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Finite-time topology identification for complex dynamical networks

Nana Wang   Dimos V. Dimarogonas

***Abstract*— This paper presents a finite-time topology identification method for complex dynamical networks. This method prevents the difficulty of verifying linear independence conditions and ensures the success of accurate topology identification. The topology identification scheme first renders the error dynamics between the networks and reference signals zero in finite time, and afterward, the topology is estimated by building an auxiliary network. The identification of topology is achieved once a relaxed excitation condition holds. The excitation condition is guaranteed by the proposed tracking control scheme. A finite-time topology identification and synchronization scheme for complex systems is further proposed where synchronization is realized by removing the exciting signals after the identification of the topology. At last, the simulation results verify the feasibility of the proposed method.**

I. Introduction

Complex dynamical networks represent many systems, ranging across social networks, distributed robots, sensor networks and biological systems [1], [2], thus attracting increasing attention. Analysis of complex dynamical networks allows us to understand their behaviors better, like how the influence spread in social networks [3] and biological systems [4]–[6]. At the design level, increasingly complex tasks have been realized by the coordination among agents [7], [8].

Most analyses and designs of dynamical networks assume access to the network topology in advance, like consensus, formation control, and distributed estimation problems [1]. The information on network topology allows us to address how to control a network efficiently, how to optimize the network based on the tasks, or analyze which node has the most influence. But in some cases, the interaction network for complex dynamical networks is unclear or unknown. For example, how genes regulate via networks in biological systems is still an open problem [5] and many methods are proposed to infer the gene regulatory networks, including decision-trees [6] and differential equations [9]. Hence, identifying the network topology for complex dynamical networks plays a key role in analyzing complex dynamical networks or making designs for coordination tasks. A wide range of methods are presented to deal with this problem for different complex networks, like knock-out method [10], adaptive control [11], [12], optimization [13]. A review can be found in [14].

This paper focuses on using adaptive control methods to address the topology identification problem, which designs one copy model to estimate the original network and identify the network topology once there is no difference between the states of the copy model and that of the original network. This method is applied to address the topology identification problem for different network models like weighted complex network [11], networks with time-delays [15] and perturbed networks [16]. One underlying assumption in the above-mentioned work is that linear independence conditions hold during the identification process, which means that the states or the output should be linearly independent. Typically, it is challenging to verify if linear independence conditions hold, causing the risk of failing in identification. Linear independence conditions are guaranteed by tracking another auxiliary network system [12], achieving topology identification when time tends to be infinite. In practice, algorithms that address this problem in finite time are needed. In addition, the existing works don't consider how to combine the information inference and actuation in order to simultaneously identify the network topology and achieve the control tasks.

Based on the above observations, we consider finite-time topology identification problem for multi-agent systems. A finite-time adaptive control method is proposed to estimate the network topology. The main idea is to drive the system to track the reference signals, providing sufficient excitation for the identification process. The controller design and parameter convergence scheme are presented to guarantee finite-time identification. In addition, finite-time topology identification and synchronization control is integrated to perform the control tasks under communication uncertainties in the dynamical network. It has been shown that topology identification is not guaranteed if the system is synchronized [17]. One solution is to achieve identification before the synchronization. Switching the exciting reference signals to the expected synchronized state renders the synchronization in finite time.

The main contribution is two-fold. On the one hand, the proposed strategy of tracking the reference signals suffices a sufficient excitation condition for topology identification. Compared to linear independence conditions [12] for the reference signals during the identification process, the excitation condition employed here has a broader family of signals to choose from and is more practical. On the other hand, the identification of network topology is achieved in finite time. Compared to the topology identification in infinite time approaches, the proposed scheme ensures topology identification in finite time. This renders the combined finite-time topology identification and synchronization for complex dynamical networks possible. Furthermore, the feasibility of the proposed scheme is guaranteed by theoretical results.

The remainder of this paper is structured as follows. In

All the authors are with the Division of Decision and Control Systems, KTH Royal Institute of Technology, SE-100 44 Stockholm, Sweden, {nanaw,dimos}@kth.se. This work was supported by the Wallenberg AI, Autonomous Systems and Software Program (WASP) and WASP-DDLS grant funded by the Knut and Alice Wallenberg (KAW) Foundation, and the Swedish Research Council (VR).

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Section II, we introduce some concepts of excitation and a relevant lemma. Section III presents the problem considered in this paper. Section IV proposes a finite-time scheme for topology identification and provides the sufficient condition under which the proposed scheme ensures finite-time topology identification. A combined scheme for topology identification and synchronization is proposed in Section V. Section VI demonstrates the the proposed scheme's effectiveness with a numerical example. Finally, Section VII concludes the paper.

Notations: R^n and $R^{n\times m}$ denote the sets of n-dimensional real vector spaces and the $n \times m$ real matrices, respectively. $R_{\geq 0}$ denotes the set of non-negative real values. For any vector $z \in R^n$, its 2-norm and infinite norm are denoted by $\|z\|_2$ and $\|z\|_\infty$. $\mathcal{L}_\infty$ denotes the set of functions $z(t)$: $R_{\geq 0} \to R^n$ with $\|z(t)\|_\infty < \infty, \forall t \geq 0$. For any matrix $M \in R^{n\times m}$, its transpose is represented by M^T. For a symmetric matrix $M \in R^{n\times n}$, $M \succ 0$ denotes that M is positive definite. We say $M \succ N$ if $M - N \succ 0$ for $M, N \in R^{n\times n}$. sign(z) is the sign function of $z \in R$. I denotes the identity matrix with appropriate dimensions.

II. Preliminaries

There are different definitions of excitation given in the literature. Here, we use the definitions proposed by [18].

Definition 1: A function $\phi : R_{\geq 0} \to R^n$ is said to be exciting over $[t, t + T]$ with $t, T > 0$ if for some $\alpha > 0$

$$\int_t^{t+T} \phi(\tau)^T \phi(\tau) d\tau \geq \alpha I.$$

Compared to excitation, a stronger property of a signal is persistent excitation, as follows.

Definition 2: A function $\phi : R_{\geq 0} \to R^n$ is said to be persistently exciting if for all $t > 0$ there exist $T > 0$ and $\alpha > 0$ such that,

$$\int_t^{t+T} \phi(\tau)^T \phi(\tau) d\tau \geq \alpha I.$$

Lemma 1 shows that the output of a strictly stable, minimum-phase filter driven by an exciting input keeps such property, referred from Lemma 4.8.3 in [19].

Lemma 1: If $z(t) : R_{\geq 0} \to R^n$ satisfies the excitation condition, $z(t), \dot{z}(t) \in \mathcal{L}_\infty$ and $H(s)$ is a strictly stable, minimum phase, proper rational transfer function, then $w = H(s)z$ also satisfies the excitation condition. □

III. Problem statement

Consider a dynamical network where the agents communicate via a topology abstracted as an undirected or directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, with a set of nodes $\mathcal{V} = 1, 2, \cdots, N$ and a set of undirected or directed edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ [1]. The dynamical network is represented as

$$\dot{x}_i(t) = f_i(t, x_i(t)) + \sum_{j=1}^{N} l_{ij} g_{ij}(x_j(t)) + u_i(t), \quad (1)$$

where $i = 1, 2, \cdots, N$. $x_i \in R^n$ is the state vector of the ith node, $f_i(t, x_i(t)) : R_{\geq 0} \times R^n \to R^n$ is a known smooth function. function $g_{ij}(x_j(t)) : R^n \to R^n$ is given as a smooth function. $u_i(t)$ is the control input and l_{ij} denotes the connection between agents i and j. If there is communication between node i and node $j (j \neq i)$, then $l_{ij} \neq 0$, otherwise, $l_{ij} = 0$, $1 \leq i \leq N$. We assume that $L = (l_{ij})_{N\times N}$ is an unknown constant matrix.

The goal of this paper is to identify the unknown topology $\mathcal{G}$, in the form of the adjacency matrix L, of system (1) in finite time.

Assumption 1: Assume the coefficients of L satisfy $|l_{ij}| < l_{\max}$, where $i, j = 1, 2, \cdots N$ and $l_{\max} > 0$ is known.

Lemma 2: [20] Consider a function $V(x) : R^n \to R$ in an open neighbourhood $\Omega \subset R^n$ and the system $\dot{x} = f(x), x \in R^n$ with $f(0) = 0$. If $V(x)$ satisfies $\dot{V}(x) \leq -\alpha V^\eta(x)$ where $\alpha > 0, 0 < \eta < 1$, then x will converge to zero in finite time $t_1 = V^{1-\eta}(0)/(\alpha(1 - \eta))$. □

Remark 1: In this paper, the maximum value $l_{\max}$ of each element of L is assumed to be known as a priori. This assumption is reasonable as the range of the elements of L is typically known when estimating its values. Even if $l_{\max}$ is unknown, a conservative guess about $l_{\max}$ can be made by using a rather large value to ensure the finite-time tracking of the reference signals. Here all state is observable. •

IV. Finite-time topology identification

We propose a method to guarantee successful topology identification in a finite interval of time. This relies on a two-step approach. First, an adaptive control-based algorithm is employed here to deal with this problem, using the concept of an excitation condition. The control for (1) is designed to fully track reference signals in finite time, providing sufficient rich information for the identification. After that, the network topology is identified in finite time.

A. Design of finite-time controller

We first design an adaptive controller for (1) to track reference signals $z_i(t) \in R^n$ for agent i in finite time, where $i = 1, 2, \cdots, N$.

Assumption 2: Assume $z_i(t), \dot{z}_i(t) \in \mathcal{L}_\infty$ for $i = 1, 2, \cdots, N$ and for $z(t) := (z_1(t), z_2(t), \cdots, z_N(t))^T$ the following excitation condition: There exist $T > 0$ and $\alpha > 0$ such that

$$\int_t^{t+T} z(\tau)^T z(\tau) d\tau \geq \alpha I. \quad (2)$$

Denote $\mu_i(t) = x_i(t) - z_i(t)$, $1 \leq i \leq N$. Then the error dynamics $\mu_i(t)$ follows

$$\dot{\mu}_i(t) = f_i(t, x_i(t)) + \sum_{j=1}^{N} l_{ij} g_{ij}(x_j(t)) + u_i - \dot{z}_i(t). \quad (3)$$

The proposed adaptive controller is given as

$$\begin{aligned} u_i(t) = &-f_i(t, x_i(t)) - \textstyle\sum_{j=1}^{N} \hat{l}_{ij} g_{ij}(x_j(t)) - k\text{sign}(\mu_i(t)) \\ &+\dot{z}_i(t) - \lambda \textstyle\sum_{j=1}^{N} (\hat{l}_{ij} + l_{\max}) \frac{\mu_i}{\|\mu_i\|_2}, \text{if } \|\mu_\text{i}\| \neq 0, \\ u_i(t) = &\ 0, \text{if } \|\mu_\text{i}\| = 0, \end{aligned} \quad (4)$$

where $\lambda = \frac{k}{\sqrt{\sigma_{ij}}}$, $\sigma_{ij} > 0$, $k > 0$ and $\hat{l}_{ij}$ is the estimation of l_{ij} for $i, j = 1, 2, \cdots, N$. Define $\tilde{l}_{ij}$ as the estimation error $\tilde{l}_{ij} := l_{ij} - \hat{l}_{ij}$.

The adaptive parameters $\hat{l}_{ij}$ are updated as follows

$$\dot{\hat{l}}_{ij} = \sigma_{ij}\mu_i^T g_{ij}(x_j), \tag{5}$$

where σ_{ij} adjusts the rate of estimating the topology parameters. Noted that the states of all agents are observable here and $g_{ij}(\dot{)}$ is known. Here $i, j = 1, 2, \cdots, N$.

Corollary 1: Under Assumptions 1 and 2, consider the control (4) and the parameter updating rules (5). Then the tracking errors μ_i for agent $i = 1, ..., N$, converge to zero in finite time t_1, where t_1 is defined by $t_1 := \frac{\sqrt{2}V^{1/2}(0)}{k}$ and $V^{1/2}(0) = \frac{1}{2}\sum_{i=1}^N \mu_i(0)^T\mu_i(0) + \frac{1}{2}\sum_{i=1}^N\sum_{j=1}^N \frac{1}{\sigma_{ij}}\tilde{l}_{ij}(0)^2$. □

Proof: Consider the function

$$V = \frac{1}{2}\sum_{i=1}^N \mu_i^T\mu_i + \frac{1}{2}\sum_{i=1}^N\sum_{j=1}^N \frac{1}{\sigma_{ij}}\tilde{l}_{ij}^2.$$

Differentiating V along the error dynamics (3), one has

$$\begin{aligned}
\dot{V} &= \textstyle\sum_{i=1}^N \mu_i^T\dot{\mu}_i - \sum_{i=1}^N\sum_{j=1}^N \frac{1}{\sigma_{ij}}\tilde{l}_{ij}\dot{\hat{l}}_{ij}, \\
&= \textstyle\sum_{i=1}^N \mu_i^T(f_i(t, x_i(t)) + \sum_{j=1}^N l_{ij}g_{ij}(x_j(t)) + u_i) \\
&\quad \textstyle - \sum_{i=1}^N \mu_i^T\dot{z}_i(t) - \sum_{i=1}^N\sum_{j=1}^N \tilde{l}_{ij}\mu_j^T g_{ij}(x_j(t)) \\
&\le \textstyle\sum_{i=1}^N \mu_i^T(\sum_{j=1}^N \tilde{l}_{ij}g_{ij}(x_j(t)) - k\text{sign}(\mu_i(t)) \\
&\quad \textstyle -\lambda\sum_{j=1}^N(\hat{l}_{ij} + l_{\max})\frac{\mu_i}{\|\mu_i\|_2}) \\
&\quad \textstyle - \sum_{i=1}^N\sum_{j=1}^N \tilde{l}_{ij}\mu_j^T g_{ij}(x_j(t)) \\
&\le \textstyle -k(\sum_{i=1}^N\|\mu_i\| + \sum_{i=1}^N\sum_{j=1}^N \frac{1}{\sqrt{\sigma_{ij}}}|\hat{l}_{ij} + l_{max}|)
\end{aligned} \tag{6}$$

The second equality above is obtained with (3) and (5). The first inequality above is deduced after applying the control (4). For $\tilde{l}_{ij} = l_{ij} - \hat{l}_{ij}$, one has $|\tilde{l}_{ij}| \le l_{\max} + |\hat{l}_{ij}|$. The last inequality is deduced from $-|\tilde{l}_{ij}| \ge -l_{\max} - |\hat{l}_{ij}|$. Replacing λ into this inequality, it yields

$$\dot{V} \le -k\left(\sum_{i=1}^N\|\mu_i\| + \sum_{i=1}^N\sum_{j=1}^N \frac{1}{\sqrt{\sigma_{ij}}}|\tilde{l}_{ij}|\right).$$

Using the following inequality

$$\|x_1\| + \|x_2\| + \cdots + \|x_n\| \ge \left(\|x_1\|^2 + \|x_2\|^2 + \cdots + \|x_n\|^2\right)^{1/2}$$

where $x_1, x_2, \cdots, x_n \in R$, it follows

$$\dot{V} \le -\sqrt{2}k\left(\frac{1}{2}\sum_{i=1}^N \mu_i^T\mu_i + \frac{1}{2}\sum_{i=1}^N\sum_{j=1}^N \frac{1}{\sqrt{\sigma_{ij}}}|\tilde{l}_{ij}|^2\right)^{1/2}.$$

From this, $\dot{V} \le -\sqrt{2}kV^{1/2}$ if $\|\mu_i\| \ne 0$ and $\dot{V} = 0$ if $\|\mu_i\| = 0$. Hence, based on Lemma 1, the error dynamics $\|\mu_i\|$ will converge to zero in finite time t_1. Let $M = \left\{\dot{V} = 0\right\} = \{\mu_i = 0, i = 1, 2, \cdots, N\}$. According to LaSalle's invariance principle [21], the trajectory of μ_i will converges to the set M in finite time t_1 and remain there.

Remark 2: Assuming a special class of reference signals guarantees linear independence conditions for the states of (1) and it has been proven that in such case the topology is identified when time tends to infinity [12]. However, it is noted that using directly this method to identify the topology in finite time, cannot guarantee finite-time topology identification, particularly by using the control input (4) if $\|\mu_i\| \ne 0$ and $u_i(t) = -f_i(t, x_i(t)) - \sum_{j=1}^N \hat{l}_{ij}g_{ij}(x_j(t)) - z_i(t)$ if $\|\mu_i\| = 0$, and the parameter updating laws (5). Using a similar method as in [11] where the topology is identified in infinite time, the proposed scheme drives the tracking errors to zero in finite time, but cannot guarantee the topology estimation errors $\tilde{l}_{ij}$ $\forall i, j = 1, 2, \cdots, N$ to converge to zero in finite time. •

B. A Finite-time Topology Identification Scheme

Based on the analysis of Corollary 1, the control input (4) and parameter updating laws (5) render the error dynamics μ_i zero in finite time t_1 for $i = 1, 2, \cdots, N$. When μ_i tends to be zero, $\tilde{l}_{ij}$ tends to be constant, but not necessarily zero. Hence, additional effort is needed to address this problem. We propose one method here to identify the network topology accurately in finite time employing a technique introduced in [22] and tailoring it to the networked problem in hand. The main idea is to obtain the error between the estimated value of the topology and the true value by establishing an auxiliary network system, and then to estimate the true value once a particular excitation condition holds.

Define the finite-time topology identification treated in this paper for the network (1).

Definition 3: We say that the topology of the network (1) is identified in a finite time if the coefficients l_{ij} in (1) are estimated accurately in a finite time t', where $0 < t' < \infty$, i.e. $\tilde{l}_{ij} \to 0$ when $t \to t'$, $\forall i, j = 1, ..., N$.

Design an auxiliary network system to estimate the state of agent i as follows:

$$\begin{aligned}
\dot{\hat{x}}_i =& f_i(t, x_i(t)) + u_i(t) + \sum_{j=1}^N \hat{l}_{ij}g_{ij}(x_j(t)) \\
&+ \sum_{j=1}^N w_{ij}\dot{\hat{l}}_{ij} + \tau e_i,
\end{aligned} \tag{7}$$

where $\tau > 0$ and $e_i := x_i - \hat{x}_i$ is the prediction error for $i = 1, 2, \cdots, N$. $u_i(t)$ is designed in (4), while w_{ij} is the output of the filter equation

$$\dot{w}_{ij} = g_{ij}(x_j(t)) - \tau w_{ij}, \qquad w_{ij}(0) = 0, \tag{8}$$

and where $i, j = 1, 2, \cdots, N$. Define $w_i := (w_{i1}, w_{i2}, \cdots, w_{iN})$.

Function $g_{ij}(x_j(t))$ is assumed to be a linear function of $x_j(t)$ here, without of generality, i.e., $g_{ij}(x_j(t)) = k_{ij}x_j(t)$, where $k_{ij} \ne 0$ is a constant. The filter (8) can be written as

$$\dot{w}_i^T = G_i x - \tau w_i^T, \qquad w_i(0) = 0, \tag{9}$$

where $x := (x_1, x_2, \cdots, x_N)^T$ and $G_i := \text{diag}(k_{i1}, k_{i2}, \cdots, k_{iN})$, since it has been assumed that

$g_{ij}(x_j) = k_{ij}x_j$. With $\tilde{l}_{ij} = l_{ij} - \hat{l}_{ij}$, we get from (1) and (7) that

$$\dot{e}_i = \sum_{j=1}^{N} \tilde{l}_{ij} g_{ij}(x_j(t)) - \tau e_i - \sum_{j=1}^{N} w_{ij} \dot{\hat{l}}_{ij}. \quad (10)$$

Define $\zeta_i := e_i - \sum_{j=1}^{N} w_{ij}\tilde{l}_{ij}$, then it evolves according to

$$\dot{\zeta}_i = -\tau \zeta_i, \qquad \zeta(0) = e_i(0). \quad (11)$$

Define l_i and g_i as $l_i := (l_{i1}, l_{i2}, \cdots, l_{iN})$ and $g_i := (g_{i1}, g_{i2}, \cdots, g_{iN})$. Similarly, let $\hat{l}_i := (\hat{l}_{i1}, \hat{l}_{i2}, \cdots, \hat{l}_{iN})$ and $\tilde{l}_i := (\tilde{l}_{i1}, \tilde{l}_{i2}, \cdots, \tilde{l}_{iN})$. Construct matrix $P_i \in R^{N\times N}$ and $H_i \in R^N$:

$$\begin{aligned} \dot{P}_i &= w_i^T w_i, & P_i(0) &= 0 \\ \dot{H}_i &= w_i^T (w_i \hat{l}_i^T + e_i - \zeta_i), & H_i(0) &= 0. \end{aligned} \quad (12)$$

The following assumption is necessary to estimate the topology matrix L accurately.

Assumption 3: Suppose there exists a time $t_c > t > 0$ and $\gamma > 0$ such that $P_i(t_c)$ satisfies the excitation condition, i.e.,

$$P_i(t_c) = \int_t^{t_c} w_i^T(\tau) w_i(\tau) d\tau \succ \gamma I, \forall i = 1, ..., N. \quad (13)$$

The following lemma presents the relation between Assumption 2 and Assumption 3.

Lemma 3: Let Assumptions 1 and 2 of Corollary 1 hold. Assuming that (1) is subject to the control (4) and the parameter updating rules (5), then the output of the filter (8) satisfies the excitation condition (13). □

Proof: Based on the analysis of the error dynamics μ_i in Corollary 1, $\mu_i \to 0$ in finite time t_1 and thus $x_i(t) \to z_i(t)$ as $t \to t_1$. Recall also that, $g_{ij}(x_j(t)) = k_{ij}x_j(t)$. Hence $g_{ij}(x_j(t)) = k_{ij}z_j(t)$ for $t \geq t_1$. Hence for agent i, (9) becomes

$$\dot{w}_i^T = G_i z - \tau w_i^T, w_i(0) = 0. \quad (14)$$

We have that, z satisfies $z, \dot{z} \in \mathcal{L}_\infty$ and z is exciting. In addition, the transfer function of filter (14) is $H(s) = G_i/(s+\tau)$, which is strictly stable and minimum-phase. Hence according to Lemma 1, the output w_i is exciting, too. Therefore, the control design (4) and the updating laws (5) with Assumptions 1 and 2 is sufficient for Assumption 3 to hold as well.

The estimation of L is obtained in the following theorem.

Theorem 1: Let Assumptions 1 and 2 of Corollary 1 hold. Assume that (1) is subject to the control (4) and the parameter updating rules (5). Further considering (7), (8), (11) and (12), the estimation of L is given by

$$l_i = P_i(t)^{-1} H_i(t) \qquad \text{for all } t \geq t_c. \quad (15)$$

where $l_i = (l_{i1}, l_{i2}, \cdots, l_{iN})$ and $i = 1, 2, \cdots, N$. □

Proof: The integral of P_i and H_i from t_0 to t is deduced from (12) as

$$\begin{aligned} P_i(t) &= \int_0^t w_i^T(\tau) w_i(\tau) d\tau \\ H_i(t) &= \int_0^t w_i^T(\tau)(w_i(\tau)\hat{l}_i^T(\tau) + e_i(\tau) - \zeta_i(\tau)) d\tau \end{aligned} \quad (16)$$

with $P_i(0) = 0$ and $H_i(0) = 0$. Using $\zeta_i = e_i - w_i\tilde{l}_i^T$ and $l_i = \hat{l}_i + \tilde{l}_i$, $H_i(t)$ is further given as

$$\begin{aligned} H_i(t) &= \int_0^t w_i^T(\tau)(w_i(\tau)\hat{l}_i^T(\tau) + w_i(\tau)\tilde{l}_i^T(\tau)) d\tau \\ &= \int_0^t w_i^T(\tau)(w_i(\tau) l_i^T) d\tau \end{aligned} \quad (17)$$

It is noted that $l_i = (l_{i1}, l_{i2}, \cdots, l_{iN})$ is an unknown but constant vector. Therefore, after moving l_i outside of the integral $\int_0^t w_i^T(\tau) w_i(\tau) d\tau$, $H_i(t)$ becomes

$$H_i(t) = \int_0^t w_i^T(\tau) w_i(\tau) d\tau \cdot l_i^T. \quad (18)$$

According to Lemma 3, Assumption 3 holds from Assumptions 1 and 2 , the control input (4) and the parameter updating laws (5). It also implies that $P_i(t)$ is invertible. Hence the value of l_i is obtained as

$$l_i^T = P_i(t)^{-1} H_i(t), \ \forall t \geq t_c. \quad (19)$$

This concludes the proof.

Then, the network topology estimation algorithm is given as

$$\begin{aligned} l_i &= \hat{l}_i, \quad \text{if} \quad t < t_c; \\ l_i &= P(t_c)^{-1} H(t_c), \quad \text{if} \quad t \geq t_c. \end{aligned} \quad (20)$$

Remark 3: Assumption 1 assuming the maximum value of matrix L is known, is necessary to ensure that the tracking of reference signals is achieved in finite time. The presented tracking scheme guarantees that the excitation condition (13) holds. Other tracking strategies can also be combined with the topology identification algorithm presented in this subsection if it ensures that w_i satisfies the excitation condition (13). •

Remark 4: To identify the network topology L, the excitation condition (13) needs to be satisfied for each agent. Compared to the standard adaptive control scheme using the persistent excitation condition, the conditions (2) and (13) are a kind of excitation condition and (2) can be guaranteed by designing specific signals over some interval. This estimation algorithm allows us to compute $\hat{L} = (\hat{L}_{ij})_{N\times N}$ once the matrix P_i are positive definite, thereby rendering the topology identification in finite time rather than in infinite time. •

Remark 5: If we have a priori knowledge about the structure of matrix L, the number of unknown coefficients of the topology is reduced using this knowledge. For example if $l_{ii} = \sum_{j=1, i\neq j}^{N} l_{ij}$ exists, we just need to calculate $N-1$ parameters of vector $l_i := (l_{i1}, l_{i2}, \cdots, l_{i(i-1)}, l_{i(i+1)}, \cdots, l_{iN})$ for agent i. Denote $w_i := (w_{i1}, w_{i2}, \cdots, w_{i(i-1)}, w_{i(i+1)}, \cdots, w_{iN})$. And the dimensions of P_i and H_i are reduced to $P_i \in R^{(N-1)\times(N-1)}$ and $H_i \in R^{(N-1)}$. Here $i = 1, 2, \cdots, N$. In total, the number of the estimated parameters of L is $N \times (N-1)$ instead of $N \times N$. If $\mathcal{G}$ is undirected, only $N(N-1)/2$ coefficients of the topology are needed to be estimated and the corresponding estimation algorithm can be adjusted. •

C. Excitation Signal Design

A design of $z(t) = (z_1(t), z_2(t), \cdots, z_N(t))^T$ is presented so that the excitation condition (2) is met. The time-varying signal $z_i(t)$ is decomposed into a constant matrix and a periodic part by computing the trigonometric (or Fourier) series expansion for each nonlinearity vector [22]. Here we use the trigonometric series to design the signal $z(t)$ as $z(t) = r + d(t)$, where r is a constant vector and $d(t)$ provides sufficient excitation. We set $d(t)$ as $d(t) = A\phi(t)$, where $A = \mathrm{diag}(a_1, a_2, \cdots, a_N)$ and $\phi(t) = (\sin(\varpi_1 t + \varphi_1), \sin(\varpi_2 t + \varphi_2), \cdots, \sin(\varpi_N t + \varphi_N)^T, \varpi_i(t) \neq \varpi_j(t)$ if $i \neq j$, where a_i, ϖ_i, φ_i are constants for $i = 1, 2, \cdots, N$.

V. Finite-time identification and synchronization

In this section, we present a finite-time identification and synchronization scheme by switching the excitation signal to zero once the topology is identified.

Definition 4 (Synchronization): A network is synchronized if the manifold of synchronized motions is stable. The manifold is $S = x_1 = x_2 = \cdots = x_N = r$, where r is the synchronized state.

In order to achieve finite-time identification and synchronization, one solution is to use the reference signals $z(t)$ to ensure that the excitation condition holds until the identification of the network is achieved. Then signals $d(t)$ in the reference signals $z(t)$ are set to zero in order to drive the network to the desired synchronization in finite time.

For the parameter identification and synchronization scheme, the amplitudes of the excitation signals are

$$\hat{a}_i(t) = \begin{cases} a_i, & \text{if } t \leq t_g \\ 0, & \text{if } t > t_g \end{cases} \tag{21}$$

where $t_g = t_c + \delta$ is the time when the excitation condition (13) is satisfied and δ is a positive constant. a_i is given in the excitation signals. Here $i = 1, 2, \cdots, N$. When the identification is achieved, the dither signals $d(t)$ are set to zero in t_g. The updating parameters $\hat{l}_{ij}$ stop updating and keep constant for $i, j = 1, 2, \cdots, N$. For $t \geq t_g$, the control input is

$$\begin{aligned} u_i(t) &= \dot{z}_i(t) - f_i(t, x_i(t)) - \textstyle\sum_{j=1}^{N} \hat{l}_{ij} g_{ij}(x_j(t)) \\ &\quad - k\mathrm{sign}(\mu_i(t)), \text{if } \|\mu_i\| \neq 0 \\ u_i(t) &= 0, \text{if } \|\mu_i\| = 0. \end{aligned} \tag{22}$$

Combing the above results, we can achieve finite-time topology identification and synchronization as follows:

Corollary 2: Let Assumptions 1 and 2 hold, and assume that (1) is subject to the control input (4) and the parameter updating rules (5). Further consider (7), (8), (11) and (12). With the control input (22), then the topology identification and synchronization for (1) is achieved in finite time $t = t_s + t_g$, where $V^{1/2}(0) = \frac{1}{2}\sum_{i=1}^{N} \mu_i(0)^T \mu_i(0)$. □

Proof: The proof that synchronization is achieved in t_s, similar to that of Corollary 1, so it is omitted here. Topology identification is guaranteed in t_g through Theorem 1.

Remark 6: In addition to the synchronization task, other control tasks, i.e. formation control, can be achieved by giving different reference signals $r(t)$. •

VI. Simulations

Consider a network of 6 agents modeled by (1), connected by a graph in Fig 1. $f_i(t, x_i(t)) = x_i(t)$ and $g_{ij}(x_j(t)) = k_{ij} x_j(t)$, where $k_{ij} = 2$ for $i, j = 1, 2, \cdots, 6$.

The topology L is

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & -1 \\ -1 & 0 & 3 & -2 & 0 & 0 \\ 0 & 0 & -2 & 3 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & -1 & 0 & 0 & -1 & 2 \end{bmatrix}.$$

The initial values of (1) are $x_0 = (1.2, 1.5, 1.4, 0.3, 0.6, 1.1)^T$. The goal is to estimate the topology l_{ij} for $i, j = 1, 2, \cdots, 6$.

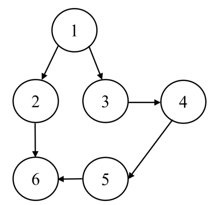

Fig. 1: The connecting graph representing the topology.

The reference signals $z(t)$ are chosen as $\dot{z}(t) = (\cos(0.5\pi t - 1/8\pi), \sin(0.7\pi t + 2/3\pi), \cos(\pi t), \sin(2\pi t), \sin(3\pi t - \pi/3), \cos(4\pi t - 1/4\pi))^T$ with zero initial values for $t \in [0, 2]s$ and $\dot{z}(t) = [0, 0, 0, 0, 0, 0]^T$ for $t \in (2, 3]s$. The maximum value $l_{\max}$ is $l_{\max} = 4$. The control input u is given by (4) and (22). And the control gains are $k = 1$ and $\lambda = 0.5$. The estimation parameter is $\sigma_{ij} = 10$ for $i, j = 1, 2, \cdots, 6$. The parameter in (8) is $\tau = 4$. The initial values of (5) and (7) are set to zero.

The simulation results are shown in Fig 2-5. The identification results are displayed in Fig 2. The estimation errors $\tilde{L} := L - \hat{L}$ first decrease and then maintain using adaptive parameters (5), which supports Corollary 1. The estimation errors are zero using the estimation equation (20) after $t = 1.13s$, which verifies Theorem 1 that the topology identification is achieved in finite time using the proposed scheme. The resulting states of all agents are depicted in Fig 3, representing that the tracking errors decrease to zero using the control input (4) after $t = 0.24s$, which supports Corollary 1. After removing the exciting signals, the synchronized result is shown in Fig 5 after switching the control input (22), which verifies Corollary 2. From this, the topology identification and synchronization is achieved within $t = 2.70s$. In conclusion, the identification and control for multi-agent systems is achieved in finite time.

VII. Conclusion

In this paper, we consider a finite-time topology identification problem for multi-agent systems. A finite-time topology identification strategy is presented to address this problem by

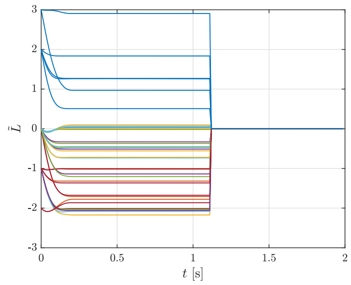

Fig. 2: Estimation errors of the topology matrix L for $t \in [0, 2]s$.

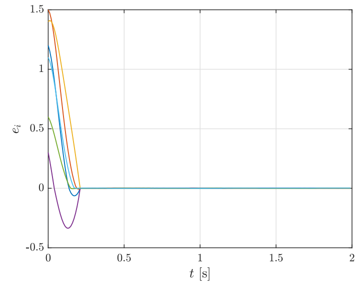

Fig. 3: Evolution of tracking error e_i for $t \in [0, 2]s$.

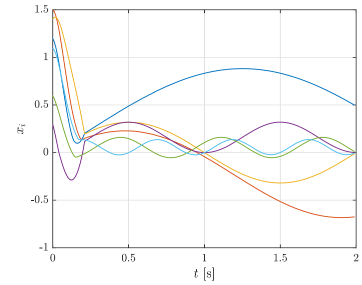

Fig. 4: Evolution of x_i of each agent for $t \in [0, 2]s$.

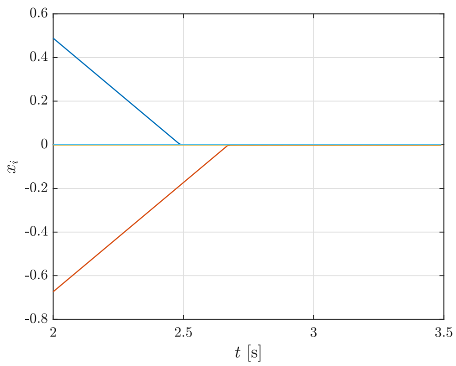

Fig. 5: Evolution of synchronized state x_i after $2s$.

employing adaptive control methods, which offers a theoretical guarantee for topology identification. The controller is designed to track reference signals in finite time, providing sufficient excitation for topology identification. The topology is accurately estimated once the excitation condition for each agent is satisfied and this estimation process ends in finite time. Moreover, a finite-time topology identification and synchronization scheme is presented, which achieves the topology identification first, and then the multi-agent system can be synchronized by switching the reference signals. This paper only solves the fixed topology identification problem and the identification of time-varying topology will be considered further. The coordination of the topology identification and control tasks for complex dynamical networks properly will also be studied further.

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