A Collision-free Communication Scheduling for Nonlinear Model Predictive Control

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Abstract: In this paper, we propose a framework to generate communication schedulings for nonlinear model predictive control. The proposed method considers the case where multiple plants share a communication network, and the goal is to pre-plan for each plant a timing to communicate with the controller to solve an optimal control problem. The desired communication schedulings are generated such that: (i) no network collisions occur; (ii) convergence to a prescribed local set around the origin is guaranteed for all plants. When formulating an algorithm, we additionally propose an optimization problem that is similar to the standard collision avoidance problem of controlling multi-agent systems. To validate our proposed scheme, a control problem of three inverted pendulums is simulated.

Keywords: Model Predictive Conrol, Networked Control Systems, Nonlinear Systems, Communication Scheduling, Event-triggered control,

1. INTRODUCTION

Networked Control Systems (NCSs) are defined as control systems where multiple sensors, actuators, and controllers are connected over a common wired/wireless network channel. Due to the progress in communication technology and many practical advantages such as low-cost maintenance and the possibility to build more complex control architectures, a wide variaty of control strategies for NCSs has been developed and investigated over the past decades (Hespanha et al. (2007)).

In NCSs, one of the most typical situations is that multiple plants share a common communication network, and these are controlled by the remote controllers (Dai et al. (2010)), see such illustration in Fig. 1. In this situation, only a limited number of plants can occupy the network at the same time due to limited communication capabilities, i.e., if too many plants transmit information at the same time, a network collision may occur which leads to a transmission failure. Therefore, it is crucial to design a communication strategy in addition to guaranteeing the desired control performance, such that the network restrictions illustrated above can be taken into account.

Communication schedulings for controlling multiple plants that have been investigated so far can be mainly divided into two categories; namely, *static scheduling* (Zhang et al. (2008); Dai et al. (2010)) and *dynamic scheduling* (Zhang et al. (2008); Reimann et al. (2012); Cervin and Alriksson (2006)). In the static scheduling case, communication schedulings over the network are determined in the off-line stage based on the knowledge of dynamics and the control strategy. In the dynamic case, on the other hand, communication times are determined on-line based on the state information and given the state feedback policy. Many criteria and design techniques for obtaining the communication strategy for multiple plants have been addressed, such as sensitivity function (Zhang et al. (2008)), infinite horizon quadratic cost (Cervin and Alriksson (2006)) and Lyapunov functions (Reimann et al. (2012)).

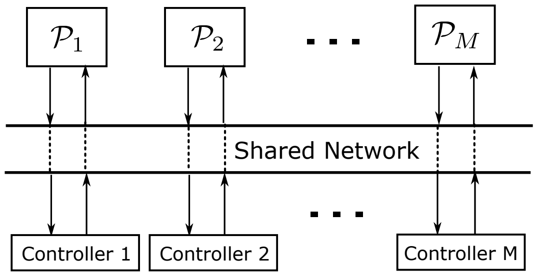


Fig. 1. Networked Control System considered in this paper. Each plant $\mathcal{P}_i$ ($i \in \mathcal{M}$) is controlled by the corresponding controller i through a shared network.

In this paper, we investigate how communication schedulings can be obtained in a Model Predictive Control (MPC) framework. MPC plays an important role for NCSs especially when one of the control objectives is to handle constraints, such as actuator saturations and physical limitations. Communication co-design techniques for MPC have been pursued for linear systems with no disturbances, see, e.g., Görges et al. (2009); Henriksson et al. (2015). In Henriksson et al. (2015), a network collision-free scheduling method has been proposed for multiple-loop linear control systems. In the proposed method, a dynamic scheduler is designed based on a self-triggered strategy. A related work is also reported in Cervin and Alriksson (2006), where a dynamic scheduling method is proposed for linear systems by evaluating an infinite horizon quadratic cost.

In this paper, we propose a new framework for generating communication schedulings for MPC. The main contribution with respect to afore-cited papers (Görges et al. (2009); Henriksson et al. (2015); Cervin and Alriksson (2006)) is to derive the network scheduling protocol for nonlinear systems, where the dynamics are perturbed by additive bounded disturbances. Our goal is to generate communication sequences for multiple plants as illustrated in Fig. 1, under the communication constraint that only a single plant can occupy the network at a certain transmission time.

The proposed framework is inspired by our previous work presented in Hashimoto et al. (2015), where an event-triggered strategy for MPC has been proposed for a single plant. While we follow a basic problem set-up, the analysis and the approach taken in this paper differs from the previous result in the following two directions:

(1) Feasibility and stability are further analyzed to design the communication strategy. In particular, stability is proven in a different manner from the approach presented in Hashimoto et al. (2015), in the sense that the maximum time of convergence is explicitly obtained here, aiming to generate the desired communication schedulings.
(2) We translate the network scheduling problem into an additional *optimization problem*, where new dynamics, cost, and constraints are defined. Since the additional problem is formulated to be similar to the standard collision avoidance problem of controlling multi-agent systems, we could utilize several existing related tools to efficiently solve the optimisation problem, such as mixed integer linear programming (see e.g., Richards et al. (2002)) or sequential linear programming (see e.g., Augugliaro et al. (2000)).

This paper is organized as follows. In Section 2, the OCP and the main problem to be solved are formulated. In Section 3, several conditions to guarantee feasibility and stability are given. In Section 4, the main strategies of determining network scheduling are proposed. In Section 5, a simulation example is provided to validate our proposed scheme. Finally, we conclude in Section 6.

Notations. $\mathbb{R}$, $\mathbb{R}_{>0}$, $\mathbb{N}_{\geq 0}$, $\mathbb{N}_{\geq 1}$ are the real, non-negative real, non-negative integers and positive integers, respectively. For a matrix Q, we use $Q \succ 0$ to denote that Q is positive definite. Given a compact set $\Phi \subseteq \mathbb{R}^n$, we denote by $\partial\Phi$ the boundary of Φ. The function $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is called Lipschitz continuous with a weighted matrix P and Lipschitz constant L_f in $x \in \mathbb{R}^n$, if $||f(x_1, u) - f(x_2, u)||_P \leq L_f ||x_1 - x_2||_P$ for all $x_1, x_2 \in \mathbb{R}^n, u \in \mathbb{R}^m$.

2. PROBLEM FORMULATION

2.1 System description and Problem statement

Consider the networked control system depicted in Fig. 1, where there exist $M \in \mathbb{N}_{\geq 1}$ number of plants $\mathcal{P}_1, \mathcal{P}_2, \cdots, \mathcal{P}_M$ in total. We assume that each i-th plant ($i \in \{1, 2, \cdots, M\}$) is controlled by the corresponding i-th controller through a shared network, and the dynamics of the i-th plant is given by the following nonlinear continuous-time system:

$$\dot{x}_i(t) = f_i(x_i(t), u_i(t)) + w_i(t), \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$ is the state, $u_i \in \mathbb{R}^{m_i}$ is the control input, and $w_i \in \mathbb{R}^{n_i}$ is the additive bounded disturbance. For notational simplicity in the sequel, we denote by $\mathcal{M}$ the set given by $\mathcal{M} = \{1, 2, \cdots, M\}$. We consider the following standard assumptions (Chen and Allgöwer (1998)):

Assumption 1. For all $i \in \mathcal{M}$: (i) The function $f_i : \mathbb{R}^{n_i} \times \mathbb{R}^{m_i} \to \mathbb{R}^{n_i}$ is twice continuously differentiable, and $f_i(0, 0) = 0$ (i.e., the origin is an equilibrium point). (ii) u_i, w_i are subject to the following constraints;

$$u_i \in \mathcal{U}_i, \qquad w_i \in \mathcal{W}_i, \tag{2}$$

where $\mathcal{U}_i$ and $\mathcal{W}_i$ are compact and convex sets containing the origin in their interiors. □

Let $t_{i,k}$, $k \in \mathbb{N}_{\geq 0}$ be the transmission time instants when the i-th ($i \in \mathcal{M}$) plant requests a communication to the i-th controller for solving an optimal control problem.

Due to the network constraint that only a limited amount of information can be transmitted over the network, we consider that only a *single* plant is capable of occupying the network at each transmission time instant; if multiple plants request communication at the same time (e.g., $t_{i,k} = t_{j,\ell}$ for some $i \neq j$ and $k, \ell \in \mathbb{N}_{\geq 0}$), then a network collision occurs leading to a communication failure. Therefore, our goal is to obtain the transmission schedulings $t_{i,0}, t_{i,1}, \cdots$ for all $i \in \mathcal{M}$, such that no network collisions arise while at the same time fulfilling the desired control performances. Regarding the network constraints, we further consider the following:

Assumption 2. Once the network is occupied by a plant at a certain transmission time, all the other plants are not allowed to use the network at least for the time interval $\delta_{\min} > 0$ afterwards.

In Assumption 2, we consider that once a plant occupies the network, say at $t_{i,k}$, the network cannot be free again at least for the time interval $[t_{i,k}, t_{i,k} + \delta_{\min}]$. This situation needs to be considered in NCSs for several reasons; for instance, when a time interval between two transmissions needs to be large enough to avoid network interferences (especially when indoor enviroments and wireless communication is used), or when it requires some time to release the network once it is occupied. Assumption 2 leads to the constraint that for every two plants i, j ($i \neq j$), every difference between two transmission times needs to be larger than $\delta_{\min}$. That is, for all $i \in \mathcal{M}$ and $j \in \mathcal{M}\backslash\{i\}$, it must hold that $|t_{i,k} - t_{j,m}| > \delta_{\min}$, $\forall k \in \mathbb{N}_{\geq 0}$, $\forall m \in \mathbb{N}_{\geq 0}$. A more specific definition of the problem statement is given later in this subsection.

For each transmission time $t_{i,k}$, the i-th controller solves an OCP based on the i-th plant's state information $x_i(t_{i,k})$, and the predictive behavior of the system from (1). We consider the following quadratic cost function to be minimized:

$$J_i(x_i(t_{i,k}), u_i(\cdot)) = \int_{t_{i,k}}^{t_{i,k}+T_p} ||\hat{x}_i(\xi)||^2_{Q_i} + ||u_i(\xi)||^2_{R_i} \mathrm{d}\xi, \tag{3}$$

where Q_i and R_i are the matrices for the stage cost satisfying $Q_i = Q_i^{\mathsf{T}} \succ 0$, $R_i = R_i^{\mathsf{T}} \succ 0$, and $\hat{x}_i(\xi)$ denotes the nominal state from $t_{i,k}$, i.e., $\dot{\hat{x}}_i(\xi) = f_i(\hat{x}_i(\xi), u_i(\xi))$, $\xi \in [t_{i,k}, t_{i,k} + T_p]$, with $\hat{x}_i(t_{i,k}) = x_i(t_{i,k})$. $T_p \in \mathbb{R}_{>0}$ denotes the prediction horizon; for simplicity, we assume that T_p is equal for all $i \in \mathcal{M}$. In order to characterize the convergence, we consider a local set Φ_i given by $\Phi_i = \{x \in \mathbb{R}^{n_i} : V_{f_i}(x) \leq \varepsilon_i^2\}$, where $V_{f_i}(x) = x^\mathsf{T} P_i x$ for given $\varepsilon_i > 0$, $P_i = P_i^\mathsf{T} \succ 0$. Following the standard MPC set-up (Chen and Allgöwer (1998)), we consider that the matrix P_i and ε_i are designed such that the following is satisfied.

Assumption 3. For all $i \in \mathcal{M}$:

(i) There exists a local state feed-back controller $\kappa_i(x) = K_i x \in \mathcal{U}_i$, satisfying

$$\frac{\partial V_{f_i}}{\partial x} f_i(x, K_i x) \leq -x^\mathsf{T}(Q_i + K_i^\mathsf{T} R_i K_i)x \tag{4}$$

for all $x \in \Phi_i$.

(ii) The nonlinear function $f_i : \mathbb{R}^{n_i} \times \mathbb{R}^{m_i} \to \mathbb{R}^{n_i}$ is Lipschitz continuous with the weighted matrix P_i and the Lipschitz constant $0 \leq L_{f_i} < \infty$ in $x \in \mathbb{R}^{n_i}$. □

Assumption (i) implies the existence of a stabilizing controller if the state is inside the local set Φ_i. If the linearized system of (1) around the origin is stabilizable, appropriate κ_i and Φ_i satisfying Assumption (i) can be found off-line by following Chen and Allgöwer (1998). The second Lipschitz assumption (Assumption (ii)) will be utilized to derive several conditions to guarantee recursive feasibility.

The main problem to be solved in this paper is now described as follows:

Problem 1. *For each $i \in \mathcal{M}$, given the initial communication time $t_{i,0}$ and the corresponding initial state $x_i(t_{i,0})$, find a communication scheduling $t_{i,0}, t_{i,1}, \cdots, t_{i,N_i}$ for some $N_i \in \mathbb{N}_{\geq 1}$, such that:*

(1) The state $x_i(t)$ is steered to the local region Φ_i, i.e., $x_i(t_{i,N_i}) \in \Phi_i$;

(2) For a given $\delta_{\min} > 0$, there exist no network collisions. That is, for all $j \in \mathcal{M}\backslash\{i\}$, it holds that

$$\begin{aligned} &|t_{i,k} - t_{j,m}| > \delta_{\min}, \\ &\qquad \forall k \in \{1, \cdots, N_i\}, \quad \forall m \in \{1, \cdots, N_j\}. \end{aligned} \tag{5}$$

□

As shown in Problem 1, our goal is to generate communication schedulings for all the plants based on their initial states, ensuring both stability (convergence to the local set Φ_i for all $i \in \mathcal{M}$) and that no network collisions arise. Note that the scheduling will be generated under a *static* fashion; communication sequences are generated off-line before the MPC implementation. In Problem 1, N_i represents the number of transmissions for the i-th plant and needs to be appropriately selected such that $x_i(t_{i,N_i}) \in \Phi_i$ holds. A more specific condition for selecting N_i will be given in later sections when presenting the communication strategy (Section 4).

2.2 Optimal Control Problem

In this subsection, we formulate the OCP for each controller as a first step to solve Problem 1. In addition to the local set Φ_i, we consider a restricted local set Φ_{f_i} given by $\Phi_{f_i} = \{x \in \mathbb{R}^{n_i} : V_{f_i}(x) \leq \varepsilon_{f_i}^2\}$, where $0 < \varepsilon_{f_i} < \varepsilon_i$. The illustration of Φ_i, Φ_{f_i} is depicted in Fig. 2. At a communication time $t_{i,k}$, the objective of the OCP given by the i-th controller is to obtain a pair of optimal control and state trajectory $(u_i^*(\xi), \hat{x}_i^*(\xi))$ for all $\xi \in [t_{i,k}, t_{i,k}+T_p]$,

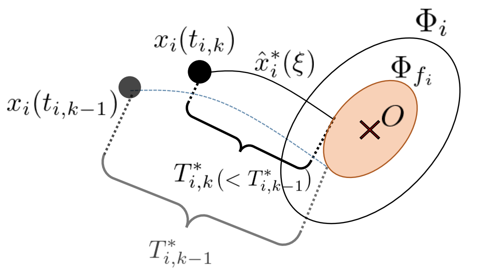


Fig. 2. Illustration of two regions Φ_i, Φ_{f_i}. $T^*_{i,k}$ denotes the time interval when the optimal state trajectory obtained at $t_{i,k}$ enters Φ_{f_i}.

such that the cost function $J_i(\cdot)$ in (3) is minimized. In this paper, we impose the constraint that the optimal state trajectory enters Φ_{f_i}; the illustration is depicted in Fig. 2. Due to this constraint, there exists a *time interval* when the optimal state trajectory reaches the boundary of Φ_{f_i}; denote by $T^*_{i,k}$ the time interval given by

$$T^*_{i,k} = \inf\{T_{i,k} \in \mathbb{R}_{>0} \ : \ \hat{x}_i^*(t_{i,k} + T_{i,k}) \in \partial\Phi_{f_i}\}. \tag{6}$$

In the following, we formulate the OCP such that this time interval is strictly decreasing over time:

Problem 2. (OCP for the i-th plant) *At any non-initial times $t_{i,k}$, $k \in \mathbb{N}_{\geq 1}$, given the state $x_i(t_{i,k})$ and $T_p > 0$, find a pair of an optimal control and the corresponding state trajectories $u_i^*(\xi)$, $\hat{x}_i^*(\xi)$ for all $\xi \in [t_{i,k}, t_{i,k} + T_p]$ by minimizing $J_i(x_i(t_{i,k}), u_i(\cdot))$ in (3), subject to:*

$$\begin{cases} \dot{\hat{x}}_i(\xi) = f_i(\hat{x}_i(\xi), u_i(\xi)), \quad \hat{x}_i(t_{i,k}) = x_i(t_{i,k}), & (7) \\ u_i(\xi) \in \mathcal{U}, \ \xi \in [t_{i,k}, t_{i,k} + T_p] & (8) \\ \hat{x}_i(t_{i,k} + T^*_{i,k-1} - \gamma_i\delta_{i,k-1}) \in \Phi_{f_i} & (9) \end{cases}$$

for given $0 < \gamma_i < 1$, where $\delta_{i,k-1} = t_{i,k} - t_{i,k-1}$. For the initial time $t_{i,0}$, minimize $J_i(x_i(t_{i,k}), u_i(\cdot))$ in (3), subject to (7), (8), and $\hat{x}_i(t_{i,0} + T_p) \in \Phi_{f_i}$. □

For the initial time $t_{i,0}$, Problem 2 is solved with a standard terminal constraint $\hat{x}_i(t_{i,0} + T_p) \in \Phi_{f_i}$. For the non-initial times, on the other hand, we impose a *terminal-like* constraint in (9); the predictive state needs to enter Φ_{f_i} within $T^*_{i,k-1} - \gamma\delta_{i,k-1}$, where $T^*_{i,k-1}$ is the time interval obtained at the previous update time $t_{i,k-1}$.

From the constraint (9) and from (6), we obtain $T^*_{i,k} \leq T^*_{i,k-1} - \gamma_i\delta_{i,k-1} < T^*_{i,k-1}$, which means that the time interval to reach Φ_{f_i} becomes strictly smaller than the previous time (see also the illustration in Fig. 2). This property is one of the important ideas to guarantee stability and obtain a suitable communication scheduling, as provided in subsequent sections.

In this paper, we consider that the controller transmits the obtained optimal control trajectory to the plant, and the plant applies it until the next update time, i.e., $u_i(t) = u_i^*(t)$, $t \in [t_{i,k}, t_{i,k+1})$.

3. GUARANTEEING FEASIBILITY AND CONVERGENCE

Before providing the scheduling algorithm, we first limit our attention to a single plant and investigate several useful properties to guarantee the control performance; namely, feasibility and stability. Regarding feasibility, we

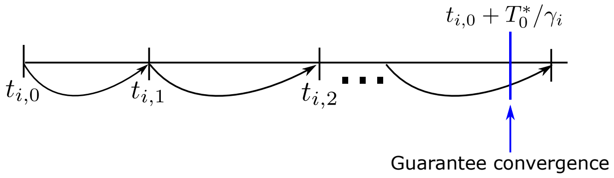


Fig. 3. We can recursively obtain a finite time sequence $t_{i,0}, t_{i,1}, t_{i,2}, \cdots$ such that both feasibility and stability are guaranteed.

will derive the following *recursive feasibility*, which states that the existence of a feasible solution to Problem 2 at the initial time $t_{i,0}$, implies the feasibility at all the time afterwards $t_{i,k}, k \in \mathbb{N}_{\geq 1}$:

Lemma 1. Consider the networked control system in Fig. 1, where the dynamics of each plant is given by (1). *For the i-the plant ($i \in \mathcal{M}$), suppose that Problem 2 has a solution at $t_{i,k}$, $k \in \mathbb{N}_{\geq 0}$, providing a pair of optimal control and the corresponding state trajectories given by $u_i^*(\xi)$, $\hat{x}_i^*(\xi)$ for all $\xi \in [t_{i,k}, t_{i,k}+T_p]$. Then, Problem 2 has a solution at the next time step $t_{i,k+1}$ ($> t_{i,k}$) if the following conditions are all satisfied;*

(1) The maximum size of the disturbance w_i satisfies $||w_i(t)||_{P_i} \leq w_{i,\max}$ for all $t \in [t_{i,k}, t_{i,k+1}]$, where

$$w_{i,\max} = \frac{\lambda_{\min}(Q_{P_i})}{2e^{L_{f_i} T^*_{i,0}}}(1-\gamma_i)\varepsilon_{f_i}, \tag{10}$$

with $Q_{P_i} = P_i^{-1/2}(Q_i + K_i^{\mathsf{T}} R_i K_i) P_i^{-1/2}$.

(2) It holds that

$$t_{i,k} < t_{i,k+1} \leq t_{i,k} + \bar{\delta}_i(t_{i,k}), \tag{11}$$

where $\bar{\delta}_i(t_{i,k})$ is given by

$$\bar{\delta}_i(t_{i,k}) = \min\left\{T_p, \quad \frac{\alpha_i\gamma_i}{1+\alpha_i}(t_{i,k} - t_{i,0}) + \frac{1}{L_{f_i}}\ln(1+\alpha_i)\right\},$$

with a given $\alpha_i = \frac{2L_{f_i}(\varepsilon_i - \varepsilon_{f_i})}{\lambda_{\min}(Q_{P_i})(1-\gamma_i)\varepsilon_{f_i}} > 0$. □

The proof is given by extending our previous result in Hashimoto et al. (2015) and is omitted for brevity. Lemma 1 states that recursive feasibility is guaranteed if the size of the disturbance is small enough to satisfy (10), and the next time to solve the OCP is upper bounded as shown in (11).

An important property of Lemma 1, is that we can recursively obtain the communication scheduling $t_{i,0}, t_{i,1}, t_{i,2}, \cdots$ off-line such that feasibility of Problem 1 is guaranteed; for a given initial time $t_{i,0}$, the feasibility at the next time step $t_{i,1}$ is guaranteed if (11) (by letting $k=0$) holds. Suppose then, that $t_{i,1}$ is selected such that (11) is satisfied. Then, for a given $t_{i,1}$, the next time $t_{i,2}$ can be selected such that (11) holds with $k=1$. By following this procedure, we can recursively generate $t_{i,0}, t_{i,1}, t_{i,2} \cdots$ such that the feasibility is guaranteed. The illustration of this recursive procedure is depicted in Fig. 3.

As another important property, we next show that the state trajectory enters Φ_i in finite time. Regarding this convergence property, the following lemma holds:

Lemma 2. For the i-th ($i \in \mathcal{M}$) plant, suppose that: (i) $||w_i(t)||_{P_i} \leq w_{i,\max}$, $\forall t \geq t_0$, where $w_{i,\max}$ is given by (10)*; (ii) Problem 2 is solved at $t_{i,k}$, $k \in \mathbb{N}_{\geq 0}$, which are all chosen such that* (11) *holds to guarantee the feasibility.*

Then, the trajectory $x_i(t)$ enters Φ_i within $t_{i,0} + T^*_{i,0}/\gamma_i$.

Proof. We prove the statement by contradiction. Assume that at $t_{i,k}$ we have $x_i(t_{i,k}) \notin \Phi_i$ and that it satisfies $t_{i,k} - t_{i,0} \geq T^*_{i,0}/\gamma_i$. As $x_i(t_{i,k}) \notin \Phi_{f_i}$, the time interval when the optimal state trajectory enters Φ_{f_i} is guaranteed to be positive, i.e., $T^*_{i,k} > 0$. Since w_i satisfies (10) and $t_{i,0}, t_{i,1}, \cdots, t_{i,k}$ are selected such that (11) holds, the feasibility of Problem 2 is guaranteed for all $t_{i,0}, t_{i,1}, \cdots, t_{i,k}$. Thus, from the constraint (9), we obtain

$$\begin{aligned} T^*_{i,k} &\leq T^*_{i,k-1} - \gamma_i\delta_{i,k-1} \\ &\leq T^*_{i,k-2} - \gamma_i(\delta_{i,k-1} + \delta_{i,k-2}) \\ &\leq \cdots \leq T^*_{i,0} - \gamma_i \sum_{l=0}^{k-1} \delta_{i,l} \\ &= T^*_{i,0} - \gamma_i(t_{i,k} - t_{i,0}). \end{aligned} \tag{12}$$

From the above in-equality, we have $T^*_{i,k} \leq T^*_{i,0} - \gamma_i(t_{i,k} - t_{i,0})$ and by assumption $t_{i,k} \geq t_{i,0} + T^*_{i,0}/\gamma_i$, we have $T^*_{i,k} \leq 0$. However, this contradicts the fact that $T^*_{i,k} > 0$. Therefore, it holds that the state enters Φ_i if $t_{i,k} \geq t_{i,0} + T^*_{i,0}/\gamma_i$. This completes the proof. □

An important property of Lemma 2 is that we can explicitly obtain the maximal convergence time when the state enters Φ_i. From Lemma 1 and Lemma 2, we can recursively obtain the communication scheduling $t_{i,0}, t_{i,1}, \cdots$, to ensure the feasibility through the above procedure, and once it attains $t_{i,k} \geq t_{i,0} + T^*_{i,0}/\gamma_i$ for some $k \in \mathbb{N}_{>0}$, then we obtain $x_i(t_{i,k}) \in \Phi_i$ (see the illustration in Fig. 3). Therefore, it is possible to generate a communication scheduling for each plant off-line, such that both feasibility and stability are guaranteed.

So far, we have limited our attention to a single plant to guarantee feasibility and stability. In the next section, we make use of the above properties to generate desired communication schedulings for all the plants, such that any network collisions can be avoided.

4. COMMUNICATION SCHEDULING VIA INTER-SAMPLING TIME ANALYSIS

Based on the analysis in the previous section, we now present a framework of generating the desired collision-free communication schedulings as a solution to Problem 1. To achieve this, we will in the following formulate a new *optimization problem*, in which dynamics, cost, and several constraints are defined.

4.1 Inter-sampling behavior as Dynamics

To ensure the feasibility of Problem 2, we require that communication scheduling times $t_{i,k}$ are generated to satisfy (11). To this end, we translate the inequality (11) into the following *equality* condition;

$$t_{i,k+1} = t_{i,k} + \delta_{i,k}, \tag{13}$$

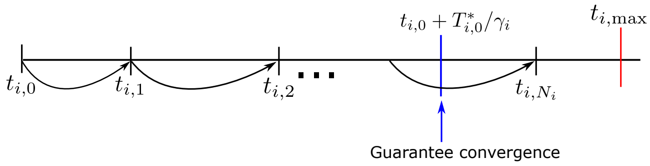


Fig. 4. The cost function is given by (15) such that $t_{i,k}$ approaches $t_{i,\max}$ as soon as possible. Due to the constraint (16), t_{i,N_i} is guaranteed to be placed between $t_{i,0} + T^*_{i,0}/\gamma_i$ and $t_{i,\max}$.

where $\delta_{i,k}$ is defined as an auxiliary variable satisfying both the following linear constraints:

$$\begin{aligned} 0 < \delta_{i,k} &< \frac{\alpha_i \gamma_i}{1+\alpha_i}(t_{i,k} - t_{i,0}) + \frac{1}{L_{f_i}} \ln(1+\alpha_i) \\ \delta_{i,k} &\leq T_p. \end{aligned} \tag{14}$$

Thus, (13) is considered as a *linear dynamics* to describe an inter-sampling time behavior, where $t_{i,k}$ represents the *state variable*, and $\delta_{i,k}$ represents the *control variable* subject to the constraints (14).

4.2 Cost to be minimized

To define a cost function, we first let $\hat{t}_{i,0}, \hat{t}_{i,1}, \hat{t}_{i,2}, \cdots$ ($i \in \mathcal{M}$) be the time sequence where $\hat{t}_{i,0} = t_{i,0}$ and $\hat{t}_{i,k+1} = \hat{t}_{i,k} + \bar{\delta}_i(\hat{t}_{i,k})$, i.e., when equality holds in (11). Then, let $N_{i,\min}$ be given by $N_{i,\min} = \min\{k \in \mathbb{N}_{\geq 1} \mid \hat{t}_{i,k} \geq \hat{t}_{i,0} + T^*_{i,0}/\gamma_i\}$. From Lemma 2, $N_{i,\min}$ represents the minimal number of transmissions to guarantee $x_i(t_{i,N_{i,\min}}) \in \Phi_i$.

Let $N_i \in \mathbb{N}_{\geq 1}$ ($i \in \mathcal{M}$) be a given number satisfying $N_i \geq N_{i,\min}$, and also let $t_{i,\max}$, ($i \in \mathcal{M}$) be a given time instant satisfying $t_{i,\max} > t_{i,0} + T^*_{i,0}/\gamma_i$. Based on these notations, we define the following quadratic cost function to be minimized:

$$J_t\left(t_{i,(1:N_i)}, \delta_{i,(1:N_i)}\right) = \sum_{i=1}^{M} \sum_{k=0}^{N_i} |t_{i,\max} - t_{i,k}|^2 \tag{15}$$

where for simplicity we denote the arguments as $t_{i,(1:N_i)} = \{t_{i,1}, \cdots, t_{i,N_i}\}$, $\delta_{i,(1:N_i)} = \{\delta_{i,1}, \cdots, \delta_{i,N_i}\}$. From above, the cost is defined by summing the difference between $t_{i,k}$ and $t_{i,\max}$ over all plants $i \in \mathcal{M}$ and the number of transmissions, i.e., the cost becomes smaller as $t_{i,k}$ gets closer to $t_{i,\max}$. This implies, that by minimizing (15) the communication sequence $t_{i,1}, \cdots, t_{i,N_i}$ will be selected to approach $t_{i,\max}$ as soon as possible to guarantee the convergence. The illustration of this interpretation is depicted in Fig. 4. As shown in (15), the parameter N_i represents the total number of transmissions for the i-th plant. Although the parameter can be arbitrary selected as long as $N_i \geq N_{i,\max}$ is fulfilled, it should not be selected too large to keep the computational complexity of the optimization problem to manageable levels.

4.3 Constraints

Finally, several constraints to formulate the optimization problem are given. To guarantee the convergence for each plant, we first impose the following terminal-like constraint for t_{i,N_i};

$$T^*_{i,0}/\gamma_i + t_{i,0} \leq t_{i,N_i} \leq t_{i,\max}, \tag{16}$$

see also the illustration in Fig. 4. Since N_i is selected in the previous subsection to satisfy $N_i \geq N_{i,\max}$, there always exists a time sequence $t_{i,0}, \cdots, t_{i,N_i}$ such that $t_{i,0} + T^*_{i,0}/\gamma_i \leq t_{i,N_i}$ holds to fulfill (16).

Additionally, we need to ensure a collision-free scheduling under $\delta_{\min}$. That is, for all $j \in \mathcal{M}\backslash\{i\}$, it must hold that

$$|t_{i,k} - t_{j,m}| > \delta_{\min}, \ \forall k \in \{1, \cdots, N_i\}, \ \forall m \in \{1, \cdots, N_j\}. \tag{17}$$

4.4 Optimal communication scheduling generation

Combining the dynamics, cost, and the constraints defined in the previous subsections, we are now ready to formulate the following optimization problem in order to find the desired collision-free communication schedulings:

Problem 3. (Communication scheduling problem): *For given $t_{i,0}$, N_i, $t_{i,\max}$ for all $i \in \mathcal{M}$, find communication schedulings $t^*_{i,0}, t^*_{i,1}, \cdots, t^*_{i,N_i}$ for all $i \in \mathcal{M}$ with $t^*_{i,0} = t_{i,0}$, by minimizing the cost J_t given by* (15)*, subject to the dynamics* (13)*, control input* (14)*, terminal constraint* (16)*, and the constraint for avoiding network collisions* (17)*.* □

Note that Problem 3 can be solved offline, since the initial times $t_{i,0}$ and all parameters used in the constraints are known. Problem 3 is indeed a *non-convex* optimization problem due to the constraint in (17). However, Problem 3 can be seen as a collision avoidance problem of controlling multi-agent systems, where each i-th plant (agent) follows the linear dynamics given by (13) subject to the input constraints in (14). Thus, one can utilize several existing tools for this problem to approximate the non-convex constraint into a convex one, such as sequential convex linear programming (see Augugliaro et al. (2000)). Moreover, one can also utilize a mixed integer linear programming approach to translate the collision avoidance constraint into the constraint involving integers, see e.g., Richards et al. (2002). If Problem 3 provides a feasible solution, then the obtained scheduling sequences $t^*_{i,0}, t^*_{i,1}, \cdots, t^*_{i,N_i}$ can ensure the feasibility of Problem 2, the convergence of the state trajectory to the local set Φ_i, and that no network collisions arise for all $i \in \mathcal{M}$. The following is thus an immediate consequence:

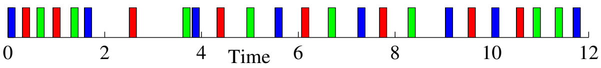
Theorem 1. Suppose that Problem 3 admits a feasible solution providing the optimal communication schedulings $t^*_{i,0}, t^*_{i,1}, \cdots, t^*_{i,N_i}$, for all $i \in \mathcal{M}$. Then, the obtained communication schedulings provide a solution to Problem 1.

5. SIMULATION RESULTS

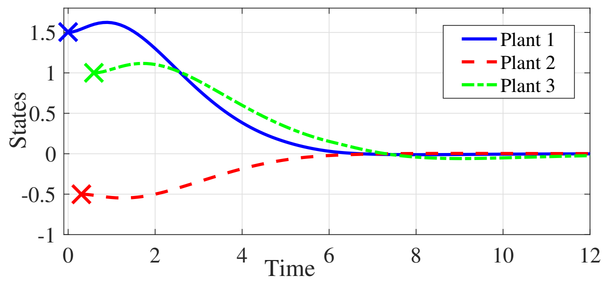
As a simulation example, we consider a problem of controlling three inverted pendulums (Wang and Lemmon (2011)). For simplicity, we linearize each system around

Table 1. Parameter settings

i	m_i	M_i	ℓ_i	$t_{i,0}$	$x_i(t_{i,0})$
1	0.2	1.0	1.5	0	[1.5; 0; 0; 0]
2	0.4	1.0	1.0	0.3	[-0.5; 0; 0; 0]
3	0.6	1.0	0.5	0.6	[1.0; 0; 0; 0]



(a) Communication schedulings for Plant 1 (blue), Plant 2 (red) and Plant 3 (green). Each color bar represents the communication time instant $t_{i,k}$ with minimal channel occupancy, i.e., $[t_{i,k}, t_{i,k} + \delta_{\min}]$, $i \in \{1, 2, 3\}$, $k \in \{1, \cdots, 8\}$.



(b) State trajectories of $x_{i,1}$ for all $i \in \{1, 2, 3\}$.

Fig. 5. Generated communication schedlings obtained by Problem 3 and the resulting state trajectories.

the origin to get $\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + w_i(t)$ where we denote $x_i = [x_{i,1};\ x_{i,2};\ x_{i,3};\ x_{i,4}]^\mathsf{T} \in \mathbb{R}^4$, $w_i \in \mathbb{R}^4$, $u_i \in \mathbb{R}$, and A_i and B_i are given by;

$$A_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -m_i g/M_i & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & g/\ell_i & 0 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 1/M_i \\ 0 \\ -1/(M_i \ell_i) \end{bmatrix}.$$

m_i, M_i, l_i, $i \in \{1, 2, 3\}$ are the point mass, mass of the cart, length of the mass-less rod respectively. We assume $\mathcal{U}_i = \{u_i \in \mathbb{R} \ : |u_i| \leq 8.0\}$ for all $i \in \{1, 2, 3\}$, and the parameters m_i, M_i, l_i, initial times $t_{i,0}$ and the initial states $x_i(t_{i,0})$ are given in Table 1. The matrices for the stage cost are $Q_i = 0.1 I_4$, $R_i = 0.05$ for all $i \in \{1, 2, 3\}$, and the prediction horizon is set to $T_p = 8$. We set $\gamma_i = 0.8$ for all $i \in \mathcal{M}$, and it is assumed that the time interval $\delta_{\min}$ is given by $\delta_{\min} = 0.15$. The matrix P_i and the local controller κ_i are computed appropriately by following the procedure presented in Chen and Allgöwer (1998). The parameters for the local sets Φ_i, Φ_{f_i} are given by $\varepsilon_i = 0.2$, $\varepsilon_{f_i} = 0.05$ for all $i \in \{1, 2, 3\}$, and we assume $\mathcal{W}_i = \{w_i \in \mathbb{R}^4 : ||w_i||_{P_i} \leq 4.2 \times 10^{-3}\}$ for all $i \in \{1, 2, 3\}$ based on the allowable size of disturbance given by (10). From Lemma 2, the maximal time when the state enters Φ_i is apriori obtained as $t_{i,0} + T^*_{i,0}/\gamma_i$, and these are illustrated in Table **??**. Based on this, we set $t_{i,\max} = 12$ and $N_i = 8$ for all $i \in \{1, 2, 3\}$.

Fig. 5(a) illustrates the communication schedulings up to the time $t_{i,\max} = 12$ obtained by solving Problem 3, where each color bar represents the time interval $[t_{i,k}, t_{i,k} + \delta_{\min}]$ in order to check no network collisions. While solving Problem 3, a sequential convex linear programming has been applied. From Fig. 5(a), the generated communication schedulings are feasible solution to Problem 3, in the sense that no network collisions are present until $t_{i,\max}$ and satisfy the terminal constraint in (16).

Fig. 5(b) represents the resulting state trajectories of $x_{i,1}, i \in \{1, 2, 3\}$ by solving Problem 2 according to the given communication schedulings. It is shown from Fig. 5(b), that all state trajectories are stabilized to Φ_i until the maximal convergence time.

6. CONCLUSIONS AND FUTURE WORK

In this paper, we proposed a scheduling algorithm for MPC of nonlinear continuous-time systems. The proposed method provides the network scheduling sequence for each plant, such that no network collisions arise, while feasibility and convergence can be guaranteed. A simulation example has validated our proposed method. Future work involves relaxing the constraint (10) to allow larger size of disturbances, and make some comparisons with other proposed schemes.

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