i - t_r}{\hat{\rho}_i})$.

The choices of $\hat{\rho}_i^{\max}$, $\hat{\rho}_i^r$, and $\hat{\rho}_i^{\psi}$ follow the same intuition as in Remark 4.

**Repair Stage 2:** The jump set
\[
D_{i,(2,3)}' := D' \cap \{(z_i, u_i^{\text{int}}, u_i^{\text{ext}}) \in \mathcal{F}_i | n_i \geq N_i \}
\]
detects repairs of the second or third stage. After $N_i$ unsuccessful repair attempts, the second stage is initiated if some timing constraints (formalized in $D_{i,2}'$) are satisfied. Collaborative control for $\phi_i$ by all agents in $V_i$ will then be initiated and guarantee that there are no further critical events. The second stage is then detected as
\[
D_{i,2}' := D_{i,(2,3)}' \cap \{(z_i, u_i^{\text{int}}, u_i^{\text{ext}}) \in \mathcal{F}_i | \forall v_j \in V_i \setminus \{v_i\}, \left( j = -1 \right) \text{ or } \left( j = 0, b_i < T_j \right) \}
\]

To use collaborative control to deal with $\phi_i$, the in $D_{i,2}'$ formalized constraints need to hold, i.e., each agent $v_j \in V_i \setminus \{v_i\}$ is either not subject to a task or there is enough time to satisfy $\phi_j$ after $\phi_i$ has been collaboratively satisfied. In this respect, the control law switches from (18a) to (18b) for agent $v_i$. Therefore, set
\[
\mathcal{G}_{i,2}'' := \{ \bar{z}_i \in Z_i | \bar{z}_i = z_i ; \hat{\rho}_i^{\max} = \rho_i^{\max} + \zeta_i^u, \hat{\rho}_i = \hat{\rho}_i^{\max,\text{new}}, \hat{\rho}_i^* = \hat{\rho}_i^{\gamma,\text{new}}, \hat{\rho}_i^* = i \}
\]
where $\hat{\rho}_i^* := i$ indicates collaborative control for $\phi_i$, while again relaxing the funnel parameters as in the first repair stage. Now changing the perspective to the participating agents $v_j \in V_i \setminus \{v_i\}$, all agents $v_j$ need to participate in collaborative control. Assume that $v_j \in \Xi_i$, then
\[
\mathcal{D}_{j,2}'' := \{(z_j, u_j^{\text{int}}, u_j^{\text{ext}}) \in \mathcal{F}_j | \epsilon_j \in \{-1, 0\}\}, \forall v_j \in \Xi_i \setminus \{v_j\}, v_j \in V_i, \epsilon_i = i \}
\]
is activated when agent $v_j$ asks agent $v_i$ for collaborative control (detected by $D_{i,2}'$). If $(z_i, u_i^{\text{int}}, u_i^{\text{ext}}) \in \mathcal{D}_{j,2}''$, the control law for each $v_j \in V_i \setminus \{v_i\}$ switches to (18b) by
\[
\mathcal{G}_{j,2}'' := \{ \bar{z}_j \in Z_j | \bar{z}_j = z_j ; \hat{\rho}_j = \hat{\rho}_j^{\gamma,\text{new}}, \hat{\rho}_j^* = i \}
\]
where $\epsilon_i = i$ and $\hat{\rho}_j = \hat{\rho}_j^{\gamma}$ enforce that all conditions in Theorem 2 or Corollary 1 hold after the jump.

**Remark 6** The second repair stage is initiated solely based on $\epsilon_i$ and $T_j$. Note that no future state predictions can be made since $f_j(x_j)$ and $f_j^* (x)$ are unknown ($f_i(x_i)$ and $f_i^* (x)$ are in general unknown) and that agent $v_i$ only has knowledge of $\epsilon_i$ and $a_i$, $a_j$, or $b_j + a_j$ depending on whether an always, eventually, or always-eventually task is considered, so that ahead planning is not possible.

**Repair Stage 3:** If the timing constraints in $D_{i,2}'$ do not apply, repairs of the third stage are initiated by
\[
D_{i,3}' := D_{i,(2,3)}' \setminus D_{i,2}'.
\]

Agent $v_i$ reacts in this case by reducing the robustness $r_i$ by $\delta_i > 0$ as illustrated in Example 2 and according to
\[
G_{i,3} := \{ \bar{z}_j \in Z_j | \bar{z}_j = z_j ; \hat{\rho}_j^{\max} = \rho_j^{\max} + \zeta_i^u, \hat{\rho}_j = \rho_j^{\opt} + \sigma, \hat{\rho}_j^* = \hat{\rho}_j^{\gamma,\text{new}} \}
\]
where $\gamma_i^u := \hat{\rho}_i^{\max} - \rho_i^\psi(\bar{x}_i) + \delta_i$ is used to calculate $\hat{\rho}_i^{\gamma,\text{new}}$ and $\sigma > 0$ is a small constant that avoids Zeno behavior.

**The Overall System:** It needs to be detected when $r_i \leq \rho_i^\psi(\bar{x}_i) \leq \rho_i^{\max}$. Define $\nu_i := \{ \epsilon_i : \epsilon_i > 0 \}
\]

\[
D_{i,\text{sat}} := \{(z_i, u_i^{\text{int}}, u_i^{\text{ext}}) \in \mathcal{F}_i | \forall v_i \in (D' \cup D_{i,2}' \cup D_{i,3}') \},
\]

with
\[
T_{i}^{\text{sat}} := \left\{ \begin{array}{ll}
[a_i, b_i] & \text{if } a_i = F_{[a_i, b_i]}(\epsilon_i) = F_{[a_i, b_i]}(\epsilon_i) \\
b_i & \text{if } a_i = G_{[a_i, b_i]}(\epsilon_i) \\
\end{array} \right.
\]
and where the set subtraction of $D' \cup D_{i,2}'$ excludes the case where $D_{i,2}'$ or $D_{i,3}'$ apply simultaneously with $D_{i,\text{sat}}$. If $(z_i, u_i^{\text{int}}, u_i^{\text{ext}}) \in D_{i,\text{sat}}$ and in case of collaborative control, the agents $v_j \in V_i \setminus \{v_i\}$ are then either not subject to a task or need to continue with $\phi_j$. Hence, set
\[
\mathcal{G}_{i,\text{sat}} := \{ \bar{z}_i \in Z_i | \bar{z}_i = z_i ; \hat{\rho}_i = \hat{\rho}_i^{\max}, \hat{\rho}_i^* = \hat{\rho}_i^{\gamma,\text{new}}, \hat{\rho}_i^* = \hat{\rho}_i^{\gamma,\text{new}} \}
\]

where $\rho_{\text{max}}$, $r_i$, $\tilde{\rho}_{\text{max}}$, and $\tilde{r}_i$ are according to (7), (8), (12), and (13) but evaluated with $\bar{x}_i(t)$ instead of $\bar{x}_i(0)$. Finally, $H_i$ is given by $D_i := D_{i,1} \cup D_{i,2} \cup D_{i,\text{sat}}$, $C_i := \mathcal{Z}_i \setminus D_i$, and $F_i(z_i, u_{i,\text{int}}, u_{i,\text{ext}})$ as defined before. The corresponding jump map $G_i(z_i, u_{i,\text{int}}, u_{i,\text{ext}})$ is $G_{i,1}(z_i, u_{i,\text{int}}, u_{i,\text{ext}})$ if $(z_i, u_{i,\text{int}}, u_{i,\text{ext}}) \in D_{i,1}$, $G_{i,2}(z_i, u_{i,\text{int}}, u_{i,\text{ext}})$ if $(z_i, u_{i,\text{int}}, u_{i,\text{ext}}) \in D_{i,2}$, and $G_{i,\text{sat}}(z_i, u_{i,\text{int}}, u_{i,\text{ext}})$ if $(z_i, u_{i,\text{int}}, u_{i,\text{ext}}) \in D_{i,\text{sat}}$. It is crucial that the behavior of $H_i$ does not exhibit two or more jump options at the same time, i.e., $H_i$ should be deterministic with respect to jumps permitted by $H_i$. Note that $D_i = D_{i,1} \cup D_{i,2} \cup D_{i,3}$ and that $D_{i,1}$, $D_{i,2}$, and $D_{i,3}$ are non-intersecting. Note also that the sets $D_i$ and $D_{i,\text{sat}}$ as well as $D_{i,1}$ and $D_{i,\text{sat}}$ are non-intersecting. However, $D_i$ and $D_{i,2}$ are intersecting. Therefore, if $(z_i, u_{i,\text{int}}, u_{i,\text{ext}}) \in D_i \cap D_{i,2}$, we only execute the jump induced by $D_{i,2}$ to account for the logic modeled by the hybrid system. This can be achieved by modifying $D_i$ to $D_i' \cap D_{i,2}'$.

**Theorem 3** Let each agent $v_j \in \mathcal{V}$ be controlled by $H_i := (C_i, F_i, D_i, G_i)$, while Assumptions 1-3 and 5 are satisfied. For $v_i \in \mathcal{V}_i$ it holds that $\rho_i^0(x_i,0) > r_i$ where either $r_i := r_i(0,0)$ (initial robustness) if Case A applies or $r_i$ is lower bounded and maximized up to a precision of $\delta_i > 0$ if Case B applies. Zeno behavior is excluded.

**Proof.** Without critical events, it is guaranteed that either $\phi_i$ is locally satisfied if $\rho_i^0 > 0$ or a least violating solution with a given gap of $\rho_i^{\text{gap}}$ is found if $\rho_i^0 \leq 0$ due to Theorem 2 and Corollary 1, respectively. In the first repair stage, the parameters $t_i^*, \rho_i^{\text{max}}, r_i, \gamma_i, \gamma_i^\infty, \lambda_i$ are repaired in a way that still guarantees local satisfaction of $\phi_i$ if $\rho_i^0 > 0$ or, otherwise, $r_i$ is reduced by $\delta_i$. Zeno behavior is excluded for this stage since only a finite number of jumps, i.e., $N_i$ jumps, are permitted. For the second repair stage, collaborative control for $\phi_i$ guarantees achieving the task $\bar{\phi}_i$ with a robustness of $r_i$ by Theorem 2 and Corollary 1. Afterwards, participating agents $v_j \in \mathcal{V}_i \setminus \{v_i\}$ have enough time to deal with their own local task $\phi_j$, which is guaranteed by the timing constraints in $D_{i,2}'$ that need to hold in order to initiate collaborative control. The third repair stage successively decreases $r_i$ by $\delta_i$. Note that $r_i$ has to be lower bounded due to Assumption 2, which states that for local satisfaction of $\phi_i$ the state $x_i$ is bounded. Hence, all agents aim to stay within a bounded set. Consequently, successively reducing $r_i$ will eventually lead to $\rho_i^0(x_i,0) \geq r_i$, i.e., maximizing $\rho_i^0(x_i,0)$ up to a precision of $\delta_i$. This again means that only a finite number of jumps is possible when the lower funnel is touched. Touching the upper funnel will also only lead to a finite number of jumps since $\rho_i^{\text{max}} = \rho_i^{\text{opt}} + \sigma$ in $G_{i,3}'$, excluding Zeno behavior.

4 Simulations

Consider $M := 9$ agents represented by three-wheeled omni-directional mobile robots as in Liu et al. (2008) with two states $x_1$ and $x_2$ indicating the robot’s position and one state $x_3$ indicating the robot’s orientation with respect to the $x_1$-axis. We denote $x_{i,k}$ with $k \in \{1, 2, 3\}$ as the $k$-th element of agent $v_i$’s state and define $p_i := [x_{i,1} \ x_{i,2}]$. Let $x_i := [p_{i,1} \ p_{i,2} \ p_{i,3}] \in \mathbb{R}^3$ so that $x := [x_1 \ldots x_M] \in \mathbb{R}^{27}$. As in Remark 2, we use induced dynamical couplings $f_i^0(x) := [f_{i,1}^0(x) \ f_{i,2}^0(x) \ 0]$ as a means of collision avoidance with $f_{i,k}^0(x) := \sum_{j=1,j\neq i}^{N_i} k_{i_j} \frac{x_{i,j} - p_{i,k}}{||x_{i,j} - p_{i,k}|| + 0.000001}$ for $k \in \{1, 2\}$ and where $k_{i_j} := 10$. By choosing $f_i^0(x)$ as above, each agent $v_i$ needs knowledge of the states of all agents, which can be prevented by only including the states of agents in the proximity of agent $v_i$. The dynamics are given by $\dot{x}_i = f_i^0(x) + \frac{\cos(x_{i,3}) - \sin(x_{i,3})}{\sin(x_{i,3})} (B_i^T)^{-1} R_i v_i + w_i$, where $R_i := 0.02$ is the wheel radius and $B_i := \begin{bmatrix} 0 & -\cos(\pi/6) & -\cos(\pi/6) \\ -\sin(\pi/6) & \sin(\pi/6) & 1 \end{bmatrix}$ describes geometrical constraints with $L_i := 0.2$ (radius of the robot body). The simulations have been performed in real-time on a two-core 1.8 GHz CPU with 4 GB of RAM. Using the forward Euler method with a sampling frequency of 500 Hz, calculations of the local control laws took on average 50 μs. The noise $w_i$ is drawn from a truncated normal distribution with mean 0 and variance 100. The simulation example resembles Examples 2 and 3. We additionally add requirements on the robot’s orientation. For the first cluster (as in Example 2), here denoted by $\mathcal{Z}_i$, we let $\phi_i := G_{i,15} \left( (||p_1 - p_2|| \leq 25) \land (||p_1 - p_4|| \leq 25) \land (||p_1 - p_4|| \leq 25) \right)$, $\phi_2 := G_{i,10,15} \left( (||p_2 - [10 \ 90]|| \leq 5) \land (||x_{2,3} - 45|| \leq 7.5) \right)$, $\phi_3 := G_{i,10} \left( (||p_3 - [10 \ 10]|| \leq 5) \land (||x_{3,3} - 45|| \leq 7.5) \right)$, and $\phi_4 := G_{i,15} \left( (||p_4 - [45 \ 20]|| \leq 5) \land (||x_{4,3} - 45|| \leq 7.5) \right)$. We added the requirements that agents $v_2$, $v_3$, and $v_4$ should eventually be oriented with $-45$, 45, and 135 degrees and remain with this orientation from then on. For the second cluster (as in Example 3), denoted by $\mathcal{Z}_2$, let $\phi_5 := F_{i,10} \left( (||p_5 - p_7|| \leq 10) \land (||p_5 - p_9|| \leq 10) \land (||p_5 - [110 \ 20]|| \leq 5) \right)$, $\phi_6 := F_{i,15} \left( (||p_6 - [50 \ 20]|| \leq 5) \land (||x_{6,3} - 45|| \leq 7.5) \right)$, and $\phi_7 := F_{i,15} \left( (||p_7 - [110 \ 80]|| \leq 5) \land (||x_{7,3} + 135|| \leq 7.5) \right)$. We have added the requirements that agents $v_5$ and $v_7$ should eventually be oriented with 45 and $-135$ degrees; $\mathcal{Z}_1$ and $\mathcal{Z}_2$ correspond to Case B in Problem 1. For the third cluster, denoted by $\mathcal{Z}_3$, let $\phi_8 := \phi_9 := F_{i,15} \left( (x_{8,1} - x_{9,1}) \leq 10 \land (x_{8,1} - x_{9,1}) \geq 10 \land (x_{8,1} - x_{9,1}) \leq 10 \land (x_{8,1} - x_{9,1}) \geq 10 \right)$.
Fig. 3. State trajectory for the clusters $\Xi_1$, $\Xi_2$, and $\Xi_3$. Note that the triangles indicate the orientation of each agent.

$$5) \wedge (x_{8,2} - x_{9,2} \leq 10) \wedge (x_{8,2} - x_{9,2} \geq 5),$$

hence satisfying Case A in Problem 1. We remark that all agents have been equipped with a formula $\psi^{\text{Ass}2}$ as mentioned in Remark 3. We select $\eta := 1$ for which it holds that $\rho_1^{\text{opt}} = 23.9$, $\rho_2^{\text{opt}} = \rho_3^{\text{opt}} = \rho_5^{\text{opt}} = 4.92$, $\rho_5^{\text{opt}} = 4.97$, $\rho_6^{\text{opt}} = \rho_7^{\text{opt}} = 4.92$, and $\rho_8^{\text{opt}} = \rho_9^{\text{opt}} = 1.11$ so that we initially choose $\delta_i := 0.5$ for all $i \in \{1, \ldots, M\}$ and $\rho_1^{\text{max}} := 20$, $\rho_2^{\text{max}} := \rho_3^{\text{max}} := \rho_5^{\text{max}} := \rho_6^{\text{max}} := \rho_7^{\text{max}} := 4.9$ and $\rho_8^{\text{max}} := \rho_9^{\text{max}} := 1.1$. For the parameters of the hybrid system, we set $\delta_i := 1.5$ and $N_i := 1$. The resulting trajectory is shown in Fig. 3. For $\Xi_1$, it can be seen that $v_1$ does not satisfy $\phi_1$, but finds a least violating solution by staying as close as possible to $v_2$, $v_3$, and $v_4$. The latter agents independently satisfy their own formulas. For $v_1$ and $v_2$, the corresponding funnels are shown in Fig. 4a and 4b. It can be seen that $v_1$ first tries to repair the parameters in the first repair stage and then successively decreases the robust $\rho_i$ by $\delta_i$ in the third repair stage. For $v_5$, $v_6$, and $v_7$, it can be seen that all agents satisfy their formulas. In particular, $v_5$ uses collaborative control together with $v_6$ and $v_7$ in the second repair stage after an unsuccessful repair attempt in the first stage. This can be seen in Fig. 4d, while Fig. 4e and 4f show the behavior of the collaborating agents $v_6$ and $v_7$. The third cluster $\Xi_3$ with $v_8$ and $v_9$ satisfies collaboratively their formulas according to Theorem 2. The funnel for $v_8$ can be seen in Fig. 4c. All tasks, except of $\phi_1$, are satisfied with a robustness of $\rho_1 := 0.5$. Our method is hence robust with respect to additive noise and with respect to the formula, where the designer can impose a robustness $\rho_i$. Note also in Fig. 3 that collisions are avoided. A comparison with the methods in Raman et al. (2014) and Pant et al. (2018) was not possible due to their high computational complexity rendering the entailed optimization programs intractable. Note that Raman et al. (2014) and Pant et al. (2018) focus on single-agent systems so that a high-dimensional centralized multi-agent system with 27 states had to be used.

5 Conclusion

A framework for the control of multi-agent systems under local signal temporal logic tasks has been presented. The local tasks may depend on the behavior of other agents and may hence be conflicting. In a first step, we identified conditions under which a local feedback control law guarantees satisfaction of the local tasks if they are satisfiable. For not satisfiable tasks, a least violating solution can be found. If the identified conditions do not hold, we proposed to combine the previously developed local feedback control law with an online detection & repair scheme. This detection & repair scheme is expressed as a local hybrid control system. Critical events are detected and repaired in a three-stage procedure so that cases with locally conflicting formulas can be resolved.

References


First jump induced by $D_{\rho_{max}1}$, who requests collaborative help from $v_6$. (d) Funnel repairs for agent $v_5$, who requests collaborative help from $v_6$ and $v_7$. Agent $v_7$ starts collaborating with $v_6$ at around 2.3 seconds.

Fig. 4. Funnel repairs for the agents $v_1$, $v_2$, $v_5$, $v_6$, and $v_7$.


