Topology Estimation for Open Multi-Agent Systems ⋆

Nana Wang *, **Pelin Sekercioglu** *, **Dimos V. Dimarogonas** *

* *KTH Royal Institute of Technology, SE-100 44 Stockholm, Sweden (e-mail: {nanaw, pelinse, dimos}@kth.se).*

Abstract: We address the problem of interaction topology identification in open multi-agent systems (OMAS) with dynamic node sets and fast switching interactions. In such systems, new agents join and interactions change rapidly, resulting in intervals with short dwell time and rendering conventional segment-wise estimation and clustering methods unreliable. To overcome this, we propose a projection-based dissimilarity measure derived from a consistency property of local least-squares operators, enabling robust mode clustering. Aggregating intervals within each cluster yields accurate topology estimates. The proposed framework offers a systematic solution for reconstructing the interaction topology of OMAS subject to fast switching. Finally, we illustrate our theoretical results via numerical simulations.

Keywords: Topology estimation, Open multi-agent systems, Time segment clustering

1. INTRODUCTION

Open multi-agent systems (OMAS) are networks, where agents can join or leave and modify their interaction over time. They naturally encompass a broad family of physical and biological systems, including social (Hendrickx and Martin, 2016; Franceschelli and Frasca, 2020), biological (Dimitrov et al., 2024), and engineered systems (Restrepo et al., 2022). In several works on OMAS (see *e.g.,* Xue et al. (2022); Restrepo et al. (2022); Şekercioğlu et al. (2025)), openness is captured using the switched system representation, where the system switches modes whenever at least one node or edge is added or removed. The underlying topological structure plays a central role in shaping system-level properties and its collective behaviors. However, all of the previously cited works assume that the network is *known*. This motivates the need for topology estimation methods capable of handling uncertain, time-varying, and structurally evolving networks.

Relevant insights stem from the literature on identification of switched linear systems, which provides a conceptual framework for analyzing systems characterized by multiple (interaction) modes. In this context, a mode represents a communication graph, and the switch occurs among different modes. The works in (Massucci et al., 2021) and (Massucci et al., 2022) investigated switched discrete-time linear systems and derived identification guarantees using statistical learning theory. In their approach, the system matrix is first estimated for each time segment (when one mode dwells consecutively until the next mode is executed) and the resulting matrices are clustered based on their pairwise distances. The system matrix for each mode is then re-estimated using all associated segments, and this procedure is repeated until convergence. However, since short time segments contain limited information, estimating the system matrix directly from each time segment is inaccurate. Hence, if two time segments share the same system matrix, their limited data can produce substantially different least-squares estimates, leading to incorrect estimation and mode assignments, and causing poor performance. Several works (Sun et al., 2023) has further examined the identifiability and recovery of switching network topologies. These works established conditions under which the topology in each mode can be uniquely identified. (Rey et al., 2025) studied the online estimation problem for expanding graphs via regularized least-squares methods promoting temporal smoothness. Although these contributions significantly advance the understanding of time-varying and switching networks, they generally assume fixed or monotonically expanding node sets, or rely on slowly varying topology changes. Such assumptions limit their applicability in OMAS characterized by rapid agent arrivals, departures, and abrupt interaction changes.

Motivated by the need to develop new topology estimation methods for OMAS, we simultaneously consider dynamic node-set variations and fast switching interactions. A key challenge in this setting is that each short time segment contains limited information, rendering direct segment-wise topology estimation unreliable. Thus, our contributions are threefold. First, we propose algorithms that estimate each mode by aggregating information across all time segments in which the same mode occurs, in contrast to (Sun et al., 2023), which relies solely on one time segment estimates obtained. Second, we introduce a new distance criterion, called projection-based dissimilarity measure, that compares time segments without requiring accurate topology estimates for each time segment. This criterion enables robust clustering of time segments belonging to the same mode, even under sparse observations and rapidly changing agent sets. Third, building on these ideas, we develop a two-stage topology reconstruction framework that first clusters time segments according to their latent modes and then aggregates information within each cluster

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to estimate the topology associated with each mode. This framework provides a resilient solution for reconstructing the interaction topologies in OMAS with dynamic and evolving population.

Notations. $\mathbb{R}^{n\times m}$, $\mathbb{R}_{\geq 0}$, and $\mathbb{N}^{+}$ denote the sets of real matrices, non-negative real numbers, and positive integers, respectively. For $x \in \mathbb{R}$, $\lfloor x \rfloor$ denotes the largest integer less than or equal to x. For matrices X and Y, $X \succ 0$ indicates that X is positive definite, and $X \succ Y$ means $X - Y \succ 0$. For $A \in \mathbb{R}^{n\times m}$, $\ker(A)$ and $\mathrm{ran}(A)$ denote its null space and range, respectively. $A^\top$ denotes the transpose of A, and $\mathrm{vec}(A)$ stacks its columns into a vector. $\|\cdot\|_F$ denotes the Frobenius norm, and $|S|$ denotes the cardinality of a set S.

2. PROBLEM FORMULATION

Let $\mathcal{G} = \big(\mathcal{V}, \mathcal{E}, W\big)$ be a directed weighted graph representing a mode, where $\mathcal{V} = \{1, 2, \ldots, N\}$ is the set of nodes corresponding to the agents, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set, and W denotes the edge weights. We consider a multi-agent system with time-varying interactions, where agents may join or leave and interconnections can change over time. The time-varying interactions are represented by a switched system representation, in which each mode corresponds to a fixed topology and switching occurs when the interactions change. The switching instants $0 = t^0 < t^1 < \cdots < t^\kappa < t^{\kappa+1} = +\infty$ correspond to events where (i) a new agent joins, (ii) an existing agent leaves, or (iii) the interaction among agents changes (*i.e.,* a new edge is created between agents, an existing edge disappears, or the edge weight changes). The dwell time of the k-th interval is defined as $\tau^k = t^k - t^{k-1}$, where $k = 1, \ldots, \kappa + 1$.

We denote the mode set by $\mathcal{P} = \{1, \ldots, M\}$, where each mode corresponds to a distinct interaction topology of the network. The value M represents the total number of modes that appear in the OMAS (See Remark 3, which discusses the potential infiniteness of M and κ). We define the mapping $\mathcal{T} : \{0, 1, \ldots, \kappa\} \to \mathcal{P}$. Now consider $k = 0, 1, \cdots, \kappa$. During the interval $[t^k, t^{k+1})$, the system operates in mode $\mathcal{T}(k)$; we say that this interval belongs to mode $\mathcal{T}(k)$. For $t \in [t^k, t^{k+1})$, the system evolves according to

$$\dot{x}(t) = f(x(t)) + L^{\mathcal{T}(k)} G(x(t)) + u(t), \tag{1}$$

where $x(t) \in \mathbb{R}^{N_{\mathcal{T}(k)}}$ denotes the collective state of the agents in the active node set $\mathcal{V}_{\mathcal{T}(k)}$, and $N_{\mathcal{T}(k)} = |\mathcal{V}_{\mathcal{T}(k)}|$. The function $f(x(t))$ represents the known internal dynamics, with $f(x(t)) = [f_1(x_1(t)), \cdots, f_{N_{\mathcal{T}(k)}}(x_{N_{\mathcal{T}(k)}}(t))]^\top$. The matrix $L^{\mathcal{T}(k)} \in \mathbb{R}^{N_{\mathcal{T}(k)} \times N_{\mathcal{T}(k)}}$ is the connectivity matrix representing the unknown weighted interaction topology associated with mode $\mathcal{T}(k)$, $G(x(t)) = x(t)$ specifies the interaction protocol (e.g., consensus or formation control (Mesbahi and Egerstedt, 2010)), and $u(t) \in \mathbb{R}^{N_{\mathcal{T}(k)}}$ denotes the externally applied control input. We emphasize that both the dimension $N_{\mathcal{T}(k)}$ and the connectivity matrix $L^{\mathcal{T}(k)}$ depend on the mode $\mathcal{T}(k)$ —see Figure 1 for an example of different modes with different networks sizes. If $L_{il}^{\mathcal{T}(k)} \neq 0$, then there is a directed edge $e_k \in \mathcal{E}_{\mathcal{T}(k)}$ from node l to node i with weight $L_{il}^{\mathcal{T}(k)}$ for all $i, l \in \{1, \ldots, N_{\mathcal{T}(k)}\}$ with $i \neq l$. If $L_{ii}^{\mathcal{T}(k)} \neq 0$, then node i has a self-loop, which belongs to $\mathcal{E}_{\mathcal{T}(k)}$. Thus, $\mathcal{V}_{\mathcal{T}(k)}$ consists of all active nodes of the system (1), while the edge set $\mathcal{E}_{\mathcal{T}(k)}$ and the edge weight set $W_{\mathcal{T}(k)}$ are derived from $L^{\mathcal{T}(k)}$. We denote the communication graph of (1) as a directed weighted graph $\mathcal{G}_{\mathcal{T}(k)} = (\mathcal{V}_{\mathcal{T}(k)}, \mathcal{E}_{\mathcal{T}(k)}, W_{\mathcal{T}(k)})$, where $\mathcal{V}_{\mathcal{T}(k)}$ is the node set, $\mathcal{E}_{\mathcal{T}(k)} \subseteq \mathcal{V}_{\mathcal{T}(k)} \times \mathcal{V}_{\mathcal{T}(k)}$ is the edge set, and $\mathcal{W}_{\mathcal{T}(k)}$ denotes the associated edge weights.

Objective. Given the observed state $x(t)$, the applied input $u(t)$, and the switching sequence $\{t^k\}_{k=0}^{\kappa}$, the goal is to reconstruct the sequence of the topologies: $\big\{L^{\mathcal{T}(k)}\big\}_{k=0}^{\kappa}$.

3. ESTIMATION OVER SINGLE AND MULTIPLE INTERVALS

In this section, we present a method to excite the network through the external control input $u(t)$ and then estimate the interaction topology. We first propose Algorithm 1 that estimates the interaction topology by comparing an auxiliary system $\hat{x}(t)$ with the original network $x(t)$, and by estimating the connectivity matrix over each interval. Then, we propose a more flexible method, presented in Algorithm 2, that aggregates information across multiple intervals to estimate the connectivity matrix. Throughout this section, we assume that the mode labels are known, an assumption that will be relaxed in Section 4.

3.1 Control Design and Estimation for Every Interval

To estimate the topology of a mode that may appear across multiple intervals with short dwell time, we begin by recalling classical excitation definitions for system identification. We adopt the notions of persistence of excitation and sufficient richness, both standard in adaptive control (Ioannou and Sun, 1996).

Definition 1. (PE condition (Ioannou and Sun, 1996)) A signal $\phi : \mathbb{R}_{\geq 0} \to \mathbb{R}^{n\times m}$ is said to be persistently exciting (PE) if, for any $t \geq 0$, there exist $\gamma > 0$ and $T > t$ such that $\int_t^{t+T} \phi(\tau)\phi(\tau)^\top d\tau \succ \gamma I$.

Definition 2. (Sufficiently rich of order n (Ioannou and Sun, 1996)) A function $\phi : \mathbb{R}_{\geq 0} \to \mathbb{R}^{n\times m}$ is said to be sufficiently rich of order n if it consists of at least of $n/2$ distinct frequencies.

To actively inject sufficient excitation into the network (1), we design the control input as

$$u_i(t) = \hat{u}_i(t) - f_i(x_i(t)), \quad \forall i\{1, 2, \ldots, N_{\mathcal{T}(k)}\} \tag{2}$$

where $\hat{u}_i(t)$ is designed to be sufficiently rich of order $N_{\mathcal{T}(k)}$, satisfying Definition 2. To this end, we propose Algorithm 1, where we reconstruct the connectivity matrix $L^{\mathcal{T}(k)}$ on each interval $[t^k, t^{k+1})$ by implementing an auxiliary system $\hat{x}(t)$, described below in (3a), provided that the output $w(t)$ of the filter in (3b) satisfies an excitation condition. To write the overall dynamics compactly, let $u = [u_1,\ u_2,\ \cdots,\ u_{N_{\mathcal{T}(k)}}]^\top$ and $\hat{u}(t) = [\hat{u}_1,\ \hat{u}_2,\ \cdots,\ \hat{u}_{N_{\mathcal{T}(k)}}]^\top$. We then define the augmented state $s(t) = \big(\hat{x}(t),\, w(t),\, \mathrm{vec}(Y(t)),\, \mathrm{vec}(Z(t))\big)$, which collects the auxiliary state, the filter state, and the integral quantities involved in the estimation. Its dynamics is given by

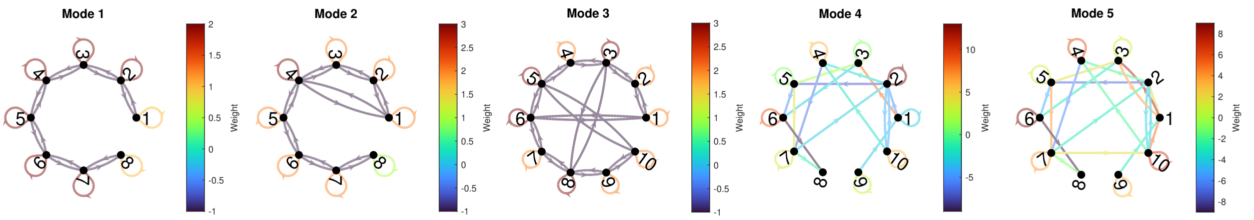

Fig. 1. Example of directed weighted graphs for each $L^{\mathcal{T}(k)}$. Edge colors indicate weights (see color bar). Modes 1 and 2 correspond to the modes 8-agent network, while Modes $3-5$ correspond to the 10-agent network.

Algorithm 1 Topology Identification Algorithm over Single Interval

1: Initialize the parameters $\tau > 0$ and $\gamma > 0$; the switching time sequence $\{t^k\}_{k=0}^{\kappa}$ with initial and final times t_0 and t_κ; and the step size $h > 0$.
2: **for** $k = 0, 1, \ldots, \kappa$ **do**
3: Initialize the matrix $L_m \in \mathbb{R}^{N_{\mathcal{T}(k)} \times N_{\mathcal{T}(k)}}$; $\hat{x}(t^k) = w(t^k) = 0$; $Y(t^k) = Z(t^k) = 0$; $\tilde{x}(t^k) = x(t^k)$.
4: **for** $\ell = 0, 1, 2, \ldots$ such that $t_\ell = t^k + \ell h < t^{k+1}$ **do**
5: Design $u(t_\ell) \in \mathbb{R}^{N_{\mathcal{T}(k)}}$ using (2) and update the state $x(t_\ell) \in \mathbb{R}^{N_{\mathcal{T}(k)}}$ according to the dynamics (1).
6: Compute $\zeta(t_\ell) = e^{-\tau(t_\ell - t^k)}\tilde{x}(t^k)$, where $\tilde{x}(t^k) := x(t^k) - \hat{x}(t^k)$.
7: Update the augmented state $s(t_\ell)$ according to the dynamics (3).
8: Perform one Runge–Kutta[1] step with step size h on (3): $s(t_{\ell+1}) = \mathrm{RK}(s(t_\ell), t_\ell, h)$.
9: Extract $(\hat{x}(t_{\ell+1}), w(t_{\ell+1}), Y(t_{\ell+1}), Z(t_{\ell+1}))$ from $s(t_{\ell+1})$.
10: Update time: $t_{\ell+1} = t_\ell + h$.
11: **if** $Y(t_{\ell+1}) \succ \gamma I$ **then**
12: Calculate the estimated topology matrix
$$\hat{L}(t_{\ell+1}) = Z(t_{\ell+1})\,Y(t_{\ell+1})^{-1}. \tag{4}$$
13: **Go to Line 2**.
14: **end if**
15: **end for**
16: **end for**

$$\dot{\hat{x}}(t) = f(x(t)) + L_m x(t) + u(t) + \tau(x(t) - \hat{x}(t)), \tag{3a}$$
$$\dot{w}(t) = x(t) - \tau w(t), \tag{3b}$$
$$\dot{Y}(t) = w(t)w(t)^\top, \tag{3c}$$
$$\dot{Z}(t) = (L_m w(t) + x(t) - \hat{x}(t) - \zeta(t))w(t)^\top, \tag{3d}$$

where $\tau > 0$ and $L_m \in \mathbb{R}^{N_{\mathcal{T}(k)} \times N_{\mathcal{T}(k)}}$ represents the connectivity matrix of the auxiliary system $\hat{x}(t)$ on each interval and can be initialized, *e.g.,* as the zero or the identity matrix. The step size h in Algorithm 1 is chosen to be small to ensure higher accuracy of system (3).

Remark 3. For Algorithms 1 and 2, the switching times κ and the number of modes M can be infinite since the model label for each interval is assumed to be known. In contrast, Algorithm 3, which determines the mode labels (see Section 4), requires κ and M to be finite in order to guarantee that mode matching across time segments is trackable.

In the following, we provide theoretical guarantees for Algorithm 1, ensuring that the OMAS maintains certain excitation and allows accurate topology estimation under a dwell time assumption. We now pose the following assumptions for system (1).

Assumption 4. $L^{\mathcal{T}(n)}$ is Hurwitz on an interval $[t^n, t^{n+1})$, where $n \in \{0, 1, \cdots, \kappa\}$.

Assumption 5. The dwell time of the interval $[t^n, t^{n+1})$ for mode $\mathcal{T}(n)$, where $n \in \{0, 1, \cdots, \kappa\}$, is sufficiently long to ensure that the filtered signal $w(t)$ is PE.

We first recall the following result from (Ioannou and Sun, 1996, Lemma 4.8.3), which states that persistence of excitation is preserved through a stable, minimum-phase filter.

Lemma 6. If $x(t)$ is bounded and PE, then $w(t)$, with the update law (3), is also PE.

Proof. The proof follows from (Wang et al., 2026, Lemma 4).

Lemma 7. Assume that the network states $x(t)$ in (1) are bounded. Under Assumptions 4 and 5, for the network modeled by the dynamics (1), the control input (2) ensures that w is PE for mode $\mathcal{T}(n)$ over the interval $[t^n, t^{n+1})$.

Proof. The proof follows from (Wang et al., 2026, Lemma 7).

The following theorem provides an original contribution of this paper regarding the identification of $L^{\mathcal{T}(n)}$.

Theorem 8. Consider the network dynamics (1) evolving under a constant interaction topology defined by $L^{\mathcal{T}(k)}$ on the interval $[t^k, t^{k+1})$. Moreover, the auxiliary system (3) is simulated by implementing Algorithm 1. Then, for $t \in [t^k, t^{k+1})$, the matrices $Y(t)$ and $Z(t)$, defined in (3), satisfy
$$Z(t) = L^{\mathcal{T}(k)}\,Y(t), \quad k = 1, \ldots, \kappa. \tag{5}$$
Furthermore, under Assumptions 4 and 5, for $[t^n, t^{n+1})$, $Y(t)$ is invertible for a certain time t satisfying $t \geq t_e$, where $t_e = \{t_{n+1} > t > t^n | Y(t) > \gamma I\}$, thereby the estimator (4) recovers the true topology of mode n, i.e. $\hat{L}^{\mathcal{T}(n)}(t) = L^{\mathcal{T}(n)}$, where $\hat{L}^{\mathcal{T}(n)}(t)$ is the estimate of $L^{\mathcal{T}(n)}$.

Proof. During $[t^k, t^{k+1})$, as L_m and τ given in (3) are fixed, we can show that $\big(L_m w(\sigma) + \tilde{x}(\sigma) - \zeta(\sigma)\big)$ is equal to $L^{\mathcal{T}(k)}w(\sigma)$ for all $\sigma \in [t^k, t^{k+1})$ when (1) holds with a constant $L^{\mathcal{T}(k)}$. We use $\zeta(\sigma) = \tilde{x}(\sigma) + \tilde{L}^{\mathcal{T}(k)}w(\sigma)$ with $\tilde{L}^{\mathcal{T}(k)} = L_m - L^{\mathcal{T}(k)}$. We obtain $\dot{\zeta}(\sigma) = -\tau\zeta(\sigma)$ from the dynamics of (1) and (3). It corresponds to $\zeta(t) = e^{-\tau(t-t^k)}\tilde{x}(t^k)$. Therefore $Z(t) = \int_{t^k}^{t} L^{\mathcal{T}(k)}w(\sigma)w(\sigma)^\top d\sigma = L^{\mathcal{T}(k)}\int_{t^k}^{t} w(\sigma)w(\sigma)^\top d\sigma = L^{\mathcal{T}(k)}Y(t)$, which proves (5). This holds for all $k = 0, 1, \ldots, \kappa$. From Lemma 7, w is PE during $[t^n, t^{n+1})$. Thereby, $Y(t) > \gamma I$ for $t > t_e$. It also

[1] A standard explicit numerical integration method for solving ordinary differential equations (See (Hairer et al., 1993)).

means that $Y(t)$ is invertible for $t > t_e$, the estimator (4) recovers the true mode.

Remark 9. The set-up of the network (1) requires controlling all nodes. This requirement can be relaxed using the framework presented in (Wang and Dimarogonas, 2025), which selects a small subset of nodes and applies the control input only to these nodes to excite the network.

3.2 Estimation across Multiple Intervals for a Single Mode

By Theorem 8, accurate identification of one mode requires that it possesses at least one interval where persistent excitation of the filtered state holds. However, in many OMAS, certain modes may reappear frequently but only for short durations, making such a requirement unrealistic. To overcome this limitation, we adopt a more flexible condition utilizing all occurrences of a mode in this section.

Formally, let $\mathcal{P}_j = \{k : \mathcal{T}(k) = j\}$ denote the index set of all the intervals during which mode j is active. Algorithm 2 is proposed to estimate the connectivity matrix L^j for each mode j, which combines different intervals for each mode j. For $k \in \{0, \dots, \kappa\}$, denote $\mathcal{Y}_k := \{ Y(t_\ell) \mid \ell \in \mathbb{N},\ t^k + \ell h < t^{k+1} \}$ and $\mathcal{Z}_k := \{ Z(t_\ell) \mid \ell \in \mathbb{N},\ t^k + \ell h < t^{k+1} \}$. Denote $t_{\mathcal{P}_j(i)} = t^{\mathcal{P}_j(i)-1} + \left\lfloor \frac{t^{\mathcal{P}_j(i)} - t^{\mathcal{P}_j(i)-1}}{h} \right\rfloor h$ if $\mathcal{P}_j(i) \neq 0$ and $t_{\mathcal{P}_j(i)} = 0$ if $\mathcal{P}_j(i) = 0$, where $i = 1, 2, \cdots, |\mathcal{P}_j|$.

Algorithm 2 Topology Identification Algorithm over Multiple Intervals

Input: $\{\mathcal{Y}_k\}_{k\in\{0,\dots,\kappa\}}$, $\{\mathcal{Z}_k\}_{k\in\{0,\dots,\kappa\}}$ and step size h from Algorithm 1.
Initialize $Y_1(t_0) = Y(t_0)$ and $Z_1(t_0) = Y(t_0)$.
for $i = 2, 3, \dots, |\mathcal{P}_j|$ **do**
  for $\ell = 0, 1, 2, \dots,$ such that $t_\ell = t^{\mathcal{P}_j(i)} + \ell h < t^{\mathcal{P}_j(i)+1}$ **do**
    Update the accumulated matrices:
$$Y_1(t_\ell) = Y_1\big(t_{\mathcal{P}_j(i-1)}\big) + Y(t_\ell),$$
$$Z_1(t_\ell) = Z_1\big(t_{\mathcal{P}_j(i-1)}\big) + Z(t_\ell).$$
    if $Y_1(t_\ell) \succ \gamma I$, **then**
      Compute $\hat{L}^j = Z_1(t_\ell)\, Y_1(t_\ell)^{-1}$.
      Return $\hat{L}^j$ and **Terminate**.
    end if
  end for
end for

Instead of relying on the PE condition within a single interval, we aggregate the filtered data from all intervals indexed by $\mathcal{P}_j$. If the cumulative excitation across these intervals is sufficient, then the topology corresponding to mode j can be uniquely recovered. We define the set of total intervals of mode j as $\{[t^{\mathcal{P}_j(i)}, t^{\mathcal{P}_j(i)+1})\}$ for all $i = 1, \cdots, |\mathcal{P}_j|$, that is, the union of all the intervals in which mode j is active. The corresponding dwell time is defined as $\sum_{k\in\mathcal{P}_j} \tau^k$.

Assumption 10. The dwell time of the total intervals associated with each mode j is sufficiently long to ensure that the aggregated matrix $Y_2 := \sum_{i\in\mathcal{P}_j} Y(t_{\mathcal{P}_j(i)})$ satisfies $Y_2 \succ \gamma I$, where I is the identity matrix with appropriate dimensions.

Remark 11. Assumption 4 states that there exists a single interval of a mode which can last sufficiently long for topology estimation, similarly to (Sun et al., 2023). Assumption 5 is significantly less restrictive than Assumption 4, as it does not require any single interval of a mode to be long, but only that the *sum* of all the intervals in which the mode is active provides adequate excitation. It is therefore well suited to settings with fast switching or intermittent agent participation.

We now present our result on aggregating all occurrences of a mode to estimate its connectivity matrix.

Theorem 12. Assume that the network states $x(t)$ in (1) remain bounded. Under Assumptions 5 and 10, for the network modeled by the dynamics (1) the control input (2), implementing Algorithms 1 and 2 guarantees that the estimation errors $\tilde{L}^j = \hat{L}^j - L^j$ are zero for each mode j.

Proof. From Theorem 8, for each interval $[t^k, t^{k+1})$ satisfying $\mathcal{T}(k) = j$, we have $Z(t) = L^j Y(t)$. Summing over all the intervals indexed by $\mathcal{P}_j$, we obtain $\sum_{i\in\mathcal{P}_j} Z(t_{\mathcal{P}_j(i)}) = L^j \sum_{i\in\mathcal{P}_j} Y(t_{\mathcal{P}_j(i)})$. These correspond to $Y_1(t)$ and $Z_1(t)$ in Algorithm 2. Assumption 5 ensures that the aggregated matrix $Y_2 \succ 0$, and therefore invertible. The topology for mode j is thus recovered via $\hat{L}^j = \left(\sum_{i\in\mathcal{P}_j} Z(t_{\mathcal{P}_j(i)})\right)\left(\sum_{i\in\mathcal{P}_j} Y(t_{\mathcal{P}_j(i)})\right)^{-1}$, thereby $\tilde{L}^j = 0^{N_j\times N_j}$.

Remark 13. If agent arrivals and departures cease after a finite time, Algorithm 2 can be applied directly. In contrast, when structural changes continue indefinitely, the method can be executed online: newly appearing vertex sets are treated as new modes, and their topologies are estimated once sufficient dwell time of the total interval has been accumulated.

4. TOPOLOGY ESTIMATION IN OMAS

In this section, we develop a method to determine the mode label for each time segment, which is the basis for identifying switching interaction topologies in OMAS. The key idea is to first provide a projected-based measure between time segments and then cluster time segments that correspond to the same mode. Afterwards, all data associated with each cluster are aggregated to estimate the topology of that mode.

4.1 Time Sequence Clustering

We assume the switching instants are observable—either directly from agent arrival/exit events or through change detection mechanisms applied to the data. Our goal is therefore to group time segments that share the same underlying mode.

A straightforward idea is to estimate the connectivity matrix for each time segment and cluster the matrices using the difference among the estimated connectivity matrices, as done in existing statistical-learning approaches for switched system identification (Massucci et al., 2022). In particular, (Massucci et al., 2022) uses an expectation–maximization–like procedure that iterates between clustering and parameter estimation. However, this pipeline perform poorly in OMAS. Each short time segment contains limited information, and thus directly estimating the connectivity matrix L^j from each time segment is inaccurate. Even if two time segments share the same matrix L^j, their limited data can produce substantially different least-squares estimates $\hat{L}^j$, resulting in incorrect mode assignments.

To address this challenge, we provide a projection-based distance to measure the distance among the rough topology estimations for each time segment. Let $I_s = [t^s, t^{s+1})$ and $I_r = [t^r, t^{r+1})$ be two intervals, where $s, r \in \{0, 1, \cdots, \kappa\}$ and $s \neq r$. Denote $t_s = t^s + \left\lfloor \frac{t^{s+1} - t^s}{h} \right\rfloor h$ and $t_r = t^r + \left\lfloor \frac{t^{r+1} - t^r}{h} \right\rfloor h$. Denote the corresponding terminal matrices from Algroithm 1 by $Y_s := Y(t_s), Z_s := Z(t_s), Y_r := Y(t_r), Z_r := Z(t_r)$, and define the corresponding least-squares estimates and projection matrices as $\hat{L}^s := Z_s Y_s^\dagger, \hat{L}^r := Z_r Y_r^\dagger, P_s := Y_s Y_s^\dagger, P_r := Y_r Y_r^\dagger$, where $(\cdot)^\dagger$ denotes the Moore–Penrose pseudoinverse, and P_s and P_s are the orthogonal projection onto range(Y_s) and range(Y_r), correspondingly.

Lemma 14. Assume that on both intervals I_s and I_r the system has the same connectivity matrix L, i.e., $Z_s = LY_s, Z_r = LY_r$. Then $\hat{L}^s P_r = \hat{L}^r P_r$ and $\hat{L}^r P_s = \hat{L}^s P_s$.

Proof. From $Z_s = LY_s$ we obtain $\hat{L}^s P_r = Z_s Y_s^\dagger P_r = LY_s Y_s^\dagger P_r = LP_s P_r$. Similarly, from $\tilde{Z}_r = LY_r$ we have $\hat{L}^r P_r = Z_r Y_r^\dagger P_r = LY_r Y_r^\dagger = LP_r$. Since $P_s P_r = P_r$ on ran(Y_r), it follows that $\hat{L}^s P_r = \hat{L}^r P_r$. The second identity $\hat{L}^r P_s = \hat{L}^s P_s$ follows by symmetry, interchanging s and r.

Remark 15. Lemma 14 implies that $\hat{L}^s$ and $\hat{L}^r$ induce the same linear map on each other's excitation subspaces.

Using Lemma 14, we define the following projection-based dissimilarity measure among two time-segments:

$$d(s, r) := \left\| \hat{L}^s P_r - \hat{L}^r P_r \right\|_F + \left\| \hat{L}^r P_s - \hat{L}^s P_s \right\|_F. \tag{6}$$

If two intervals share the same mode and are sufficiently excited, then $d(s, r)$ is small; otherwise, the projection identities fail and $d(s, r)$ is large. Clustering time segments based on $d(s, r)$ therefore yields groups corresponding approximately to distinct modes. We only compare the distances among time segments when the vertex set is the same. Denote by $p \in \mathbb{N}^+$ the number of the vertex sets among all time segments. Hence we propose Algorithm 3 for time segment clustering.

4.2 Clustering of Time Segments into Modes

After computing the pairwise dissimilarity $d(s, r)$ for all time segments, we cluster them into groups, each corresponding to a single mode. Intuitively, time segments sharing the same underlying topology yield similar projection identities (Lemma 14), resulting in small dissimilarities, whereas segments from different modes violate these identities and yield large dissimilarities. Applying agglomerative clustering to the dissimilarity matrix partitions the switching sequence into mode-consistent clusters.

Once clustering is complete, all the time segments assigned to the same mode are merged into a unified dataset. This aggregation compensates for the limited excitation available within each short time segment and provides a sufficiently rich dataset for reliable topology identification. Given the merged dataset for a mode, we apply Algorithm 2 to estimate the corresponding connection matrix (i.e., the graph associated with that mode). In this way, each cluster yields one reconstructed interaction topology.

Algorithm 3 Mode Clustering via Projection-Based Dissimilarity

Input: Time partition $\{[t^s, t^{s+1})\}_{s=0}^{\kappa}$, terminal matrices $\{Y_s, Z_s\}_{s=0}^{\kappa}$ obtained from Algorithm 1, number of modes M^i for each vertex set $\mathcal{V}_i$, $i = 1, \ldots, p$.

Step 1: Local operators and projections

for $s = 0, \ldots, \kappa$ **do**
  Compute local LS estimate $\hat{L}^s := Z_s Y_s^\dagger$, where $Y_s^\dagger$ is the Moore–Penrose pseudoinverse.
  Compute excitation projector $P_s := Y_s Y_s^\dagger$.
end for

Step 2: Pairwise dissimilarity matrix

Classify time segments according to the size of vertex set as $\mathcal{V}_1, \ldots, \mathcal{V}_p$.
Initialize $D^i \in \mathbb{R}^{|\mathcal{V}_i| \times |\mathcal{V}_i|}$ for each $\mathcal{V}_i$ with $D^i_{s,s} = 0$.
for $i = 1, \ldots, p$ **do**
  for $s = 0, \ldots, |\mathcal{V}_i|$ **do**
    for $r = s + 1, \ldots, |\mathcal{V}_i|$ **do**
      Compute projection-based dissimilarity measure given in (6).
      Set $D^i_{s,r} = D^i_{r,s} = d(s, r)$.
    end for
  end for
end for

Step 3: Mode clustering

Apply agglomerative clustering with M^i clusters to the distance matrix D^i.
Obtain mode labels $\ell_s \in \{1, \ldots, M\}$ for each time segment $s = 0, \ldots, \kappa$ from the chosen clustering method.
return $\{\ell_s\}_{s=0}^{\kappa}$.

5. SIMULATION RESULTS

To illustrate our theoretical findings, we present a numerical example of an OMAS modeled by (1), initially represented by a directed graph of 8 agents, to which two additional agents join over time. We consider two modes for the 8-agent network, corresponding to modes 1-2, and three modes for 10-agent network, corresponding to modes 3-5 in Figure 1. The switching sequence among these modes, during which agents may join or leave the system, is shown in Figure 2. The initial conditions of the agents are set randomly, and whenever new agents join, their initial states are also assigned randomly. The control input $u(t)$ is chosen as a combination of sinusoid signals with different frequencies. We set $\tau = 0.05$ in (3) and different L_m are selected as the connectivity matrices of the auxiliary system $\hat{x}(t)$ for the 8- and 10-agent networks. The state trajectories $x(t)$ remains bounded during simulation. We first cluster time segments according to the network size using Algorithm 3. The dendrograms for the 8- and 10-agent cases are presented in Fig. 3 and Fig. 4, respectively, where agglomerative clustering correctly identifies the modes for each time segment. Then, using Algorithm 2, we obtain the

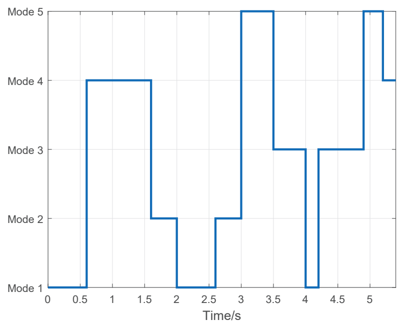

Fig. 2. The evolution of the switching sequence with time.

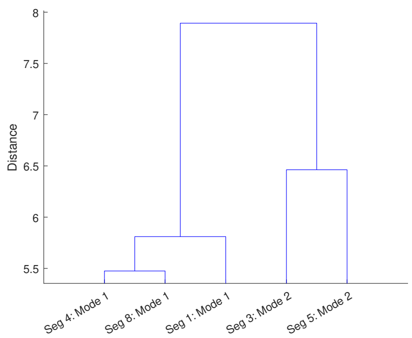

Fig. 3. Dendrogram (8 agents) showing projection-based dissimilarity among time segments, consistent with Fig. 2. Two modes are correctly identified.

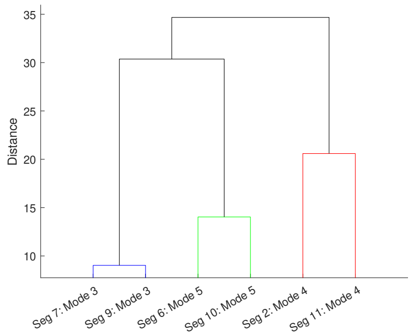

Fig. 4. Dendrogram (10 agents) showing projection-based dissimilarity among time segments, consistent with Fig. 2. Three modes are correctly identified.

topology estimation errors for the five modes, as reported in Table 1, which verifies Theorem 12.

Table 1. Estimation Errors ins Different Modes

Modes number	1	2	3	4	5
$\|\tilde{L}\|_F$	1.8e-11	2.1e-8	1.6e-8	1.0e-8	3.1e-7

6. CONCLUSION

This paper presents a topology identification framework for OMAS characterized by dynamic node sets and fast switching interactions. By introducing a projection-based dissimilarity measure, we enable reliable clustering of time segments and accurate mode-wise topology estimation without requiring long dwell time. Future research will focus on developing clustering algorithms that operate without prior knowledge of the number of modes, allowing data-driven mode discovery. Another important direction is to further relax the dwell time requirements, enabling identification under even more rapid and irregular switching conditions.

REFERENCES

Dimitrov, D., Schäfer, P.S.L., Farr, E., Rodriguez-Mier, P., Lobentanzer, S., Badia-i Mompel, P., Dugourd, A., Tanevski, J., Ramirez Flores, R.O., and Saez-Rodriguez, J. (2024). Liana+ provides an all-in-one framework for cell–cell communication inference. *Nature Cell Biology*, 26(9), 1613–1622.

Franceschelli, M. and Frasca, P. (2020). Stability of open multiagent systems and applications to dynamic consensus. *IEEE Trans. Autom. Control*, 66(5), 2326–2331.

Hairer, E., Wanner, G., and Nørsett, S.P. (1993). *Solving ordinary differential equations I: Nonstiff problems.* Springer.

Hendrickx, J.M. and Martin, S. (2016). Open multi-agent systems: Gossiping with deterministic arrivals and departures. In *In Proc. 54th Allerton Conf.*, 1094–1101.

Ioannou, P.A. and Sun, J. (1996). *Robust adaptive control*, volume 1. PTR Prentice-Hall Upper Saddle River, NJ.

Massucci, L., Lauer, F., and Gilson, M. (2021). Regularized switched system identification: a statistical learning perspective. *IFAC-PapersOnLine*, 54(5), 55–60.

Massucci, L., Lauer, F., and Gilson, M. (2022). A statistical learning perspective on switched linear system identification. *Automatica*, 145, 110532.

Mesbahi, M. and Egerstedt, M. (2010). *Graph theoretic methods in multiagent networks.* Princeton University Press.

Restrepo, E., Loría, A., Sarras, I., and Marzat, J. (2022). Consensus of open multi-agent systems over dynamic undirected graphs with preserved connectivity and collision avoidance. In *Proc. IEEE CDC*, 4609–4614.

Rey, S., Das, B., and Isufi, E. (2025). Online learning of expanding graphs. *IEEE Open Journal of Signal Processing.*

Şekercioğlu, P., Fontan, A., and Dimarogonas, D.V. (2025). Stability of open multi-agent systems over dynamic signed graphs. In *Proc. IEEE Conf. on Dec. and Control*, 7172–7177.

Sun, W., Xu, J., and Chen, J. (2023). Identifiability and identification of switching dynamical networks: A data-based approach. *IEEE Trans. Control Netw. Syst.*, 11(3), 1177–1189.

Wang, N. and Dimarogonas, D.V. (2025). Leader selection and control design for topology estimation of dynamical networks. In *In Proc. IEEE CDC*, 8058–8063. IEEE.

Wang, N., Sekercioglu, P., and Dimarogonas, D.V. (2026). Topology estimation for open multi-agent systems. *arXiv preprint arXiv:2604.13628.*

Xue, M., Tang, Y., Ren, W., and Qian, F. (2022). Stability of multi-dimensional switched systems with an application to open multi-agent systems. *Automatica*, 146, 110644.