Communication-Constrained STL Task Decomposition through Convex Optimization

Gregorio Marchesini, Siyuan Liu, Lars Lindemann and Dimos V. Dimarogonas

Abstract— **We propose a method to decompose signal temporal logic tasks for multi-agent systems under communication constraints. Specifically, given a task graph representing task dependencies among couples of agents in the system, we propose to decompose tasks assigned to couples of agents not connected in the communication graph by a set of sub-tasks assigned to couples of communicating agents over the communication graph. To this end, we parameterize the predicates' level set of tasks to be decomposed as hyper-rectangles with parametric centres and dimensions. Convex optimization is then leveraged to find optimal parameters maximising the volume of the predicate's level sets. Moreover, a formal treatment of conflicting conjunctions of formulas in the considered STL fragment is introduced, including sufficient conditions to avoid the insurgence of such conflicts in the final decomposition.**

I. INTRODUCTION

Temporal logics have recently received increased attention in the control community. Among the different temporal logics, signal temporal logic (STL) has been successfully applied for real-time reactive control and planning of multi-agent systems (MAS) leveraging its abstraction-free and continuous-time nature [1], [2], [3], [4]. Notably, control frameworks like prescribed performance control [5], predictive control with integer variables [6], [7], and time-varying control barrier functions (CBF) [3] were employed to satisfy complex high-level task express as STL formulae. The common underlying assumption for these methods is that state information (communicated or achieved by perception systems) from agents involved in collaborative tasks, is available. Nevertheless, this assumption is often not met when communication/perception is range-limited or special communication topology is enforced. We propose a first step toward relaxing such an assumption by decomposing task dependencies to match the communication topology.

Previously, [8] employed convex optimization to decompose global STL tasks for a MAS into local tasks independently satisfiable by distinct sub-clusters of agents in the MAS. However, full communication connectivity of each sub-cluster is still assumed. In [9], [10] a similar decomposition is achieved employing Mixed-Integer Linear Programming (MILP) for an STL fragment including conjunctions and disjunctions, but the interplay between task dependencies and communication is again not considered here. In [11], communication constraints are considered at the planning level, by solving a complex MILP yielding state trajectories for each agent in the MAS such that a certain quality of communication is preserved during task satisfaction. Nevertheless, communication is here enforced to match the collaborative tasks' dependencies. On the contrary, the current work suggests adapting the task dependencies to the communication topology.

The main contributions of this work are twofold. First, we propose a communication-constrained task decomposition for an STL fragment defined over the absolute and relative state of the agents in the MAS. Namely, we exploit the communication graph of the MAS to decompose tasks depending on the relative state of agents separated by multiple hops in the communication graph, into a conjunction of sub-tasks defined over the relative state of agents having 1-hop communication distance. The predicate function of each newly introduced sub-tasks is parametrised as a hyper-rectangle such that optimal parameters (namely the centre and dimensions) are optimised to maximise the volume of the super-level set of the predicate. After decomposition, only local state information from neighbours in the communication graph is required to satisfy the global task. Second, we formally derive a set of conditions leading to un-satisfiable conjunctions of tasks in the considered STL fragments. A set of convex constraints over the parameters of the newly introduced subtasks is then provided to exclude such un-satisfiable conjunctions from the solution set of our task decomposition. The paper is organized as follows: Sec. II presents preliminaries and Sec. III introduces the problem definition. Sec. IV proposes our main task decomposition result. In Sec. V, we provide a formal definition of conflicting conjunctions of tasks and then propose a strategy to avoid such conflicts. A relevant simulation example and conclusions are provided in Sec. VI and Sec. VII.

Notation: Bold letters denote vectors. Capital letters indicate matrices. Vectors are considered to be column vectors and $\boldsymbol{x}[k]$ indicates the k-th element of $\boldsymbol{x}$. Let the element-wise minimum for a vector $\boldsymbol{a} \in \mathbb{R}^n$ be $\overset{\star}{\min}(\boldsymbol{a}) := \min_{k=1,\dots n}\{\boldsymbol{a}[k]\}$. The notation $|\mathcal{A}|$ denotes the cardinality of the set $\mathcal{A}$. The symbols $\oplus/\ominus$ indicate the Minkowski sum/difference. Let $\prod_{i=1}^{k} \mathcal{A}_i$ represents the Cartesian product of $\mathcal{A}_i$ and $-\mathcal{A} = \{x|x = -v \ \forall v \in \mathcal{A}\}$. The identity matrix of dimension n is denoted as I_n. The set $\mathbb{R}_+$ denotes

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the non-negative real numbers.

II. PRELIMINARIES

Let $\mathcal{V} = \{1, \dots N\}$ be the set of indices assigned to each agent in a multi-agent system with dynamics

$$\dot{\boldsymbol{x}}_i = \mathfrak{f}_i(\boldsymbol{x}_i) + \mathfrak{g}_i(\boldsymbol{x}_i)\boldsymbol{u}_i \tag{1}$$

where $\boldsymbol{x}_i \in \mathbb{X}_i \subset \mathbb{R}^{n_i}$ is the state of the i-th agent and $\boldsymbol{u}_i \in \mathbb{U}_i \subset \mathbb{R}^{m_i}$ is the associated bounded control input. We assume, without loss of generality, that $n_i = n$, $\forall i \in \mathcal{V}$. Let $\mathfrak{f}_i : \mathbb{R}^n \to \mathbb{R}^n$ and $\mathfrak{g}_i : \mathbb{R}^n \to \mathbb{R}^{n \times m_i}$ be locally Lipschitz continuous functions of the agent state. We denote the full state of the system as $\boldsymbol{x} := [\boldsymbol{x}_1^T, \boldsymbol{x}_2^T, \dots \boldsymbol{x}_N^T]^T$. Given a control input $\boldsymbol{u}_i : [t_0, t_1] \to \mathbb{U}_i$, we define the state trajectory $\boldsymbol{x}_i : [t_0, t_1] \to \mathbb{X}_i$ for agent $i \in \mathcal{V}$ if $\boldsymbol{x}_i(t)$ satisfies (1) for all $t \in [t_0, t_1]$. We also define the *relative* state vector $\boldsymbol{e}_{ij} := \boldsymbol{x}_j - \boldsymbol{x}_i$ for $i, j \in \mathcal{V}$.

A. Signal Temporal Logic

STL is a predicate logic applied to define spatio-temporal properties of real-valued continuous-time signals [12]. The atomic elements of STL are Boolean predicates $\mu : \mathbb{R} \to \{\top, \bot\}$ defined as $\mu := \begin{cases} \top & \text{if } h(\boldsymbol{x}) \geq 0 \\ \bot & \text{if } h(\boldsymbol{x}) < 0, \end{cases}$ where $h(\boldsymbol{x}) : \mathbb{R}^n \to \mathbb{R}$ is a *predicate function*. The function h generally depends on the state of all or some of the agents in the MAS. The STL grammar is recursively defined as $\phi ::= \top | \mu | \neg\phi | \phi_1 \wedge \phi_2 | F_{[a,b]}\phi | G_{[a,b]}\phi | \phi_1 U_{[a,b]} \phi_2$ where $F_{[a,b]}$, $G_{[a,b]}$ and $U_{[a,b]}$ are the temporal *eventually*, *always* and *until* operators over the time interval $[a, b] \subset \mathbb{R}_+$. We indicate that a state trajectory $\boldsymbol{x}(t)$ satisfies task ϕ at time t as $\boldsymbol{x}(t) \models \phi$. The STL *semantics* define the conditions such that $\boldsymbol{x}(t) \models \phi$ [12]. We adopt the robust quantitative STL semantics: $\rho^{\mu}(\boldsymbol{x}, t) = h(\boldsymbol{x}(t))$, $\rho^{\neg\phi}(\boldsymbol{x}, t) = -\rho^{\phi}(\boldsymbol{x}, t)$, $\rho^{\phi_1 \wedge \phi_2}(\boldsymbol{x}, t) = \min\left(\rho^{\phi_1}(\boldsymbol{x}, t), \rho^{\phi_2}(\boldsymbol{x}, t)\right)$, $\rho^{\phi_1 \mathcal{U}_{[a,b]} \phi_2}(\boldsymbol{x}, t) = \max_{t_1 \in [t+a, t+b]} \min\left(\rho^{\phi_2}(\boldsymbol{x}, t_1), \min_{t_2 \in [t, t_1]} \rho^{\phi_1}(\boldsymbol{x}, t_2)\right)$, $\rho^{F_{[a,b]}\phi}(\boldsymbol{x}, t) = \max_{t_1 \in [t+a, t+b]} \rho^{\phi}(\boldsymbol{x}, t_1)$, $\rho^{G_{[a,b]}\phi}(\boldsymbol{x}, t) = \min_{t_1 \in [t+a, t+b]} \rho^{\phi}(\boldsymbol{x}, t_1)$, and we recall that $\rho^{\phi}(\boldsymbol{x}, 0) > 0 \Rightarrow \boldsymbol{x}(t) \models \phi$ [13].The STL fragment employed in the current work is then given as:

$$\phi_i := F_{[a,b]}\mu_i | G_{[a,b]}\mu_i | \phi_i^1 \wedge \phi_i^2, \tag{2a}$$

$$\phi_{ij} := F_{[a,b]}\mu_{ij} | G_{[a,b]}\mu_{ij} | \phi_{ij}^1 \wedge \phi_{ij}^2, \tag{2b}$$

where $\mu_i := \begin{cases} \top & \text{if } h_i(\boldsymbol{x}_i) \geq 0 \\ \bot & \text{if } h_i(\boldsymbol{x}_i) < 0 \end{cases}$, $\mu_{ij} := \begin{cases} \top & \text{if } h_{ij}(\boldsymbol{e}_{ij}) \geq 0 \\ \bot & \text{if } h_{ij}(\boldsymbol{e}_{ij}) < 0, \end{cases}$ with predicate functions $h_i(\boldsymbol{x}_i)$ and $h_{ij}(\boldsymbol{e}_{ij})$. We refer to (2a) as *independent* tasks and (2b) as *collaborative* tasks, and let the respective super-level sets be given by

$$\begin{aligned} \mathcal{B}_i &= \{\boldsymbol{x}_i \in \mathbb{X}_i | h_i(\boldsymbol{x}_i) \geq 0\}; \\ \mathcal{B}_{ij} &= \{\boldsymbol{e}_{ij} \in \mathbb{X}_j \ominus \mathbb{X}_i) | h_{ij}(\boldsymbol{e}_{ij}) \geq 0\}. \end{aligned} \tag{3}$$

B. Communication and Task Graphs

Let $\mathcal{G}(\mathcal{V}, \mathcal{E}) \in \Gamma$ define an *undirected* graph over the set of agents $\mathcal{V}$, where $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of undirected edges and Γ is the set of undirected graphs over $\mathcal{V}$. Let $\mathcal{N}(i) = \{j | (i, j) \in \mathcal{E} \wedge i \neq j\}$ be the neighbours set of i and let $\bar{\mathcal{N}}(i) := \mathcal{N} \cup \{i\}$ such that self-loops are considered.

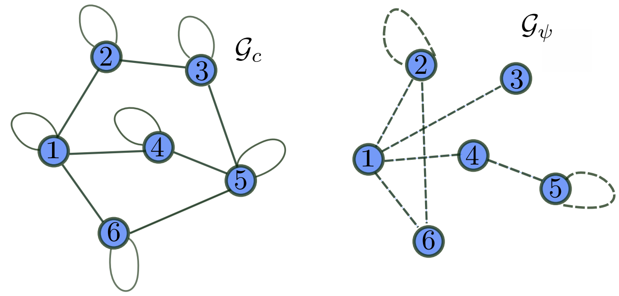


Fig. 1: Simple example of communication (left) and task graph (right) for a multi-agent system with 6 agents. The task and communication graph are mismatching in their case.

Furthermore, let the graph-valued function $\mathbf{add}(\cdot, \cdot) : \Gamma \times \mathcal{V} \times \mathcal{V} \to \Gamma$ be such that $\mathcal{G}'(\mathcal{E}', \mathcal{V}) = \mathbf{add}(\mathcal{G}, \mathcal{Q})$ with $\mathcal{E}' = \mathcal{E} \cup \mathcal{Q}$ for a given set of edges $\mathcal{Q} \subset \mathcal{V} \times \mathcal{V}$. Similarly let $\mathbf{del}(\cdot, \cdot) : \Gamma \times \mathcal{V} \times \mathcal{V} \to \Gamma$ such that $\mathcal{G}'(\mathcal{E}', \mathcal{V}) = \mathbf{del}(\mathcal{G}, \mathcal{Q})$ and $\mathcal{E}' = \mathcal{E} \setminus \mathcal{Q}$. Let the vector $\boldsymbol{\pi}_i^j \in \mathcal{V}^l$ represent a *directed* path of length l of non-repeating indices in $\mathcal{V}$ such that $\boldsymbol{\pi}_i^j[k] \in \mathcal{V}\ \forall k = 1, \dots l$, $\boldsymbol{\pi}_i^j[r] \neq \boldsymbol{\pi}_i^j[s]\ \forall s \neq r$, $(\boldsymbol{\pi}_i^j[r], \boldsymbol{\pi}_i^j[r+1]) \in \mathcal{E}$ and $(\boldsymbol{\pi}_i^j[1], \boldsymbol{\pi}_i^j[l]) = (i, j)$. Let $\boldsymbol{\omega} \in \mathcal{V}^l$ represent a *directed* closed cycle path, such that $\boldsymbol{\omega}$ is a *directed* path and $\boldsymbol{\omega}[1] = \boldsymbol{\omega}[l]$. Let $\epsilon : \mathcal{V}^l \to 2^{\mathcal{E}}$ be a set-valued function such that $\epsilon(\boldsymbol{\pi}_i^j) = \{(\boldsymbol{\pi}_i^j[k], \boldsymbol{\pi}_i^j[k+1]) | k = 1, \dots l-1\}$. The following relations hold

$$\boldsymbol{e}_{ij} = \sum_{(r,s) \in \epsilon(\boldsymbol{\pi}_i^j)} \boldsymbol{e}_{rs}, \quad \mathbf{0} = \sum_{(r,s) \in \epsilon(\boldsymbol{\omega})} \boldsymbol{e}_{rs}. \tag{4}$$

We further distinguish between *communication graph* $\mathcal{G}_c(\mathcal{V}, \mathcal{E}_c) \in \Gamma$ such that $(i, j) \in \mathcal{E}_c \subset \mathcal{V} \times \mathcal{V}$ if i and j can communicate their respective state to each other; and *task* graph $\mathcal{G}_\psi(\mathcal{V}, \mathcal{E}_\psi) \in \Gamma$ such that $(i, j) \in \mathcal{E}_\psi$ if there exists a collaborative task ϕ_{ij} as per (2b) between agent i and j. Independent tasks ϕ_i as per (2a) induce self-loops in the task graph (see in Fig. 1). We repeatedly adopt the subscript c and ψ to differentiate among properties of the communication and task graph, respectively. For instance, $\mathcal{N}_c(i)$ and $\mathcal{N}_\psi(i)$ indicate the neighbour set in the communication and task graph, respectively. The *global* task ψ is then given as

$$\psi := \bigwedge_{i=1}^{N} \big(\phi_i \wedge \bigwedge_{j \in \mathcal{N}_\psi(i)} \phi_{ij}\big). \tag{5}$$

Tasks according to (5) are particularly suitable for, e.g. time-varying relative formations. An example is proposed next to clarify our notation.

Example 1: Consider the graphs $\mathcal{G}_c$ and $\mathcal{G}_\psi$ in Fig 1. Independent task are ϕ_2, ϕ_5, while $\phi_{12}, \phi_{13}, \phi_{14}, \phi_{16}, \phi_{56}, \phi_{26}$ are collaborative tasks. The path $\boldsymbol{\pi}_1^3 = [1, 2, 3]$ connects 1 to 3 with length $l = 3$ and $\epsilon(\boldsymbol{\pi}_1^3) = \{(1, 2), (2, 3)\}$. Likewise, $\boldsymbol{\pi}_6^2 = [6, 1, 2]$ and $\epsilon(\boldsymbol{\pi}_6^2) = \{(6, 1), (1, 2)\}$. Agents 1, 2, 6 form a cycle $\boldsymbol{\omega} = [1, 2, 6, 1]$ in $\mathcal{G}_\psi$.

Three main assumptions are considered:

Assumption 1: (Connectivity) The communication graph $\mathcal{G}_c$ is a time-invariant connected graph.

Assumption 2: (Concavity) The predicate functions h_{ij} and h_i are concave functions of $\boldsymbol{e}_{ij}$ and $\boldsymbol{x}_i$ respectively.

Assumption 3: (Task symmetry) For each STL task ϕ_{ij} in (2b), we have that $\phi_{ij} = \phi_{ji}\ \forall (i,j) \in \mathcal{E}_\psi$.
Assmp. (1) is required to obtain a decomposition and ensure that such decomposition remains valid over time. The case of time-varying $\mathcal{G}_c$ is given as future work. Assmp. 2 is required to maintain the super-level sets $\mathcal{B}_{ij}$ in (3) convex. Similar assumptions are considered in [8]. Note that Assmp. 3 is consistent with $\mathcal{G}_\psi$ being undirected.

III. PROBLEM FORMULATION

Decentralised controllers to satisfy ψ, like the one proposed in [3], are effectively applied when $\mathcal{N}_\psi(i) \subseteq \mathcal{N}_c(i)$. However, this is not the case when $\mathcal{N}_\psi(i) \not\subseteq \mathcal{N}_c(i)$ due to missing state information. Thus, we propose to exploit paths of communicating agents in $\mathcal{G}_c$ to construct a new task $\bar{\psi}$ in as per (5) and a new graph $\mathcal{G}_{\bar{\psi}}$ such that $\mathcal{N}_{\bar{\psi}}(i) \subseteq \mathcal{N}_c(i)$ and $(\boldsymbol{x}, t) \models \bar{\psi} \Rightarrow (\boldsymbol{x}, t) \models \psi$. We state the problem as follows:

Problem 1: Consider the multi-agent system with agents in $\mathcal{V}$, communication graph $\mathcal{G}_c$ and task graph $\mathcal{G}_\psi$ such that ψ is according to (5) and $\mathcal{E}_\psi \setminus \mathcal{E}_c \neq \emptyset$. Then, define a new global task $\bar{\psi}$ in the form

$$\bar{\psi} := \bigwedge_{i=1}^{N} \left(\phi_i \wedge \bigwedge_{j\in\mathcal{N}_\psi(i)\cap\mathcal{N}_c(i)} \phi_{ij} \wedge \bigwedge_{j\in\mathcal{N}_\psi(i)\setminus\mathcal{N}_c(i)} \phi^{\pi_i^j} \right), \tag{6}$$

with task graph $\mathcal{G}_{\bar{\psi}}$, such that $\forall (i,j) \in \mathcal{E}_\psi \setminus \mathcal{E}_c$; and let

$$\phi^{\pi_i^j} := \bigwedge_{(r,s)\in\epsilon(\pi_i^j)} \bar{\phi}_{rs}^{\pi_i^j}, \tag{7}$$

where $\boldsymbol{\pi}_i^j$ are paths in $\mathcal{G}_c$ with $\epsilon(\pi_i^j) \subset \mathcal{E}_c$. Furthermore, $\bar{\phi}_{rs}^{\pi_i^j}$ are tasks of type (2b) to be appropriately defined such that $\boldsymbol{x}(t) \models \bar{\psi}$ implies $\boldsymbol{x}(t) \models \psi$.

Example 2: Consider Fig. 1 such that Agents 2 and 6 share task ϕ_{26} (dashed graph), but $(1,4) \notin \mathcal{E}_c$. Task ϕ_{26} is then replaced by $\phi^{\pi_2^6} = \bar{\phi}_{61}^{\pi_2^6} \wedge \bar{\phi}_{16}^{\pi_2^6}$ where $\pi_2^6 = [2,1,6]$.
In the next sections, we will develop how the tasks $\bar{\phi}_{rs}^{\pi_i^j}$ are defined according to $\mathcal{G}_c$ and the temporal properties of the collaborative tasks ϕ_{ij} with $(i,j) \in \mathcal{E}_\psi \setminus \mathcal{E}_c$.

IV. TASK DECOMPOSITION

In this section, we define how a task $\phi^{\pi_i^j}$ as per (7), is obtained by decomposition of a collaborative task $\phi_{ij} = T_{[a,b]}\mu_{ij}$ with $T \in \{G, F\}$ as defined in (2b). In the case $\phi_{ij} = \bigwedge_k \phi_{ij}^k$, the method developed in this section is applied for each ϕ_{ij}^k, leading to ϕ_{ij} being decomposed as $\phi^{\pi_i^j} = \bigwedge_k (\phi^{\pi_i^j})^k$. Each $(\phi^{\pi_i^j})^k$ is computed with the same approach explained in this section.

A. Path decomposition of STL tasks

Constructing task $\phi^{\pi_i^j}$ from $\phi_{ij} = T_{[a,b]}\mu_{ij}$ requires two steps: 1) find a path $\boldsymbol{\pi}_i^j$ from i to j through $\mathcal{G}_c$, 2) find a family of tasks $\bar{\phi}_{rs}^{\pi_i^j}$ to construct $\phi^{\pi_i^j}$ as per (7) such that $\boldsymbol{x}(t) \models \phi^{\pi_i^j} \Rightarrow \boldsymbol{x}(t) \models \phi_{ij}$. A solution to step 1) is readily available since several algorithms to find paths among nodes in a connected graph already exist [14]. In the current work, we adopt the Dijkstra algorithm, noting that finding the shortest path connecting two nodes is not a requirement for our work. On the other hand, for step 2), we leverage axis-aligned n-dimensional hyper-rectangles as suitable predicate functions for the tasks $\bar{\phi}_{rs}^{\pi_i^j}$ [15], [16]

Definition 1: ([15, Ch .1.1][16, Def 3.6]) Given $\boldsymbol{\nu} \in \mathbb{R}^n$ such that $\boldsymbol{\nu}[k] \in \mathbb{R}_+ \quad \forall k \in 1, \dots n$ and $\boldsymbol{p} \in \mathbb{R}^n$, an axis-aligned hyper-rectangle $\mathcal{H}(\boldsymbol{p}, \boldsymbol{\nu})$ is defined as $\mathcal{H}(\boldsymbol{p}, \boldsymbol{\nu}) = \prod_{k=1}^{n} [\boldsymbol{p}[k] - \boldsymbol{\nu}[k]/2, \boldsymbol{p}[k] + \boldsymbol{\nu}[k]/2]$. Equivalently, $\mathcal{H}(\boldsymbol{p}, \boldsymbol{\nu}) = \{\boldsymbol{\zeta} \in \mathbb{R}^n | A(\boldsymbol{\zeta} - \boldsymbol{p}) - \boldsymbol{b}(\boldsymbol{\nu}) \geq \boldsymbol{0}\}$ such that $\boldsymbol{b}(\boldsymbol{\nu}) = [\boldsymbol{\nu}^T/2, -\boldsymbol{\nu}^T/2]$ and $A = [I_n, -I_n]^T$.

Proposition 1: ([15, Ch. 1.1]) Any point $\boldsymbol{\zeta} \in \mathcal{H}(\boldsymbol{p}, \boldsymbol{\nu})$ is a convex combination of the set of vertices $\mathcal{P}(\boldsymbol{p}, \boldsymbol{\nu}) := \{\boldsymbol{v} \in \mathbb{R}^n | \boldsymbol{v}[s] = \boldsymbol{p}[s] + \boldsymbol{\nu}[s]/2 \text{ or } \boldsymbol{v}[s] = \boldsymbol{p}[s] - \boldsymbol{\nu}[s]/2\ \forall s = 1, \dots n\}$, where $|\mathcal{P}(\boldsymbol{p}, \boldsymbol{\nu})| = 2^n$, such that $\boldsymbol{\zeta} = \sum_{i=1}^{2n} \lambda_i \boldsymbol{v}_i$, with $\sum_i^{2n} \lambda_i = 1$, $0 \geq \lambda_i \geq 1$ and $\boldsymbol{v}_i \in \mathcal{P}(\boldsymbol{p}, \boldsymbol{\nu})$.

Proposition 2: ([16]) Let two axis aligned hyper-rectangles $\mathcal{H}^1(\boldsymbol{p}_1, \boldsymbol{\nu}_1)$, $\mathcal{H}^2(\boldsymbol{p}_2, \boldsymbol{\nu}_2)$, then the Minkowski sum $\mathcal{H}^3(\boldsymbol{p}_3, \boldsymbol{\nu}_3) = \mathcal{H}^1(\boldsymbol{p}_1, \boldsymbol{\nu}_1) \oplus \mathcal{H}^2(\boldsymbol{p}_2, \boldsymbol{\nu}_2)$ is an axis aligned hyper-rectangle such that $\boldsymbol{p}_3 = \boldsymbol{p}_1 + \boldsymbol{p}_2$ and $\boldsymbol{\nu}_3 = \boldsymbol{\nu}_1 + \boldsymbol{\nu}_2$.

Proof: From Def. 1 $\mathcal{H}^3(\boldsymbol{p}_3, \boldsymbol{\nu}_3) = \big(\prod_{k=1}^{n} [\boldsymbol{p}^1[k] - \boldsymbol{\nu}^1[k], \boldsymbol{p}^1[k] + \boldsymbol{\nu}^1[k]]\big) \oplus \big(\prod_{k=1}^{n} [\boldsymbol{p}^2[k] - \boldsymbol{\nu}^2[k], \boldsymbol{p}^2[k] + \boldsymbol{\nu}^2[k]]\big) = \prod_{k=1}^{n} \big([\boldsymbol{p}^1[k] - \boldsymbol{\nu}^1[k], \boldsymbol{p}^1[k] + \boldsymbol{\nu}^1[k]] \oplus [\boldsymbol{p}^2[k] - \boldsymbol{\nu}^2[k], \boldsymbol{p}^2[k] + \boldsymbol{\nu}^2[k]]\big) = \prod_{k=1}^{n} \big([\boldsymbol{p}^1[k] + \boldsymbol{p}^2[k] - (\boldsymbol{\nu}^1[k] + \boldsymbol{\nu}^2[k]), \boldsymbol{p}^1[k] + \boldsymbol{p}^2[k] + (\boldsymbol{\nu}^1[k] + \boldsymbol{\nu}^2[k])]\big) = \prod_{k=1}^{n} ([\boldsymbol{p}^3[k] - \boldsymbol{\nu}^3[k], \boldsymbol{p}^3[k] + \boldsymbol{\nu}^3[k]])$. ■

Proposition 3: ([15]) Consider a concave scalar-valued function $g : \mathbb{R}^n \to \mathbb{R}$ and hyper-rectangle $\mathcal{H}(\boldsymbol{p}, \boldsymbol{\nu})$. If $g(\boldsymbol{v}_i) \geq 0\ \forall \boldsymbol{v}_i \in \mathcal{P}(\boldsymbol{p}, \boldsymbol{\nu})$ then $g(\boldsymbol{\zeta}) \geq 0\ \forall \boldsymbol{\zeta} \in \mathcal{H}(\boldsymbol{p}, \boldsymbol{\nu})$.

Proof: By Prop. 1 we know that $\boldsymbol{\zeta} = \sum_{i=1}^{2n} \lambda_i \boldsymbol{v}_i$ with $\sum_i^{2n} \lambda_i = 1, 0 \geq \lambda_i \geq 1$ and $\boldsymbol{v}_i \in \mathcal{P}(\boldsymbol{p}, \boldsymbol{\nu})$. By Jensen's inequality $g(\boldsymbol{\zeta}) = g(\sum_{i=1}^{2n} \lambda_i \boldsymbol{v}_i) \geq \sum_{i=1}^{2n} \lambda_i g(\boldsymbol{v}_i) \geq 0$. ■

If for each $\bar{\phi}_{rs}^{\pi_i^j}$ with $(r,s) \in \epsilon(\boldsymbol{\pi}_i^j)$ we define a centre $\boldsymbol{p}_{rs}^{\pi_i^j}$ and a dimension vector $\boldsymbol{\nu}_{rs}^{\pi_i^j}$, the following family of concave predicate functions can be employed for our decomposition:

$$\bar{h}_{rs}^{\pi_i^j}(\boldsymbol{e}_{rs}, \boldsymbol{\eta}_{rs}^{\pi_i^j}) := \overset{\star}{\min}(A(\boldsymbol{e}_{rs} - \boldsymbol{p}_{rs}^{\pi_i^j}) - \boldsymbol{b}(\boldsymbol{\nu}_{rs}^{\pi_i^j})) \geq 0, \tag{8a}$$

$$\bar{\mathcal{B}}_{rs}^{\pi_i^j}(\boldsymbol{\eta}_{rs}^{\pi_i^j}) := \{\boldsymbol{e}_{rs} \in \mathbb{X}_r \oplus (-\mathbb{X}_s) | \bar{h}_{rs}^{\pi_i^j}(\boldsymbol{e}_{rs}, \boldsymbol{\eta}_{rs}^{\pi_i^j}) \geq 0\}, \tag{8b}$$

$$\bar{\mu}_{rs}^{\pi_i^j}(\boldsymbol{\eta}_{rs}^{\pi_i^j}) := \begin{cases} \top & \text{if} \quad \bar{h}_{rs}^{\pi_i^j}(\boldsymbol{e}_{rs}, \boldsymbol{\eta}_{rs}^{\pi_i^j}) \geq 0 \\ \bot & \text{if} \quad \bar{h}_{rs}^{\pi_i^j}(\boldsymbol{e}_{rs}, \boldsymbol{\eta}_{rs}^{\pi_i^j}) < 0, \end{cases} \tag{8c}$$

where $\boldsymbol{\eta}_{rs}^{\pi_i^j} := [(\boldsymbol{p}_{rs}^{\pi_i^j})^T, (\boldsymbol{\nu}_{rs}^{\pi_i^j})^T]^T$ are free parameters. The vector $\boldsymbol{\eta}_{rs}^{\pi_i^j}$ is computed as a result of the convex program defined in Theorem 1. The set $\bar{\mathcal{B}}_{rs}^{\pi_i^j}$ in (8b) is a convex parametric hyper-rectangle by Def. 1 with volume $\prod_{s=1}^{n} \boldsymbol{\nu}_{rs}^{\pi_i^j}[s]$ [17]. Hyper-rectangles are particularly suitable for our decomposition due to the efficient vertex representation and Minkowski sum computation (Prop. 2) [17, Sec. 3.6]. Our main result is next presented:

Lemma 1: Consider a task $\phi_{ij} = T_{[a,b]}\mu_{ij}$ with $T \in \{G, F\}$ according to (2b), the corresponding predicate function h_{ij} satisfying Assmp. 2 and $\mathcal{B}_{ij}$ according to (3). Further consider a path $\boldsymbol{\pi}_i^j$ through the communication graph $\mathcal{G}_c$ and

$\phi^{\pi_i^j} = \bigwedge_{(r,s)\in\epsilon(\pi_i^j)} \bar{\phi}_{rs}^{\pi_i^j}$ such that each $\bar{\phi}_{rs}^{\pi_i^j}$ is defined as

$$\bar{\phi}_{rs}^{\pi_i^j} := \begin{cases} F_{[a^*,b^*]}\bar{\mu}_{rs}^{\pi_i^j} \text{ if } T = F & \text{(9a)} \\ G_{[a^*,b^*]}\bar{\mu}_{rs}^{\pi_i^j} \text{ if } T = G,, & \text{(9b)} \end{cases}$$

with $[a^*, b^*]$ defined as

$$[a^*, b^*] := \begin{cases} [\bar{t}, \bar{t}] \text{ with } \bar{t} \in [a, b] \text{ if } T = F & \text{(10a)} \\ [a, b] \text{ if } T = G, & \text{(10b)} \end{cases}$$

where $\bar{\mu}_{rs}^{\pi_i^j}(\boldsymbol{\eta}_{rs}^{\pi_i^j})$, $\bar{h}_{rs}^{\pi_i^j}(\boldsymbol{e}_{rs}, \boldsymbol{\eta}_{rs}^{\pi_i^j})$ and $\bar{\mathcal{B}}_{rs}^{\pi_i^j}(\boldsymbol{\eta}_{rs}^{\pi_i^j})$ are as per (8). If $\bar{\phi}_{rs}^{\pi_i^j}$ are defined according to (9)-(10) and

$$\bigoplus_{(r,s)\in\epsilon(\boldsymbol{\pi}_i^j)} \bar{\mathcal{B}}_{rs}^{\pi_i^j}(\boldsymbol{\eta}_{rs}^{\pi_i^j}) \subseteq \mathcal{B}_{ij}; \tag{11}$$

then $\boldsymbol{x}(t) \models \phi^{\pi_i^j} \Rightarrow \boldsymbol{x}(t) \models \phi_{ij}$.

Proof: We prove the lemma for $\phi_{ij} := F_{[a,b]}\mu_{ij}$ while the case of $\phi_{ij} := G_{[a,b]}\mu_{ij}$ follows a similar procedure. We omit the dependency of $\bar{h}_{rs}^{\pi_i^j}, \mathcal{B}_{rs}^{\pi_i^j}$ from $\boldsymbol{\eta}_{rs}^{\pi_i^j}$ to reduce the burden of notation. Given the path $\boldsymbol{\pi}_i^j$ over $\mathcal{G}_c$ we define $\phi^{\pi_i^j} = \bigwedge_{(r,s)\in\epsilon(\boldsymbol{\pi}_i^j)} \bar{\phi}_{rs}^{\pi_i^j} = \bigwedge_{(r,s)\in\epsilon(\boldsymbol{\pi}_i^j)} F_{[\bar{t},\bar{t}]}\bar{\mu}_{rs}^{\pi_i^j}$ according to (9)-(10), where $[\bar{t}, \bar{t}] \subseteq [a, b]$. It is known that $\boldsymbol{x}(t) \models \phi^{\pi_i^j} \Rightarrow \rho^{\phi^{\pi_i^j}}(\boldsymbol{x}, 0) = \min_{(r,s)\in\epsilon(\boldsymbol{\pi}_i^j)}\{\rho^{\bar{\phi}_{rs}^{\pi_i^j}}(\boldsymbol{x}, 0)\} > 0$. By definition of robust semantics for the F operator we know that $\rho^{\bar{\phi}_{rs}^{\pi_i^j}}(\boldsymbol{x}, 0) > 0 \Rightarrow \exists t_{rs} \in [\bar{t}, \bar{t}]$ such that $\bar{h}_{rs}^{\pi_i^j}(\boldsymbol{e}_{rs}(t_{rs})) > 0 \ \forall (r, s) \in \epsilon(\pi_i^j)$. Since the interval $[\bar{t}, \bar{t}]$ only contains $\bar{t}$, then $t_{rs} = \bar{t} \ \forall (r, s) \in \epsilon(\boldsymbol{\pi}_i^j)$. Hence $\rho^{\bar{\phi}_{rs}^{\pi_i^j}}(\boldsymbol{x}, 0) > 0 \Rightarrow \bar{h}_{rs}^{\pi_i^j}(\boldsymbol{e}_{rs}(\bar{t})) > 0 \Rightarrow \boldsymbol{e}_{rs}(\bar{t}) \underset{\text{(8b)}}{\in} \bar{\mathcal{B}}_{rs}^{\pi_i^j} \ \forall (r, s) \in \epsilon(\pi_i^j)$. From (4) and (11) we have $\boldsymbol{e}_{ij}(\bar{t}) \underset{(4)}{=} \sum_{(r,s)\in\epsilon(\boldsymbol{\pi}_i^j)} \boldsymbol{e}_{rs}(\bar{t}) \in \bigoplus_{(r,s)\in\epsilon(\boldsymbol{\pi}_i^j)} \bar{\mathcal{B}}_{rs}^{\pi_i^j} \underset{(11)}{\subseteq} \mathcal{B}_{ij} \ \Rightarrow \boldsymbol{e}_{ij}(\bar{t}) \in \mathcal{B}_{ij}] \Rightarrow h_{ij}(\boldsymbol{e}_{ij}(\bar{t})) > 0$. We thus arrived at the conclusion since $\bar{t} \in [a, b]$ and the robust semantics for the F operator we have $\rho^{\phi_{ij}}(\boldsymbol{x}, 0) = \max_{t\in[a,b]}(h_{ij}(\boldsymbol{e}_{ij}(t))) > 0 \Rightarrow \boldsymbol{x}(t) \models \phi_{ij}$. ■

We highlight that $\bigoplus_{(r,s)\in\epsilon(\boldsymbol{\pi}_i^j)} \bar{\mathcal{B}}_{rs}^{\pi_i^j}$ is an axis-aligned hyper-rectangles according to Prop. 2. Moreover, from Assmp. 2, each h_{ij} is concave, such that satisfying (11) consists in verifying the 2^n convex relations $-h_{ij}(\boldsymbol{v}) \leq 0$ over the vertices $\boldsymbol{v} \in \mathcal{P}(\boldsymbol{p}^{\pi_i^j}, \boldsymbol{\nu}^{\pi_i^j})$, where $\boldsymbol{p}^{\pi_i^j} = \sum_{(r,s)\in\epsilon(\boldsymbol{\pi}_i^j)} \boldsymbol{p}_{rs}^{\pi_i^j}$ and $\boldsymbol{\nu}^{\pi_i^j} = \sum_{(r,s)\in\epsilon(\boldsymbol{\pi}_i^j)} \boldsymbol{\nu}_{rs}^{\pi_i^j}$ as per Prop. 2.

B. Computing optimal parameters

In Lemma 1 we showed that (9), (10) and (11) imply $\boldsymbol{x}(t) \models \phi^{\pi_i^j} \Rightarrow \boldsymbol{x}(t) \models \phi_{ij}$. In Thm. 1, we present a procedure to compute the parameters $\boldsymbol{\eta}_{rs}^{\pi_i^j}$ for each task $\bar{\phi}_{rs}^{\pi_i^j}$ such that the (9), (10) and (11) are satisfied and the volumes of the super-level set $\bar{\mathcal{B}}_{rs}^{\pi_i^j}$ are maximized. Namely, given a single hyper-rectangle $\bar{\mathcal{B}}_{rs}^{\pi_i^j}$, it is possible to maximize its volume $\prod_{k=1}^n \boldsymbol{\nu}_{rs}^{\pi_i^j}[k]$, by minimizing $(\prod_{k=1}^n \boldsymbol{\nu}_{rs}^{\pi_i^j}[k])^{-1}$, which is a convex function since $\boldsymbol{\nu}_{rs}^{\pi_i^j}[k] > 0 \ \forall k = 1, \ldots n$.

We introduce the set $\Theta^{\pi_i^j} := \{\boldsymbol{\eta}_{rs}^{\pi_i^j} | \forall (r, s) \in \epsilon(\pi_i^j)\}$ which gathers all the parameter along a path $\boldsymbol{\pi}_i^j$ applied for the decomposition of a given task ϕ_{ij} and $\Theta = \bigcup_{(i,j)\in\mathcal{E}_\psi\setminus\mathcal{E}_c} \Theta^{\pi_i^j}$ is then the set of all the parameters applied for the decomposition. The second main result is presented next.

Theorem 1: Consider a multi-agent system with agents in $\mathcal{V}$ and subject to a global task ψ according to (2). Further consider the associated task and communication graphs $\mathcal{G}_c$, $\mathcal{G}_\psi$ such that $\mathcal{E}_\psi \setminus \mathcal{E}_c \neq \emptyset$ and $\mathcal{G}_c$ respects Assmp. 1. For all $(i, j) \in \mathcal{E}_\psi \setminus \mathcal{E}_c$ consider the paths $\boldsymbol{\pi}_i^j$ in $\mathcal{G}_c$ and the corresponding task $\phi^{\pi_i^j}$ satisfying the conditions in Lemma 1. Define the following convex optimization problem

$$\min_{\boldsymbol{\eta}_{rs}^{\pi_i^j}\in\Theta} \sum_{(i,j)\in\mathcal{E}_\psi\setminus\mathcal{E}_c} \sum_{(r,s)\in\epsilon(\pi_i^j)} \Big(\prod_{k=1}^n \boldsymbol{\nu}_{rs}^{\pi_i^j}[k]\Big)^{-1} \tag{12a}$$

$$\bigoplus_{(r,s)\in\epsilon(\boldsymbol{\pi}_i^j)} \bar{\mathcal{B}}_{rs}^{\pi_i^j}(\boldsymbol{\eta}_{rs}^{\pi_i^j}) \subseteq \mathcal{B}_{ij} \ \forall (i, j) \in \mathcal{E}_\psi \setminus \mathcal{E}_c. \tag{12b}$$

Assuming (12) is feasible and that there exists $\boldsymbol{x}(t)$ such that $\boldsymbol{x}(t) \models \bar{\psi}$ with $\bar{\psi}$ according to (6), then $\boldsymbol{x}(t) \models \psi$.

Proof: Given that a solution to (12) exists, then the satisfaction of the constraints set (12b) implies that condition (11) is satisfied for all $(i, j) \in \mathcal{E}_\psi \setminus \mathcal{E}_c$. The conditions from Lemma 1 are then satisfied for all $\phi^{\pi_i^j}$ with $(i, j) \in \mathcal{E}_\psi \setminus \mathcal{E}_c$. We now analyse the satisfaction of ψ and $\bar{\psi}$ through the definition of the robust semantics such that $\rho^\psi = \min\{\min_{i\in\mathcal{V}}\{\rho^{\phi_i}\}, \min_{(i,j)\in\mathcal{E}_\psi}\{\rho^{\phi_{ij}}\}\}$ and $\rho^{\bar{\psi}} = \min\{\min_{i\in\mathcal{V}}\{\rho^{\phi_i}\}, \min_{(i,j)\in\mathcal{E}_\psi\cap\mathcal{E}_c}\{\rho^{\phi_{ij}}\} \min_{(i,j)\in\mathcal{E}_\psi\setminus\mathcal{E}_c}\{\rho^{\phi^{\pi_i^j}}\}\}$; where we have omitted the dependency from $(\boldsymbol{x}, 0)$. Assuming that $\boldsymbol{x}(t) \models \bar{\psi}$, then $\rho^{\phi_{ij}}(\boldsymbol{x}, 0) > 0 \, \forall (i, j) \in \mathcal{E}_\psi \cap \mathcal{E}_c$ and $\rho^{\phi_i}(\boldsymbol{x}, 0) > 0$. Furthermore, we know from Lemma 1 that $\rho^{\phi^{\pi_i^j}}(\boldsymbol{x}, 0) > 0 \Rightarrow \rho^{\phi_{ij}}(\boldsymbol{x}, 0) > 0 \ \forall (i, j) \in \mathcal{E}_\psi \setminus \mathcal{E}_c$ and eventually $\rho^{\phi_{ij}}(\boldsymbol{x}, 0) > 0 \, \forall (i, j) \in \mathcal{E}_\psi \Rightarrow \boldsymbol{x}(t) \models \bar{\psi} \Rightarrow \boldsymbol{x}(t) \models \psi$. ■

If we define the set of edges involved in the decomposition of ψ as $\mathcal{Q} := \bigcup_{(i,j)\in\mathcal{E}_\psi\setminus\mathcal{E}_c} \epsilon(\boldsymbol{\pi}_i^j)$ then we can write $\mathcal{G}_{\bar{\psi}}$ as a function of $\mathcal{G}_\psi$ as $\mathcal{G}_{\bar{\psi}} = \textbf{add}\big(\textbf{del}(\mathcal{G}_\psi, \mathcal{E}_\psi \setminus \mathcal{E}_c), \mathcal{Q}\big)$, which correspond to deleting all the edges in $\mathcal{G}_\psi$ not corresponding to an edge in $\mathcal{G}_c$, while we add all the edges from $\mathcal{G}_c$ that are introduced by the paths $\boldsymbol{\pi}_i^j$ during the decomposition. We have thus deduced a procedure that solves Problem 1.

Remark 1: Problem (12) handles cases in which ϕ_{ij} has conjunctions. Indeed, if $\phi_{ij} = \wedge_{k=1}^p \phi_{ij}^k$ for some $p \geq 1$ then we define a task $(\phi^{\pi_i^j})^k$ as per Lemma 1 and a constraint as per (12b) for each $k = 1, \ldots p$ has to be introduced in (12).

Remark 2: Although any type of zonotope can be employed for the decomposition, the number of constraints (12b) increases with the number of vertices defining the Minkowski sum in the left-hand side of (12b), thus increasing the computational cost of the proposed approach.

While (12) might yield a solution, there is no guarantee that the tasks $\phi^{\pi_i^j}$ computed from (12) in Thm. 1 can be satisfied in conjunction with each other or in conjunction with the undecomposed formulas $\phi_{ij} \ \forall (i, j) \in \mathcal{E}_\psi \cap \mathcal{E}_c$. In other words, the new task $\bar{\psi}$ obtained from the optimization problem presented in Thm. 1 is not guaranteed to be satisfiable. We analyse this problem in the next section.

V. Dealing with conflicts

A. Conflicting conjuncitons

We consider the following notion of conflicting conjunction (CC) for formulas defined by the STL fragment (2):

Definition 2: A conjunction of formulas $\bigwedge_k \phi_{ij}^k$, where ϕ_{ij}^k is as per (2b), is a *conflicting conjunction* (CC) if there does not exist a state trajectory $\boldsymbol{x}(t)$ for the multi-agent system such that $\boldsymbol{x}(t) \models \bigwedge_k \phi_{ij}^k$.

We state four types of CCs for fragment (2) and we conjecture that these are the only 4 possible types. Sufficient conditions enforcing the exclusion of such CCs from the solution set of (12) are then provided in the form of additional convex constraints to be included in (12). It is assumed that none of these conflicts arise in the original task in ψ. For clarity, in this section, we drop the notation $\bar{\phi}_{rs}^{\pi_i^j}$ and we rewrite tasks ϕ_{ij} in $\mathcal{G}_{\bar{\psi}}$ for a single edge $(i,j) \in \mathcal{E}_{\bar{\psi}}$ as

$$\phi_{ij} = (\wedge_{k\in\mathcal{I}_{ij}^g}\phi_{ij}^k) \wedge (\wedge_{k\in\mathcal{I}_{ij}^f}\phi_{ij}^k) \tag{13a}$$

$$\phi_{ij}^k = G_{[a^k,b^k]}\mu_{ij}^k \text{ if } k \in \mathcal{I}_{ij}^g, \tag{13b}$$

$$\phi_{ij}^k = F_{[a^k,b^k]}\mu_{ij}^k \quad \text{if } k \in \mathcal{I}_{ij}^f. \tag{13c}$$

such that $\mathcal{I}_{ij} := \mathcal{I}_{ij}^g \cup \mathcal{I}_{ij}^f$. Differently from (2b), each edge $(i,j) \in \mathcal{E}_{\bar{\psi}}$ can now contain parametric tasks as per (9) and un-decomposed tasks from ψ as per (2b) that are directly inherited by $\bar{\psi}$. To differentiate among the formers and the latters, we introduce the sets $\bar{\mathcal{F}}$ and $\mathcal{F}$ such that $\phi_{ij}^k \in \bar{\mathcal{F}}$ if ϕ_{ij}^k is a task as per (9), while $\phi_{ij}^k \in \mathcal{F}$ if ϕ_{ij}^k is defined as per (2b). For any task ϕ_{ij}^k in (13b)-(13c) we define the predicate function $h_{ij}^k(\boldsymbol{e}_{ij})$, predicate $\mu_{ij}^k(\boldsymbol{e}_{ij}) := \begin{cases} \top & \text{if} \quad h_{ij}^k(\boldsymbol{e}_{ij},\boldsymbol{\eta}_{ij})\geq 0 \\ \bot & \text{if} \quad h_{ij}^k(\boldsymbol{e}_{ij},\boldsymbol{\eta}_{ij})<0, \end{cases}$ and super-level set $\mathcal{B}_{ij}^k = \{\boldsymbol{e}_{ij} | h_{ij}^k(\boldsymbol{e}_{ij}) \geq 0\}$, which is an hyper-rectangle if $\phi_{ij}^k \in \bar{\mathcal{F}}$.

Fact 1: (Conflict of type 1) Consider two tasks ϕ_{ij}^k and ϕ_{ij}^q defined over the edge $(i,j) \in \mathcal{E}_{\bar{\psi}}$ and such that $k, q \in \mathcal{I}_{ij}^g$ If $[a^q, b^q] \cap [a^k, b^k] \neq \emptyset$ and $\mathcal{B}_{ij}^k \cap \mathcal{B}_{ij}^q = \emptyset$ then $\phi_{ij}^k \wedge \phi_{ij}^q$ is a conflicting conjunction.

Proof: We prove the fact by contradiction. Suppose there exists $\boldsymbol{x}(t)$ such that $\boldsymbol{x}(t) \models \phi_{ij}^q \wedge \phi_{ij}^k$. Then $\rho^{\phi_{ij}^q \wedge \phi_{ij}^k}(\boldsymbol{x}, 0) = \min\{\rho^{\phi_{ij}^q}(\boldsymbol{x}, 0), \rho^{\phi_{ij}^k}(\boldsymbol{x}, 0)\} > 0 \Rightarrow \rho^{\phi_{ij}^q}(\boldsymbol{x}, 0) > 0 \wedge \rho^{\phi_{ij}^k}(\boldsymbol{x}, 0) > 0$. Recalling the definition of robust semantics for the G operator we have $h_{ij}^q(\boldsymbol{e}_{ij}(t)) > 0 \ \forall t \in [a^q, b^q] \Rightarrow \boldsymbol{e}_{ij}(t) \in \mathcal{B}_{ij}^q \ \forall t \in [a^q, b^q]$ and $h_{ij}^k(\boldsymbol{e}_{ij}(t)) > 0 \ \forall t \in [a^k, b^k] \Rightarrow \boldsymbol{e}_{ij}(t) \in \mathcal{B}_{ij}^k \ \forall t \in [a^k, b^k]$. Since $[a^k, b^k] \cap [a^q, b^q] \neq \emptyset$ then there exists $\bar{t} \in [a^k, b^k] \cap [a^q, b^q]$ for which $\boldsymbol{e}_{ij}(\bar{t}) \in \mathcal{B}_{ij}^k \wedge \boldsymbol{e}_{ij}(\bar{t}) \in \mathcal{B}_{ij}^q$. We thus arrived at a contradiction since $\mathcal{B}_{ij}^k \cap \mathcal{B}_{ij}^q = \emptyset$. ■

Fact 2: (Conflict of type 2) Consider two tasks ϕ_{ij}^k and ϕ_{ij}^q defined over the edge $(i,j) \in \mathcal{E}_{\bar{\psi}}$ and such that $k \in \mathcal{I}_{ij}^g$, $q \in \mathcal{I}_{ij}^f$. If $[a^q, b^q] \subseteq [a^k, b^k]$ and $\mathcal{B}_{ij}^q \cap \mathcal{B}_{ij}^k = \emptyset$ then $\phi_{ij}^k \wedge \phi_{ij}^q$ is a conflicting conjunction.

Proof: We prove the fact by contradiction. Suppose there exists $\boldsymbol{x}(t)$ such that $\boldsymbol{x}(t) \models \phi_{ij}^q \wedge \phi_{ij}^k$. Then $\rho^{\phi_{ij}^q \wedge \phi_{ij}^k}(\boldsymbol{x}, 0) = \min\{\rho^{\phi_{ij}^q}(\boldsymbol{x}, 0), \rho^{\phi_{ij}^k}(\boldsymbol{x}, 0)\} > 0 \Rightarrow \rho^{\phi_{ij}^q}(\boldsymbol{x}, 0) > 0 \wedge \rho^{\phi_{ij}^k}(\boldsymbol{x}, 0) > 0$. Recalling the definition of robust semantics for the F and G operators we have $h_{ij}^q(\boldsymbol{e}_{ij}(t)) > 0 \ \forall t \in [a^q, b^q] \Rightarrow \boldsymbol{e}_{ij}(t) \in \mathcal{B}_{ij}^q \ \forall t \in [a^q, b^q]$ and there exist $\bar{t} \in [a^k, b^k]$ such that $h_{ij}^k(\boldsymbol{e}_{ij}(\bar{t})) > 0 \Rightarrow \boldsymbol{e}_{ij}(\bar{t}) \in \mathcal{B}_{ij}^k$. Since $[a^k, b^k] \subset [a^q, b^q]$ then $\bar{t} \in [a^q, b^q]$ and $\boldsymbol{e}_{ij}(\bar{t}) \in \mathcal{B}_{ij}^k \wedge \boldsymbol{e}_{ij}(\bar{t}) \in \mathcal{B}_{ij}^q$. We thus arrived at the contradiction as $\mathcal{B}_{ij}^k \cap \mathcal{B}_{ij}^q = \emptyset$. ■

CCs over cycles of tasks in $\mathcal{G}_{\bar{\psi}}$ are defined next. Such conflicts arise if the cycle closure relation (4) is unsatisfiable under a cycle of tasks in $\mathcal{G}_{\bar{\psi}}$. Namely, we consider the conjunction $\wedge_{(r,s)\in\epsilon(\boldsymbol{\omega})}\phi_{rs}$ where $\boldsymbol{\omega}$ is a cycle of length l in $\mathcal{G}_{\bar{\psi}}$ and ϕ_{rs} is a task of type (13b) or (13c). The case for which each ϕ_{rs} is a task of type (13a) that contains conjunctions, is a generalization of this simpler case. For clarity of presentation, let the notation $\phi_{\omega[k,k+1]}$ indicate a task ϕ_{rs} such that $(r,s) = (\omega[k], \omega[k+1])$ for $k \in 1, \ldots l-1$. Likewise, we denote the time interval, super-level set, predicate function and predicate of $\phi_{\omega[k,k+1]}$ as $[a,b]_{\omega[k,k+1]}$, $\mathcal{B}_{\omega[k,k+1]}$, $h_{\omega[k,k+1]}$ and $\mu_{\omega[k,k+1]}$ respectively, with the notational equivalence $\wedge_{(r,s)\in\epsilon(\omega)}\phi_{rs} = \wedge_{k=1}^{l-1}\phi_{\omega[k,k+1]}$. We now present the next two types of CC.

Fact 3: (Conflict of type 3) Consider a cycle $\boldsymbol{\omega}$ of length l in $\mathcal{G}_{\bar{\psi}}$ such that each edge $(\boldsymbol{\omega}[k], \boldsymbol{\omega}[k+1])$ corresponds to a unique task $\phi_{\omega[k,k+1]} = G_{[a,b]}\mu_{\omega[k,k+1]}$. If $\bigcap_{k=1}^{l-1}[a,b]_{\omega[k,k+1]} \neq \emptyset$ and

$$\bigoplus_{k=1}^{p} \mathcal{B}_{\omega[k,k+1]} \cap \Big(-\bigoplus_{k=p+1}^{l-1} \mathcal{B}_{\omega[k,k+1]}\Big) = \emptyset \tag{14}$$

for some $1 \leq p \leq l-1$ then $\bigwedge_{k=1}^{l-1} \phi_{\omega[k,k+1]}$ is a CC.

Proof: We prove the theorem by contradiction. Assume that there exists a state trajectory $\boldsymbol{x}(t)$ such that $\boldsymbol{x}(t) \models \bigwedge_{k=1}^{l-1} \phi_{\omega[k,k+1]}$. Then we know from the robust satisfaction of such conjunction that $\rho^{\bigwedge_{k=1}^{l-1} \phi_{\omega[k,k+1]}}(\boldsymbol{x}, 0) = \min_{k=1,\ldots l-1}\{\rho^{\phi_{\omega[k,k+1]}}(\boldsymbol{x}, 0)\} > 0 \Rightarrow \rho^{\phi_{\omega[k,k+1]}}(\boldsymbol{x}, 0) > 0 \ \forall k = 1, \ldots l-1$. Recalling the definition of robust semantics for the always formulas we can then write $h_{\omega[k,k+1]}(\boldsymbol{e}_{\omega[k,k+1]}(t)) > 0, \forall t \in [a,b]_{\omega[k,k+1]}, \forall k = 1, \ldots l-1$ and thus $\boldsymbol{e}_{\omega[k,k+1]}(t) \in \mathcal{B}_{\omega[k,k+1]}, \forall t \in [a,b]_{\omega[k,k+1]}, \forall k = 1, \ldots l-1$. We now recall from (4) that for a cycle of edges, we have $\sum_{k=1}^{l-1} \boldsymbol{e}_{\omega[k,k+1]}(t) = 0 \Rightarrow \sum_{k=1}^{p} \boldsymbol{e}_{\omega[k,k+1]}(t) = -\sum_{k=p+1}^{l-1} \boldsymbol{e}_{\omega[k,k+1]}(t)$ for any $1 \leq p \leq l-2$. Since we know that $\bigcup_{k=1}^{l-1}[a,b]_{\omega[k,k+1]} \neq \emptyset$ then there exist a time instant $\bar{t} \in \bigcup_{k=1}^{l-1}[a,b]_{\omega[k,k+1]}$ such that the three relations $\sum_{k=1}^{p} \boldsymbol{e}_{\omega[k,k+1]}(\bar{t}) \in \bigoplus_{k=1}^{p} \mathcal{B}_{\omega[k,k+1]}$, $-\sum_{k=p+1}^{l-1} \boldsymbol{e}_{\omega[k,k+1]}(\bar{t}) \in \big(-\bigoplus_{k=p+1}^{l-1} \mathcal{B}_{\omega[k,k+1]}\big)$ and $\sum_{k=1}^{p} \boldsymbol{e}_{\omega[k,k+1]}(\bar{t}) = -\sum_{k=p+1}^{l-1} \boldsymbol{e}_{\omega[k,k+1]}(\bar{t})$ must hold. We thus arrived at a contradiction as the three aforementioned conditions can not hold jointly if $\bigoplus_{k=1}^{p} \mathcal{B}_{\omega[k,k+1]} \cap \Big(-\bigoplus_{k=p+1}^{l-1} \mathcal{B}_{\omega[k,k+1]}\Big) = \emptyset$. Since the argument is independent of the index p chosen, the proof is valid for any chosen $1 \leq p \leq l-2$. ■

Fact 4: (Conflict of type 4) Consider a cycle $\boldsymbol{\omega}$ of length l in $\mathcal{G}_{\psi}$ such that for each edge $(\boldsymbol{\omega}[k], \boldsymbol{\omega}[k+1])$ there corresponds a unique task $\phi_{\omega[k,k+1]}$, for which we have $\phi_{\omega[k,k+1]} = F_{[a,b]}\mu_{\omega[k,k+1]} \forall k = 1,\dots q$ and $\phi_{\omega[k,k+1]} = G_{[a,b]}\mu_{\omega[k,k+1]} \forall k = q+1,\dots l-1$ for some $1 \leq q \leq l-1$. If $\bigoplus_{k=1}^{p} \mathcal{B}_{\omega[k,k+1]} \cap \left(-\bigoplus_{k=p+1}^{l-1} \mathcal{B}_{\omega[k,k+1]}\right) \neq \emptyset$ for some $1 \leq p \leq l-1$ and if either 1) $q = 1$, $\bigcap_{k=q+1}^{l-1}[a,b]_{\omega[k,k+1]} \neq \emptyset$, $[a,b]_{\omega[1,2]} \subseteq \bigcap_{k=q+1}^{l-1}[a,b]_{\omega[k,k+1]}$ or 2) $q \geq 1$, $\bigcap_{k=q+1}^{l-1}[a,b]_{\omega[k,k+1]} \neq \emptyset$, $[a,b]_{\omega[k,k+1]} = [\bar{t},\bar{t}] \forall k = 1,\dots q$, $\bar{t} \in \bigcap_{k=q+1}^{l-1}[a,b]_{\omega[k,k+1]}$; then $\bigwedge_{k=1}^{l-1} \phi_{\omega[k,k+1]}$ is a CC.

Proof: The proof is similar to the proof of Fact 3 and it is not reported due to space limitation. ■

Intuitively, CCs of type 3 correspond to a cycles of tasks to be satisfied *always* within $[a,b]_{\omega[k,k+1]}$, but the cycle closure relation (4) is not satisfiable over the intersection of intervals $\bigcap_{k=1}^{l-1}[a,b]_{\omega[k,k+1]}$ when all tasks are active simultaneously. CCs of type 4 follow a similar intuition.

B. Resolving conflicting conjunctions

To avoid the insurgence of CCs of type 1-4 in (12), sufficient conditions are introduced by Lemmas 2-5 on the parameters of the tasks in $\bar{\mathcal{F}}$ such that CCs are excluded from the solution set of (12).

Lemma 2: Let $(i,j) \in \mathcal{E}_{\bar{\psi}}$ with task $\phi_{ij} = \phi_{ij}^s \wedge \phi_{ij}^r$ such that $s, r \in \mathcal{I}_{ij}^g$ and $[a^r, b^r] \cap [a^s, b^s] \neq \emptyset$. Then, the conditions

$$\mathcal{B}_{ij}^s \subseteq \mathcal{B}_{ij}^r \text{ if } \quad (b^r - a^r) \geq (b^s - a^s) \wedge \phi_{ij}^r, \phi_{ij}^s \in \bar{\mathcal{F}} \tag{15a}$$
$$\mathcal{B}_{ij}^r \subseteq \mathcal{B}_{ij}^s \text{ if } \quad (b^r - a^r) < (b^s - a^s) \wedge \phi_{ij}^r, \phi_{ij}^s \in \bar{\mathcal{F}} \tag{15b}$$
$$\mathcal{B}_{ij}^s \subseteq \mathcal{B}_{ij}^r \text{ if } \quad \phi_{ij}^r \in \mathcal{F}, \phi_{ij}^s \in \bar{\mathcal{F}} \tag{15c}$$

where $\mathcal{F}$,$\bar{\mathcal{F}}$ are the sets of non-parametric and parametric tasks respectively, ensures $\phi_{ij}^s \wedge \phi_{ij}^r$ is not a CC of type 1.

Proof: Let $[a^r, b^r] \cap [a^s, b^s] \neq \emptyset$, then (15) contradicts the conditions of CC of type 1 as per Fact 1. ■

Constraints (15a)-(15b) specify that when two parametric formulas could be in conflict as per Fact 1, then we decide to include the super-level set of tasks with shorter time intervals into the ones with longer time intervals.

Lemma 3: Let $(i,j) \in \mathcal{E}_{\bar{\psi}}$ with $\phi_{ij} = \phi_{ij}^s \wedge \phi_{ij}^r$ such that $s \in \mathcal{I}_{ij}^g$, $r \in \mathcal{I}_{ij}^f$ and $[a^r, b^r] \subseteq [a^s, b^s]$. Then, the conditions

$$\mathcal{B}_{ij}^r \subseteq \mathcal{B}_{ij}^s \quad \text{if} \quad \phi_{ij}^s \in \mathcal{F}, \phi_{ij}^r \in \bar{\mathcal{F}} \tag{16a}$$
$$\mathcal{B}_{ij}^s \subseteq \mathcal{B}_{ij}^r \quad \text{if} \quad \phi_{ij}^s \in \bar{\mathcal{F}}, \phi_{ij}^r \in \mathcal{F} \tag{16b}$$
$$\mathcal{B}_{ij}^r \subseteq \mathcal{B}_{ij}^s \quad \text{if} \quad \phi_{ij}^s \in \bar{\mathcal{F}}, \phi_{ij}^r \in \bar{\mathcal{F}} \tag{16c}$$

with $\mathcal{F}$, $\bar{\mathcal{F}}$ being the sets of non-parametric and parametric tasks respectively, ensure $\phi_{ij}^s \wedge \phi_{ij}^r$ is not a CC of type 2.

Proof: Let $[a^r, b^r] \subseteq [a^s, b^s]$, then (16) contradicts the conditions of CC of type 2 as per Fact 2. ■

When more than two tasks are considered in conjunction, then Lemma 2-3 are applied to every possible couple of tasks in the conjunction.

Lemma 4: Let $\boldsymbol{\omega}$ be a cycle of length l in $\mathcal{G}_{\bar{\psi}}$ such that for each $(\boldsymbol{\omega}[k], \boldsymbol{\omega}[k+1])$ there corresponds a unique task $\phi_{\omega[k,k+1]} = G_{[a,b]}\mu_{\omega[k,k+1]}$. If $\bigcap_{k=1}^{l-1}[a,b]_{\omega[k,k+1]} \neq \emptyset$ then imposing the constraints

$$\bigoplus_{k=1}^{p} \mathcal{B}_{\omega[k,k+1]} \subseteq \left(-\bigoplus_{k=p+1}^{l-1} \mathcal{B}_{\omega[k,k+1]}\right) \tag{17}$$

for some $1 \leq p \leq l-1$, ensures that $\bigwedge_{k=1}^{l-1} \phi_{\omega[k,k+1]}$ is not a CC of type 3.

Proof: The relation (17) is a special case (inclusion) of (14). Since we considered $\bigcap_{k=1}^{l-1}[a,b]_{\omega[k,k+1]} \neq \emptyset$, then (17) contradicts (14) as per Fact 3. ■

Lemma 5: Let $\boldsymbol{\omega}$ be a cycle of length l in $\mathcal{G}_{\psi}$ such that for each edge $(\boldsymbol{\omega}[k], \boldsymbol{\omega}[k+1])$ there corresponds a unique task $\phi_{\omega[k,k+1]}$. Assume $\phi_{\omega[k,k+1]} = F_{[a,b]}\mu_{\omega[k,k+1]} \forall k = 1,\dots q$ and $\phi_{\omega[k,k+1]} = G_{[a,b]}\mu_{\omega[k,k+1]} \forall k = q+1,\dots l-1$ for some $1 \leq q \leq l-1$. If either 1) $q = 1$, $\bigcap_{k=q+1}^{l-1}[a,b]_{\omega[k,k+1]} \neq \emptyset$, $[a,b]_{\omega[1,2]} \subseteq \bigcap_{k=q+1}^{l-1}[a,b]_{\omega[k,k+1]}$, or 2) $q \geq 1$, $\bigcap_{k=q+1}^{l-1}[a,b]_{\omega[k,k+1]} \neq \emptyset$, $[a,b]_{\omega[k,k+1]} = [\bar{t},\bar{t}] \forall k = 1,\dots q$, $\bar{t} \in \bigcap_{k=q+1}^{l-1}[a,b]_{\omega[k,k+1]}$; then imposing (17) ensures that $\bigwedge_{k=1}^{l-1} \phi_{\omega[k,k+1]}$ is not a CC of type 4.

Proof: The proof is similar to the proof of Lemma 4 and it is not reported due to space limitations. ■

Remark 3: In the statement of Lemmas 4-5 we considered that no edge of a cycle $\boldsymbol{\omega}$ contains conjunctions. Nevertheless, in the case $\phi_{\omega[k,k+1]}$ contains conjunctions as $\phi_{\omega[k,k+1]} = \bigwedge_{s \in \mathcal{I}_{\omega[k,k+1]}^g} \phi_{\omega[k,k+1]}^s \wedge \bigwedge_{s \in \mathcal{I}_{\omega[k,k+1]}^f} \phi_{\omega[k,k+1]}^s$ as per (13a), it is possible to define the combination set $C = \prod_{k=1}^{l-1} \mathcal{I}_{\omega[k,k+1]}$ where $\mathcal{I}_{\omega[k,k+1]}$ contains all the tasks indices for edge $(\boldsymbol{\omega}[k], \boldsymbol{\omega}[k+1])$ as per (13a). Then, for each combination of indices $c \in \mathcal{C}$ we can check if the conjunction $\wedge_{k=1}^{l-1} \phi_{\omega[k,k+1]}^{c[k]}$ satisfies the conditions for CC of type 3-4 and introduce new constraints of type (17) accordingly.

Remark 4: The computation of the Minkowski sum in (17) can possibly become a complex computation as not all the tasks defined over a cycle $\boldsymbol{\omega}$ are parametric as per (8). Nevertheless, we can under-approximate the sets $\mathcal{B}_{\omega[k,k+1]}$ such that $\phi_{\omega[k,k+1]} \in \mathcal{F}$ by a hyper-rectangle that can be computed offline when ψ is defined. [15].

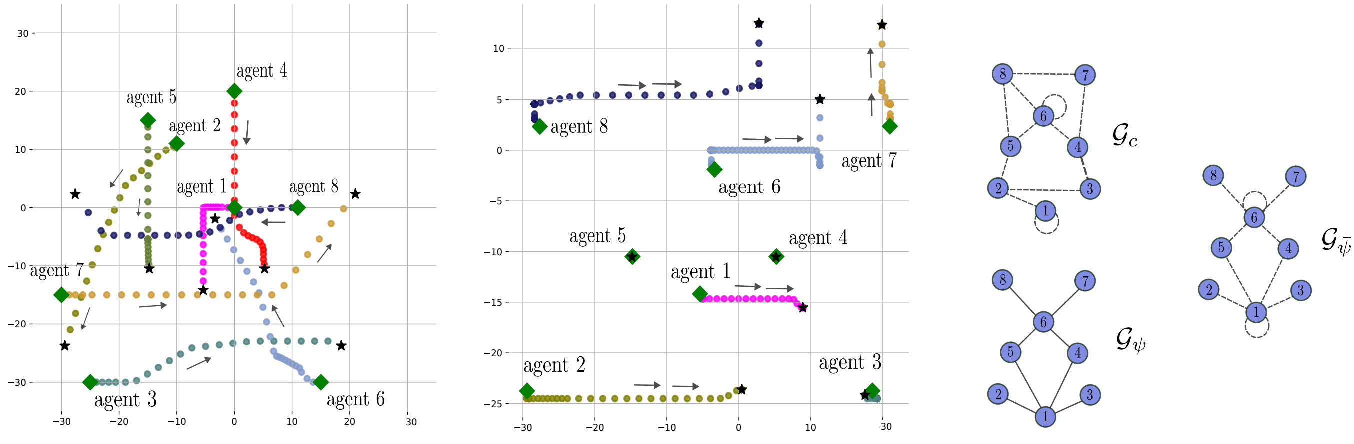
We conclude this section with the final result of our work.

Theorem 2: Let the conditions of Thm. 1 hold. If a solution to (12) exists after including constraints (15), (16), (17), then the resulting global formula $\bar{\psi}$ defined as in (9) from Thm. 1 does not contain CCs as per Fact 1-4.

Thus, Thm. 2 resolves the feasibility issues not considered in Thm. 1, so that $\bar{\psi}$ does not suffer from CCs of type 1-4.

VI. SIMULATIONS

We consider a formation problem with agents $\mathcal{V} := \{1,\dots 8\}$ and single-integrator dynamics $\dot{\boldsymbol{x}}_i = \boldsymbol{u}_i$ where $\boldsymbol{x}_i = [x_i, y_i]^T \in \mathbb{R}^2$ represents the position of the agents in a 2-dimensional space. We consider a time-varying formation task divided into two parts. First, a star formation tasks is given: $\phi_{85} = G_{[10,15]}(||\boldsymbol{e}_{85} - [-15,15]^T|| \leq 3)$, $\phi_{52} = G_{[10,15]}(||\boldsymbol{e}_{25} - [-15,-15]^T|| \leq 3)$, $\phi_{34} = G_{[10,15]}(||\boldsymbol{e}_{34} - [15,-15]^T|| \leq 3)$, $\phi_{74} = G_{[10,15]}(||\boldsymbol{e}_{74} - [15,15]^T|| \leq 3)$, $\phi_{46} = G_{[10,15]}(||\boldsymbol{e}_{46} - [10,-10]^T|| \leq 2)$, $\phi_{56} = G_{[10,15]}(||\boldsymbol{e}_{56} - [-10,-10]^T|| \leq 2)$. Second, agents in $\mathcal{V}_1 = \{6,7,8\}$ and $\mathcal{V}_2 = \{1,2,3\}$ get detached from the star formation. Namely, agents 1 and 6 move independently toward the region $x > 5, y > 5$ and $x > 5, y < -5$ respectively while teams $\mathcal{V}_1$, $\mathcal{V}_2$ achieve a triangle formation from $t = 25s$ to $t = 28s$. This



(a) (b) (c)

Fig. 2: Trajectory evolution of the agents from time $t = 0s$ to $t = 15s$ (a) and from time $t = 15s$ to $t = 28s$ (b); Short solid arrows represent the direction of movement of the agents; green lozenges represent the starting positions of the agents, while black stars represent the final positions. Communication graph $\mathcal{G}_c$, initial task graph $\mathcal{G}_\psi$ and final task graph $\mathcal{G}_{\bar{\psi}}$ (c).

task is given by the sub-tasks $\phi_1 = F_{[25,28]}([-1,0](\boldsymbol{x}_1 - [5,0]^T) \leq 0) \wedge ([0,-1](\boldsymbol{x}_1 - [0,-5]^T) \leq 0$, $\phi_6 = F_{[25,28]}([-1,0](\boldsymbol{x}_6 - [5,0]^T) \leq 0) \wedge ([0,-1](\boldsymbol{x}_6 - [0,5]^T) \leq 0$, $\phi_{32} = G_{[25,28]}(||\boldsymbol{e}_{32} - [16,0]^T|| \leq 2\sqrt{2})$, $\phi_{87} = G_{[25,28]}(||\boldsymbol{e}_{87} - [16,0]^T|| \leq 2\sqrt{2})$, $\phi_{21} = G_{[25,28]}(||\boldsymbol{e}_{21} - [-8,-8]^T|| \leq 2\sqrt{2})$, $\phi_{68} = G_{[25,28]}(||\boldsymbol{e}_{68} - [-8,8]^T|| \leq 2\sqrt{2})$. The graphs $\mathcal{G}_\psi$, $\mathcal{G}_c$ are represented Fig. 2(c) such that edge $(7,8),(5,8),(5,2),(5,3) \in \mathcal{E}_\psi$ are to be decomposed. We use the open-source library *CasADi* with Ipopt to solve (12) adding constraints (17) for the newly formed cycle $[5,6,4,1]$ and constraints (15)-(16) for the edges $(5,1),(2,1),(4,1),(3,1),(5,6),(6,8),(6,4),(6,7)$. The algorithm finds a decomposition solution in $0.019s$ running on an Intel-Core i7-1265U × 12 with 32 GB of RAM. To satisfy $\bar{\psi}$, we employ the distributed controller in [3]. The graphs $\mathcal{G}_\psi$ is transformed into $\mathcal{G}_{\bar{\psi}}$ Fig. 2(c) and the optimal parameters for the decomposition of tasks ϕ_{25}, ϕ_{32}, ϕ_{34}, ϕ_{74}, ϕ_{85} and ϕ_{78} can be found in [18, Table I]. The MAS' trajectories are simulated from $t = 0$ to $t = 28$ in Fig. 2(a)-2(b).

VII. CONCLUSIONS

A framework for STL task decomposition was presented based on the communication graph of multi-agent systems. We showed that the satisfaction of tasks deriving from our decomposition implies the original one a valid solution is obtained. Furthermore, four possible conflicting conjunctions related to the applied STL fragment were presented, and sufficient conditions to avoid the insurgence of those in the obtained decomposition were given. Future work will investigate time-varying communication graphs and different parameterizations for the task decomposition.

REFERENCES

[1] D. Sun, J. Chen, S. Mitra, and C. Fan, "Multi-agent motion planning from signal temporal logic specifications," *IEEE Robotics and Automation Letters*, vol. 7, no. 2, pp. 3451–3458, 2022.

[2] F. S. Barbosa, D. Duberg, P. Jensfelt, and J. Tumova, "Guiding autonomous exploration with signal temporal logic," *IEEE Robotics and Automation Letters*, vol. 4, no. 4, pp. 3332–3339, 2019.

[3] L. Lindemann and D. V. Dimarogonas, "Control barrier functions for multi-agent systems under conflicting local signal temporal logic tasks," *IEEE Control Systems Letters*, vol. 3, no. 3, pp. 757–762, 2019.

[4] D. Gundana and H. Kress-Gazit, "Event-based signal temporal logic synthesis for single and multi-robot tasks," *IEEE Robotics and Automation Letters*, vol. 6, no. 2, pp. 3687–3694, 2021.

[5] M. Sewlia, C. K. Verginis, and D. V. Dimarogonas, "Cooperative object manipulation under signal temporal logic tasks and uncertain dynamics," *IEEE Robotics and Automation Letters*, vol. 7, no. 4, pp. 11561–11568, 2022.

[6] N. Mehdipour, C.-I. Vasile, and C. Belta, "Arithmetic-geometric mean robustness for control from signal temporal logic specifications," in *2019 American Control Conference (ACC)*, pp. 1690–1695.

[7] V. Raman, A. Donzé, M. Maasoumy, R. M. Murray, A. Sangiovanni-Vincentelli, and S. A. Seshia, "Model predictive control with signal temporal logic specifications," in *53rd IEEE Conference on Decision and Control*, pp. 81–87, 2014.

[8] M. Charitidou and D. V. Dimarogonas, "Signal temporal logic task decomposition via convex optimization," *IEEE Control Systems Letters*, vol. 6, pp. 1238–1243, 2021.

[9] K. Leahy, A. Jones, and C.-I. Vasile, "Fast decomposition of temporal logic specifications for heterogeneous teams," *IEEE Robotics and Automation Letters*, vol. 7, no. 2, pp. 2297–2304, 2022.

[10] K. Leahy, M. Mann, and C.-I. Vasile, "Rewrite-based decomposition of signal temporal logic specifications," in *NASA Formal Methods Symposium*, pp. 224–240, Springer, 2023.

[11] Z. Liu, J. Dai, B. Wu, and H. Lin, "Communication-aware motion planning for multi-agent systems from signal temporal logic specifications," in *2017 American Control Conference (ACC)*, pp. 2516–2521.

[12] O. Maler and D. Nickovic, "Monitoring temporal properties of continuous signals," in *International Symposium on Formal Techniques in Real-Time and Fault-Tolerant Systems*, pp. 152–166, Springer, 2004.

[13] S. Sadraddini and C. Belta, "Robust temporal logic model predictive control," in *53rd Annual Allerton Conference on Communication, Control, and Computing*, pp. 772–779, IEEE, 2015.

[14] S. M. LaValle, *Planning algorithms*. Cambridge university press, 2006.

[15] G. M. Ziegler, *Lectures on polytopes*, vol. 152. Springer Science & Business Media, 2012.

[16] S. Froitzheim, *Efficient conversion of geometric state set representations for hybrid systems*. PhD thesis, Bachelor's thesis. RWTH Aachen University, 2016.

[17] S. S. Farahani, V. Raman, and R. M. Murray, "Robust model predictive control for signal temporal logic synthesis," *IFAC*, vol. 48, no. 27, pp. 323–328, 2015.

[18] G. Marchesini, S. Liu, L. Lindemann, and D. V. Dimarogonas, "Communication-constrained stl task decomposition through convex optimization," *arXiv preprint arXiv:2402.17585*, 2024.