Hybrid Control of Multi-Agent Systems under Local Temporal Tasks and Relative-Distance Constraints

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Abstract— **In this paper, we propose a distributed hybrid control strategy for multi-agent systems where each agent has a local task specified as a Linear Temporal Logic (LTL) formula and at the same time is subject to relative-distance constraints with its neighboring agents. The local tasks capture the temporal requirements on individual agents' behaviors, while the relative-distance constraints impose requirements on the collective motion of the whole team. The proposed solution relies only on relative-state measurements among the neighboring agents without the need for explicit information exchange. It is guaranteed that the local tasks given as syntactically co-safe or general LTL formulas are fulfilled and the relative-distance constraints are satisfied at all time. The approach is demonstrated with computer simulations.**

I. Introduction

Cooperative control of multi-agent systems generally focuses on designing local control laws to achieve a global control objective, such as reference-tracking [8], consensus [18], or formation [9]. In addition to these objectives, various relative-motion constraints are often imposed to ensure stability, safety and integrity of the overall system, such as collision avoidance [2], network connectivity [9], [23], or relative velocity constraints [8]. In order to specify and achieve more structured and complex team behaviors than the listed ones, we consider Linear Temporal Logic (LTL) formulas as suitable descriptions of desired high-level goals, including periodic surveillance, sequencing, request-response, and their combinations. Furthermore, a generic hierarchical approach that allows for the correct-by-design control has been formulated and largely employed during the last decade or so in single-agent as well as multi-agent settings. In particular, task specifications are expressed as LTL formulas for a single dynamical system in [3] and an automated framework is proposed to translate the task directly into a hybrid controller, which drives the system to fulfill this task. For multi-agent systems, LTL formulas have been used to specify complex high-level global tasks [11], [13]–[15], [17], [20], [22], local tasks [5], [21].

In temporal logic-based multi-agent control, two different points of view can be taken: a top-down and a bottom-up. In the former one, a global specification captures requirements on the overall team behavior. Typically, the focus of synthesizing a control strategy is on decomposing the global specification into smaller local tasks to be executed by the individual agents in a synchronized or partially synchronized manner [11], [22]. A central monitoring unit is often crucial to ensure that the composition of the local plans yields the satisfaction of the global goal. In contrast, in the bottom-up approach, each agent is assigned a local task. These tasks can be fully independent [5] or partially dependent, involving requests for collaboration with the others [20], [21].

In this work, we tackle the multi-agent control problem under local LTL tasks from the bottom-up perspective. The local tasks are mutually independent but the agents are subject to relative-distance constraints with their neighbors. Thus integration of the continuous motion control with the high-level discrete network structure control is essential. Relative-distance constraints are closely related to the connectivity of the multi-agent network in robotic tasks [9]. As pointed out in [8], [23], maintaining this connectivity is of great importance for the stability, safety and integrity of the overall team. We addressed a version of this problem in [6], where we proposed a dynamic leader-follower coordination and control scheme. In this work, however, we aim for a decentralized and communication-free solution that is applicable to low-cost robotic systems equipped with range and angle sensors, but without communication capabilities.

Main contributions of the proposed hybrid control strategy lie in two aspects: (i) the proposed distributed motion controller guarantees almost global convergence and the satisfaction of relative-distance constraints at all time, for an *arbitrary* number of leaders with different local goals in the team. Most related literature in the area of distributed control of multi-agent systems only allows for a single leader [6], [18] or multiple leaders with the same global goal as the team [16]; (ii) two different local coordination and control-law switching policies are proposed depending on the types of local tasks assigned, which can be applied to communication-less agents in a distributed way.

The rest of the paper is organized as follows. Section II introduces necessary preliminaries. In Section III, we formalizes the considered problem. Section IV presents our solution in details. Section V demonstrates the feasibility of the results by numerical simulations. We conclude and discuss about future directions in Section VI. The extended version of this paper can be found in [7].

II. Preliminaries

A. Linear Temporal Logic (LTL)

A *Linear Temporal Logic (LTL)* formula over a set of *atomic propositions* Σ that can be evaluated as true or false is defined inductively according to the following rules [1]:

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an atomic proposition $\sigma \in \Sigma$ is an LTL formula; if φ and ψ are LTL formulas, then also $\neg\varphi$, $\varphi \wedge \psi$, $\bigcirc\varphi$, $\varphi \,\mathsf{U}\, \psi$, $\Diamond\,\varphi$, and $\Box\,\varphi$ are LTL formulas, where $\neg$ (*negation*), $\wedge$ (*conjunction*) are standard Boolean connectives and $\bigcirc$ (*next*), U (*until*), $\Diamond$ (*eventually*), and $\Box$ (*globally*) are temporal operators. The semantics of LTL is omitted here due to limited space. For full details, see e.g., [1]. *Syntactically co-safe LTL (sc-LTL)* is a subclass of LTL built without the operators $\Box$ and $\neg$ can be applied to atomic propositions only [12]. Each word over 2^{Σ} satisfying an sc-LTL formula φ consists of a satisfying prefix that can be followed by an arbitrary suffix.

B. Weighted Graph

An undirected weighted graph is a tuple $G = (\mathcal{N}, E, h)$, where $\mathcal{N} = \{1, \ldots, N\}$ is a set of nodes; $E \subseteq \mathcal{N} \times \mathcal{N}$ is a set of *edges*; and $h : E \to \mathbb{R}^+$ is the weight function, which can be omitted if the weight is uniform over all edges. Each node i has a set of *neighbors* $\mathcal{N}_i = \{j \in \mathcal{N} \,|\, (i, j) \in E\}$. A path from node i to j is a sequence of nodes starting with i and ending with j such that the consecutive nodes are neighbors. G is *connected* if there is a path between any two nodes and G is *complete* if $E = \mathcal{N} \times \mathcal{N}$. The Laplacian matrix $\mathbf{H}$ of G is an $N \times N$ positive semidefinite matrix: $\mathbf{H}(i,i) = \sum_{j \in \mathcal{N}_i} h(i,j), \forall i \in \mathcal{N}$; $\mathbf{H}(i,j) = -h(i,j)$, $\forall (i,j) \in E$, and $\mathbf{H}(i,j) = 0$, otherwise. For a connected graph G, $\mathbf{H}$ has nonnegative eigenvalues [18] and a *single* zero eigenvalue with eigenvector $\mathbf{1}_N$, where $\mathbf{1}_N = [1, \ldots, 1]^T$.

In this paper, each vector norm over $\mathbb{R}^n$ is the Euclidean norm. $|S|$ denotes the cardinality of a set S and $v[i]$ denotes the i-th element of a vector or a sequence v.

III. Problem Formulation

A. Agent Dynamics and Network Structure

We consider a team of N autonomous agents with unique identities (IDs) $i \in \mathcal{N} = \{1, \ldots, N\}$. They all satisfy the single-integrator dynamics below:

$$\dot{x}_i(t) \triangleq u_i(t), \qquad i \in \mathcal{N} \tag{1}$$

where $x_i(t)$, $u_i(t) \in \mathbb{R}^2$ are the respective state and the control input of agent i at time $t > 0$. Let $x_i(0)$ be the given initial state. The agents are modeled as point masses without volume, i.e., inter-agent collisions are not considered. Each agent has a sensing radius $r > 0$, which is assumed to be identical for all agents. Namely, each agent can only observe another agent's state if their relative distance is less than r. Thus, given $\{x_i(0), i \in \mathcal{N}\}$, we define the undirected graph $G_0(t) \triangleq (\mathcal{N}, E_0(t))$, where $(i, j) \in E_0(t)$ if $\|x_i(t) - x_j(t)\| < r$. We assume that $G_0(0)$ is connected.

B. Task Specifications

Within the 2D workspace, each agent $i \in \mathcal{N}$ has a set of $M_i \geq 1$ regions of interest, denoted by $\Pi_i \triangleq \{\pi_{i1}, \ldots, \pi_{iM_i}\}$. These regions can be of different shapes, such as spheres, triangles, or polygons. For simplicity of presentation, $\pi_{i\ell} \in \Pi_i$ is here represented by a circular area around a point of interest: $\pi_{i\ell} = \mathcal{B}(c_{i\ell}, r_{i\ell}) = \{y \in \mathbb{R}^2 \mid \|y - c_{i\ell}\| \leq r_{i\ell}\}$, where $c_{i\ell} \in \mathbb{R}^2$ is the center; $r_{i\ell} \geq r_{\min}$ is the radius and $r_{\min} > 0$ is a given minimal radius for all regions. We assume that their centers do not overlap and that the workspace is bounded:

Assumption 1: (I) $\|c_{i\ell_i} - c_{j\ell_j}\| > 2\,r_{\min}$, $\forall i, j \in \mathcal{N}$, $\forall \pi_{i\ell_i} \in \Pi_i$ and $\forall \pi_{j\ell_j} \in \Pi_j$. (II) $\|c_{i\ell}\| < c_{\max}$, $\forall i \in \mathcal{N}$ and $\forall \pi_{i\ell} \in \Pi_i$, where $c_{\max} > 0$ is a given constant. ■

Moreover, there is a set of atomic propositions known to agent i, denoted by Σ_i. Each region of interest is associated with a subset of Σ_i through the labeling function $L_i : \Pi_i \to 2^{\Sigma_i}$. We assume that $\Sigma_i \cap \Sigma_j = \emptyset$, for all $i, j \in \mathcal{N}$ such that $i \neq j$. $L_i(\pi_{i\ell})$ is a set of *services* that agent i can provide when being present in region $\pi_{i\ell} \in \Pi_i$. Hence, upon the visit to $\pi_{i\ell}$, the agent i chooses among $L_i(\pi_{i\ell})$ the subset of services it provides among the available ones.

We denote by $\mathbf{x}_i(T)$ the *trajectory* of agent i during the time interval $[0, T)$, where $T > 0$ and T can be infinity. The trajectory $\mathbf{x}_i(T)$ is associated with a unique finite or infinite sequence, called a *path*, $\mathbf{p}_i(T) \triangleq \pi_{i1}\pi_{i2}\ldots$ of regions in Π_i that agent i crosses, and with a finite or infinite sequence of time instants $t'_{i0}t_{i1}t'_{i1}t_{i2}t'_{i2}\ldots$ when i enters or leaves the respective regions. Formally, for all $k \geq 1$: $0 = t'_{i0} \leq t_{ik} \leq t'_{ik} < t_{ik+1} < T$, $x_i(t) \in \pi_{ik}$, for $\pi_{ik} \in \Pi_i$, $\forall t \in [t_{ik}, t'_{ik}]$, and $x_i(t) \notin \pi_{i\ell}$, $\forall \pi_{i\ell} \in \Pi_i$ and $\forall t \in (t'_{ik-1}, t_{ik})$. Agent i may choose to provide services only at some regions along the path $\mathbf{p}_i$. Denote by $\overline{\mathbf{p}}_i(T) = \pi_{i\ell_1}\pi_{i\ell_2}\ldots$ the *effective path* as a subsequence of $\mathbf{p}_i$ such that $\ell_k < \ell_{k+1}$, $\forall k \geq 1$ and $\pi_{i\ell_k} \in \mathbf{p}_i(T)$, $\forall \pi_{i\ell_k} \in \overline{\mathbf{p}}_i(T)$. The *word* produced by agent i is given by the provided services along the sequence of regions in $\overline{\mathbf{p}}_i$. In particular, at region for $\pi_{i\ell_k} \in \overline{\mathbf{p}}_i(T)$, the agent i chooses to provide a set of services w_{ℓ_k}, where $w_{\ell_k} \neq \emptyset$ and $w_{\ell_k} \subseteq L_i(\pi_{i\ell_k})$ is a subset of services available at the region $\pi_{i\ell_k}$. In other words, the produced word $\mathtt{word}_i(T) = w_{\ell_1}w_{\ell_2}\ldots$ *complies with* $\overline{\mathbf{p}}_i(T)$ by satisfying the property that $\emptyset \subset w_{\ell_k} \subseteq L_i(\pi_{i\ell_k})$, $\forall \pi_{i\ell_k} \in \overline{\mathbf{p}}_i(T)$.

The specification of the local task for each agent $i \in \mathcal{N}$ is given as a general LTL or an sc-LTL formula φ_i over Σ_i and captures requirements on the services to be provided by agent i. Thus agent i's trajectory $\mathbf{x}_i(T)$ satisfies φ_i if there exists an effective path $\overline{\mathbf{p}}_i(T)$ and a compliant word $\mathtt{word}_i(T)$ such that $\mathtt{word}_i(T) \models \varphi_i$. At last, the problem we consider in this work is stated below:

Problem 1: Given a team of N agents and their tasks as in Section III-B, design a distributed control law u_i such that for $T = \infty$: (1) $\mathbf{x}_i(T)$ satisfies φ_i; and (2) $\|x_i(t) - x_j(t)\| < r$, $\forall (i, j) \in E_0(0)$, $\forall t \in [0, T)$. ■

IV. Solution

The proposed solution consists of three layers: (i) an offline synthesis algorithm for the discrete plan of each agent; (ii) a continuous control scheme that guarantees one of the agents reaches its progressive goal region in finite time while the relative-distance constraints are fulfilled; (iii) a hybrid control layer that coordinates the discrete plan execution and the continuous control law switching in running time.

A. Discrete Plan Synthesis

We aim to find an effective path for agent $i \in \mathcal{N}$ that there exists a compliant word satisfying φ_i. The discrete plan can be generated using standard techniques leveraging ideas from automata-based formal verification [1]. Loosely speaking, an LTL or an sc-LTL formula φ_i is first translated into a Büchi or a finite automaton, respectively. The automaton is viewed as a graph and analyzed using graph search algorithms. As a result, a word that satisfies φ_i is obtained and mapped onto the sequence of regions to be visited. Current temporal logic-based discrete plan synthesis algorithms can accommodate various environmental constraints and advanced plan optimality criteria [5], [11], [22]. Recall that the labeling function L_i defines the available services at each region of Π_i. Then the accepting run of $\mathcal{A}_i$ can be naturally translated into the *discrete plan* of agent i in the prefix-suffix form: $\tau_i = \tau_{i,\text{pre}}(\tau_{i,\text{suf}})^{\omega}$, where $\tau_{i,\text{pre}} = (\pi_{i1}, w_{i1})\ldots(\pi_{ik_i}, w_{ik_i})$ is the plan prefix, and $\tau_{i,\text{suf}} = (\pi_{ik_i+1}, w_{ik_i+1})\ldots(\pi_{iK_i}, w_{iK_i})$ is the periodical plan suffix; $\pi_{ik} \in \Pi_i$ and $\emptyset \subset w_{ik} \subseteq L_i(\pi_{ik})$, $\forall k = 1, \ldots, K_i$. Thus the word corresponding to τ_i is given by its projection onto Σ_i, namely $\texttt{word}_i(T) = \tau_i|_{\Sigma_i} = w_{i1}\ldots w_{ik_i}(w_{ik_i+1}\ldots w_{iK_i})^{\omega}$; then the effective path $\overline{\mathbf{p}}_i$ is given as the projection of τ_i onto Π_i, namely $\overline{\mathbf{p}}_i(T) = \tau_i|_{\Pi_i} = \pi_{i1}\ldots\pi_{ik_i}(\pi_{ik_i+1}\ldots\pi_{iK_i})^{\omega}$. We denote by $\overline{\mathbf{p}}_{i,\text{pre}}(T) = \pi_{i1}\ldots\pi_{ik_i}$ the prefix of the effective path and by $\overline{\mathbf{p}}_{i,\text{suf}}(T) = \pi_{ik_i+1}\ldots\pi_{iK_i}$ the suffix.

B. Continuous Controller Design

Let us first introduce the notion of *connectivity graph*, which allows us to handle the relative-distance constraints. Let $\delta \in (0, r)$ be a given constant.

Definition 1: Denote by $G(t) \triangleq (\mathcal{N}, E(t))$ the undirected time-varying connectivity graph at time $t \geq 0$, where $E(t) \subseteq \mathcal{N} \times \mathcal{N}$ is the set of edges. (I) $G(0) = G_0(0)$; (II) At time $t > 0$, $(i, j) \in E(t)$ iff one of conditions below hold: (i) $\|x_i(t) - x_j(t)\| \leq r - \delta$; or (ii) $r - \delta < \|x_i(t) - x_j(t)\| \leq r$ and $(i, j) \in E(t^-)$, where $t^- < t$ and $|t - t^-| \to 0$. ■

Note that the condition (II) above guarantees that a new edge will only be added when the distance between two previously-unconnected agents decreases below $r - \delta$. In other words, there is a hysteresis effect when adding new edges to the connectivity graph. Consequently, each agent $i \in \mathcal{N}$ has a time-varying set of neighbors $\mathcal{N}_i(t) = \{j \in \mathcal{N} \,|\, (i, j) \in E(t)\}$. Let the progressive goal region of agent $i \in \mathcal{N}$ at time t be given by $\pi_{ig} = \mathcal{B}(c_{ig}, r_{ig}) \in \Pi_i$. We propose the following two different control modes:

$$\mathbf{C}_{act}: \quad u_i(t) \triangleq -d_i\, p_i - \sum_{j \in \mathcal{N}_i(t)} h_{ij}\, x_{ij}, \tag{2}$$

$$\mathbf{C}_{pas}: \quad u_i(t) \triangleq - \sum_{j \in \mathcal{N}_i(t)} h_{ij}\, x_{ij}, \tag{3}$$

where $\mathbf{C}_{act}$ is the *active* control mode; $\mathbf{C}_{pas}$ is the *passive* control mode; $x_{ij} \triangleq x_i - x_j$; $p_i \triangleq x_i - c_{ig}$; and

$$d_i \triangleq \frac{\varepsilon^3}{(\|p_i\|^2 + \varepsilon)^2} + \frac{\varepsilon^2}{2\,(\|p_i\|^2 + \varepsilon)}; h_{ij} \triangleq \frac{r^2}{(r^2 - \|x_{ij}\|^2)^2},$$

where $\varepsilon > 0$ is a key design parameter to be appropriately tuned, as shown in the sequel. Note that both controllers above are nonlinear and rely on only locally-available states.

Assume that $G(T_s)$ is connected at time $T_s > 0$. Moreover, assume that there are $N_a \geq 1$ agents within $\mathcal{N}$ that are in the *active* mode obeying (2) with its goal region as $\pi_{ig} = \mathcal{B}(c_{ig}, r_{ig}) \in \Pi_i$; and the rest $N_p = N - N_a$ agents that are in the *passive* mode obeying (3). For simplicity, denote by the group of active and passive agents $\mathcal{N}_a, \mathcal{N}_p \subseteq \mathcal{N}$ respectively. We show now that for *any* allowed combination of $N_a \geq 1$ and $N_p \leq N - 1$, by following the control laws (2) and (3), *one* active agent can reach its goal region within finite time $T_f \in (T_s, +\infty)$, while the relative distance $\|x_i(t) - x_j(t)\| < r$, $\forall (i, j) \in E(T_s)$ and $\forall t \in [T_s, T_f]$.

1) Relative-Distance Maintenance: We consider the following potential-field function:

$$V(t) \triangleq \frac{1}{2} \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}_i(t)} \phi_c(x_{ij}) + b_i \sum_{i \in \mathcal{N}} \phi_g(x_i) \tag{4}$$

where $\phi_c(\cdot)$ stands for an attractive potential to agent i's neighbors and is defined by:

$$\phi_c(x_{ij}) \triangleq \frac{1}{2}\, \frac{\|x_{ij}\|^2}{r^2 - \|x_{ij}\|^2}, \qquad \|x_{ij}\| \in [0,\, r - \delta); \tag{5}$$

while $\phi_g(\cdot)$ is an attractive force to agent i's goal:

$$\phi_g(x_i) \triangleq \frac{\varepsilon^2}{2}\, \frac{\|p_i\|^2}{\|p_i\|^2 + \varepsilon} + \frac{\varepsilon^2}{4}\, \ln(\|p_i\|^2 + \varepsilon), \tag{6}$$

where function $\ln(\cdot)$ is the natural logarithm; $b_i \in \mathbb{B}$ indicates the agent i's control mode. Namely, $b_i = 1$, $\forall i \in \mathcal{N}_a$ and $b_i = 0$, $\forall i \in \mathcal{N}_p$. It can be verified that the gradient of $V(t)$ from (4) with respect to x_i is given by $\nabla_{x_i} V = \frac{\partial V}{\partial x_i} = \nabla_{x_i} \phi_g(x_i) + \sum_{j \in \mathcal{N}_i} \nabla_{x_i} \phi_c(x_{ij}) = b_i\, d_i\, p_i + \sum_{j \in \mathcal{N}_i(t)} h_{ij}\, x_{ij} = -u_i$, $\forall i \in \mathcal{N}$.

Theorem 1: $G(t)$ remains connected and no existing edges within $E(T_s)$ will be lost, $\forall t \geq T_s$.

Proof: (Sketch) If the network $G(t)$ remains *invariant* during the time period $[t_1, t_2) \subseteq [T_s, \infty)$, we can show that $\dot{V}(t) \leq 0$ given the closed-loop dynamics. Moreover, if a set of new edges $\widehat{E} \subset \mathcal{N} \times \mathcal{N}$ is added to $G(t)$ at $t = t_2$. Let $V(t_2^-)$ and $V(t_2^+)$ be the value of $V(t)$ before and after adding the new edges. We get $V(t_2^+) = V(t_2^-) + \sum_{(p,\, q) \in \widehat{E}} \phi_c(x_{pq}(t_2)) \leq V(t_2^-) + |\widehat{E}|\, \frac{(r-\delta)^2}{\delta(2r-\delta)} < +\infty$. As a result, $V(t) < +\infty$ for $t \in [T_s, \infty)$. By contradiction, new edges might be added into $E(t)$ but no existing edges within $E(t)$ will be lost, namely $E(T_s) \subseteq E(t)$, $\forall t \geq T_s$. Detailed proof can be found in the proof of Theorem 1 of [7]. ■

2) Convergence Analysis: We have shown that the potential function $V(t)$ is lower-bounded and non-increasing if $G(t)$ remains static. By LaSalle's invariance principle [10] we only need to find out the largest invariant set within $\{x_i,\, \forall i \in \mathcal{N} \,|\, \dot{V}(t) = 0\}$. By enforcing $\dot{V}(t) = 0$, it implies:

$$b_i\, d_i\, p_i + \sum_{j \in \mathcal{N}_i(t)} h_{ij}\, x_{ij} = 0, \quad \forall i \in \mathcal{N}. \tag{7}$$

Then we can construct one $N \times N$ diagonal matrix $\mathbf{D}$ that $\mathbf{D}(i, i) = b_i\, d_i$, $\forall i \in \mathcal{N}$ and $\mathbf{D}(i, j) = 0$, $i \neq j$ and $i, j \in \mathcal{N}$. and another $N \times N$ matrix $\mathbf{H}$ that $\mathbf{H}(i, i) = \sum_{j \in \mathcal{N}_i} h_{ij}$, $\forall i \in \mathcal{N}$ and $\mathbf{H}(i, j) = -h_{ij}$, $i \neq j$ and $\forall (i, j) \in E(t)$ while

$\mathbf{H}(i,j) = 0$, $\forall (i,\, j) \notin E(t)$. Note that $h_{ij} > 0$ as $\|x_{ij}\| \in [0,\, r)$, $\forall (i,\, j) \in E(t)$. As a result, $\mathbf{H}$ is the Laplacian matrix of the graph $G(t) = (\mathcal{N}, E(t), h)$, where $h(i,\, j) = h_{ij}$, $\forall (i,\, j) \in E(t)$. Then (7) can be re-written as: $\mathbf{H} \otimes \mathbf{I}_2 \cdot \mathbf{x} + \mathbf{D} \otimes \mathbf{I}_2 \cdot (\mathbf{x} - \mathbf{c}) = 0$, where $\otimes$ is the Kronecker product; $\mathbf{x}$ is the stack vector for x_i, $i \in \mathcal{N}$ and $\mathbf{x}[i] = x_i$; $\mathbf{I}_2$ is the 2×2 identity matrix; $\mathbf{c}$ is the stack vector for c_{ig} and $\mathbf{c}[i] = c_{ig}$ if $i \in \mathcal{N}_a$ and $\mathbf{c}[i] = \mathbf{0}_2$ if $i \in \mathcal{N}_p$, where $\mathbf{0}_2$ is a 2×1 zero vector. Let $\mathcal{C}$ be the set of critical points of $V(t)$, i.e., $\mathcal{C} \triangleq \{x \in \mathbb{R}^{2N} \,|\, \mathbf{H} \otimes \mathbf{I}_2 \cdot \mathbf{x} + \mathbf{D} \otimes \mathbf{I}_2 \cdot (\mathbf{x} - \mathbf{c}) = 0\}$.

Lemma 2: For all critical points $\mathbf{x}_c \in \mathcal{C}$, (I) $\|x_{ij}\|$ can be made arbitrarily small by reducing ε, $\forall (i,\, j) \in E(t)$; (II) there exists $\varepsilon_0 > 0$ such that if $\varepsilon < \varepsilon_0$, then the connectivity graph $G(t)$ is complete.

Proof: (Sketch) (I) Consider the following equation for a critical point $\mathbf{x}_c \in \mathcal{C}$, it holds that $\sum_{(i,j) \in E(t)} h_{ij} \|x_{ij}\|^2 = \mathbf{x}_c^T \cdot (\mathbf{H} \otimes \mathbf{I}_2) \cdot \mathbf{x}_c$. Since it can be verified that $d_i\, \|p_i\| < \varepsilon \sqrt{\varepsilon}$ for $\|p_i\| \geq 0$ and $\|c_{i\ell}\| < c_{\max}$ is given in Assumption 1, we get $\sum_{(i,j) \in E(t)} h_{ij} \|x_{ij}\|^2 < N_a\, c_{\max}\, \varepsilon \sqrt{\varepsilon} \leq N\, c_{\max}\, \varepsilon \sqrt{\varepsilon}$, where we use the fact that $b_i = 0$, for $i \in \mathcal{N}_p$ and $\mathcal{N}_a \leq N$. Thus $\forall (i,\, j) \in E(t)$, it holds that $\|x_{ij}\|^2 \leq \varepsilon \sqrt{\varepsilon}\, \xi$, where $\xi \triangleq r^2\, N\, c_{\max}$. Thus $\|x_{ij}\|$ can be made arbitrarily small by reducing ε. (II) Moreover, let ε_0 satisfy the condition

$$(N-1)\sqrt{\varepsilon_0 \sqrt{\varepsilon_0}\, \xi} < r - \delta. \tag{8}$$

If $\varepsilon < \varepsilon_0$, then for *any* pair $(p,\, q) \in \mathcal{N} \times \mathcal{N}$, $\|x_{pq}\|$ satisfies $\|x_{pq}\| = |x_p - x_1 + x_1 - x_2 + \cdots - x_q| \leq (N-1)\sqrt{\varepsilon \sqrt{\varepsilon}\, \xi} < r - \delta$. As there exists a path in $G(t)$ of maximal length N from any node $p \in \mathcal{N}$ to another node q as $G(t)$ remains connected for $t > T_s$ by Theorem 1. By Definition 1, this implies $(p,\, q) \in E(t)$. Thus $G(t)$ is a complete graph. Detailed proof can be found in Lemma 2 of [7]. ■

We first define the following sets for all $i \in \mathcal{N}_a$:

$$\mathcal{S}_i \triangleq \{\mathbf{x} \in \mathbb{R}^{2N} \,|\, \|\mathbf{x} - \mathbf{1}_N \otimes c_{ig}\| \leq r_S(\varepsilon)\}, \tag{9}$$

where $r_S(\varepsilon) \triangleq \sqrt{3N\varepsilon} + \sqrt{(N-1)\varepsilon\sqrt{\varepsilon}\,\xi}$ and ξ is defined above.$\mathcal{S}_i$ represents the neighbourhood around the goal region center of an active agent $i \in \mathcal{N}_a$. Furthermore, let $\mathcal{S} \triangleq \cup_{i \in \mathcal{N}_a} \mathcal{S}_i$ and $\mathcal{S}^{\neg} \triangleq \mathbb{R}^{2N} \setminus \mathcal{S}$. The second partial derivatives of $V(t)$ with respect to x_i are given by

$$\begin{aligned} \frac{\partial^2 V}{\partial x_i \partial x_i} &= b_i\, d_i \otimes \mathbf{I}_2 + b_i\, d_i'\, p_i \cdot p_i^T \\ &\quad + \sum_{j \in \mathcal{N}_i(t)} \big(h_{ij} \otimes \mathbf{I}_2 + h_{ij}'\, x_{ij} \cdot x_{ij}^T\big) \end{aligned} \tag{10}$$

$$\frac{\partial^2 V}{\partial x_i \partial x_j} = -h_{ij} \otimes \mathbf{I}_2 - h_{ij}'\, x_{ij} \cdot x_{ij}^T, \qquad \forall j \neq i, \tag{11}$$

where $d_i' = \frac{-4\,\varepsilon^3}{(\|p_i\|^2 + \varepsilon)^3} + \frac{-\,\varepsilon^2}{(\|p_i\|^2 + \varepsilon)^2}$, and $h_{ij}' = \frac{4\, r^2}{(r^2 - \|x_{ij}\|^2)^3}$.

Lemma 3: There exists $\varepsilon_1 > 0$ such that if $\varepsilon < \varepsilon_1$, all critical points of V in $\mathcal{S}^{\neg}$ are non-degenerate saddle points.

Proof: (Sketch) To show that V is Morse we use Lemma 3.8 from [19], which states that the non-singularity of a linear operator follows from the fact that its associated quadratic form is sign definite on complementary subspaces. First of all we show that for any vector $v \in \mathcal{Q}$, where $\mathcal{Q} = \{v \in \mathbb{R}^{2N} \,|\, v = \mathbf{1}_N \otimes z,\, z \in \mathbb{R}^2\}$, it holds that $v^T \nabla^2 V v < 0$. It means that for any critical point $\mathbf{x}_c \in \mathcal{C}$, if $\mathbf{x}_c \in \mathcal{S}^{\neg}$ then $\mathbf{x}_c$ is not a local minimum. Then we show that $\nabla^2 V$ is positive definite in $\mathcal{P}$, where $\mathcal{P}$ is defined by that if $\mathbf{z} \in \mathcal{P}$, then $\mathbf{z} \triangleq \mathbf{e}_N \otimes z \triangleq [z_1^T, z_2^T, \ldots, z_n^T]^T$, where $z \in \mathbb{R}^2$, $\mathbf{e}_N \in \mathbb{R}^N$, $\mathbf{e}_N^T \perp \mathbf{1}_N$, $z_i \in \mathbb{R}^2$, $\forall i \in \mathcal{N}$, if ε satisfies

$$\varepsilon < \min\{\varepsilon_0,\, \frac{N}{0.1 r^2}\} \triangleq \varepsilon_1. \tag{12}$$

By Lemma 3.8 from [19], we conclude that $\nabla^2 V$ is non-singular at the saddle points $\mathbf{x}_c \in \mathcal{S}^{\neg}$. Thus all critical points within $\mathcal{S}^{\neg}$ are non-degenerate saddle points if $\varepsilon < \varepsilon_1$. More details can be found in Lemmas 3 and 4 of [7]. ■

Lemma 4: There exists $\varepsilon_{\min} > 0$ such that if $\varepsilon < \varepsilon_{\min}$, all critical points of V within $\mathcal{S}$ are local minima.

Proof: (Sketch) The proof is based on the fact that if

$$\varepsilon < \min\{\varepsilon_1,\, \varepsilon_2,\, \varepsilon_6,\, \varepsilon_7\} \triangleq \varepsilon_{\min}, \tag{13}$$

where ε_1 is given by (12) and ε_2 is determined by $\sqrt{3N\,\varepsilon_2} + \sqrt{(N-1)\varepsilon_2\sqrt{\varepsilon_2}\,\xi} \triangleq r_{\min}$, and $\varepsilon_6 \triangleq \min\{\varepsilon_3, \varepsilon_4, \varepsilon_5\}$, where $\varepsilon_3 \triangleq 0.07\, r_{\min}^2$, $\varepsilon_4 \triangleq 4.1/\xi^2$, $\varepsilon_5 \triangleq 0.8\, r_{\min}^2/(N-1)^2$; and

$$\varepsilon_7 \triangleq \frac{\sqrt{(\frac{N}{0.08\, r^2})^2 + \frac{4}{r^2 |\hat{g}|}} - \frac{N}{0.08\, r^2}}{2}, \tag{14}$$

where $\hat{g} \triangleq -2/r_{\min}^2$, then $\nabla^2 V$ is positive definite at all critical points $\mathbf{x}_c \in \mathcal{S}$. In other words, all local minima within $\mathcal{S}$ are stable if $\varepsilon < \varepsilon_{\min}$. We refer the readers to the proofs of Lemmas 5, 6 and 7 of [7] for details. ■

Theorem 5: Assume that $G(T_s)$ is connected and $\varepsilon < \varepsilon_{\min}$ by (13). Then starting from anywhere in the workspace except a set of measure zero, there exists a finite time $T_f \in [T_s, \infty)$ and one agent $i^\star \in \mathcal{N}_a$, such that $x_j(T_f) \in \pi_{i^\star \mathrm{g}}$, $\forall j \in \mathcal{N}$, while at the same time $\|x_i(t) - x_j(t)\| < r$, $\forall (i,\, j) \in E(T_s)$ and $\forall t \in [T_s,\, T_f]$.

Proof: (Sketch) By LaSalle's invariance principle [10] we only need to find out the largest invariant set within $\dot{V}(t) = 0$. Lemma 4 ensures that $V(t)$ has only local minima inside $\mathcal{S}$ and saddle points outside $\mathcal{S}$. These saddle points have attractors of measure zero by Lemma 3. Thus starting from anywhere in the workspace except a set of measure zero, the system converges to the set of local minima, i.e., within $\mathcal{S}_{i^\star}$ for one active agent $i^\star \in \mathcal{N}_a$. Consequently, by continuity there exists a finite time $T_f < \infty$ that $x_j(T_f) \in \pi_{i^\star \mathrm{g}}$, $\forall j \in \mathcal{N}$, for exactly one active agent $i^\star \in \mathcal{N}_a$. ■

C. Hybrid Control Structure

In this part, we propose *two* different switching protocols for each agent to decide on its own activity or passivity under different cases, such that all agents can fulfill their local tasks and at the same time satisfy the relative-distance constraints.

1) Switching Protocol for sc-LTL: Let us first focus on a case when each local task φ_i, $i \in \mathcal{N}$ is an sc-LTL formula. The discrete plan τ_i for agent i can be represented by a finite satisfying prefix of progressive goal regions and the set of services to provide at each region: $\tau_{i,\text{pre}} = (\pi_{i1}, w_{i1}) \ldots (\pi_{ik_i}, w_{ik_i})$, where $\pi_{i1}, \pi_{i2}, \ldots, \pi_{ik_i} \in \Pi_i$ and

$w_{i1}, w_{i2}, \ldots, w_{ik_i} \in 2^{\Sigma_i}$. We propose the following *activity switching protocol* for each agent $i \in \mathcal{N}$, denoted by $\mathbf{P_{sc}}$:

(I) At time $t = 0$, agent i sets $\varkappa_i := 1$ and itself as active and sets $\pi_{ig} := \pi_{i\varkappa_i}$, namely the first goal region by $\tau_{i,\text{pre}}$. The *active* controller (2) is applied to agent i, where the progressive goal region is π_{ig}, i.e., $c_{ig} = c_{i\ell_1}$; (II) Whenever agent i reaches its current progressive goal region $\pi_{ig} = \pi_{i\varkappa_i}$ and $\varkappa_i < k_i$, it provides the prescribed set of services $w_{i\varkappa_i}$ by $\tau_{i,\text{pre}}$ and it sets $\varkappa_i := \varkappa_i + 1$ and $\pi_{ig} := \pi_{i\varkappa_i}$. Then the controller (2) for agent i is updated accordingly by setting $c_{ig} = c_{i\ell_{k+1}}$; (III) Whenever agent i reaches its last goal region $\pi_{ig} = \pi_{ik_i}$, it provides the set of services w_{ik_i} by which it finishes the execution of its finite discrete plan $\tau_{i,\text{pre}}$. Afterwards agent i remains *passive* by controller (3).

Theorem 6: By following the protocol $\mathbf{P_{sc}}$, it is guaranteed that $\forall i \in \mathcal{N}$, φ_i is satisfied by $\mathbf{x}_i(T)$, and $\|x_i(t) - x_j(t)\| < r$, $\forall (i, j) \in E_0(0)$ and $\forall t \geq 0$, where $T = \infty$.

Proof: (Sketch) Initially all agents are active by (2). By Theorem 5, all agents converge to one agent's goal region at a finite time $t_1 > 0$. Denote by $i \in \mathcal{N}$ this agent. Then either agent i updates its active control law by setting $\pi_{ig} = \pi_{i2}$, or it has completed its plan $\tau_{i,\text{pre}}$ and becomes passive. Since all agents' plans are finite and Theorem 5 holds for any number of active agents, there exists a finite time instant T_{f_j}, such that all agents complete their plans. The second part of the theorem follows directly from Theorem 5. ■

2) Switching Protocol for General LTL: If the task specification φ_i is given as a general LTL formula, then the plan τ_i is given as an infinite sequence of goal regions and services:

$$\tau_i = \tau_{i,\text{pre}}(\tau_{i,\text{suf}})^\omega = (\pi_{i1}, w_{i1})(\pi_{i2}, w_{i2}) \ldots,$$

where $\tau_{i,\text{pre}} = (\pi_i, w_{i1}) \ldots (\pi_{ik_i}, w_{ik_i}), k_i > 0$ and $\tau_{i,\text{suf}} = (\pi_{ik_i+1}, w_{ik_i+1}) \ldots (\pi_{iK_i}, w_{iK_i})$. The main challenge here is to ensure that each agent executes its plan suffix infinitely often. Thereto, we introduce the *reaching-event detector* [4] for agent $i \in \mathcal{N}$ to detect when it reaches its own goal region π_{ig} and when its neighbour $j \in \mathcal{N}_i$ reaches π_{jg}.

Let $\Omega_i(j, t) \in \mathbb{B}$ be a Boolean variable indicating that agent i detects its neighboring agent $j \in \mathcal{N}_i(t)$ reaching the goal region π_{jg} at time $t > 0$. Simply speaking, the detector checks if within a short time period $[t - \Delta_t, t]$, there exists $j \in \mathcal{N}_i(t)$, such that $u_j(t)$ has changed from a relatively small value (below a given Δ_u) by a difference larger than certain Δ_d. If so, it means that the agent j has reached its goal region π_{jg}. This design is motivated by the fact that when the system is at a local minimum whenever an active agent is in its goal region. Thus, when the agent j reaches π_{jg} at time t, all control inputs $u_i(t)$ are close to zero for all $i \in \mathcal{N}$ by (7). Afterwards, the switching protocol below guarantees that *only* agent j switches its control law either to (2) to navigate to the next goal region or to (3) to become passive. This change is lower-bounded by constant Δ_d derived using control law (2) and as $\Delta_d \triangleq |f(r_{\min}) - f(\sqrt{0.4\varepsilon})|$, where $f(\|p_j\|) = d_j(\|p_j\|)\|p_j\|$ is a scalar function. In contrast, for the other agents $i \neq j$, $i \in \mathcal{N}$, the control input $u_i(t)$ remains unchanged and close to zero. Then we define a *round* as the time period during which each agent has reached at least one of its goal regions.

Definition 2: For all $m \geq 1$, the *m-th round* is defined as the time interval $[T_{\circlearrowleft_{m-1}}, T_{\circlearrowleft_m})$, where $T_{\circlearrowleft_0} = 0$, $T_{\circlearrowleft_{m-1}} < T_{\circlearrowleft_m}$ and for all $m \geq 1$, $T_{\circlearrowleft_m}$ is the smallest time satisfying that for all $i \in \mathcal{N}$: $\texttt{word}_i(T_{\circlearrowleft_m}) = w_{i1}w_{i2} \ldots w_{i\ell}$ for some $\ell \geq 1$ and $\texttt{word}_i(T_{\circlearrowleft_m}) \neq \texttt{word}_i(T_{\circlearrowleft_{m-1}})$. ■

To recognize a round completion, we introduce: $\chi_i \geq 0$ that indicates the starting time of the current round, and $\Upsilon_i \in \mathbb{Z}^N$ that records how many goal regions each agent has reached within one round since χ_i. We propose the following *activity switching protocol*, referred by $\mathbf{P_{ge}}$:

(I) At time $t = 0$, $\Upsilon_i := \mathbf{0}_N$, $\chi_i := 0$, $\varkappa_i := 1$. The agent i is active and follows (2), where $\pi_{ig} := \pi_{i\varkappa_i}$; (II) Whenever the agent i reaches $\pi_{ig} = \pi_{i\varkappa_i}$, it provides the services $w_{i\varkappa_i}$ by τ_i and updates the goal region accordingly: If $\varkappa_i < K_i$ then $\varkappa_i := \varkappa_i + 1$, and if $\varkappa_i = K_i$ then $\varkappa_i := k_i + 1$. Furthermore, $\pi_{ig} := \pi_{i\varkappa_i}$, and finally $\Upsilon_i[i] := \Upsilon_i[i] + 1$. Agent i stays active or becomes passive based on the probability function: $\mathbf{Pr}(b_i = 1) = f_{\text{prob}}(\cdot)$ if $f_{\text{cond}}(\cdot) = \texttt{True}$; and $\mathbf{Pr}(b_i = 1) = 0$, otherwise, where $f_{\text{prob}}(\cdot) \in [0, 1]$ and $f_{\text{cond}}(\cdot) \in \{\texttt{True}, \texttt{False}\}$ are functions of time t such that Υ_i and χ_i satisfy: given that it is the m-th round, there exists a time $T \in (T_{\circlearrowleft_{m-1}}, T_{\circlearrowleft_m})$, such that $f_{\text{cond}}(\cdot) = \texttt{False}$ for all $t \in [T, T_{\circlearrowleft_m})$. Whenever $b_i = 1$, agent i keeps following (2). Otherwise, it becomes passive by (3); (III) Whenever agent i detects that $\Omega_i(j, t) = \texttt{True}$, for some $j \neq i \in \mathcal{N}$, it sets $\Upsilon_i[j] = \Upsilon_i[j] + 1$; (IV) Whenever it holds that $\Upsilon_i[j] \geq 1, \forall j \in \mathcal{N}$, then agent i sets $\Upsilon_i := \mathbf{0}_N$, $\chi_i := t$ and follows (2) to its goal π_{ig}.

Lemma 7: The round $[T_{\circlearrowleft_{m-1}}, T_{\circlearrowleft_m})$ is finite, $\forall m \geq 1$.

Proof: Let $t = T_{\circlearrowleft_{m-1}} = 0$, and thus $\Upsilon_i[j] = 0$, for all $i, j \in \mathcal{N}$ by step (I). By Theorem 5, one active agent reaches its goal region in finite time at $t_1 \geq T_{\circlearrowleft_{j-1}}$. Due to the properties of f_{cond}, there exists a finite time $T_{f_j} \geq 0$, when either the step (IV) applies or when one of the agents $j \in \mathcal{N}_a$ necessarily becomes passive by $\mathbf{Pr}(\cdot)$ and remains passive till the end of the round. In the former case, $T_{\circlearrowleft_m} = T_{f_j}$. In the latter case, the same argument applies to the $N - 1$ active agents such that one of them will become passive in finite time. Inductively, we derive that the every round is finite. ■

Theorem 8: By following the protocol $\mathbf{P_{ge}}$ above, it is guaranteed that $\forall i \in \mathcal{N}$, φ_i is satisfied by $\mathbf{x}_i(T)$ and $\|x_i(t) - x_j(t)\| < r$, $\forall (i, j) \in E_0(0)$ and $\forall t > 0$, where $T = \infty$.

Proof: (Sketch) The satisfaction of φ_i follows from the correctness of its discrete plan and the fact that each round is finite by Lemma 7. At last, the relative distance constraints are always maintained as shown in Theorem 6. ■

V. SIMULATION

In the case study, we simulate a team of four autonomous robots $\mathcal{N} = \{\mathfrak{R}_1, \ldots, \mathfrak{R}_4\}$ subject to the dynamics (1) in a bounded, obstacle-free workspace of $40m \times 40m$. Each robot $\mathfrak{R}_i$ is given a local task specified as LTL formulas φ_i.

As shown in Figure 1, several sphere regions of interest for each agent are placed in top-left, top-right, bottom-right, and bottom-left corners of the workspace and they all satisfy

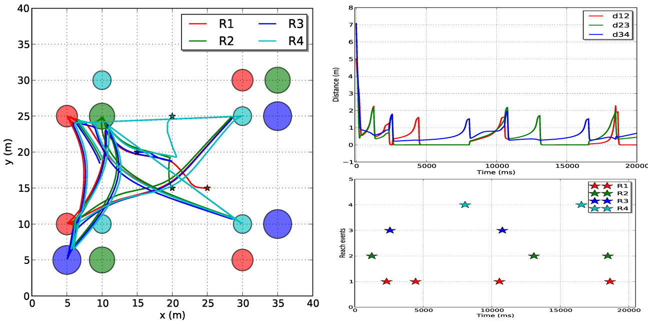

Fig. 1: Agents' trajectories and pair-wise distances for simulation time $20s$. The switching protocol $\mathbf{P_{ge}}$ from Section IV-C.2 is used.

Assumption 1 with $c_{\max} = 40$ and $r_{\min} = 2$. Particularly, they are given by: $\Pi_1 = \{\pi_{1\mathtt{tl}}, \pi_{1\mathtt{tr}}, \pi_{1\mathtt{br}}, \pi_{1\mathtt{bl}}\}$ shown in red; $\Pi_2 = \{\pi_{2\mathtt{tl}}, \pi_{2\mathtt{tr}}, \pi_{2\mathtt{bl}}\}$ shown in green; $\Pi_3 = \{\pi_{3\mathtt{tr}}, \pi_{3\mathtt{br}}, \pi_{3\mathtt{bl}}\}$ shown in blue; $\Pi_4 = \{\pi_{4\mathtt{tl}}, \pi_{4\mathtt{tr}}, \pi_{4\mathtt{br}}, \pi_{4\mathtt{bl}}\}$ shown in cyan. The respective sets of atomic propositions (services) are $\Sigma_1 = \{\sigma_{11}, \sigma_{12}\}$; $\Sigma_2 = \{\sigma_{21}, \sigma_{22}, \sigma_{23}\}$; $\Sigma_3 = \{\sigma_{31}, \sigma_{32}, \sigma_{33}\}$; and $\Sigma_4 = \{\sigma_{41}, \sigma_{42}\}$. The regions are labeled as follows: $L_1(\pi_{1\mathtt{tl}}) = L_1(\pi_{1\mathtt{br}}) = \{\sigma_{11}\}$, $L_1(\pi_{1\mathtt{tr}}) = L_1(\pi_{1\mathtt{bl}}) = \{\sigma_{12}\}$; $L_2(\pi_{2\mathtt{tl}}) = \{\sigma_{21}\}$, $L_2(\sigma_{2\mathtt{tr}}) = \{\sigma_{22}\}$, $L_2(\pi_{2\mathtt{bl}}) = \{\sigma_{23}\}$; $L_3(\pi_{3\mathtt{tr}}) = \{\sigma_{31}\}$, $L_3(\pi_{3\mathtt{br}}) = \{\sigma_{32}\}$, $L_3(\pi_{3\mathtt{bl}}) = \{\sigma_{33}\}$; and finally $L_4(\pi_{4\mathtt{tl}}) = L_4(\pi_{4\mathtt{tr}}) = \{\sigma_{41}\}$, $L_4(\pi_{4\mathtt{bl}}) = L_4(\pi_{4\mathtt{br}}) = \{\sigma_{42}\}$. The agents start from 2-D coordinates $(25, 15)$, $(20, 15)$, $(15, 20)$, and $(20, 25)$, respectively. We set $r = 8m$ and $\delta = 0.5m$. The edge set of $G(0)$ is hence $E_0(0) = \{(\mathfrak{R}_1, \mathfrak{R}_2), (\mathfrak{R}_2, \mathfrak{R}_3), (\mathfrak{R}_3, \mathfrak{R}_4)\}$. The upper bound by (13) is $\varepsilon < \varepsilon_{\min} \approx 0.031$ and we choose $\varepsilon = 0.03$.

We consider the case of agent task specifications given as general LTL formulas here only, due to limited space. The task of agent $\mathfrak{R}_1$ to periodically provide both services σ_{11} and σ_{12}, specified as the general LTL formula $\phi_1 = \Box\Diamond\sigma_{11} \wedge \Box\Diamond\sigma_{12}$. The task of agent $\mathfrak{R}_2$ is to periodically provide one of the services σ_{21} or σ_{22} or σ_{23}, formalized as $\phi_2 = \Box\Diamond(\sigma_{21} \vee \sigma_{22} \vee \sigma_{23})$. The tasks of agents $\mathfrak{R}_3$ and $\mathfrak{R}_4$ are defined in a similar way that $\phi_3 = \Box\Diamond(\sigma_{31} \vee \sigma_{32} \vee \sigma_{33})$, and $\phi_4 = \Box\Diamond\sigma_{41} \wedge \Box\Diamond\sigma_{42}$, respectively. The synthesized discrete plans are: $\tau_1 = \big((\pi_{1\mathtt{bl}}, \{\sigma_{12}\})\big((\pi_{1\mathtt{tl}}, \{\sigma_{11}\})\big)^{\omega}$; $\tau_2 = (\pi_{2\mathtt{tl}}, \{\sigma_{21}\})^{\omega}$; $\tau_3 = (\pi_{3\mathtt{bl}}, \{\sigma_{33}\})^{\omega}$; $\tau_4 = \big((\pi_{4\mathtt{br}}, \{\sigma_{41}\})(\pi_{4\mathtt{tr}}, \{\sigma_{42}\})\big)^{\omega}$. The simulation results of $20s$ are illustrated in Figure 1. The functions f_{prob} and f_{cond} were chosen as follows: $\mathbf{Pr}(b_i = 1) = e^{-\alpha_i \Upsilon_i[i](t-\chi_i)}$, if $\Upsilon_i[i] \cdot (t - \chi_i) < \bar{\chi}_i$; and $\mathbf{Pr}(b_i = 1) = 0$, if $\Upsilon_i[i] \cdot (t - \chi_i) \geq \bar{\chi}_i$, where $\bar{\chi}_i = 5$, and $\alpha_i = 1$. Figure 1 shows the complete agent trajectories for simulation time $20s$. It can be seen that the relative distance constraints are satisfied for all time and each agent is making progress in its plan execution by following the protocol $\mathbf{P_{ge}}$.

VI. Conclusion and Future Work

We proposed a distributed communication-free hybrid control scheme for multi-agent systems to fulfil locally-assigned tasks as general or sc-LTL formulas, while at the same time subject to relative-distance constraints. Future work plans include handling uncertainties in the relative state measurements and more complex agent dynamics.

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