Robust Distance-Based Formation Control of Multiple Rigid Bodies with Orientation Alignment ⋆

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Abstract: This paper addresses the problem of distance- and orientation-based formation control of a class of second-order nonlinear multi-agent systems in 3D space, under static and undirected communication topologies. More specifically, we design a decentralized model-free control protocol in the sense that each agent uses only local information from its neighbors to calculate its own control signal, without incorporating any knowledge of the model nonlinearities and exogenous disturbances. Moreover, the transient and steady state response is solely determined by certain designer-specified performance functions and is fully decoupled by the agents' dynamic model, the control gain selection, the underlying graph topology as well as the initial conditions. Additionally, by introducing certain inter-agent distance constraints, we guarantee collision avoidance and connectivity maintenance between neighboring agents. Finally, simulation results verify the performance of the proposed controllers.

Keywords: Multi-agent systems, Cooperative systems, Distributed nonlinear control, Nonlinear cooperative control, Robust control.

1. INTRODUCTION

During the last decades, decentralized control of networked multi-agent systems has gained a significant amount of attention due to the great variety of its applications. The main focus of multi-agent systems is the design of distributed control protocols in order to achieve global tasks, such as consensus, network connectivity and collision avoidance.

A particular multi-agent problem that has been considered in the literature is the formation control problem, where the agents represent robots that aim to form a prescribed geometrical shape, specified by a certain set of desired relative configurations between the agents. The main categories of formation control that have been studied in the related literature are ([Oh et al., 2015]) position-based control, displacement-based control, distance-based control and orientation-based control. In distance-based formation control, inter-agent distances are actively controlled to achieve a desired formation, dictated by desired inter-agent distances. Each agent is assumed to be able to sense the relative positions of its neighboring agents, without the need of orientation alignment of the local coordinate systems. When orientation alignment is considered as a control design goal, the problem is known as orientation-based (or bearing-based) formation control. The desired formation is then defined by relative inter-agent orientations.

The literature in distance-based formation control is rich, and is traditionally categorized in single or double integrator agent dynamics and directed or undirected communication topologies (see e.g. [Olfati-Saber and Murray, 2002, Smith et al., 2006, Hendrickx et al., 2007, Anderson et al., 2007, 2008, Dimarogonas and Johansson, 2008, Krick et al., 2009, Dorfler and Francis, 2010, Cao et al., 2011, Park et al., 2012, Belabbas et al., 2012, Oh and Ahn, 2014]) Orientation-based formation control has been addressed in [Eren, 2012, Trinh et al., 2014, Zhao and Zelazo, 2016], whereas the authors in [Trinh et al., 2014, Bishop et al., 2015, Fathian et al., 2016] have considered the combination of distance- and orientation-based formation.

In most of the aforementioned works in formation control, the two-dimensional case with simple dynamics and point-mass agents has been dominantly considered. In real applications, however, the engineering systems have nonlinear second order dynamics and are usually subject to exogenous disturbances and modeling errors. Another important issue concerns the connectivity maintenance, the collision avoidance between the neighboring agents and the transient and steady state response of the closed loop system, which have not been taken into account in the majority of related woks. Thus, taking all the above into consideration, the design of robust distributed control schemes for the multi-agent formation control problem becomes a challenging task.

Motivated by this, we aim to address here the distance-based formation control problem with orientation alignment for a team of rigid bodies operating in 3D space, with unknown second-order nonlinear dynamics and external disturbances. We propose a purely decentralized control protocol that guarantees distance formation, orientation alignment as well as collision avoidance and connectivity maintenance between neighboring agents and in parallel ensures the satisfaction of prescribed transient and

⋆ This work was supported by the H2020 ERC Starting Grand BUCOPHSYS, the Swedish Research Council (VR), the Knut och Alice Wallenberg Foundation and the European Union's Horizon 2020 Research and Innovation Programme under the Grant Agreement No. 644128 (AEROWORKS).

steady state performance. The prescribed performance control framework has been incorporated in multi-agent systems in [Karayiannidis et al., 2012, Bechlioulis and Kyriakopoulos, 2014], where first order dynamics have been considered. Furthermore, the first one only addresses the consensus problem, whereas the latter solves the position based formation control problem, instead of the distance- and orientation-based problem treated here. Due to space constraints, a more detailed version of this paper that contains the extended derivations of the proofs and omitted calculations, can be found in [Nikou et al., 2016].

The remainder of the paper is structured as follows. In Section 2 notation and preliminary background is given. Section 3 provides the system dynamics and the formal problem statement. Section 4 discusses the technical details of the solution and Section 5 is devoted to a simulation example. Finally, the conclusion and future work directions are discussed in Section 6.

2. NOTATION AND PRELIMINARIES

2.1 Notation

The set of positive integers is denoted as $\mathbb{N}$. The real n-coordinate space, with $n \in \mathbb{N}$, is denoted as $\mathbb{R}^n$; $\mathbb{R}^n_{\geq 0}$ and $\mathbb{R}^n_{>0}$ are the sets of real n-vectors with all elements nonnegative and positive, respectively. Given a set S, we denote as $|S|$ its cardinality. The notation $\|x\|$ is used for the Euclidean norm of a vector $x \in \mathbb{R}^n$. Given a symmetric matrix $A, \lambda_{\min}(A) = \min\{|\lambda| : \lambda \in \sigma(A)\}$ denotes the minimum eigenvalue of A, where $\sigma(A)$ is the set of all the eigenvalues of A and $rank(A)$ is its rank; $A \otimes B$ denotes the Kronecker product of matrices $A, B \in \mathbb{R}^{m\times n}$, as was introduced in [Horn and Johnson, 2012]. Define by $I_n \in \mathbb{R}^{n\times n}, 0_{m\times n} \in \mathbb{R}^{m\times n}$ the unit matrix and the $m \times n$ matrix with all entries zero, respectively; $\mathcal{B}_c(r) = \{x \in \mathbb{R}^3 : \|x - c\| \leq r\}$ is the 3D sphere of radius $r \geq 0$ and center $c \in \mathbb{R}^3$. The vector connecting the origins of coordinate frames $\{A\}$ and $\{B\}$ expressed in frame $\{C\}$ coordinates in 3D space is denoted as $p^C_{B/A} \in \mathbb{R}^3$. Given $a \in \mathbb{R}^3$, $S(a)$ is the skew-symmetric matrix defined according to $S(a)b = a \times b$. We further denote as $q_{B/A} \in \mathbb{T}^3$ the Euler angles representing the orientation of frame $\{B\}$ with respect to frame $\{A\}$, where $\mathbb{T}^3$ is the 3D torus. The angular velocity of frame $\{B\}$ with respect to $\{A\}$, expressed in frame $\{C\}$ coordinates, is denoted as $\omega^C_{B/A} \in \mathbb{R}^3$. We also use the notation $\mathbb{M} = \mathbb{R}^3 \times \mathbb{T}^3$. For notational brevity, when a coordinate frame corresponds to an inertial frame of reference $\{0\}$, we will omit its explicit notation (e.g., $p_B = p^0_{B/0}, \omega_B = \omega^0_{B/0}$ etc.). All vector and matrix differentiations are derived with respect to an inertial frame $\{0\}$, unless otherwise stated.

2.2 Dynamical Systems

Consider the initial value problem:

$$\dot{\psi} = H(t, \psi), \psi(0) = \psi^0 \in \Omega_\psi, \tag{1}$$

with $H : \mathbb{R}_{\geq 0} \times \Omega_\psi \to \mathbb{R}^n$, where $\Omega_\psi \subseteq \mathbb{R}^n$ is a non-empty open set.

Definition 1. ([Sontag, 2013]) A solution $\psi(t)$ of the initial value problem (1) is maximal if it has no proper right extension that is also a solution of (1).

Theorem 1. ([Sontag, 2013]) Consider the initial value problem (1). Assume that $H(t, \psi)$ is: a) locally Lipschitz in ψ for almost all $t \in \mathbb{R}_{\geq 0}$, b) piecewise continuous in t for each fixed $\psi \in \Omega_\psi$ and c) locally integrable in t for each fixed $\psi \in \Omega_\psi$. Then, there exists a maximal solution $\psi(t)$ of (1) on the time interval $[0, \tau_{\max})$, with $\tau_{\max} \in \mathbb{R}_{>0}$ such that $\psi(t) \in \Omega_\psi, \forall t \in [0, \tau_{\max})$.

Proposition 1. ([Sontag, 2013]) Assume that the hypotheses of Theorem 1 hold. For a maximal solution $\psi(t)$ on $[0, \tau_{\max})$ with $\tau_{\max} < \infty$ and for any compact set $\Omega'_\psi \subseteq \Omega_\psi$, there exists a $t' \in [0, \tau_{\max})$ such that $\psi(t') \notin \Omega'_\psi$.

2.3 Graph Theory

An *undirected graph* $\mathcal{G}$ is a pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is a finite set of nodes, representing a team of agents, and $\mathcal{E} \subseteq \{\{i, j\} : i, j \in \mathcal{V}, i \neq j\}$, with $M = |\mathcal{E}|$, is the set of edges that model the communication capability between neighboring agents. For each agent, its neighbors' set $\mathcal{N}_i$ is defined as $\mathcal{N}_i = \{j_1, \ldots, j_{|\mathcal{N}_i|}\} = \{j \in \mathcal{V} \text{ s.t. } \{i, j\} \in \mathcal{E}\}$.

If there is an edge $\{i, j\} \in \mathcal{E}$, then i, j are called *adjacent*. A *path* of length r from vertex i to vertex j is a sequence of $r + 1$ distinct vertices, starting from i and ending to j, such that consecutive vertices are adjacent. For $i = j$, the path is called a cycle. If there is a path between any two vertices of the graph $\mathcal{G}$, then $\mathcal{G}$ is called *connected*. A connected graph is called a tree if it contains no cycles.

The *adjacency matrix* $A(\mathcal{G}) = [a_{ij}] \in \mathbb{R}^{N\times N}$ of a graph $\mathcal{G}$ is defined by $a_{ij} = a_{ji} = 1$, if $\{i, j\} \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. The *degree* $d(i)$ of vertex i is defined as the number of its neighboring vertices, i.e. $d(i) = |\mathcal{N}_i|, i \in \mathcal{V}$. Let also $\Delta(\mathcal{G}) = \text{diag}\{[d(i)]_{i\in\mathcal{V}}\} \in \mathbb{R}^{N\times N}$ be the *degree matrix* of the system. Consider an arbitrary orientation of $\mathcal{G}$, which assigns to each edge $\{i, j\} \in \mathcal{E}$ precisely one of the ordered pairs (i, j) or (j, i). When selecting the pair (i, j), we say that i is the tail and j is the head of the edge $\{i, j\}$. By considering a numbering $k \in \mathcal{M} = \{1, ..., M\}$ of the graph's edge set, we define the $N \times M$ *incidence matrix* $D(G)$ as it was given in [Mesbahi and Egerstedt, 2010]. The *Laplacian matrix* $L(\mathcal{G}) \in \mathbb{R}^{N\times N}$ of the graph $\mathcal{G}$ is defined as $L(\mathcal{G}) = \Delta(\mathcal{G}) - A(\mathcal{G}) = D(\mathcal{G})D(\mathcal{G})^\tau$.

Lemma 1. [Dimarogonas and Johansson, 2008, Section III] Assume that the graph $\mathcal{G}$ is a tree. Then, $D^\tau(\mathcal{G})D(\mathcal{G})$ is positive definite.

3. PROBLEM FORMULATION

Consider a set of N rigid bodies, with $\mathcal{V} = \{1, 2, \ldots, N\}$, $N \geq 2$, operating in a workspace $W \subseteq \mathbb{R}^3$, with coordinate frames $\{i\}, i \in \mathcal{V}$, attached to their centers of mass. Each agent occupies a sphere $\mathcal{B}_{r_i}(p_i(t))$, where $p_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^3$ is the position of the agent's center of mass and r_i is the agent's radius. We also denote as $q_i : \mathbb{R}_{\geq 0} \to \mathbb{T}^3, i \in \mathcal{V}$, the Euler angles representing the agents' orientation with respect to an inertial frame $\{0\}$, with $q_i = [\phi_i, \theta_i, \psi_i]^\tau$. By defining $x_i : \mathbb{R}_{\geq 0} \to \mathbb{M}, v_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^6$, with $x_i = [p_i^\tau, q_i^\tau]^\tau, v_i = [\dot{p}_i^\tau, \omega_i^\tau]^\tau$, we model each agent's motion with the 2nd order dynamics:

$$\dot{x}_i(t) = J_i(x_i)v_i(t), \tag{2a}$$

$$M_i(x_i)\dot{v}_i(t) + C_i(x_i, \dot{x}_i)v_i(t) + g_i(x_i) + w_i(x_i, \dot{x}_i, t) = u_i, \tag{2b}$$

where $J_i : \mathbb{M} \to \mathbb{R}^{6\times 6}$ is a Jacobian matrix given by $J_i(x_i) = \text{diag}\{I_3, J_q(x_i)\}$, with

$$J_q(x_i) = \begin{bmatrix} 1 & \sin(\phi_i)\tan(\theta_i) & \cos(\phi_i)\tan(\theta_i) \\ 0 & \cos(\phi_i) & -\sin(\phi_i) \\ 0 & \dfrac{\sin(\phi_i)}{\cos(\theta_i)} & \dfrac{\cos(\phi_i)}{\cos(\theta_i)} \end{bmatrix}, \quad (3)$$

for which we make the following assumption:

Assumption 1. The angle θ_i satisfies the inequality $-\frac{\pi}{2} < \theta_i(t) < \frac{\pi}{2}, \forall i \in \mathcal{V}, t \in \mathbb{R}_{\geq 0}$.

The aforementioned assumption guarantees that J_i is always well-defined and invertible, since $\det(J_i) = \frac{1}{\cos\theta_i}$. Furthermore, $M_i : \mathbb{M} \to \mathbb{R}^{6\times 6}$ is the positive definite inertia matrix, $C_i : \mathbb{M}\times\mathbb{R}^6 \to \mathbb{R}^{6\times 6}$ is the Coriolis matrix, $g_i : \mathbb{M} \to \mathbb{R}^6$ is the gravity vector, and $w_i : \mathbb{M}\times\mathbb{R}^6\times\mathbb{R}_{\geq 0} \to \mathbb{R}^6$ is a bounded vector representing model uncertainties and external disturbances. We consider that the aforementioned vector fields are unknown and continuous. Finally, $u_i \in \mathbb{R}^6$ is the control input vector representing the 6D generalized force acting on the agent.

The dynamics (2) can be written in vector form as:

$$\dot{x}(t) = J(x)v(t), \quad (4a)$$

$$\bar{M}(x)\dot{v}(t) + \bar{C}(x,\dot{x})v(t) + \bar{g}(x) + \bar{w}(x,\dot{x},t) = u, \quad (4b)$$

where $x = [x_1^\tau, \ldots, x_N^\tau]^\tau : \mathbb{R}_{\geq 0} \to \mathbb{M}^N, v = [v_1^\tau, \ldots, v_N^\tau]^\tau : \mathbb{R}_{\geq 0} \to \mathbb{R}^{6N}, u = [u_1^\tau, \ldots, u_N^\tau]^\tau \in \mathbb{R}^{6N}$, and $J = \mathrm{diag}\{[J_i]_{i\in\mathcal{V}}\} \in \mathbb{R}^{6N\times 6N}, \bar{M} = \mathrm{diag}\{[M_i]_{i\in\mathcal{V}}\} \in \mathbb{R}^{6N\times 6N}$, $\bar{C} = \mathrm{diag}\{[C_i]_{i\in\mathcal{V}}\} \in \mathbb{R}^{6N\times 6N}, \bar{g} = [g_1^\tau, \ldots, g_N^\tau]^\tau \in \mathbb{R}^{6N}, \bar{w} = [w_1^\tau, \ldots, w_N^\tau]^\tau \in \mathbb{R}^{6N}$.

It is also further assumed that each agent can measure its own $p_i, q_i, \dot{p}_i, v_i, i \in \mathcal{V}$, and has a limited sensing range of $s_i > \max\{r_i + r_j : i,j \in \mathcal{V}\}$. Therefore, by defining the neighboring set $\mathcal{N}_i(t) = \{j \in \mathcal{V} : p_j(t) \in \mathcal{B}_{s_i}(p_i(t))\}$, agent i also knows at each time instant t all $p^i_{j/i}(t), q_{j/i}(t)$ and, since it knows its own $p_i(t), q_i(t)$, it can compute all $p_j(t), q_j(t), \forall j \in \mathcal{N}_i(t), t \in \mathbb{R}_{\geq 0}$.

The topology of the multi-agent network is modeled through the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, with $\mathcal{V} = \{1, \ldots, N\}$ and $\mathcal{E} = \{\{i,j\} \in \mathcal{V}\times\mathcal{V} \text{ s.t. } j \in \mathcal{N}_i(0) \text{ and } i \in \mathcal{N}_j(0)\}$. The latter implies that at $t = 0$ the graph is undirected, i.e.,

$$\|p_{\ell_k}(0) - p_{m_k}(0)\| < d_{k,\mathrm{con}}, \forall\{\ell_k, m_k\} \in \mathcal{E}, \quad (5)$$

with $d_{k,\mathrm{con}} = \min\{s_{\ell_k}, s_{m_k}\}, \ell_k, m_k \in \mathcal{V}, \forall k \in \mathcal{M}$. We also consider that $\mathcal{G}$ is static in the sense that no edges are added to the graph. We do not exclude, however, edge removal through connectivity loss between initially neighboring agents, which we guarantee to avoid, as presented in the sequel. It is also assumed that at $t = 0$ the neighboring agents are at a collision-free configuration, i.e., $d_{k,\mathrm{col}} < \|p_{\ell_k}(0) - p_{m_k}(0)\|, \forall\{\ell_k, m_k\} \in \mathcal{E}$, with $d_{k,\mathrm{col}} = r_{\ell_k} + r_{m_k}$. Hence, we conclude that

$$d_{k,\mathrm{col}} < \|p_{\ell_k}(0) - p_{m_k}(0)\| < d_{k,\mathrm{con}}, \forall\{\ell_k, m_k\} \in \mathcal{E}. \quad (6)$$

Moreover, given the desired formation constants $d_{k,\mathrm{des}}$, $q_{k,\mathrm{des}}$ for the edge $k \in \mathcal{M}$, the formation configuration is called *feasible* if the set $\Phi = \{x \in \mathbb{M}^N : \|p_{\ell_k} - p_{m_k}\| = d_{k,\mathrm{des}}, q_{\ell_k} - q_{m_k} = q_{k,\mathrm{des}}, \forall\{\ell_k, m_k\} \in \mathcal{E}\}$, with $\ell_k, m_k \in \mathcal{V}, \forall k \in \mathcal{M}$, is nonempty. Due to the fact that the agents are not dimensionless and their communication capabilities are limited, the control protocol, except from achieving a desired inter-agent formation, should also guarantee for all $t \in \mathbb{R}_{\geq 0}$ that (i) the neighboring agents avoid collision with each other and (iii) all the initial edges are maintained, i.e., connectivity maintenance. Therefore, all pairs $\{\ell_k, m_k\} \in \mathcal{V}\times\mathcal{V}$ of agents that initially form an edge must remain within distance greater than $d_{k,\mathrm{col}}$ and less than $d_{k,\mathrm{con}}$. We also make the following assumptions that are required on the graph topology:

Assumption 2. The graph $\mathcal{G}$ is initially a tree.

Formally, the robust formation control problem under the aforementioned constraints is formulated as follows:

Problem 1. Given N agents governed by the dynamics (2), under the Assumptions 1, 2 and given the desired inter-agent distances and angles $d_{k,\mathrm{des}}, q_{k,\mathrm{des}}$, with $d_{k,\mathrm{col}} < d_{k,\mathrm{des}} < d_{k,\mathrm{con}}, \forall\{\ell_k, m_k\} \in \mathcal{E}, \ell_k, m_k \in \mathcal{V}, \forall k \in \mathcal{M}$, design decentralized control laws $u_i \in \mathbb{R}^6, i \in \mathcal{V}$ such that $\forall\ \{\ell_k, m_k\} \in \mathcal{E}, k \in \mathcal{M}$, the following hold:

(1) $\lim_{t\to\infty} \|p_{\ell_k}(t) - p_{m_k}(t)\| = d_{k,\mathrm{des}}$,
(2) $\lim_{t\to\infty} [q_{m_k}(t) - q_{\ell_k}(t) - q_{k,\mathrm{des}}] = 0_{3\times 1}$,
(3) $d_{k,\mathrm{col}} < \|p_{\ell_k}(t) - p_{m_k}(t)\| < d_{k,\mathrm{con}}, \forall\ t \in \mathbb{R}_{\geq 0}$.

4. PROBLEM SOLUTION

Let $p = [p_1^\tau, \ldots, p_N^\tau]^\tau : \mathbb{R}_{\geq 0} \to \mathbb{R}^{3N}, q = [q_1^\tau, \ldots, q_N^\tau]^\tau : \mathbb{R}_{\geq 0} \to \mathbb{T}^{3N}$ be the stacked vectors of all the agent positions and Euler angles. We further denote as $\tilde{p}, \tilde{q} : \mathbb{R}_{\geq 0} \to \mathbb{R}^{3M}$, with $\tilde{p}(t) = [p^T_{\ell_1,m_1}(t), \ldots, p^T_{\ell_M,m_M}(t)]^T, \tilde{q}(t) = [q^T_{\ell_1,m_1}(t), \ldots, q^T_{\ell_M,m_M}(t)]^T$ and $p_{\ell_k,m_k}(t) = p_{\ell_k}(t) - p_{m_k}(t)$, $q_{\ell_k,m_k}(t) = q_{\ell_k}(t) - q_{m_k}(t)$, $\forall\{\ell_k, m_k\} \in \mathcal{E}$, with the edges ordered as in the case of the incidence matrix $D(\mathcal{G})$. Thus, the following holds:

$$\tilde{p}(t) = (D^\tau(\mathcal{G}) \otimes I_3)\, p(t), \quad (7a)$$

$$\tilde{q}(t) = (D^\tau(\mathcal{G}) \otimes I_3)\, q(t). \quad (7b)$$

Next, let us introduce the errors $e_k^p : \mathbb{R}_{\geq 0} \to \mathbb{R}, e_k^p(t) = \|p_{\ell_k,m_k}(t)\|^2 - d^2_{k,\mathrm{des}}$ and $e_k^q = [e^q_{k_1}, e^q_{k_2}, e^q_{k_3}]^\tau : \mathbb{R}_{\geq 0} \to \mathbb{T}^3, e_k^q(t) = q_{m_k}(t) - q_{\ell_k}(t) - q_{k,\mathrm{des}}$, for all distinct edges $\{\ell_k, m_k\} \in \mathcal{E}, k \in \mathcal{M}$, as well as the stack vectors $e^p(t) = [e_1^p(t), \ldots, e_M^p(t)]^\tau \in \mathbb{R}^M, e^q(t) = [(e_1^q(t))^\tau, \ldots, (e_M^q(t))^\tau]^\tau \in \mathbb{T}^{3M}$. By taking the time derivative of e^p, e^q, we obtain:

$$\dot{e}^p(t) = \mathbb{F}_p(x)\,(D^\tau(\mathcal{G}) \otimes I_3)\,\dot{p}, \quad (8a)$$

$$\dot{e}^q(t) = (D^\tau(\mathcal{G}) \otimes I_3)\,\dot{q}, \quad (8b)$$

where $\mathbb{F}_p : \mathbb{M}^N \to \mathbb{R}^{M\times 3M}$, with

$$\mathbb{F}_p(x) = 2\begin{bmatrix} p^\tau_{\ell_1,m_1}(t) & \cdots & 0_{1\times 3} \\ \vdots & \ddots & \vdots \\ 0_{1\times 3} & \cdots & p^\tau_{\ell_M,m_M}(t) \end{bmatrix}.$$

By introducing the stack error vector $e(t) = [(e^p(t))^\tau, (e^q(t))^\tau]^\tau \in \mathbb{R}^{4M}$, (8) can be written as:

$$\dot{e}(t) = \bar{\mathbb{F}}_p(x)\bar{D}^\tau(\mathcal{G})[\dot{p}^\tau, \dot{q}^\tau]^\tau, \quad (9)$$

where

$$\bar{\mathbb{F}}_p(x) = \begin{bmatrix} \mathbb{F}_p(x) & 0_{M\times 3M} \\ 0_{3M\times 3M} & I_{3M} \end{bmatrix} \in \mathbb{R}^{4M\times 6M}, \quad (10a)$$

$$\bar{D}(\mathcal{G}) = \begin{bmatrix} D(\mathcal{G})\otimes I_3 & 0_{3N\times 3M} \\ 0_{3N\times 3M} & D(\mathcal{G})\otimes I_3 \end{bmatrix} \in \mathbb{R}^{6N\times 6M}. \quad (10b)$$

Finally, we obtain from (4a):

$$\begin{bmatrix}\dot{p}\\ \dot{q}\end{bmatrix} = \underbrace{\left[\begin{array}{ccc|ccc} I_3 & \dots & 0_{3\times3} & 0_{3\times3} & \dots & 0_{3\times3} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0_{3\times3} & \dots & I_3 & 0_{3\times3} & \dots & 0_{3\times3} \\ \hline 0_{3\times3} & \dots & 0_{3\times3} & J_q(x_1) & \dots & 0_{3\times3} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0_{3\times3} & \dots & 0_{3\times3} & 0_{3\times3} & \dots & J_q(x_N) \end{array}\right]}_{\underline{J}(x)} \underbrace{\begin{bmatrix}\dot{p}_1\\ \vdots \\ \dot{p}_N \\ \omega_1 \\ \vdots \\ \omega_N\end{bmatrix}}_{\underline{v}(t)}$$

$$= \underline{J}(x)\underline{v}(t), \tag{11}$$

and thus, (9) can be written as:

$$\dot{e}(t) = \bar{\mathbb{F}}_p(x)\bar{D}^\tau(\mathcal{G})\underline{J}(x)\underline{v}(t). \tag{12}$$

The concepts and techniques of prescribed performance control, originally proposed in [Bechlioulis and Rovithakis, 2008], are adapted in this work in order to: a) achieve predefined transient and steady state response for the distance and orientation errors $e^p_k, e^q_k, \forall k \in \mathcal{M}$ as well as ii) avoid the violation of the collision and connectivity constraints between neighboring agents, as presented in Section 3. As stated in [Bechlioulis and Rovithakis, 2008], prescribed performance characterizes the behavior where the aforementioned errors evolve strictly within a predefined region that is bounded by absolutely decaying functions of time, called performance functions. The mathematical expressions of prescribed performance are given by the inequality objectives:

$$-C_{k,\text{col}}\rho^p_k(t) < e^p_k(t) < C_{k,\text{con}}\rho^p_k(t),\ -\rho^q_k(t) < e^q_{k_n}(t) < \rho^q_k(t), \tag{13}$$

$\forall k \in \mathcal{M}, n \in \{1,2,3\}$, where the functions $\rho^q_k(t) = (\rho^q_{k,0} - \rho^q_{k,\infty})e^{-l^q_k t} + \rho^q_{k,\infty}$, $\rho^p_k(t) = (1 - \rho^p_{k,\infty}\bar{c})e^{-l^p_k t} + \rho^p_{k,\infty}\bar{c}$, with $\bar{c} = (\max\{C_{k,\text{con}}, C_{k,\text{col}}\})^{-1}$, are designer-specified, smooth, bounded, and decreasing functions of time, where $l^p_k, l^q_k, \rho^p_{k,\infty}, \rho^q_{k,\infty} \in \mathbb{R}_{>0}, \forall k \in \mathcal{M}$, incorporate the desired transient and steady state performance specifications respectively, and $C_{k,\text{col}}$, $C_{k,\text{con}} \in \mathbb{R}_{>0}, \forall k \in \mathcal{M}$, are associated with the collision and connectivity constraints. In particular, we select

$$C_{k,\text{col}} = d_k^2 - d^2_{k,\text{col}},\quad C_{k,\text{con}} = d^2_{k,\text{con}} - d^2_k, \tag{14}$$

$\forall k \in \mathcal{M}$, which, since the desired formation is compatible with the collision and connectivity constraints (i.e., $d_{k,\text{col}} < d_{k,\text{des}} < d_{k,\text{con}}, \forall k \in \mathcal{M}$), ensures that $C_{k,\text{col}}, C_{k,\text{con}} \in \mathbb{R}_{>0}, \forall k \in \mathcal{M}$ and, in view of (6) that $-C_{k,\text{col}}\rho^p_k(0) < e^p_k(0) < \rho^p_k(0)C_{k,\text{con}}$, $\forall k \in \mathcal{M}$. Moreover, by choosing

$$\rho^q_{k,0} = \rho^q_k(0) > \max_{n\in\{1,2,3\}} |e^q_{k_n}(0)|, \tag{15}$$

it is also guaranteed that $-\rho^q_k(0) < e^q_{k_n}(0) < \rho^q_k(0)$. $\forall k \in \mathcal{M}, n \in \{1,2,3\}$. Hence, if we guarantee prescribed performance via (13), by employing the decreasing property of $\rho^p_k(t), \rho^q_k(t), \forall k \in \mathcal{M}$, we obtain $-C_{k,\text{col}} < e^p_k(t) < C_{k,\text{con}}, -\rho^q_k(t) < e^q_{k_n}(t) < \rho^q_k(t)$, and, consequently, owing to (14) we have $d_{k,\text{col}} < \|p_{\ell_k}(t) - p_{m_k}(t)\| < d_{k,\text{con}}, \forall k \in \mathcal{M}, t \in \mathbb{R}_{\geq 0}$, and, hence, a solution to problem 1.

In the sequel, we propose a decentralized control protocol that does not incorporate any information on the agents' dynamic model and guarantees (13) for all $t \in \mathbb{R}_{\geq 0}$. Given the errors $e^p(t), e^q(t)$:

Step I-a: Select the corresponding functions $\rho^p_k(t), \rho^q_k(t)$ and positive parameters $C_{k,\text{con}}, C_{k,\text{col}}, k \in \mathcal{M}$, following (13), (15), and (14), respectively, in order to incorporate the desired transient and steady state performance specifications as well as the collision and connectivity constraints. In addition, define the normalized errors $\xi^p_k : \mathbb{R}_{\geq 0} \to \mathbb{R}, \xi^q_k = [\xi^q_{k_1}, \xi^q_{k_2}, \xi^q_{k_3}]^\tau : \mathbb{R}_{\geq 0} \to \mathbb{R}^3$ with $\xi^p_k(t) = (\rho^p_k(t))^{-1}e^p_k(t), \xi^q_k(t) = (\rho^q_k(t))^{-1}e^q_k(t)$, $\forall k \in \mathcal{M}$, as well as the stack vector forms $\xi^p = [\xi^p_1, \dots, \xi^p_M]^\tau = (\rho^p)^{-1}e^p, \xi^q = [(\xi^q_1)^\tau, \dots, (\xi^q_M)^\tau]^\tau = (\rho^q)^{-1}e^q$, and $\xi = [(\xi^p)^\tau, (\xi^q)^\tau]^\tau = (\rho)^{-1}e \in \mathbb{R}^{4M}$, where $\rho^p = \text{diag}\{[\rho^p_k]_{k\in\mathcal{M}}\} \in \mathbb{R}^{M\times M}, \rho^q = \text{diag}\{[\rho^q_k I_3]_{k\in\mathcal{M}}\} \in \mathbb{R}^{3M\times 3M}, \rho = \text{diag}\{\rho^p, \rho^q\}$.

Step I-b: Define the transformed errors $\varepsilon^p_k : \mathbb{R} \to \mathbb{R}, \varepsilon^q_k : \mathbb{R}^3 \to \mathbb{R}^3$ and the signals $r^p_k : \mathbb{R} \to \mathbb{R}, r^q_k : \mathbb{R}^3 \to \mathbb{R}^{3\times 3}$ as

$$\varepsilon^p_k(\xi^p_k) = \ln\left(\left(1 + \frac{\xi^p_k}{C_{k,\text{col}}}\right)\left(1 - \frac{\xi^p_k}{C_{k,\text{con}}}\right)^{-1}\right), \tag{16a}$$

$$\varepsilon^q_k(\xi^q_k) = \left[\ln\left(\frac{1+\xi^q_{k_1}}{1-\xi^q_{k_1}}\right), \ln\left(\frac{1+\xi^q_{k_2}}{1-\xi^q_{k_2}}\right), \ln\left(\frac{1+\xi^q_{k_3}}{1-\xi^q_{k_3}}\right)\right]^\tau, \tag{16b}$$

$$\begin{aligned} r^p_k(\xi^p_k) &= \frac{\partial \varepsilon^p_k(\xi^p_k)}{\partial \xi^p_k} = \frac{C_{k,\text{col}} + C_{k,\text{con}}}{(C_{k,\text{col}} + \xi^p_k)(C_{k,\text{con}} - \xi^p_k)}, \\ r^q_k(\xi^q_k) &= \frac{\partial \varepsilon^q_k(\xi^q_k)}{\partial \xi^q_k} = \text{diag}\left\{\left[r^p_{k_n}(\xi^p_{k_n})\right]_{n\in\{1,2,3\}}\right\} \\ &= \text{diag}\left\{\left[\frac{2}{1-(\xi^q_{k_n})^2}\right]_{n\in\{1,2,3\}}\right\}, \end{aligned}$$

and design the decentralized reference velocity vector for each agent $v_{i,\text{des}} = [\dot{p}^\tau_{i,\text{des}}, \omega^\tau_{i,\text{des}}]^\tau : \mathbb{R}^{4M} \times \mathbb{R}_{\geq 0} \to \mathbb{R}^6$ as:

$$v_{i,\text{des}}(\xi,t) = -J_i^{-1}(x_i)\begin{bmatrix} \sum\limits_{j\in\mathcal{N}_i(0)} (\rho^p_{k_{ij}}(t))^{-1} r^p_{k_{ij}}(\xi^p_{k_{ij}})\varepsilon^p_{k_{ij}}(\xi^p_{k_{ij}})p_{i,j}(t) \\ \sum\limits_{j\in\mathcal{N}_i(0)} (\rho^q_{k_{ij}}(t))^{-1} r^q_{k_{ij}}(\xi^q_{k_{ij}})\varepsilon^q_{k_{ij}}(\xi^q_{k_{ij}}) \end{bmatrix} \tag{17}$$

where $k_{ij} \in \mathcal{M}$ is the edge of agents $i, j \in \mathcal{N}_i(0)$, i.e., $\{\ell_{k_{ij}}, m_{k_{ij}}\} \in \mathcal{E}$ and $\ell_{k_{ij}} = i, m_{k_{ij}} = j$. We write (17) in vector form $\underline{v}_\text{des}(\xi, t) = [\dot{p}^\tau_\text{des}(\xi^p, t), \omega^\tau_\text{des}(\xi^q, t)]^\tau$:

$$\underline{v}_\text{des}(\xi,t) = -\underline{J}^{-1}(x)\bar{D}(\mathcal{G})\bar{\mathbb{F}}^\tau_p(x)r(\xi)(\rho(t))^{-1}\varepsilon(\xi), \tag{18}$$

where $\dot{p}_\text{des} = [\dot{p}^\tau_{1,\text{des}}, \dots, \dot{p}^\tau_{N,\text{des}}]^\tau, \omega_\text{des} = [\omega^\tau_{1,\text{des}}, \dots, \omega^\tau_{N,\text{des}}]^\tau \in \mathbb{R}^{3N}, \varepsilon = [(\varepsilon^p)^\tau, (\varepsilon^q)^\tau]^\tau = [\varepsilon^p_1, \dots, \varepsilon^p_M, (\varepsilon^q_1)^\tau, \dots, (\varepsilon^q_M)^\tau]^\tau \in \mathbb{R}^{4M}$ and $\underline{J}(x), \bar{D}(\mathcal{G}), \bar{\mathbb{F}}_p$ as they were defined in (10) and (11), respectively. Moreover, $r = \text{diag}\{r^p, r^q\} \in \mathbb{R}^{4M\times 4M}, r^p = \text{diag}\{[r^p_k]_{k\in\mathcal{M}}\} \in \mathbb{R}^{M\times M}$ and $r^q = \text{diag}\{[r^q_k]_{k\in\mathcal{M}}\} \in \mathbb{R}^{3M\times 3M}$. It should be noted that $\underline{J}^{-1}(x)$ is always well-defined due to Assumption 1.

Step II-a: Define the errors $e^v : \mathbb{R}^{4M} \times \mathbb{R}_{\geq 0} \to \mathbb{R}^{6N}$, with $e^v(\xi, t) = [(e^v_1)^\tau(\xi,t), \dots, (e^v_N)^\tau(\xi,t)]^\tau = \underline{v}(t) - v_\text{des}(\xi,t)$, where $e^v_i(\xi,t) = [e^v_{i_1}(\xi,t), \dots, e^v_{i_6}(\xi,t)]^\tau = [\dot{p}^\tau_i(t) - \dot{p}^\tau_{i,\text{des}}(\xi^p,t), \omega^\tau_i(t) - \omega^\tau_{i,\text{des}}(\xi^q,t)]^\tau = v_i(t) - v_{i,\text{des}}(\xi,t), i \in \mathcal{V}$, and select the corresponding performance functions $\rho^v_{i_m} : \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$, with $\rho^v_{i_m}(t) = (\rho^v_{i_m,0} - \rho^v_{i_m,\infty})e^{-l^v_{i_m}t} + \rho^v_{i_m,\infty}$ and $\rho^v_{i_m,0} = \rho^v_{i_m}(0) > |e^v_{i_m}(0)|, l^v_{i_m}, \rho^v_{i_m,\infty} \in \mathbb{R}_{>0}, \rho^v_{i_m,\infty} < \rho^v_{i_m,0}, \forall i \in \mathcal{V}, m \in \{1, \dots, 6\}$.

Moreover, define the normalized errors $\xi^v_i = [\xi^v_{i_1}, \dots, \xi^v_{i_6}]^\tau : \mathbb{R}^{4M} \times \mathbb{R}_{\geq 0} \to \mathbb{R}^6$ with $\xi^v_i(\xi,t) = (\rho^v_i(t))^{-1}e^v_i(\xi,t)$, $\rho^v_i = \text{diag}\{[\rho^v_{i_m}]_{m\in\{1,\dots,6\}}\} \in \mathbb{R}^{6\times 6}$, which is written as $\xi^v(\xi,t) = [(\xi^v_1(\xi,t))^\tau, \dots, (\xi^v_N(\xi,t))^\tau]^\tau = (\rho^v(t))^{-1}e^v(\xi,t) \in \mathbb{R}^{6N}$, with $\rho^v(t) = \text{diag}\left\{[\rho^v_i(t)]_{i\in\mathcal{V}}\right\} \in \mathbb{R}^{6N\times 6N}$.

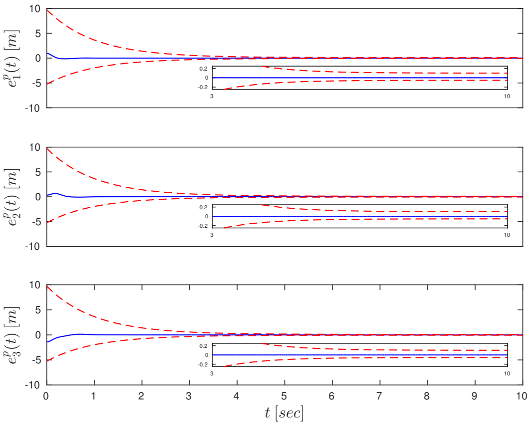

Fig. 1. The evolution of the distance errors $e_k^p(t)$ with the performance functions $\rho_k^p(t), \forall k \in \{1,2,3\}$.

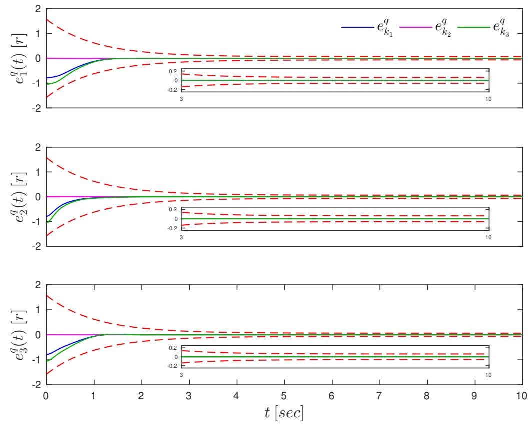

Fig. 2. The evolution of the orientation errors $e_{k_n}^q(t)$ with the performance functions $\rho_k^q(t), \forall k, n \in \{1,2,3\}$.

Step II-b: Define the transformed velocity errors $\varepsilon_i^v : \mathbb{R}^6 \to \mathbb{R}^6$ and the signals $r_i^v : \mathbb{R}^6 \to \mathbb{R}^{6\times 6}$ as:

$$\varepsilon_i^v(\xi_i^v) = \left[\ln\left(\frac{1+\xi_{i_1}^v}{1-\xi_{i_1}^v}\right), \dots, \ln\left(\frac{1+\xi_{i_6}^v}{1-\xi_{i_6}^v}\right)\right]^\tau, \tag{19a}$$

$$r_i^v(\xi_i^v) = \frac{\partial \varepsilon_i^v(\xi_i^v)}{\partial \xi_i^v} = \mathrm{diag}\{\left[r_{i_m}^v(\xi_{i_m}^v)\right]_{m\in\{1,\dots,6\}}\}$$
$$= \mathrm{diag}\left\{\left[\frac{2}{(1-(\xi_{i_m}^v)^2)}\right]_{m\in\{1,\dots,6\}}\right\}, \tag{19b}$$

and design the decentralized control protocol for each agent $i \in \mathcal{V}$ as $u_i : \mathbb{R}^6 \times \mathbb{R}_{\geq 0} \to \mathbb{R}^6$:

$$u_i(\xi_i^v, t) = -\gamma_i(\rho_i^v(t))^{-1} r_i^v(\xi_i^v)\varepsilon_i^v(\xi_i^v), \tag{20}$$

with $\gamma_i \in \mathbb{R}_{>0}, \forall i \in \mathcal{V}$, which can be written in vector form as:

$$u(\xi^v, t) = -\Gamma(\rho^v(t))^{-1} r^v(\xi^v)\varepsilon^v(\xi^v), \tag{21}$$

where $\Gamma = \mathrm{diag}\{[\gamma_i I_6]_{i\in\mathcal{V}}\} \in \mathbb{R}^{6N\times 6N}$, $\varepsilon^v = [(\varepsilon_1^v)^\tau, \dots, (\varepsilon_N^v)^\tau]^\tau \in \mathbb{R}^{6N}$ and $r^v = \mathrm{diag}\{[r_i^v]_{i\in\mathcal{V}}\} \in \mathbb{R}^{6N\times 6N}$.

Remark 1. Notice by (17) and (20) that the proposed control protocols are distributed in the sense that each agent uses only local information to calculate its own signal. In that respect, regarding every edge k_{ij}, with $\{\ell_{k_{ij}}, m_{k_{ij}}\} = \{i, j\}$, the parameters $\rho_{k_{ij},\infty}^p, \rho_{k_{ij},\infty}^q, l_{k_{ij}}^p, l_{k_{ij}}^q$, as well as the sensing radii $s_j, \forall j \in \mathcal{N}_i(0)$, which are needed for the calculation of the functions $\rho_{k_{ij}}^p, \rho_{k_{ij}}^q$, can be transmitted off-line to each agent $i \in \mathcal{V}$. Moreover, the proposed control law does not incorporate any prior knowledge of the model nonlinearities/disturbances, enhancing thus its robustness.

The main results of this work are summarized in the following theorem.

Theorem 2. Consider a system of N agents, subject to (2), aiming at establishing a formation described by the distances $d_{k,\mathrm{des}}$ and orientation angles $q_{k,\mathrm{des}}, k \in \mathcal{M}$, while satisfying the collision and connectivity constraints between neighboring agents, represented by $d_{k,\mathrm{col}}$ and $d_{k,\mathrm{con}}$, respectively, with $d_{k,\mathrm{col}} < d_{k,\mathrm{des}} < d_{k,\mathrm{con}}, k \in \mathcal{M}$. Then, under Assumptions 1, 2, the control protocol (17)-(21) guarantees: $-C_{k,\mathrm{col}}\rho_k^p(t) < e_k^p(t) < C_{k,\mathrm{con}}\rho_k^p(t), -\rho_k^q(t) < e_{k_n}^q(t) < \rho_k^q(t), \forall k \in \mathcal{M}, n \in \{1,2,3\}, t \geq 0$, as well as the boundedness of all closed loop signals.

Proof. The proof can be found in [Nikou et al., 2016].

Remark 2. The transient and steady state performance of the closed loop system is explicitly and solely determined by the parameters $l_k^p, l_k^q, \rho_{k,\infty}^p, \rho_{k,\infty}^q, \rho_{k,0}^p$ and $C_{k,\mathrm{col}}, C_{k,\mathrm{con}}$, $k \in \mathcal{M}$. Nonetheless, fine tuning might be needed in real-time scenarios, to retain the required control input signals within the feasible range that can be implemented by real actuators. Similarly, the input constraints impose an upper bound on the speed of convergence of $\rho_k^p(t)$ and $\rho_k^q(t), k \in \mathcal{M}$, as obtained by the exponentials $e^{-l_k^p t}, e^{-l_k^q t}$.

Remark 3. Regarding Assumption 1, we stress that, by choosing the initial conditions $\theta_i(0), \forall i \in \mathcal{V}$, and the desired formation constants $\theta_{k,\mathrm{des}} = q_{k_2,\mathrm{des}}, \forall k \in \mathcal{M}$ close to zero, the condition $-\frac{\pi}{2} < \theta_i(t) < \frac{\pi}{2}$ will not be violated, since the agents will be mostly operating near the point $\theta_i = 0, \forall i \in \mathcal{V}$. This is a reasonable assumption for real applications, since θ_i represents pitch angles that are desired to be close to zero (consider, e.g., aerial vehicles). Furthermore, notice that the proposed control scheme guarantees collision avoidance only for the initially neighboring agents (at $t = 0$), since that is how the edge set $\mathcal{E}$ is defined. Inter-agent collision avoidance with all possible agent pairs is left as future work.

5. SIMULATION RESULTS

To demonstrate the efficiency of the proposed control protocol, we considered a simulation example with $N = 4, \mathcal{V} = \{1,2,3,4\}$ spherical agents of the form (2), with $r_i = 1$m and $s_i = 4$m$, \forall i \in \{1,\dots,4\}$. We selected the exogenous disturbances as $w_i = A_i \sin(\omega_{c,i}t)(a_{i_1}x_i - a_{i,2}\dot{x}_i)$, where the parameters $A_i, \omega_{c,i}, a_{i_1}, a_{i_2}$ as well as the dynamic parameters of the agents were randomly chosen in $[0,1]$. The initial conditions were taken as $p_1(0) = [0,0,0]^T$ m$, p_2(0) = [2,2,2]^T$ m$, p_3(0) = [2,4,4]^T$ m$, p_4(0) = [2,3,2.5]^T$ m$, q_1(0) = q_2(0) = q_3(0) = q_4(0) = [0,0,0]^T$ r, which give the initial edge set $\mathcal{E} = \{\{1,2\},\{2,3\},\{2,4\}\}$. The desired graph formation was defined by the constants $d_{k,\mathrm{des}} = 2.5$m$, q_{k,\mathrm{des}} = [\frac{\pi}{4}, 0, \frac{\pi}{3}]^T$ r$, \forall k \in \{1,2,3\}$. Invoking (14), we also chose $C_{k,\mathrm{col}} = 5.25$m and $C_{k,\mathrm{con}} = 10.75$m. Moreover, the parameters of the performance functions were chosen as $\rho_{k,\infty}^p = 0.1, \rho_{k,0}^q = \frac{\pi}{2} > \max\{e_{k_1}^q(0), e_{k_2}^q(0), e_{k_3}^q(0)\} = \frac{\pi}{3}$ and $l_k^p = l_k^q = 1, \forall k \in \{1,2,3\}$. In addition, we chose $\rho_{i_m,0}^v = 2|e_{i_m}^v(0)| + 0.5, l_{i_m}^v = 1$ and $\rho_{i_m,\infty}^v = 0.1$. Finally, γ_i was set to 5. The simulation results are depicted in Fig. 1-4. In particular, Fig. 1 and 2 show the evolution of $e_k^p(t)$ and $e_{k_n}^q(t)$ along with $\rho_k^p(t)$ and $\rho_k^q(t)$, respectively, $\forall k \in \{1,2,3\}, n \in \{1,2,3\}$. The velocity errors $e_{i_m}^v(t)$

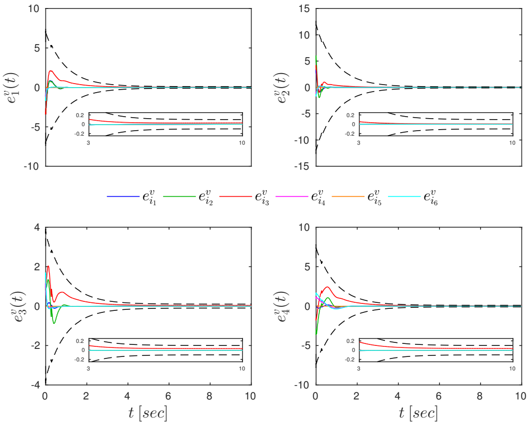

Fig. 3. The evolution of the velocity errors $e^v_{i_m}(t)$ with the functions $\rho^v_{i_m}(t), \forall i \in \{1,\dots,4\}, m \in \{1,\dots,6\}$.

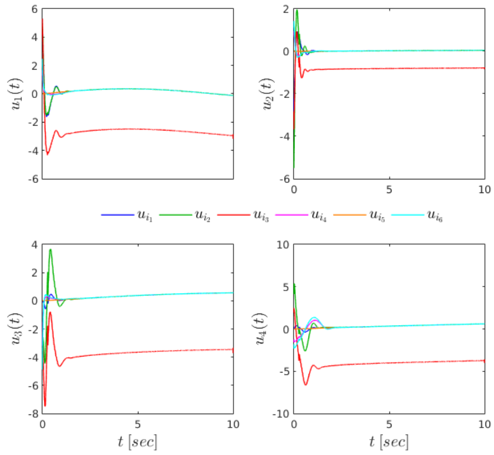

Fig. 4. The resulting input signals $u_i(t), i \in \{1,\dots,4\}$.

along $\rho^v_{i_m}(t)$ and the control signals u_i are illustrated in Figs. 3 and 4, respectively. As it was predicted by the theoretical analysis, the formation control problem with prescribed transient and steady state performance is solved with bounded closed loop signals, despite the unknown agent dynamics and the presence of external disturbances.

6. CONCLUSIONS AND FUTURE WORK

In this work we proposed a robust decentralized control protocol for distance- and orientation-based formation control, collision avoidance and connectivity maintenance of multiple rigid bodies with unknown dynamic models. Future efforts will be devoted towards extending the current results to directed as well as time-varying communication graph topologies.

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