Leader-follower Coordinated Tracking of Multiple Heterogeneous Lagrange Systems Using Continuous Control

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***Abstract*—In this paper, we study the coordinated tracking problem of multiple heterogeneous Lagrange systems with a dynamic leader. Only nominal parameters of Lagrange dynamics are assumed to be available. Under the local interaction constraints, *i.e.*, the followers only have access to their neighbors' information and the leader being a neighbor of only a subset of the followers, continuous coordinated tracking algorithms with adaptive coupling gains are proposed. Except for the benefit of the chattering-free control achieved, the proposed algorithm also has the attribute that it does not require the neighbors' generalized coordinate derivatives. Global asymptotic coordinated tracking is guaranteed and the tracking errors between the followers and the leader are shown to converge to zero. Examples are given to validate the effectiveness of the proposed algorithms.**

***Index Terms*—Coordinated tracking, Multiple heterogeneous Lagrange systems, Continuous control algorithms**

I. INTRODUCTION

Coordination of multi-agent systems has been extensively studied for the past two decades due to its broad range of applications. One fundamental problem is coordinated tracking with a time-varying global objective [1], [3]. The goal is to control a group of followers to track a time-varying global objective function (often denoted a leader) by using only local information interactions [23]. The coordinated tracking problem was introduced and studied in [12], where the followers were modeled as single integrators and the input delays were considered. With the emphasis on the delay effect analysis, ref. [21] studied the stability conditions for the leader-follower tracking problem for both single integrator networks and double integrator networks. Recently, the authors of [4] proposed algorithms using variable structure approaches. Both the case of multiple single integrators and that of multiple double integrators were considered and the tracking errors were shown to be zero using the proposed discontinuous control algorithms.

In this paper, instead of modeling the follower dynamics as single or double integrators, we study the coordinated tracking problem of multiple heterogeneous Lagrange systems with a dynamic leader. Here, a Lagrange system is used to represent a mechanical system, such as autonomous vehicles, robotic manipulators, and walking robots [27]. Therefore, the study on the coordination control of multiple Lagrange systems may provide some basic ideas for the applications on the formation control of multiple mobile robots and the coordinated object grabbing of multiple robot manipulators. Existing works on the coordination control of multiple Lagrange systems include [11], [25], [6], [13], [8], [17], [5], [14], [22], [7], [20] with different emphasis. For example, time-varying delays, limited communication rates and non-vanishing bounded disturbances were considered in [11], coordinated tracking with finite-time convergence was studied in [13], and a class of nonlinear function was introduced in [8] to alleviate the chattering issues raised by the discontinuous coordinated tracking algorithm. The influence of communication delays was studied in [14], [22], a flocking behavior was guaranteed in [7], and the containment control with group dispersion and group cohesion behaviors was reconstructed in [20]. In addition, the applications of coordination algorithms of multiple Lagrange systems on the shape and formation control were given in [10], and the application to task-space synchronization of multiple robotic manipulators was given in [15].

In this paper, by focusing on the leader-follower coordinated tracking problem of multiple Lagrange systems, we improve the existing works in three aspects. First, the proposed zero-error coordinated tracking algorithm is distributed, continuous, and guaranteeing zero-error tracking. Note that discontinuous control algorithms were proposed in [13], [17], [20] to ensure zero-error coordinated tracking, the leader is assumed to be available to all the followers in [22], and the tracking errors were shown to be bounded instead of approaching zero in [5] although the proposed algorithms are continuous. Second, in contrast to [17], [18], where the eigenvalues of the interaction Laplacian matrix and the upper bound of states of the bounded time-varying leader are assumed to be available to all the followers, the proposed algorithm in the current paper is purely distributed in the sense that both the control input and coupling gain depend only on local information. Third, the neighbors' generalized coordinate derivative information is not required to be available in the proposed algorithm. Thus, such an approach may provide a solution to the case when the agents are not equipped with the sensors capable of obtaining relative generalized coordinate derivative information (*e.g.,* relative velocity measurements). Moveover, since we do not need the neighbors' generalized coordinate derivative information, the communication capacities may be reduced. This is particularly important when the number of agents is large and the com-

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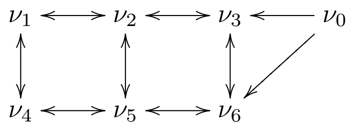


Fig. 1. Information flow associated with the leader and the six followers

munication structure is complex.

The outline of the paper is as follows. In Section II, we formulate the problem of coordinated tracking of multiple Lagrange systems and give some basic notations and definitions. The main results are presented in Section III. Numerical studies are carried out in Section IV to validate the theoretical results and a brief concluding remark is given in Section V.

II. Problem Statement and Preliminaries

A. Problem Statement

Suppose that there are n follower agents in the group, labeled by $\nu_1, \nu_2, \ldots, \nu_n$. The system dynamics of the followers are described by the Lagrange equations

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i, \; i = 1, 2, \ldots, n, \quad (1)$$

where $q_i \in \mathbb{R}^p$ is the vector of generalized coordinates, $M_i(q_i) \in \mathbb{R}^{p\times p}$ is the $p \times p$ inertia (symmetric) matrix, $C_i(q_i, \dot{q}_i)\dot{q}_i$ is the Coriolis and centrifugal terms, $g_i(q_i)$ is the vector of gravitational force, and $\tau_i \in \mathbb{R}^p$ is the control force. The dynamics of a Lagrange system satisfies the following properties [27]:

1. $0 < k_{\underline{M}} I_p \le M_i(q_i) \le k_{\overline{M}} I_p$, $\|C_i(x, y)\| \le k_C \|y\|$ for all vectors $x, y \in \mathbb{R}^p$, and $\|g_i(q_i)\| \le k_g$, where $k_{\underline{M}}$, $k_{\overline{M}}$, k_C, and k_g are positive constants.

2. $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew symmetric.

3. The left-hand side of the dynamics can be parameterized, i.e., $M_i(q_i)y + C_i(q_i, \dot{q}_i)x + g_i(q_i) = Y_i(q_i, \dot{q}_i, x, y)\theta_i$, $\forall x, y \in \mathbb{R}^p$, where $Y_i \in \mathbb{R}^{p\times p_\theta}$ is a regression matrix with a constant parameter vector $\theta_i \in \mathbb{R}^{p_\theta}$.

From Property 3, we know that the nominal dynamics (available dynamics) satisfy

$$\widehat{M}_i(q_i)\ddot{q}_i + \widehat{C}_i(q_i, \dot{q}_i)\dot{q}_i + \widehat{g}_i(q_i) = Y_i(q_i, \dot{q}_i, \dot{q}_i, \ddot{q}_i)\widehat{\theta}_i,$$

where $\widehat{M}_i(q_i)$, $\widehat{C}_i(q_i, \dot{q}_i)$, $\widehat{g}_i(q_i)$, and $\widehat{\theta}_i$ are nominal dynamics terms.

In addition to the n followers, we denote the global information as a leader agent in the group, labeled as agent ν_0 with the desired time-varying generalized coordinate $q_0 \in \mathbb{R}^p$ and the desired time-varying generalized coordinate derivative $\dot{q}_0 \in \mathbb{R}^p$. The objective of this paper is to design *continuous* coordinated tracking algorithms for follower dynamics (1) such that $q_i(t) \to q_0(t)$ and $\dot{q}_i(t) \to \dot{q}_0(t)$ as $t \to \infty$ by using only local interactions, *i.e.*, the leader's states q_0 and $\dot{q}_0$ are only available to a subset of the followers and the followers only have access to their local neighbors' information.

Considering that there are six followers ($n = 6$) in the group, Fig. 1 gives an example of information flow among the leader and six followers. Note that the leader's states are only available to followers ν_3 and ν_6 and the followers only have access to their neighbors' information.

B. Basic Definitions in Graph Theory

We use graphs to represent the communication topology among agents. A directed graph $\mathcal{G}_n$ consists of a pair $(\mathcal{V}_n, \mathcal{E}_n)$, where $\mathcal{V}_n = \{\nu_1, \nu_2, \ldots, \nu_n\}$ is a finite, nonempty set of nodes and $\mathcal{E}_n \subseteq \mathcal{V}_n \times \mathcal{V}_n$ is a set of ordered pairs of nodes. An edge (ν_i, ν_j) denotes that node ν_j has access to the information from node ν_i. An undirected graph is defined such that $(\nu_j, \nu_i) \in \mathcal{E}_n$ implies $(\nu_i, \nu_j) \in \mathcal{E}_n$. A directed path in a directed graph or an undirected path in an undirected graph is a sequence of edges of the form $(\nu_i, \nu_j), (\nu_j, \nu_k), \ldots$. The neighbors of node ν_i are defined as the set $N_i := \{\nu_j | (\nu_j, \nu_i) \in \mathcal{E}_n\}$.

For a follower graph $\mathcal{G}_n$, its adjacency matrix $\mathcal{A}_n = [a_{ij}] \in \mathbb{R}^{n\times n}$ is defined such that a_{ij} is positive if $(\nu_j, \nu_i) \in \mathcal{E}_n$ and $a_{ij} = 0$ otherwise. Here we assume that $a_{ii} = 0$, $\forall i = 1, 2, \ldots, n$ and $a_{ij} = a_{ji}$, $\forall i, j = 1, 2, \ldots, n$. The Laplacian matrix $\mathcal{L}_n = [l_{ij}] \in \mathbb{R}^{n\times n}$ associated with $\mathcal{A}_n$ is defined as $l_{ii} = \sum_{j\ne i} a_{ij}$ and $l_{ij} = -a_{ij}$, where $i \ne j$. For the leader-follower graph $\mathcal{G}_{n+1} := (\mathcal{V}_{n+1}, \mathcal{E}_{n+1})$, the adjacency matrix $\mathcal{A}_{n+1} = [a_{ij}] \in \mathbb{R}^{(n+1)\times(n+1)}$ is defined such that a_{i0} is positive if $(\nu_0, \nu_i) \in \mathcal{E}_{n+1}$ and $a_{i0} = 0$ otherwise, $\forall i = 1, 2, \ldots, n$.

Assumption 1. *The global information q_0 and $\dot{q}_0$ are available to at least one follower,* i.e, $a_{i0} > 0$ *for at least one* i, $i = 1, 2, \ldots, n$. *In addition, the follower graph $\mathcal{G}_n$ is undirected and connected.*

Note that Figure 1 is an example that satisfies Assumption 1. Letting $\mathcal{M} = \mathcal{L}_n + \mathrm{diag}(a_{10}, a_{20}, \ldots, a_{n0})$ ($\mathcal{L}_n$ is the Laplacian matrix associated with $\mathcal{G}_n$), we recall the following result.

Lemma 1. *[12] Under Assumption 1, $\mathcal{M}$ is positive definite (symmetric).*

C. Filippov Solution and Nonsmooth Analysis

Consider the vector differential equation

$$\dot{x} = f(x, t), \quad (2)$$

where $f : \mathbb{R}^p \times \mathbb{R} \to \mathbb{R}^p$ is measurable and essentially locally bounded. A vector function $x(t)$ is called a solution of (2) on $[t_0, t_1]$ if $x(t)$ is absolutely continuous on $[t_0, t_1]$ and for almost all $t \in [t_0, t_1]$, $\dot{x} \in \mathbb{K}[f](x, t)$. Here $\mathbb{K}[f](x, t) = \bigcap_{\delta>0} \bigcap_{\mu\overline{N}=0} \overline{co} f(B(x, \delta) \setminus \overline{N}, t)$, $\bigcap_{\mu\overline{N}=0}$ denotes the intersection over all sets $\overline{N}$ of Lebesgue measure zero, $\overline{co}(X)$ is the convex closure of X, and $B(x, \delta)$ denotes the open ball of radius δ centered at x.

For a locally Lipschitz function $V : \mathbb{R}^p \times \mathbb{R} \to \mathbb{R}$, the generalized gradient of V at (x, t) is defined by $\partial V(x, t) = \overline{co}\{\lim \nabla V(x, t) | (x_i, t_i) \to (x, t), (x_i, t_i) \notin \Omega_V\}$, where Ω_V is the set of measure zero where the gradient of V is not defined. The generalized time derivative of V with respect to (2) is defined as $\dot{\tilde{V}} := \bigcap_{\zeta\in\partial V} \zeta^{\mathrm{T}} \begin{pmatrix} \mathbb{K}[f](x, t) \\ 1 \end{pmatrix}$. In addition, $f(x, t) : \mathbb{R}^p \times \mathbb{R} \to \mathbb{R}$ is called regular if for all ψ, the usual one-sided directional derivative $f'(x; \psi)$ exists, and $f'(x; \psi) = f^o(x; \psi)$, where $f^o(x; \psi) = \lim_{y\to x, t\downarrow 0} \sup \frac{f(y+t\psi)-f(y)}{t}$ [24], [26].

Lemma 2. *[9] Let* (2) *be essentially locally bounded and* $0 \in \mathbb{K}[f](x,t)$ *in a region* $\mathbb{R}^p \times [0,\infty)$*. Furthermore, suppose that* $f(0,t)$ *is uniformly bounded for all* $t \geq 0$*. Let* $V : \mathbb{R}^p \times [0,\infty) \to \mathbb{R}$ *be locally Lipschitz in* t*, and regular such that* $\forall t \geq 0, W_1(x) \leq V(t,x) \leq W_2(x), \dot{\tilde{V}}(x,t) \leq -W(x)$*, where* $W_1(x)$ *and* $W_2(x)$ *are continuous positive definite functions and* $W(x)$ *is a continuous positive semidefinite function. Here* $\dot{\tilde{V}}(x,t) \leq -W(x)$ *means that* $\psi \leq -W, \forall \psi \in \dot{\tilde{V}}$*. Then all Filippov solutions of* (2) *are bounded and satisfy* $W(x(t)) \to 0$*, as* $t \to \infty$*.*

D. Other Notation

Given a vector $x = [x_1, x_1, \ldots, x_n]^{\mathrm{T}}$, we define $\mathrm{sgn}(x) = [\mathrm{sgn}(x_1), \mathrm{sgn}(x_2), \ldots, \mathrm{sgn}(x_n)]^{\mathrm{T}}$, and $|x| = [|x_1|, |x_2|, \ldots, |x_n|]^{\mathrm{T}}$. In addition, $\mathrm{diag}(x)$ denotes the diagonal matrix of a vector x, $\|x\|_1 = \sum_{i=1}^n |x_i|$ denotes 1-norm of a vector x, $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ denote respectively the minimum and maximum eigenvalues of the matrix P, and $P > 0$ and $P \geq 0$ mean that P is positive definite and positive semidefinite, respectively.

III. MAIN RESULT

The objective here is to drive the states of the followers to converge to those of the global objective. Note that the global objective is available to only a portion of the followers and we use nominal parameters of Lagrange dynamics. We also assume that the neighbors' generalized coordinate derivative information is not available. The following *continuous* control algorithm is proposed for each follower,

$$\tau_i = Y_i(q_i, \dot{q}_i, \dot{q}_{ri}, \dot{\widehat{v}}_i)\widehat{\theta}_i - \alpha_i(t) s_i, \quad i = 1, 2, \ldots, n, \tag{3}$$

where Y_i is defined in Sections II-A and $\dot{\alpha}_i = \overline{\alpha}_i s_i^{\mathrm{T}} s_i$, with $\overline{\alpha}_i > 0$, $i = 1, 2, \ldots n$, being an arbitrary positive constant. The sliding surface and the adaptive control term are designed by

$$s_i = \dot{q}_i - \dot{q}_{ri}, \tag{4}$$

$$\dot{\widehat{\theta}}_i = -\kappa_i Y_i^{\mathrm{T}}(q_i, \dot{q}_i, \dot{q}_{ri}, \dot{\widehat{v}}_i) s_i. \tag{5}$$

where $\kappa_i > 0$, $i = 1, 2, \ldots n$, is an arbitrary positive constant, and, motivated by [17], [18], the virtual reference trajectory $\dot{q}_{ri}$ and the leader's generalized coordinate derivative estimator $\widehat{v}_i$ are proposed, respectively, as

$$\dot{q}_{ri} = \widehat{v}_i - \left(\sum_{j=1}^n a_{ij}(q_i - q_j) + a_{i0}(q_i - q_0) \right), \tag{6}$$

$$\begin{aligned} \dot{\widehat{v}}_i(t) = &- 2\widehat{v}_i(t) - \int_0^t \left(k_{2i}(\tau) \sum_{j=0}^n a_{ij}(\widehat{v}_i(\tau) - \widehat{v}_j(\tau)) \right. \\ &\left. + \beta_i(\tau) \mathrm{sgn}\left(\sum_{j=0}^n a_{ij}(\widehat{v}_i(\tau) - \widehat{v}_j(\tau)) \right) \right) \mathrm{d}\tau, \end{aligned} \tag{7}$$

where $\widehat{v}_0(t) = \dot{q}_0(t)$, a_{ij}, $i, j = 1, 2, \ldots n$, is the (i,j)th entry of $\mathcal{A}_n$ associated with $\mathcal{G}_n$ defined in Section II-B, $a_{i0} > 0$ if the follower i has access to the global information ν_0 and $a_{i0} = 0$ otherwise,

$$\begin{aligned} k_{2i}(t) = &\frac{1}{2}\overline{k}_{2i} \left(\sum_{j=0}^n a_{ij}(\widehat{v}_i(t) - \widehat{v}_j(t)) \right)^{\mathrm{T}} \left(\sum_{j=0}^n a_{ij}(\widehat{v}_i(t) - \widehat{v}_j(t)) \right) \\ &+ \overline{k}_{2i} \int_0^t \left(\sum_{j=0}^n a_{ij}(\widehat{v}_i(\tau) - \widehat{v}_j(\tau)) \right)^{\mathrm{T}} \\ &\times \left(\sum_{j=0}^n a_{ij}(\widehat{v}_i(\tau) - \widehat{v}_j(\tau)) \right) \mathrm{d}\tau, \end{aligned} \tag{8}$$

and

$$\begin{aligned} \beta_i(t) = &\overline{\beta}_i \left\| \sum_{j=0}^n a_{ij}(\widehat{v}_i(t) - \widehat{v}_j(t)) \right\|_1 \\ &+ \overline{\beta}_i \int_0^t \left\| \sum_{j=0}^n a_{ij}(\widehat{v}_i(\tau) - \widehat{v}_j(\tau)) \right\|_1 \mathrm{d}\tau, \end{aligned} \tag{9}$$

with $\overline{k}_{2i} > 0$ and $\overline{\beta}_i > 0$, $i = 1, 2, \ldots n$, being arbitrary positive constants.

Before moving on, we need the following assumption and lemmas.

Assumption 2. *$\dot{q}_0$ is bounded up to its third derivative.*

Note that the assumption on that $\dot{q}_0$, $\ddot{q}_0$ are bounded is a necessary assumption to ensure zero-error tracking of generalized coordinates and generalized coordinate derivatives for the adaptive case. The assumption on $\dddot{q}_0$, $\ddddot{q}_0$ being bounded is necessary to ensure the convergence for the leader's generalized coordinate derivative estimator. Also note that in contrast to [13], [17], [18], [20], the upper bound on any derivative of q_0 is not assumed to be available in the design of the controllers. Generally speaking, Assumption 2 is a mild assumption.

Lemma 3. *[2] Let S be a symmetric matrix partitioned as* $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^{\mathrm{T}} & S_{22} \end{bmatrix}$*, where* S_{22} *is square and nonsingular. Then* $S > 0$ *if and only if* $S_{22} > 0$ *and* $S_{11} - S_{12} S_{22}^{-1} S_{12}^{\mathrm{T}} > 0$*.*

Lemma 4. *[28] Define* $\xi(t) \in \mathbb{R}$ *as* $\xi = (\mu + \dot{\mu})^{\mathrm{T}}(-\overline{\beta}\mathrm{sgn}(\mu) + N_d)$*, where* $\mu(t) \in \mathbb{R}^p$*,* $\overline{\beta}$ *is a positive constant, and* $N_d(t) \in \mathbb{R}^p$ *is a bounded disturbance. Then we have that* $\int_0^t \xi(\tau)\mathrm{d}\tau \leq \mathcal{B}$*, if* $\overline{\beta} > \sup_t\{\|N_d(t)\|_\infty + \|\dot{N}_d(t)\|_\infty\}$*, where* $\mathcal{B} = \overline{\beta}\|\mu(0)\|_1 - \mu^{\mathrm{T}}(0) N_d(0) > 0$*.*

Theorem 1. *Let Assumptions 1 and 2 hold. Under the local continuous coordinated tracking algorithm* (3)-(9)*, the states of the followers governed by the Lagrange dynamics* (1) *globally asymptotically converge to those of the leader,* i.e., $\lim_{t\to\infty}(q_i(t) - q_0(t)) = 0$ *and* $\lim_{t\to\infty}(\dot{q}_i(t) - \dot{q}_0(t)) = 0$, $\forall i = 1, 2, \ldots, n$.

Proof: It follows from Property 3 of Lagrange dynamics in Section II-A that $M_i(q_i)\ddot{q}_{ri} + C_i(q_i, \dot{q}_i)\dot{q}_{ri} + g_i(q_i) = Y_i(q_i, \dot{q}_i, \dot{q}_{ri}, \dot{\widehat{v}}_i)\theta_i - M_i(q_i)\sum_{j=0}^n a_{ij}(\dot{q}_i - \dot{q}_j)$. We then further have that $M_i(q_i)\dot{s}_i + C_i(q_i,\dot{q}_i)s_i = Y_i(q_i,\dot{q}_i,\dot{q}_{ri},\dot{\widehat{v}}_i)\triangle\theta_i + M_i(q_i)\sum_{j=0}^n a_{ij}(\dot{q}_i-\dot{q}_j) - \alpha_i s_i$, where $\triangle\theta_i = \widehat{\theta}_i - \theta_i$.

It also follows from (7) that

$$\ddot{\overline{v}}_i = -2\dot{\overline{v}}_i - k_{2i}\left(\sum_{j=1}^n a_{ij}(\overline{v}_i-\overline{v}_j) + a_{i0}\overline{v}_i\right) - \beta_i \mathrm{sgn}\left(\sum_{j=1}^n a_{ij}(\overline{v}_i-\overline{v}_j)+a_{i0}\overline{v}_i\right) + N_{di},$$

where $\overline{v}_i = \widehat{v}_i - \dot{q}_0$, $N_{di} = -2\ddot{q}_0 - \dddot{q}_0$, for all $i=1,2,\ldots,n$. We then have that for $i=1,2,\ldots,n$,

$$\ddot{\overline{v}}_i(t) = -2\dot{\overline{v}}_i(t) - k_{2i}(t)\left(\sum_{j=1}^n m_{ij}\overline{v}_j(t)\right) - \beta_i(t)\mathrm{sgn}\left(\sum_{j=1}^n m_{ij}\overline{v}_j(t)\right) + N_{di}(t), \tag{10}$$

where m_{ij} denotes the (i,j)th entry of $\mathcal{M}$ defined after Assumption 1. Note that the right-hand side of (10) is discontinuous. Because the signum function sgn is measurable and essentially locally bounded, we can rewrite (10) in terms of differential inclusions as

$$\ddot{\overline{v}}_i \in^{a.e.} \mathbb{K}\left[-2\dot{\overline{v}}_i - k_{2i}\left(\sum_{j=1}^n m_{ij}\overline{v}_j\right) - \beta_i\mathrm{sgn}\left(\sum_{j=1}^n m_{ij}\overline{v}_j\right) + N_{di}\right], \tag{11}$$

where *a.e.,* stands for "almost everywhere" and $\mathbb{K}$ is defined in Section II-C. Define $\eta_i = \overline{v}_i + \dot{\overline{v}}_i$. It also follows from (8) and (9) that for $i=1,2,\ldots,n$,

$$\dot{k}_{2i} = \overline{k}_{2i}\left(\sum_{j=1}^n m_{ij}\overline{v}_j\right)^{\mathrm{T}}\left(\sum_{j=1}^n m_{ij}\eta_j\right), \tag{12}$$

and from the fact that the signum function sgn is measurable and locally essentially bounded

$$\dot{\beta}_i \in^{a.e.} \mathbb{K}\left[\overline{\beta}_i\left(\sum_{j=1}^n m_{ij}\eta_j\right)^{\mathrm{T}}\mathrm{sgn}\left(\sum_{j=1}^n m_{ij}\overline{v}_j\right)\right]. \tag{13}$$

We then construct a Lyapunov function candidate as,

$$V = V_0 + \frac{1}{2}\sum_{i=1}^n s_i^T M_i(q_i)s_i + \sum_{i=1}^n \frac{1}{2\kappa_i}(\triangle\theta_i)^{\mathrm{T}}\triangle\theta_i + \frac{1}{2}\overline{q}^{\mathrm{T}}\overline{q} + \frac{1}{2}\eta^{\mathrm{T}}(\mathcal{M}\otimes I_p)\eta + \frac{1}{2}\overline{k}\overline{v}^{\mathrm{T}}(\mathcal{M}^2\otimes I_p)\overline{v} + \sum_{i=1}^n \frac{1}{2\overline{k}_{2i}} \times (k_{2i}-\overline{k})^2 + \sum_{i=1}^n \frac{1}{2\overline{\beta}_i}(\beta_i-\overline{\beta})^2 + \sum_{i=1}^n\frac{1}{2\overline{\alpha}_i}(\alpha_i-\overline{\alpha})^2,$$

where

$$V_0 = \sum_{i=1}^n \mathcal{B}_i - \sum_{i=1}^n\int_0^t\left(\sum_{j=1}^n m_{ij}\eta_j(\tau)\right)^{\mathrm{T}} \times\left(-\overline{\beta}\mathrm{sgn}\left(\sum_{j=0}^n a_{ij}(\widehat{v}_i(\tau)-\widehat{v}_j(\tau))\right) + N_{di}(\tau)\right)\mathrm{d}\tau,$$

$\eta = [\eta_1^{\mathrm{T}},\eta_2^{\mathrm{T}},\ldots,\eta_n^{\mathrm{T}}]$, $\overline{v} = [\overline{v}_1^{\mathrm{T}},\overline{v}_2^{\mathrm{T}},\ldots,\overline{v}_n^{\mathrm{T}}]$, $\overline{q}_i = q_i - q_0$, $\overline{q} = [\overline{q}_1^{\mathrm{T}},\overline{q}_2^{\mathrm{T}},\ldots,\overline{q}_n^{\mathrm{T}}]^{\mathrm{T}}$, $\mathcal{B}_i = \overline{\beta}\|\sum_{j=1}^n m_{ij}\overline{v}_j(0)\|_1 - \left(\sum_{j=1}^n m_{ij}\overline{v}_j(0)\right)^{\mathrm{T}} N_{di}(0)$. In addition, we select $\overline{\beta}$ and $\overline{k}$ as two positive constants satisfying that $\overline{\beta} > \sup_t\{2\|\ddot{q}_0(t)\|_\infty + 3\|\dddot{q}_0(t)\|_\infty + \|\ddddot{q}_0(t)\|_\infty\}$ and $\overline{k} > \overline{b} + \frac{1}{4\lambda_{\min}(\mathcal{M})}$ and $\overline{b} > \frac{1}{4\lambda^3_{\min}(\mathcal{M})}$. Also, $\overline{\alpha}$ is a constant to be determined later. It follows from Lemma 4 that $V_0 > 0$ when $\overline{\beta} > \sup_t\{2\|\ddot{q}_0(t)\|_\infty + 3\|\dddot{q}_0(t)\|_\infty + \|\ddddot{q}_0(t)\|_\infty\}$. It follows that the generalized time derivative of V (see the definition of $\dot{\tilde{V}}$ in Section II-C) can be evaluated as

$$\begin{aligned}
\dot{\tilde{V}} = & \bigcap_{\xi\in\partial\|\mu\|_1} -((\mathcal{M}\otimes I_p)\eta)^{\mathrm{T}}\left(-\overline{\beta}\xi + N_d\right) \\
& + \mathbb{K}\Bigg[\sum_{i=1}^n s_i^{\mathrm{T}}\Bigg(Y_i(q_i,\dot{q}_i,\dot{q}_{ri},\dot{\widehat{v}}_i)\triangle\theta_i - \alpha_i s_i + M_i(q_i) \\
& \times\sum_{j=0}^n a_{ij}(\dot{q}_i-\dot{q}_j)\Bigg) - \sum_{i=1}^n(\triangle\theta_i)^{\mathrm{T}}Y_i^{\mathrm{T}}(q_i,\dot{q}_i,\dot{q}_{ri},\dot{\widehat{v}}_i)s_i \\
& + \sum_{i=1}^n\left(\sum_{j=1}^n m_{ij}\eta_j\right)^{\mathrm{T}}\Bigg(-k_{2i}\sum_{j=1}^n m_{ij}\overline{v}_j - \dot{\overline{v}}_i - \overline{v}_i \\
& + \overline{v}_i + N_{di} - \beta_i\mathrm{sgn}\left(\sum_{j=1}^n m_{ij}\overline{v}_j\right)\Bigg) \\
& + \overline{k}\overline{v}^{\mathrm{T}}(\mathcal{M}^2\otimes I_p)(\eta-\overline{v}) + \sum_{i=1}^n(k_{2i}-\overline{k})\left(\sum_{j=1}^n m_{ij}\overline{v}_j\right)^{\mathrm{T}} \\
& \times\left(\sum_{j=1}^n m_{ij}\eta_j\right) + \sum_{i=1}^n(\beta_i-\overline{\beta})\left(\sum_{j=1}^n m_{ij}\eta_j\right)^{\mathrm{T}} \\
& \times\mathrm{sgn}\left(\sum_{j=1}^n m_{ij}\overline{v}_j\right) + \sum_{i=1}^n(\alpha_i-\overline{\alpha})s_i^{\mathrm{T}}s_i + \overline{q}^{\mathrm{T}}\dot{\overline{q}}\Bigg] \\
= & \bigcap_{\xi\in\partial\|\mu\|_1} -((\mathcal{M}\otimes I_p)\eta)^{\mathrm{T}}\left(-\overline{\beta}\xi + N_d\right) \\
& + ((\mathcal{M}\otimes I_p)\eta)^{\mathrm{T}}\left(-\overline{\beta}\partial\|\mu\|_1 + N_d\right) \\
& + \sum_{i=1}^n s_i^{\mathrm{T}}\left(-\alpha_i s_i + M_i(q_i)\sum_{j=0}^n a_{ij}(\dot{q}_i-\dot{q}_j)\right) \\
& - \eta^{\mathrm{T}}(\mathcal{M}\otimes I_p)\eta + \eta^{\mathrm{T}}(\mathcal{M}\otimes I_p)\overline{v} - \overline{k}\overline{v}^{\mathrm{T}}(\mathcal{M}^2\otimes I_p) \\
& \times\overline{v} + \sum_{i=1}^n(\alpha_i-\overline{\alpha})s_i^{\mathrm{T}}s_i + \overline{q}^{\mathrm{T}}(s - (\mathcal{M}\otimes I_p)\overline{q} + \overline{v}),
\end{aligned}$$

where $N_d = [N_{d1}^{\mathrm{T}}, N_{d2}^{\mathrm{T}}, \ldots, N_{dn}^{\mathrm{T}}]^{\mathrm{T}}$, $\mu = (\mathcal{M} \otimes I_p)\overline{v}$, $\partial|\mu_k| = \begin{cases} \{-1\}, & \mu_k \in \mathbb{R}^- \\ \{1\}, & \mu_k \in \mathbb{R}^+ \\ [-1,1], & \mu_k = 0, \end{cases}$ and μ_k is kth entry of μ. In addition, we have used (11), (12), (13), Property 2 of Lagrange dynamics in Section II-A and the fact that $\mathbb{K}[f] = \{f\}$ if f is continuous [24].

If $\dot{\tilde{V}} \neq \emptyset$, suppose that $\phi \in \dot{\tilde{V}}$. By following a similar analysis as the one given in the example in Section II of [26] and noting that $\bigcap_{\xi_2 \in [-1,1]} [\xi_2 - 1, \xi_2 + 1] = 0$, we know that

$$\begin{aligned}\phi =& \sum_{i=1}^{n} s_i^{\mathrm{T}} \left(-\alpha_i s_i + M_i(q_i) \sum_{j=0}^{n} a_{ij}(\dot{q}_i - \dot{q}_j) \right) \\ & - \eta^{\mathrm{T}}(\mathcal{M} \otimes I_p)\eta + \eta^{\mathrm{T}}(\mathcal{M} \otimes I_p)\overline{v} - \overline{k}\overline{v}^{\mathrm{T}}(\mathcal{M}^2 \otimes I_p) \\ & \times \overline{v} + \sum_{i=1}^{n} (\alpha_i - \overline{\alpha}) s_i^{\mathrm{T}} s_i + \overline{q}^{\mathrm{T}}(s - (\mathcal{M} \otimes I_p)\overline{q} + \overline{v}).\end{aligned}$$

It is clear to see that $\dot{\tilde{V}}$ is a singleton. We then have that

$$\begin{aligned}\dot{\tilde{V}} \leq & -\overline{\alpha} \sum_{i=1}^{n} s_i^{\mathrm{T}} s_i + s^{\mathrm{T}} M(q)(\mathcal{M} \otimes I_p)s \\ & - s^{\mathrm{T}} M(q)(\mathcal{M}^2 \otimes I_p)\overline{q} + s^{\mathrm{T}} M(q)(\mathcal{M} \otimes I_p)\overline{v} \\ & - \overline{q}^{\mathrm{T}}(\mathcal{M} \otimes I_p)\overline{q} + \overline{q}^{\mathrm{T}} s + \overline{q}^{\mathrm{T}} \overline{v} - \overline{b}\overline{v}^{\mathrm{T}}(\mathcal{M}^2 \otimes I_p)\overline{v} \\ & - (\overline{k} - \overline{b})\overline{v}^{\mathrm{T}}(\mathcal{M}^2 \otimes I_p)\overline{v} - \eta^{\mathrm{T}}(\mathcal{M} \otimes I_p)\eta \\ & + \eta^{\mathrm{T}}(\mathcal{M} \otimes I_p)\overline{v},\end{aligned}$$

where $M(q) = \mathrm{diag}(M_1(q_1), M_2(q_2), \ldots, M_n(q_n))$, $\overline{b} > \frac{1}{4\lambda_{\min}^3(\mathcal{M})}$ is a constant and we have used the fact that $\sum_{j=0}^{n} a_{ij}(\dot{q}_i - \dot{q}_j) = \sum_{j=1}^{n} m_{ij}(\dot{q}_j - \dot{q}_0) = \sum_{j=1}^{n} m_{ij}(\dot{q}_j - \widehat{v}_j + \overline{v}_j) = \sum_{j=1}^{n} m_{ij}(s_j - \sum_{j=1}^{n} m_{ij}\overline{q}_j + \overline{v}_j)$. It then follows that

$$\begin{aligned}\dot{\tilde{V}} \leq & -\begin{bmatrix} \eta^{\mathrm{T}} & \overline{v}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \mathcal{M} \otimes I_p & -\frac{\mathcal{M} \otimes I_p}{2} \\ -\frac{\mathcal{M} \otimes I_p}{2} & (\overline{k} - \overline{b})\mathcal{M}^2 \otimes I_p \end{bmatrix} \begin{bmatrix} \eta \\ \overline{v} \end{bmatrix} \\ & - \begin{bmatrix} s^{\mathrm{T}} & \overline{q}^{\mathrm{T}} & \overline{v}^{\mathrm{T}} \end{bmatrix} \Omega \begin{bmatrix} s \\ \overline{q} \\ \overline{v} \end{bmatrix} \\ \triangleq & -W(\eta, \overline{q}, \overline{v}, s),\end{aligned}$$

where $\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} \\ \Omega_{12}^{\mathrm{T}} & \mathcal{M} \otimes I_p & -\frac{1}{2} I_{pn} \\ \Omega_{13}^{\mathrm{T}} & -\frac{1}{2} I_{pn} & \overline{b}\mathcal{M}^2 \otimes I_p \end{bmatrix}$, $\Omega_{11} = \overline{\alpha} I_{pn} - \frac{1}{2}\left(M(q)(\mathcal{M} \otimes I_p) + (\mathcal{M} \otimes I_p)M(q)\right)$, $\Omega_{12} = \frac{M(q)(\mathcal{M}^2 \otimes I_p)}{2} - \frac{I_{pn}}{2}$ and $\Omega_{13} = -\frac{M(q)(\mathcal{M} \otimes I_p)}{2}$. Note that $\overline{b} > \frac{1}{4\lambda_{\min}^3(\mathcal{M})}$ guarantees that $\begin{bmatrix} \mathcal{M} \otimes I_p & -\frac{1}{2} I_{pn} \\ -\frac{1}{2} I_{pn} & \overline{b}\mathcal{M}^2 \otimes I_p \end{bmatrix}$ is positive definite. Then it follows that Ω is positive definite from Lemma 3 when $\overline{\alpha}$ is chosen large enough satisfying $\overline{\alpha} > k_{\overline{M}}\overline{\lambda} + \frac{(1 + k_{\overline{M}}\overline{\lambda}^2)^2 \overline{b}\underline{\lambda}^2 + k_{\overline{M}}\overline{\lambda}(1 + k_{\overline{M}}\overline{\lambda}^2) + (k_{\overline{M}}\overline{\lambda})^2 \underline{\lambda}}{4\overline{b}\underline{\lambda}^3 - 1}$, where $\overline{\lambda}$ and $\underline{\lambda}$ denote, respectively, $\lambda_{\max}(\mathcal{M})$ and $\lambda_{\min}(\mathcal{M})$. Therefore, $W(\eta, \overline{q}, \overline{v}, s) \geq 0$ when $\overline{k} > \overline{b} + \frac{1}{4\lambda_{\min}(\mathcal{M})}$. It follows that $\int_0^t W(\eta(\tau), \overline{q}(\tau), \overline{v}(\tau), s(\tau))\mathrm{d}\tau$ is bounded. Thus, we know that V is bounded and therefore s_i, $\triangle\theta_i$, $\forall i = 1, 2, \ldots, n$, $\overline{v}$, $\dot{\overline{v}}$, η, and $\overline{q}$ are bounded. It then follows that $\dot{q}_{ri}$, $\forall i = 1, 2, \ldots, n$ are bounded from (6) and the facts that $\overline{v}_i$, $\forall i = 1, 2, \ldots, n$, $\dot{q}_0$, $\overline{q}$ are bounded and $\mathcal{M}$ is positive definite. This in turn shows that $\dot{q}_i$, $\forall i = 1, 2, \ldots, n$, are bounded from (4). This further implies that $\ddot{q}_{ri}$, $\forall i = 1, 2, \ldots, n$, are bounded since $\dot{\widehat{v}}_i$, $\forall i = 1, 2, \ldots, n$, and $\ddot{q}_0$ are bounded. Also, based on the first property of Lagrange dynamics given in Section II-A and the relationship of $M_i(q_i)\ddot{q}_{ri} + C_i(q_i, \dot{q}_i)\dot{q}_{ri} + g_i(q_i) = Y_i(q_i, \dot{q}_i, \dot{q}_{ri}, \dot{\widehat{v}}_i)\theta_i - M_i(q_i)\sum_{j=0}^{n} a_{ij}(\dot{q}_i - \dot{q}_j)$, $\forall i = 1, 2, \ldots, n$, we know that $Y_i(q_i, \dot{q}_i, \dot{q}_{ri}, \dot{\widehat{v}}_i)$ is bounded, $\forall i = 1, 2, \ldots, n$. It therefore shows that $\dot{s}_i$, $\forall i = 1, 2, \ldots, n$, are bounded. Also, we know that $\dot{\eta}_i$, $\forall i = 1, 2, \ldots, n$, are bounded based on the fact that $\dddot{q}_0$ is bounded and (11). We then know that $s_i(t)$, $\eta_i(t)$, $\overline{q}_i(t)$, and $\overline{v}_i(t)$, $\forall i = 1, 2, \ldots, n$ are uniformly continuous in t. This shows that $W(\eta(t), \overline{q}(t), \overline{v}(t), s(t))$ is uniformly continuous in t. Therefore, it follows from Lemma 2 that $W(\eta(t), \overline{q}(t), \overline{v}(t), s(t)) \to 0$, as $t \to \infty$. This shows that $\eta(t) \to 0$, $\overline{v}(t) \to 0$, $\overline{q}(t) \to 0$ and $s(t) \to 0$, as $t \to \infty$. It follows from (4) and (6) that $\dot{\overline{q}}_i = -\sum_{j=1}^{n} m_{ij}\overline{q}_j + s_i + \overline{v}_i$. We can then easily have that $\lim_{t\to\infty}(q_i(t) - q_0(t)) = 0$ and $\lim_{t\to\infty}(\dot{q}_i(t) - \dot{q}_0(t)) = 0$, $\forall q_i(0) \in \mathbb{R}^p$, $\forall i = 1, 2, \ldots, n$. ∎

Remark 1. *The proposed algorithm possesses the following attributes. First, it is distributed, i.e., the leader's information is available to only a portion of the followers and the followers only have local interactions. This is a rather mild communication topology assumption compared to those in existing works, such as [6], [22], where the leader's information is assumed to be available to all the followers. Second, the proposed algorithm is continuous and the tracking errors are shown to converge to zero even when the leader's generalized coordinate derivative is time-varying. This improves the ultimate boundedness results reported in [5] and avoids the chattering phenomenon in the discontinuous designs of [13], [17], [20]. Third, by introducing an adaptive gain scheduling technique, the coupling gain no longer relies on a certain bound relevant to the global information and the exact value of the upper bound of states of the time-varying leader is not required to be available. Therefore, in contrast to [17], [18], the proposed algorithm is purely distributed in the sense that both the control input and the coupling gains depend only on the local information interactions and is feasible as long as that the leader's generalized coordinate derivative is bounded up to its third derivative. Fourth, the neighbors' generalized coordinate derivative information is not required to be available. This reduces the communication as the relative velocity measurements do not need to be exchanged between neighbors.*

Remark 2. *Coordination algorithms without using neighbors' generalized coordinate derivative information were proposed in [16] for static leader-follower regulation and leaderless synchronization of multiple Lagrange systems. One necessary assumption of [16] is that the target generalized coordinate derivative is constant. In contrast, the proposed algorithm* (3)-(9) *in this paper can be applied to the case when the leader's generalized coordinate derivative is time-varying.*

IV. Simulation Results

In this section, numerical simulation results are given to validate the effectiveness of the theoretical results obtained in this paper. We assume that there exist six followers ($n = 6$) in the group. The system dynamics of the followers are given by the Lagrange dynamics of the two-link manipulators [17], [27],

$$\begin{bmatrix} M_{11,i} & M_{12,i} \\ M_{21,i} & M_{22,i} \end{bmatrix} \begin{bmatrix} \ddot{q}_{ix} \\ \ddot{q}_{iy} \end{bmatrix} + \begin{bmatrix} C_{11,i} & C_{12,i} \\ C_{21,i} & C_{22,i} \end{bmatrix} \begin{bmatrix} \dot{q}_{ix} \\ \dot{q}_{iy} \end{bmatrix} + \begin{bmatrix} g_{1,i} \\ g_{2,i} \end{bmatrix} = \begin{bmatrix} \tau_{ix} \\ \tau_{iy} \end{bmatrix}, i = 1, 2, \ldots, 6,$$

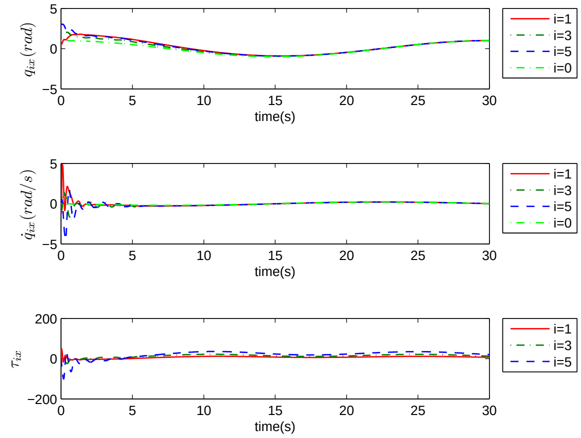
where $M_{11,i} = \theta_{1i} + 2\theta_{2i} \cos q_{iy}$, $M_{12,i} = M_{21,i} = \theta_{3i} + \theta_{2i} \cos q_{iy}$, $M_{22,i} = \theta_{3i}$, $C_{11,i} = -\theta_{2i} \sin q_{iy} \dot{q}_{iy}$, $C_{12,i} = -\theta_{2i} \sin q_{iy}(\dot{q}_{ix} + \dot{q}_{iy})$, $C_{21,i} = \theta_{2i} \sin q_{iy} \dot{q}_{ix}$, $C_{22,i} = 0$, $g_{1,i} = \theta_{4i} g \cos q_{ix} + \theta_{5i} g \cos(q_{ix} + q_{iy})$, $g_{2,i} = \theta_{5i} g \cos(q_{ix} + q_{iy})$ and $g = 9.8$. Also, $\theta_{1i} = m_{1i} l_{c1,i}^2 + m_{2i}(l_{1i}^2 + l_{c2,i}^2) + J_{1i} + J_{2i}$, $\theta_{2i} = m_{2i} l_{1i} l_{c2,i}$, $\theta_{3i} = m_{2i} l_{c2,i}^2 + J_{2i}$, $\theta_{4i} = m_{1i} l_{c1,i} + m_{2i} l_{1i}$, $\theta_{5i} = m_{2i} l_{2i}$. We choose $m_{1i} = 1 + 0.3i$, $m_{2i} = 1.5 + 0.3i$, $l_{li} = 0.2 + 0.06i$, $l_{2i} = 0.3 + 0.06i$, $l_{c1,i} = 0.1 + 0.03i$, $l_{c2,i} = 0.15 + 0.03i$, $J_{1i} = \frac{m_{1i} l_{li}^2}{12}$, $J_{2i} = \frac{m_{2i} l_{2i}^2}{12}$, $i = 1, 2, \ldots, 6$. According to property 3 of Lagrange dynamics given in Section II-A, the dynamics of the followers can be parameterized as $Y_i(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri}) = [y_{pq}]_i \in \mathbb{R}^{2\times 5}$ [27].

The initial states of the followers are given by $q_{ix}(0) = 0.6i$, $q_{iy}(0) = 0.4i - 1$, $\dot{q}_{ix}(0) = 0.05i - 0.2$, $\dot{q}_{iy}(0) = -0.05i + 0.2$, $i = 1, 2, \ldots, 6$. The leader-follower communication topology is given in Fig. 1. The adjacency matrix $\mathcal{A}_n$ of the generalized coordinate derivatives associated with $\mathcal{G}_n$ is chosen to be

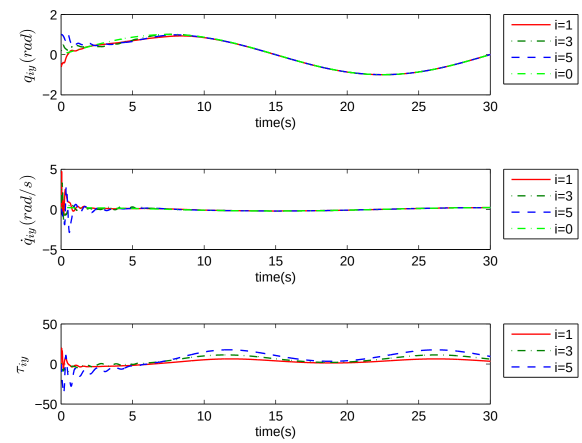
$$\mathcal{A}_n = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix},$$

and $a_{10} = 0$, $a_{20} = 0$, $a_{30} = 1$, $a_{40} = 0$, $a_{50} = 0$, $a_{60} = 1$. The initial estimations for θ_{1i}, θ_{2i}, θ_{3i}, θ_{4i}, and θ_{5i} for each follower $i = 1, 2, \ldots, 6$, are given by $\widehat{\theta}_{1i}(0) = 0$, $\widehat{\theta}_{2i}(0) = 0$, $\widehat{\theta}_{3i}(0) = 0$, $\widehat{\theta}_{4i}(0) = 0$, and $\widehat{\theta}_{5i}(0) = 0$.

For the case of coordinated tracking without using neighbors' generalized coordinate derivative information (algorithm (3)-(9)), the trajectories of the leader are given by $q_{0x}(t) = \cos(\frac{\pi}{15}t)$ and $q_{0y}(t) = \sin(\frac{\pi}{15}t)$. The constant control parameters are chosen by $\kappa_i = 2$, $\alpha_i = 1$, $\overline{k}_{2i} = 0.001$, and $\overline{\beta}_i = 0.1$, $\forall i = 1, 2, \ldots, 6$. The initial states of k_{2i} and β_i for each follower $i = 1, 2, \ldots, 6$ are given by $k_{2i}(0) = 0$ and $\beta_i(0) = 0$. The initial states of $\widehat{v}_i$ for each follower $i = 1, 2, \ldots, 6$ are given by $\widehat{v}_i(0) = \dot{\widehat{v}}_i(0) = [0, 0]^{\mathrm{T}}$ and the initial states of α_i for each follower $i = 1, 2, \ldots, 6$ are given by $\alpha_i(0) = 0$. The control parameters are chosen by $\overline{\alpha}_i = 1$, $\forall i = 1, 2, \ldots, 6$. Under the feedback algorithm (3)-(9), the generalized coordinates, the generalized coordinate derivatives, and the control torques of the followers and the leader are shown in Figs. 2(a) and 2(b). We see that the coordinated tracking is achieved for a group of heterogeneous Lagrange systems without using neighbors' generalized coordinate derivative information.



(a) The trajectories of the states and the control torques of the followers and the leader in x-coordinate



(b) The trajectories of states and the control torques of the followers and the leader in y-coordinate

Fig. 2. The states and the control torques of system (1) under algorithm (3)-(9)

V. Concluding Remarks

In this paper, we study the leader-follower coordinated tracking problem for multiple heterogeneous Lagrange systems. The continuous coordinated tracking algorithms with uncertain parameter adaptive control and the leader's generalized coordinate derivative estimator are proposed. Except for benefit of the chattering-free control, the proposed algorithm also has the attribute that does not require the neighbors' generalized coordinate derivatives. Global asymptotic coordinated tracking is guaranteed and the tracking errors between the followers and the leader are shown to converge to zero. Simulations are given to validate the effectiveness of the proposed *continuous* coordinated tracking algorithms. Further directions include the study of directed communication topology and an arbitrary varying leader for the leader-follower coordinated tracking problems of multiple Lagrange systems.

References

[1] H. Bai, M. Arcak, and J. Wen. *Cooperative control design: a systematic, passivity-based approach*. Communications and Control Engineering. Springer, New York, 2011.

[2] S. P. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan. *Linear matrix inequalities in system and control theory*. SIAM, Philadelphia, 1994.

[3] F. Bullo, J. Cortes, and S. Martinez. *Distributed control of robotic networks*. Princeton University Press, Princeton, 2009.

[4] Y. Cao and W. Ren. Distributed coordinated tracking with reduced interaction via a variable structure approach. *IEEE Transactions on Automatic Control*, 57(1):33–48, 2012.

[5] G. Chen and F. L. Lewis. Distributed adaptive tracking control for synchronization of unknown networked Lagrangian systems. *IEEE Transactions on Systems, Man and Cybernetics-Part B: Cybernetics*, 41(3):805–816, 2011.

[6] S.-J. Chung, U. Ahsun, and J.-J. E. Slotine. Cooperative robot control and concurrent synchronization of Lagrangian systems. *IEEE Transactions on Robotics*, 25(3):686–700, 2009.

[7] K. D. Do. Flocking for multiple elliptical agents with limited communication ranges. *IEEE Transactions on Robotics*, 27(5):931–942, 2011.

[8] W. Dong. On consensus algorithms of multiple uncertain mechanical systems with a reference trajectory. *Automatica*, 47(9):2023–2028, 2011.

[9] N. Fischer, R. Kamalapurkar, and W. E. Dixon. Lasalle-Yoshizawa corollaries for nonsmooth systems. *IEEE Transactions on Automatic Control*, 58(9):2333–2338, 2013.

[10] R. Haghighi and C. C. Cheah. Multi-group coordination control for robot swarms. *Automatica*, 48(10):2526–2534, 2012.

[11] P. F. Hokayem, D. M. Stipanovic, and M. W. Spong. Semiautonomous control of multiple networked Lagrangian systems. *International Journal of Robust and Nonlinear control*, 19(18):2040–2055, 2009.

[12] J. Hu and Y. Hong. Leader-following coordination of multi-agent systems with coupling time delays. *Physica A: Statistical Mechanics and its Applications*, 374(2):853–863, 2007.

[13] S. Khoo, L. Xie, and Z. Man. Robust finite-time consensus tracking algorithm for multirobot systems. *IEEE/ASME Transactions on Mechatronics*, 14(2):219–228, 2009.

[14] D. Lee and M. W. Spong. Passive bilateral teleoperation with constant time delay. *IEEE Transactions on Robotics*, 22(2):269–281, 2006.

[15] Y. Liu and N. Chopra. Controlled synchronization of heterogeneous robotic manipulators in the task space. *IEEE Transactions on Robotics*, 28(1):268–275, 2012.

[16] J. Mei, W. Ren, J. Chen, and G. Ma. Distributed adaptive coordination for multiple Lagrangian systems under a directed graph without using neighbors' velocity information. *Automatica*, 49(6):1723–1731, 2013.

[17] J. Mei, W. Ren, and G. Ma. Distributed coordinated tracking with a dynamic leader for multiple Euler-Lagrange systems. *IEEE Transactions on Automatic Control*, 56(6):1415–1421, 2011.

[18] J. Mei, W. Ren, and G. Ma. Distributed containment control for Lagrangian networks with parametric uncertainties under a directed graph. *Automatica*, 48(4):653–659, 2012.

[19] Z. Meng, Dimos V. Dimarogonas, and K. H. Johansson. Zero-error coordinated tracking of multiple heterogeneous Lagrange systems using continuous control. In *Proceedings of 52nd the Control and Decision Conference*, 2013. To appear.

[20] Z. Meng, Z. Lin, and W. Ren. Leader-follower swarm tracking for networked Lagrange systems. *Systems & Control Letters*, 61(1):117–126, 2012.

[21] Z. Meng, W. Ren, Y. Cao, and Z. You. Leaderless and leader-following consensus with communication and input delays under a directed network topology. *IEEE Transactions on Systems, Man, and Cybernetics – Part B: Cybernetics*, 41(1):75–87, 2011.

[22] E. Nuno, R. Ortega, L. Basanez, and D. Hill. Synchronization of networks of nonidentical Euler-Lagrange systems with uncertain parameters and communication delays. *IEEE Transactions on Automatic Control*, 56(4):935–941, 2011.

[23] R. Olfati-Saber, J. A. Fax, and R. M. Murray. Consensus and cooperation in networked multi-agent systems. *Proceedings of the IEEE*, 95(1):215–233, 2007.

[24] B. E. Paden and S. S. Sastry. A calculus for computing Filippov's differential inclusion with application to the variable structure control of robot manipulators. *IEEE Transactions on Circuits and Systems*, 34(1):73–82, 1987.

[25] W. Ren. Distributed leaderless consensus algorithms for networked Euler-Lagrange systems. *International Journal of Control*, 82(11):2137–2149, 2009.

[26] D. Shevitz and B. E. Paden. Lyapunov stability theory of nonsmooth systems. *IEEE Transactions on Automatic Control*, 39(9):1910–1914, 1994.

[27] M. W. Spong, S. Hutchinson, and M. Vidyasagar. *Robot dynamics and control*. John Wiley & Sons, Inc., 2006.

[28] B. Xian, D. M. Dawson, M. S. de Queiroz, and J. Chen. A continuous asymptotic tracking control strategy for uncertain nonlinear systems. *IEEE Transactions on Automatic Control*, 49(7):1206–1211, 2004.