Abstract: This paper presents a novel hybrid control framework for the motion planning of a team of $N$ autonomous agents and $M$ objects under LTL specifications. We design feedback control laws for i) the navigation of the agents and ii) the transportation of the objects by the agents, among predefined regions of interest in the workspace, while ensuring inter-agent collision avoidance. This allows us to model the coupled behavior of the agents and the objects with a finite transition system, which can be used for the design of high-level plans that satisfy the given LTL specifications.

Keywords: Multi-agent systems, Robotic manipulators, Cooperative navigation, Nonlinear cooperative control, Temporal logics.

1. INTRODUCTION

Temporal-logic based motion planning has gained significant amount of attention over the last decade, as it provides a fully automated correct-by-design controller synthesis approach for autonomous robots. Temporal logics, such as linear temporal logic (LTL), provide formal high-level languages that can describe planning objectives more complex than the well-studied navigation algorithms, and have been used extensively both in single- as well as in multi-agent setups (Fainekos et al., 2009; Lahijanian et al., 2016; Diaz-Mercedes et al., 2015; Cowling and Zhang, 2016; Belta et al., 2015; Bhatia et al., 2011; Guo and Dimarogonas, 2015).

Most works in the related literature consider temporal logic-based motion planning for fully actuated, autonomous agents. Consider, however, cases where some unactuated objects must undergo a series of processes in a workspace with autonomous agents (e.g., car factories). In such cases, the agents, except for satisfying their own motion specifications, are also responsible for coordinating with each other in order to transport the objects around the workspace. When the unactuated objects’ specifications are expressed using temporal logics, then the abstraction of the agents’ behavior becomes much more complex, since it has to take into account the objects’ goals.

Another issue regarding the temporal logic-based planning in the related literature is the non-realistic assumptions that are often considered. In particular, many works either do not take into account continuous agent dynamics or do not take into account continuous agent dynamics or adopt single or double integrators (Fainekos et al., 2009; Bhatia et al., 2011; Guo and Dimarogonas, 2015), which can deviate from the actual dynamics of the agents, leading thus to poor performance in real-life scenarios. Moreover, many works adopt dimensionless point-mass agents and can deviate from the actual dynamics of the agents, leading to a high level of high level goals.

2. NOTATION AND PRELIMINARIES

Vectors and matrices are denoted with bold lowercase letters, respectively, whereas scalars are denoted with non-bold lowercase letters. The set of positive integers is denoted as $\mathbb{N}$ and the real $n$-space, with $n \in \mathbb{N}$, as $\mathbb{R}^n$; $\mathbb{R}^n_+$ and $\mathbb{R}^n_+$ are the sets of real $n$-vectors with all elements nonnegative and positive, respectively, and $\mathbb{T}^n$ is the $n$-D torus. Given a set $S$, $S^*$ is the set of all possible subsets of $S$, $|S|$ is its cardinality, and, given a finite sequence $s_1, \ldots, s_n$ of elements in $S$, with $n \in \mathbb{N}$, we denote by $\omega^i s_1, \ldots, s_n$ the infinite sequence $s_1, \ldots, s_n, s_1, \ldots, s_n, \ldots$ created by repeating $s_1, \ldots, s_n$. The notation $\|y\|$ is used for the Euclidean norm of a vector $y \in \mathbb{R}^n$. Given $x \in \mathbb{R}$ and $y, z \in \mathbb{R}^n$, we use $\nabla_x y = \partial y / \partial x \in \mathbb{R}^n$ and $\nabla_z y = \partial y / \partial z \in \mathbb{R}^{n \times n}$. $B_r(c)$ denotes the ball of radius $r \in \mathbb{R}^n_+$ and center $c \in \mathbb{R}^n$. Finally, we use $\mathcal{N} = \{1, \ldots, N\}, M = \{1, \ldots, M\}, K = \{1, \ldots, K\}$, with $N, M, K \in \mathbb{N}$, as well as $\mathbb{M} = \mathbb{R}^3 \times \mathbb{T}^3$.

We focus on the task specification $\phi$ given as a Linear Temporal Logic (LTL) formula. The basic ingredients of a LTL formula are a set of atomic propositions $\mathcal{AP}$ and several boolean and temporal operators. LTL formulas are formed according to the following grammar (Baier et al., 2008): $\phi ::= \text{true} | a | \phi_1 \land \phi_2 | \phi | \diamond \phi_1 | \phi_1 \cup \phi_2$, where $a \in \mathcal{AP}$ and $\diamond$ (next) and $\phi$ (until). Definitions of other useful operators like $\square$ (always), $\diamond$ (eventually) and $\Rightarrow$ (implication) are omitted and can be found at (Baier et al.,...
The semantics of LTL are defined over infinite words over $2^{AP}$. Intuitively, an atomic proposition $\psi \in AP$ is satisfied on a word $w = w_1w_2\ldots$ if it holds at its first position $w_1$, i.e., $\psi \in w_1$. Formula $\emptyset \phi$ holds true if $\phi$ is satisfied on the word suffix that begins in the next position $w_2$, whereas $\phi_1 \cup \phi_2$ states that $\phi_1$ has to be true until $\phi_2$ becomes true. Finally, $\bigwedge \phi$ holds on $w$ eventually and always, respectively. For a full definition of the LTL semantics, the reader is referred to (Baier et al., 2008).

3. SYSTEM MODEL AND PROBLEM FORMULATION

Consider $N$ robotic agents operating in a workspace $W$ with $M$ objects: $W$ is a bounded sphere in 3D space, i.e., $W = B_{r_0}(p_0) = \{p \in \mathbb{R}^3 \ s.t. \|p - p_0\| \leq r_0\}$, where $p_0 \in \mathbb{R}^3$ and $r_0 \in \mathbb{R}_{>0}$ are the center and radius, respectively, of $W$. The objects are represented by rigid bodies whereas the robotic agents are fully actuated and consist of a moving part (i.e., mobile base) and a robotic arm, having, therefore, access to the entire workspace. Within $W$ there exist $K$ smaller spheres around points of interest, which are described by $\pi_k \in \pi = \{\pi_k\}_{k=1}^K \subset \mathbb{R}^3$. The regions of interest are sufficiently large to contain an object along with an agent, in a collision-free configuration.

Similarly to the robotic agents, we denote as $p_{oj} : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^3$, $\eta_{oj} : \mathbb{R}_{\geq 0} \rightarrow \mathbb{T}^3$, the position and Euler-angle orientation of object $j \in \mathcal{M}$, which obeys the second order dynamics:

$$M_{oj}(x_{oj})\ddot{x}_{oj} + C_{oj}(x_{oj}, \dot{x}_{oj})\dot{x}_{oj} + g_{oj}(x_{oj}) = f_{oj},$$

where $x_{oj} = [p_{oj}^T, \omega_{oj}^T]^T \in \mathbb{R}^6, v_{oj} = [\dot{p}_{oj}^T, \omega_{oj}^T]^T \in \mathbb{R}^6, M_{oj} : \mathbb{M} \rightarrow \mathbb{R}^{6 \times 6}$ is the positive definite inertia matrix, $C_{oj} : \mathbb{M} \times \mathbb{R}^6 \rightarrow \mathbb{R}^6$ and $g_{oj} : \mathbb{M} \rightarrow \mathbb{R}^6$ are the Coriolis and gravity terms, respectively, and $f_{oj} \in \mathbb{R}^6$ is the vector of generalized forces acting on the object’s center of mass. In case of a rigid grasp between agent $i$ and object $j$, $f_i$ and $f_{oj}$ are related through $f_i = G_{ij}(q_i)f_{oj}$, where $G_{ij} : \mathbb{R}^n \rightarrow \mathbb{R}^6$ is the full-rank grasp matrix. The aforementioned inertia and Coriolis matrices satisfy the skew-symmetric property (Siciliano et al., 2010) $\dot{B}_i \mathbf{2} \mathbf{N}_i = - (\dot{B}_i \mathbf{2} \mathbf{N}_i)^T, M_{oj} - 2C_{oj} = - (M_{oj} - 2C_{oj})^T, \forall i \in \mathcal{N}, j \in \mathcal{M}$.

We now provide the following definitions:

Definition 1. Agent $i \in \mathcal{N}$ is in region $\pi_k, k \in \mathcal{K}$, at a configuration $q_i$, denoted as $A_i(q_i) \in \pi_k$, if $\|p_i - p_k\| \leq r_k - \varepsilon, \forall p \in \Omega^{k\ast}(q_i), \varepsilon > 0$ arbitrarily small.

Definition 2. Object $j \in \mathcal{M}$ is in region $\pi_k, k \in \mathcal{K}$, at a configuration $x_{oj}$, denoted as $O_j(x_{oj}) \in \pi_k$, if $\|p - p_k\| \leq r_k - \varepsilon, \forall p \in \Omega^{k\ast}(x_{oj}), \varepsilon > 0$ arbitrarily small.

In the following, we use the notation $\Omega^{k\ast}(q_i(t_0)) \cap \Omega^{j\ast}(x_{oj}(t_0)) = \emptyset, \Omega^{k\ast}(q_i(t_0)) \cap \Omega^{k\ast, \nu}(x_{oj}(t_0)) = \emptyset, \Omega^{k\ast, \nu}(x_{oj}(t_0)) \cap \Omega^{j\ast}(q_i(t_0)) = \emptyset, i, i', j, j' \in \mathcal{N},$ with $i \neq i', j \neq j'$, to describe collision-free cases at $t_0$ between the agents and the objects. In order for our workspace discretization to be valid, we need the following assumption, which implies that all regions of interest are sufficiently large to contain an object along with an agent, in a collision-free configuration.
Assumption 1. There exist $q_k^i, x_k^j$ s. t. $A(q_k^i), O(x_k^j) \in \pi_k$ and $\Omega i, j (q_k^i, x_k^j) = \emptyset$, $\forall i \in N, j \in M, k \in K$.

We also use the boolean variable $AG_i(t^*)$ to denote whether agent $i \in N$ rigidly grasps an object $j \in M$ at the time instant $t^*$, $AG_i(t^*) = \top$ denotes that agent $i$ does not grasp any object. Note that $AG_i(t^*) = \top, \ell \in \{0\} \cup M \Leftrightarrow AG_i(t^*) = \bot, \forall j \in \{0\} \cup M \{\ell\}$ (i.e., agent $i$ can grasp at most one object at a time) and $\Omega i, j (q_k^i) \cap \Omega j, i (x_k^j) = \emptyset, \forall i \in N, j \in M$ such that $Osv(x_{Osv}(t_0)) \in \pi_k, Oxg(x_{Oxg}(t_0)) \in \pi_g, Oyq(x_{Oyq}(t_0)) \in \pi_y, sv \in AG_i(t_0), sv \neq 0, t_{m+1} \geq t_m \geq 0, \forall m \in N$. The sequences $\sigma_i, \sigma_j$ are the services provided to the agent and the object, respectively, over their trajectories, i.e., $\sigma_i, \sigma_j \in 2^{\Omega_i, k}$ with $\Omega_i(q_k(t_m)) \in \pi_k, \sigma_i \in \bar{L}_i(\pi_k), O_j(x_{O_j}(t_m)) \in \pi_j, \forall i, j \in N, k \in M, \forall l \in L_i, L_j$, as defined in Section 3.

Definition 6. The behaviors $\beta_i, \beta_j$ satisfy formulas $\phi_i, \phi_j$, iff $\sigma_i = \sigma_j = \phi_i, \phi_j$, respectively.

The control objectives are given as LTL formulas $\phi_i, \phi_j$ over $\Psi_i, \Psi_j$, respectively, $\forall i \in N, j \in M$. The LTL formulas $\phi_i, \phi_j$ are satisfied if there exist behaviors $\beta_i, \beta_j$ of agent $i$ and object $j$ that satisfy $\phi_i, \phi_j$. Formally, the problem treated in this paper is the following:

Problem 1. Consider $N$ robotic agents and $M$ objects in $\mathbb{W}$ subject to the dynamics (1) and (3), respectively, and (i) $\Omega i, j (q_k(0)) \cap \Omega j, i (x_k(0)) = \emptyset$, (ii) $q_0 = 0, \forall i \in N$, (iii) $A_i(q_0(t_m)) = \pi_k, O_i(x_{O_i}(t_m)) = \pi_g, \forall i, j \in N, k \in M$. For a given $t_m, \forall i \neq j, \exists i \neq j$, $t_m \in T$. Given the disjoint set $\Psi_i, \Psi_j$, $\forall i \in N, j \in M$, LTL formulas $\phi_i$ over $\Psi_i$ and $M$ LTL formulas $\phi_j$ over $\Psi_j$, develop a control strategy that achieves behaviors $\beta_i, \beta_j$, which yield the satisfaction of $\phi_i, \phi_j, \forall i \neq j, j \in M$.

4. MAIN RESULTS

Continuous Control Design: The first ingredient of our solution is the development of feedback control laws that establish agent transitions and object transportations as defined in Def. 3 and 5, respectively. Regarding the grasping actions of Def. 4, we assume that there exists a methodology that derives the corresponding control laws.

Transformation to Point World: In this work we employ the algorithm proposed in (Tanner et al., 2003) to create point worlds. In particular, there exists a sequence of smooth transformations on the rigid body ellipsoids, introduced in Section 3, that creates spaces where the robotic agents and the objects are represented by points. Since 3D spheres are a special case of 3D ellipsoids, we also consider the regions of interest as obstacles that will be transformed to points. For details on the transformation, the reader is referred to (Tanner et al., 2003).

Assume that the conditions of Problem 1 hold for some $t_0 \in \mathbb{R}^o$, i.e., all agents and objects are located in regions of interest and there is no more than one agent or one object at the same region. We design a control law such that a subset of agents performs a transition between two regions of interest and another subset of agents performs object transportation, according to Def. 3 and 5, respectively. Let $Z, V, G, Q \subseteq N$ denote disjoint sets of agents corresponding to transition, transportation, grasping and releasing actions, respectively, with $|V| + |G| + |Q| \leq |M|$ and $A_i(q_k(t_m)) \in \pi_k, A_2(q_k(t_m)) \in \pi_k, A_3(q_k(t_m)) \in \pi_k, A_4(q_k(t_m)) \in \pi_k, A_5(q_k(t_m)) \in \pi_k, A_6(q_k(t_m)) \in \pi_k, A_7(q_k(t_m)) \in \pi_k, A_8(q_k(t_m)) \in \pi_k, A_9(q_k(t_m)) \in \pi_k, \forall i, j \in N, k \in M, \forall l \in L_i, L_j, L_k, L_m, L_n, L_s, l \in K, \forall z \in Z, v \in V, g \in G, q \in Q$. Note that there might be idle agents in some regions, not performing any actions, i.e., $Z \cup V \cup G \cup Q \subseteq N$. Let also $S = (\{s_v\}, \mathcal{N}) = \{s_v, l, q\} \subseteq \mathcal{M}$ such that $O_k(x_{O_k}(t_0)) \in \pi_k, O_x(x_{O_x}(t_0)) \in \pi_k, \forall v \in V$.
\[ S, x_g \in X, y_g \in Y, \forall v \in V, g \in G, q \in Q, \text{i.e., there exists one object at each } \pi_{kv}, \pi_{kg}, \pi_{kq}. \text{ Moreover, assume that the conditions of Def. 3 hold for all } z \in \mathbb{Z}, \text{ the conditions of Def. 5 hold for all } v \in V \text{ and } s_i \in S, \text{ the conditions of Def. 4 hold for all } g \in G \text{ and } x_g \in X, \text{ and the corresponding release conditions (which are omitted due to space limitations), hold for all } q \in Q \text{ and } y_q \in Y. \] In the following, we design \( \tau \) and \( \tau_q \) such that \( \pi_{kv} \rightarrow \pi_{k'}v \) and \( \pi_{kg} \rightarrow \pi_{k'}g \), where \( \pi_{k'}v \) and \( \pi_{k'}g \) are \( \gamma \)-regularity ellipsoids, as analyzed above. Then, by employing the continuous dependence of rigid grasp between agent \( v \) and object \( s \) to space limitations, hold for all \( \tau \in \tau \) and the conditions of Def. 5 hold for all \( v \in V, s_i \in S \) for \( t_0 = 0, k_z, k_v \in K \). Then the control protocols (5) to form the “obstacle” function, we adopt the notion of proximity relations of (Loizou and Kyriakopoulos, 2006), which are all the possible collision schemes between the aforementioned transformed points. A measure of the distance for each \( r_v \) and its obstacles is the function \( \delta_{p,obs} : \mathbb{R}^n \rightarrow \mathbb{R} \geq 0 \) with \( \beta_{p,obs}(q) = \| h^{\beta_{p,obs}}(q) - h_{p,obs}(\bar{q}) \|^2 \), \( o \in N_{p,obs} \). By considering the relation proximity function, which represents the sum of all distance measures in a specific relation between the transformed points, we can define the relation verification function (RVF), as in (Loizou and Kyriakopoulos, 2006). Then, the total “obstacle” function \( \delta_{obs} : \mathbb{R}^n \rightarrow \mathbb{R} \geq 0 \) is the product of the RVFs for all relations and resembles the possible collision schemes between all \( r_v, i \in \{z, v\} \), \( z \in \mathbb{Z}, v \in V \), and the corresponding obstacles. For more details on the technique, the reader is referred to (Loizou and Kyriakopoulos, 2006). Regarding the workspace boundaries, we form the function \( \delta_{p} : \mathbb{R}^n \rightarrow \mathbb{R} \geq 0 \), with \( \delta_{p}(q) = \| \bar{p}_{obs}(q) - p \|^2 \), that represents an over-approximation of the distance ellipsoid \( R_{p,obs} \) from the workspace boundary, with \( p_{obs} \) being the ellipsoid’s center. Then, \( \delta^\ast : \mathbb{R}^n \rightarrow \mathbb{R} \geq 0 \), with \( \delta^\ast(q) = \sum_{p \in \mathbb{P}} \sum_{e \in \mathbb{E}} \delta_{p,obs}(q) \), encodes the distance of agent \( i \) from the workspace boundaries. We construct now the following multi-agent navigation function \( \varphi : \mathbb{R}^n \rightarrow [0,1] \) (Rimon and Koditschek, 1992; Loizou and Kyriakopoulos, 2006), that incorporates the desired behavior of the agents:

\[
\varphi(\bar{q}) = \frac{\gamma(\bar{q})}{(\gamma(\bar{q}) + \gamma_{obs}(\bar{q}) \prod_{i \in \mathcal{V} \cup \mathcal{U}} \delta^i(\bar{q})))^T},
\]

where \( \kappa \in \mathbb{R} \geq 0 \) and \( \gamma : \mathbb{R}^n \rightarrow \mathbb{R} \geq 0 \) is defined as \( \gamma(q) = \sum_{v \in \mathcal{V}} \gamma_{v,k}(q) + \sum_{e \in \mathcal{E}} \gamma_{v,e}(q) \). Note that, a sufficient condition for avoidance of the undesired regions and avoidance of collisions and singularities is \( \varphi < 1 \).

Next, we design the control protocols \( \tau : [t_0, \infty) \rightarrow \mathbb{R}^n, \tau_q : [t_0, \infty) \rightarrow \mathbb{R}^n \):

\[
\tau_z(t) = g_i(q_z) - \nabla_q \varphi(q) - K_z \dot{q_z}(t), \quad (5a)
\]

\[
\tau_v(t) = g_{v,s_v}(q_v) - \nabla_q \varphi(q) - K_v \dot{q}_v(t), \quad (5b)
\]

\( \forall z \in \mathcal{Z}, v \in \mathcal{V}, \text{ where } K_i = \text{diag}(k_i) \in \mathbb{R}^{n_i \times n_i}, k_i \in \mathbb{R}^{n_i \times n_i}, \text{ is a constant positive definite gain matrix, } k_i \in \mathbb{R}^{n_i \times n_i}, \text{ is the coupled agent-object gravity vector } \forall i \in \mathcal{N}, j \in \mathcal{M}. \text{ In the same vein, we also define the coupled matrices } B_{i,j} = B_i + G_i^T M_{ij} G_{i,j}, \quad \dot{N}_{i,j} = N_i + G_i^T M_{ij} \dot{G}_{i,j} + (G_{i,j})^T C_{ij} G_{i,j}, \quad \dot{G}_{i,j} = G_{i,j}^T J_i, \quad \forall i \in \mathcal{N}, \forall j \in \mathcal{M}. \text{ The following Proposition is needed for the subsequent analysis:}

Proposition 1. The matrix \( \tilde{B}_{ij} \) is positive definite and the matrix \( \tilde{B}_{ij} - 2 \tilde{N}_{ij} \) is skew-symmetric, \( \forall i \in \mathcal{N}, \forall j \in \mathcal{M}. \)

Proof: The proof can be found in (Verginis and Dimarogonas, 2017).

Lemma 1. Consider the sets of agent \( Z, V \) and the set of objects \( S \) as defined above, described by the dynamics (1) and (3), such that the conditions of Def. 3 hold for all \( z \in Z \) and the conditions of Def. 5 hold for all \( v \in V, s_i \in S \) for \( t_0 = 0, k_z, k_v \in K \). Then the control protocols (5)
guarantee that $π_k z →z π_{k′} z$ and $π_k v \rightarrow v_{s′} π_{k′} v_{, k′}$, $v \in V$, according to Def. 3 and 5, respectively.

**Proof.** The proof can be found in (Verginis and Dimarogonas, 2017).

**Remark 1.** During the transitions $π_k z → z π_{k′} z$, once agent $z$ leaves $π_k z$, there is no guarantee that it will not enter it again until it reaches $π_{k′} z$. The same holds for $π_k T v → v_{s′} π_{k′} v_{, k′}$, $v \in V$, as well. For that reason, we can modify (4) to include continuous switchings to a navigation controllers that avoid $π_k (or π_{k′})$, once agent $z$ (or $v$) is out of it, as in (Guo and Dimarogonas, 2015).

Considering the agents $g \in G, q \in Q$ that perform grasp and release actions, note that there exist positive and finite time instants $t_f, t' _f > t_0$ that these actions will be completed, $\forall g \in G, q \in Q$. We define $t_f = \max \{\max_{g \in G} \{t_f\}, \max_{q \in Q} \{t_f\}, \max S\}$, where $S = \{t ≥ t_0 s.t. A_i(q(t)) ∈ π_k, A_o(q(t)), O_s(x_o(t)) ∈ π_{k′}, \forall z ∈ Z, \forall v ∈ V\}$, which represents a time instant that all the agents $i ∈ I$ will have completed their respective action. Therefore, by choosing $t_f > t_0$, we can define a new set of actions to be executed by the agents, starting at $t_f$ (i.e., the conditions of Def. 3-5 hold at $t_0$ instead of $t_0$). In this way, we add a notion of synchronization to our system, since each (non-idle) agent, after completing an action, will wait for all other agents to complete their own, so that they start the next set of actions simultaneously.

**High-Level Plan Generation:** The second part of the solution is the derivation of a high-level plan that satisfies the given LTL formulas $φ_i$ and $φ_o$ and can be generated by using standard techniques from automata-based formal verification methodologies. Thanks to (i) the proposed control laws that allow agent transitions and object transportations $π_k → π_{k′}$ and $π_k → i, j π_{k′}$, respectively, (ii) the assumed control laws that guarantee grasp and release actions $i → j$ and $i → j$, respectively, and (iii) the formulation for the synchronization of actions, we can abstract the behavior of the agents using a finite transition system as presented in the sequel.

**Definition 7.** The coupled behavior of the overall system of all the $N$ agents and $M$ objects is modeled by the transition system $\mathcal{T}S = (Π, Π^∞ → \mathcal{A}, Ψ, \mathcal{L})$, where

(i) $Π = Π_N × Π_o × AG$ is the set of states; $Π = Π_1 × Π_2 × \cdots × Π_N$, where $Π_N = Π_o = Π_i = Π_o$ are the set of regions of the states and the objects can be at, respectively.

(ii) The initial set of states at $t = 0$, which, owing to (i), satisfies the conditions of Problem 1.

(iii) $π_k \rightarrow \mathcal{A}$ is a transition relation defined as follows: given the states $π_k π_{k′} ∈ Π$, with $π_k = (π_k, π_{k′})$, $π_{k′} = (π_{k′}, π_{k′′}, \cdots, π_{k′′′}, \cdots)$, $π_{k′′} = (π_{k′′}, π_{k′′′}, \cdots)$, $w_1, \cdots, w_N$, a transition $π_k → π_{k′}$ occurs iff there exist disjoint sets $\mathcal{Z}, \mathcal{Y}, \mathcal{G}, \mathcal{Q} ∈ N$ with $|\mathcal{Z}| + |\mathcal{G}| + |\mathcal{Q}| ≤ |M|$ and $\mathcal{S} = \{\{s_z\}_{z \in \mathcal{Z}}, \mathcal{X} = \{\{x_g\}_{g \in \mathcal{G}}, \mathcal{Y} = \{\{y_q\}_{q \in \mathcal{Q}}\} \in \mathcal{M}$ s.t.:

1. $w_z = w_′_z = AG.z = \bar{T}$ and $π_k z → z π_{k′} z$, $\forall z ∈ \mathcal{Z}$,
2. $π_k = π_k o_{z, q}$, $π_{k′} = π_k o_{z, q}$, $w_0 = w′_0 = AG.\mathcal{S} = \bar{T}$ and $π_k z → z π_{k′} z$, $\forall v ∈ V$.
3. $π_k = π_k o_{z, q}$, $π_{k′} = π_k o_{z, q}$, $w_0 = w′_0 = AG.\mathcal{S} = \bar{T}$ and $π_k z → z π_{k′} z$, $\forall v ∈ V$.
4. $π_k = π_k o_{z, q}$, $π_{k′} = π_k o_{z, q}$, $w_0 = w′_0 = AG.\mathcal{S} = \bar{T}$ and $q → \mathcal{Q}$, $\forall q ∈ \mathcal{Q}$.

(iv) $Ψ = Ψ_1 U Ψ_2$ with $Ψ = Ψ_1 U Ψ_2$ and $Ψ_o = Ψ_{1,M} Ψ_o$. are the atomic propositions of the agents and objects, respectively, as defined in Section 3.

(v) $\mathcal{L}: Π → 2^\mathcal{P}$ is a labeling function defined as follows: Given a state $π$ and $ψ = Ψ_1 U Ψ_2$ with $ψ_i = \varphi_i \varphi_2$, $ψ_o = \varphi_3 \varphi_4$, then $ψ = \mathcal{L}(π)$ if and only if $ψ_i(π_i)$ and $ψ_o(π_o)$, $\forall i, j ∈ M$.

Next, we form the model global LTL formula $φ = (λ_i ∈ N, φ) \land (λ_o ∈ M, φ_o)$ of the set $Ψ$. Then, we translate the $φ$ to a Buchi Automaton $\mathcal{B}A$ and we build the product $\mathcal{T}S × \mathcal{B}A$. The accepting runs of $\mathcal{T}S$ satisfy $φ$ and are directly projected to a sequence of desired states to be visited in $\mathcal{T}S$. Although the semantics of LTL are defined over infinite sequences of services, it can be proven that there always exists a high-level plan that takes the form of a finite state sequence followed by an infinite repetition of another finite state sequence. For more details on the followed technique, the reader is referred to the related literature, e.g., (Baier et al., 2008).

Following the aforementioned methodology, we obtain a high-level plan as sequences of states and atomic propositions $π = (π^p, π^m, \cdots)$ and $ψ = ψ_1ψ_2\cdots$, with $ψ_i = (z^m, \bar{z}^m, \bar{w}^m)$, $\forall i, j ∈ M$, $\forall m ∈ N, \forall j ∈ M$. The aforementioned sequences determine the behavior of agent $i ∈ N$, i.e., the sequence of actions (transit, transportation, grasp, release or stay idle) it must take.

By the definition of $\mathcal{L}$ in Def. 7, we obtain that $\psi_1 ψ_2\cdots ψ_i \in \mathcal{L}(\pi^p)$, $\psi_o \in \mathcal{L}(\pi^m)$, $\forall i, j ∈ M, m ∈ N$. Therefore, since $φ = (λ_i ∈ N, φ) \land (λ_o ∈ M, φ_o)$ is satisfied by $ψ$, we conclude that $ψ_i = (ψ_1 ψ_2\cdots ψ_i \cdots) = φ_i$ and $ψ_o = (ψ_1 ψ_2\cdots ψ_o \cdots) = φ_o$, $\forall i, j ∈ M, m ∈ N$.

The sequence of the states $(π_{k, 1}, π_{k, 2}, \cdots, π_{k, N})$ and $(\pi_{o, 1}, \pi_{o, 2}, \cdots, \pi_{o, N})$ over (II,$2^\mathcal{P}$) and (II,$2^\mathcal{P}$), respectively, produces the trajectories $q_i(t)$ and $\pi_{o, i}(t), \forall i, j ∈ M$. The corresponding behaviors are $β_i = (q_i(t_1), \sigma_i) ∈ (q_{init}, q_2, q_{init}, q_{init}, \cdots)$ and $\beta_o = (\pi_{o, 1}(t_1), \sigma_o) = (\pi_{o, 1}(t_2), \sigma_o, \cdots)$, respectively, according to Section 3, with $A_i(q_i(t_{init})), O_s(x_o(t_{init})), \sigma_o = (\pi_{o, 1}(t_{init}), \sigma_o, \cdots)$, respectively, in each region.
and consequently, the behaviors $\beta_i$ and $\beta_{Oj}$ satisfy the formulas $\phi_i$ and $\phi_{Oj}$, respectively, $\forall i \in N, j \in M$. The aforementioned reasoning is summarized as follows:

**Theorem 2.** The execution of the path (p, v) of $\mathcal{T}S$ guarantees behaviors $\beta_i, \beta_{Oj}$ that yield the satisfaction of $\phi_i$ and $\phi_{Oj}$, respectively, $\forall i \in N, j \in M$, providing, therefore, a solution to Problem 1.

**Remark 2.** Note that although the overall set of states of $\mathcal{T}S$ increases exponentially with respect to the number of agents/objects/regions (the maximum number of states is $K^{N+M}(M+1)^M$), some states are either not reachable or simply removed by the constraints set of Def. 7, reducing the state complexity.

5. SIMULATION RESULTS

To demonstrate the proposed methodology, we consider a simplified scenario involving $N = 2$ agents, $M = 1$ object in a workspace with $p_0 = [0, 0, 0]^T m, r_0 = 6m$, and $K = 2$ regions of interest $\pi_1, \pi_2$, with $p_{\pi_1} = [-2, -3, 0.2]^T m, p_{\pi_2} = [2, 3, 0.2]^T m, r_{\pi_1} = r_{\pi_2} = 1m$. The object is a rigid cube of dimensions $0.1 \times 0.1 \times 0.1 m^3$ and each agent consists of a cubic mobile base of dimensions $0.3 \times 0.3 \times 0.3 m^3$, able to move on the $x-y$ plane, and two rigid rectangular links of dimensions $0.05 \times 0.05 \times 0.3 m^3$ connected by a cylindrical joint rotating around the negative y-axis. The generalized variables for each agent are taken as $q_i = [x_{c_i}, y_{c_i}, \theta_i]^T \in \mathbb{R}^3, i \in \{1, 2\}$, where $[x_{c_i}, y_{c_i}]^T$ is the base’s center of mass and $\theta_i$ is the joint’s angle. The initial conditions are taken such that $A_1(q_1(0)), A_2(q_2(0)) \in \pi_{\pi_1}, A_2(q_3(0)) \in \pi_{\pi_2}$ and $\Omega^{-1}(x_{c_1}(0)) \cap \Omega^{-1}(x_{c_2}(0)) = \emptyset$. The resulting $\mathcal{T}S$ is depicted in Fig. 1, where we show each state $\pi_{s_j}$ in the form $(\pi_{\pi_1}, \pi_{\pi_2}, \pi_{\pi_3}, \mathcal{AG}_{1,0}, \mathcal{AG}_{2,0})$, as depicted with thick blue color in the figure, with $j \in \{0, 1\}$. The initial state is $\Pi_{s_0}^{\text{init}} = (\pi_{k_1}, \pi_{k_2}, \pi_{k_3}, w_1^1, w_2^1)$ (depicted with green color in the figure). Note that, due to our restriction that no more than one agent is allowed to be in the same region, the number of states is reduced from $K^{N+M}(M+1)^N \geq 32$ to 16. Moreover, since an agent $i$ cannot have a grasp with object $j$ if $\pi_{s_j} \neq \pi_{s_{Oj}}, \forall m \in N$, some states are not reachable, and thus the number of states is further reduced to 8. We also consider the atomic propositions $\Psi_1 = \{\text{“red”, “blue”}, \Psi_2 = \{\text{“green”, “yellow”}\}$ and $\Psi_{Oj} = \{\text{“Goal}_1, \text{“Goal}_2\}$, with $L_1(\pi_1) = \{\text{“red”}\}, L_1(\pi_2) = \{\text{“blue”}\}, L_2(\pi_1) = \{\text{“green”}\}, L_2(\pi_2) = \{\text{“yellow”}\}$, and $L_{Oj}(\pi_1) = \{\text{“Goal}_1\}$, $L_{Oj}(\pi_2) = \{\text{“Goal}_2\}$. The formulas to be satisfied by the agents and the object are the following: $\phi_1 = \Box (\text{“red”} \land \Box \text{“blue”}), \phi_2 = \Box (\text{“green”} \land \Box \text{“red”}), \phi_{Oj} = \Box (\text{“Goal}_1 \land \Box \text{“Goal}_2$) by following the procedure of Section 4, we obtain a path satisfying $\phi = \phi_1 \land \phi_2 \land \phi_{Oj}$ as $\pi_1^s \rightarrow \pi_1^\omega$, $\pi_2^s \rightarrow \pi_2^\omega$, and $\pi_{s_{Oj}}^s \rightarrow \pi_{s_{Oj}}^\omega$, which includes transitions, grasping/releasing as well as transportation actions from both agents. Fig. 2 depicts two indicative transitions, namely, $\pi_1^s \rightarrow \pi_1^\omega, \pi_2^s \rightarrow \pi_2^\omega, \pi_{s_{Oj}}^s \rightarrow \pi_{s_{Oj}}^\omega$.

6. CONCLUSION

We have presented a novel hybrid control framework for the motion planning of a system comprising of $N$ agents and $M$ objects. Future works will address decentralization of the framework as well as cooperative transportation of the objects by agents with limited sensing information.

REFERENCES


![Fig. 1. The transition system $\mathcal{T}S$. The information in each state is depicted according to the state with thick blue color. The initial state is colored with green and the unreachable states are omitted.](image1)

![Fig. 2. The transitions $\pi_2^s \rightarrow \pi_2^\omega$ and $\pi_3^s \rightarrow \pi_3^\omega$ in (a) and (b), respectively.](image2)