Collaborative Satisfaction of Long-Term Spatial Constraints in Multi-Agent Systems: A Distributed Optimization Approach

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Abstract— **This paper addresses the problem of collaboratively satisfying long-term spatial constraints in multi-agent systems. Each agent is subject to spatial constraints, expressed as inequalities, which may depend on the positions of other agents with whom they may or may not have direct communication. These constraints need to be satisfied asymptotically or after an unknown finite time. The agents' objective is to collectively achieve a formation that fulfills all constraints. The problem is initially framed as a centralized unconstrained optimization, where the solution yields the optimal configuration by maximizing an objective function that reflects the degree of constraint satisfaction. This function encourages collaboration, ensuring agents help each other meet their constraints while fulfilling their own. When the constraints are infeasible, agents converge to a least-violating solution. A distributed consensus-based optimization scheme is then introduced, which approximates the centralized solution, leading to the development of distributed controllers for single-integrator agents. Finally, simulations validate the effectiveness of the proposed approach.**

I. Introduction

Control and coordination of Multi-Agent Systems (MAS) have been a major research focus in the past decade, driven by tasks that require collaboration which are otherwise nearly impossible to achieve. Traditional MAS problems include consensus, rendezvous, flocking, formation, coverage, and containment [1]–[3]. However, recent research has shifted toward new demands, such as distributed optimal coordination [4] and handling high-level spatiotemporal (i.e., space and time) specifications in multi-robot systems [5]–[8], which do not explicitly lie within the classical MAS problems.

Distributed Optimization (DO) often involves minimizing a joint objective function using algorithms deployed across a network of communicating computation nodes (agents), where each agent knows only a small part of the problem and can communicate with a limited number of neighbors [9]–[12]. In multi-robot systems, DO offers a framework for developing local decision-making rules, addressing various challenges in cooperative robotics, such as surveillance, task allocation, optimal consensus, cooperative motion planning, self-organization, cooperative estimation, target tracking, and distributed SLAM. For a detailed exploration of DO's applications in multi-robot networks, see [11], [13]–[15].

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This paper introduces the problem of collaborative coordination in multi-agent systems under long-term spatial constraints. Each agent's position is subject to inequality type constraints that may depend on the positions of other agents, with whom they may or may not have direct communication. The objective is to collectively achieve a desired formation that satisfies all constraints. These constraints are long-term, as they only need to be satisfied asymptotically or after an unknown finite time. Furthermore, each agent must meet its own constraints while helping others satisfy theirs, despite lacking explicit knowledge of the other agents' constraints.

Unlike distributed aggregative optimization in cooperative robotics [15]–[17], where each agent's local objective function must depend on the positions of all other agents, our formulation does not require this global dependency, thereby addressing a broader class of coordination problems.

We first reformulate the problem as a centralized optimization task, where maximizing the objective function results in a desired configuration that satisfies all constraints. The objective function is designed such that its positive values represent the satisfaction of the multi-agent constraints, with larger values indicating better overall constraints fulfillment. In cases where the constraints are collectively infeasible, solving the optimization problem provides a least-violating solution, reflected by a negative optimal value of the objective function. Next, we develop a novel multi-agent objective function as a sum of agents' local (private) objective functions, which depend solely on each agent's spatial constraints. We demonstrate that minimizing the multi-agent objective function yields an approximate solution to the centralized optimization problem. This enables the design of distributed control protocols for single-integrator agents using distributed continuous-time consensus-based optimization algorithms. Additionally, we explore the sufficient conditions for convexity and strictly convexity of the multi-agent system's global objective function.

Due to space constraints, detailed proofs, additional remarks, and extended simulation results are provided in [18].

II. Problem Formulation

Consider a multi-agent system operating in a d-dimensional space, with $N \in \mathbb{N}$ agents, each governed by the single integrator dynamics:

$$\dot{\boldsymbol{x}}_i = \boldsymbol{u}_i, \quad i = 1, \dots, N, \tag{1}$$

where $\boldsymbol{x}_i \in \mathbb{R}^d$ represents the position of agent i, and $\boldsymbol{u}_i \in \mathbb{R}^d$ is its velocity control input. Define the stacked vector of all agents' positions as $\boldsymbol{x} := [\boldsymbol{x}_1^\top, \boldsymbol{x}_2^\top, \dots, \boldsymbol{x}_N^\top]^\top \in \mathbb{R}^{Nd}$.

Let $\mathcal{V} \coloneqq \{1, \ldots, N\}$ represent the index set of agents, and define $\mathcal{I}_i \subseteq \mathcal{V} \setminus \{i\}$ as the set of agents with which agent i has spatial coupling constraints. Furthermore, let $n_i \in \mathbb{N}$ denote the number of agents in $\mathcal{I}_i$ (i.e., $|\mathcal{I}_i| = n_i$). Let $\mathrm{col}(\cdot)$ be the vector concatenation operator and define $\boldsymbol{x}_{\mathcal{I}_i} \coloneqq \mathrm{col}(\boldsymbol{x}_j)_{j\in\mathcal{I}_i} \in \mathbb{R}^{n_i d}$ as the stacked positions of the agents that are involved in the spatial constraints of agent i. Given this notation, assume agent i is subject to $m_i \in \mathbb{N}$ spatial *long-term constraints*, formulated as inequalities:

$$\psi_{i,k}(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i}) > 0, \quad k = 1, \ldots, m_i, \quad i = 1, \ldots, N, \quad (2)$$

where $\psi_{i,k} : \mathbb{R}^d \times \mathbb{R}^{n_i d} \to \mathbb{R}$ are continuously differentiable constraint (or task) functions. These constraints are termed long-term because they are required to hold only after an unknown but finite time, or as $t \to +\infty$.

Objective: Develop a distributed control protocol that guides agents to a desired spatial multi-agent formation, $\boldsymbol{x}_1^*, \boldsymbol{x}_2^*, \ldots, \boldsymbol{x}_N^*$, satisfying all constraints $\psi_{i,k}(\boldsymbol{x}_i^*, \boldsymbol{x}_{\mathcal{I}_i}^*) > 0$, for $k = 1, \ldots, m_i$ and $i = 1, \ldots, N$, assuming the constraints are feasible. If any constraints are infeasible, the protocol should instead direct the agents toward a formation in $\mathbb{R}^d$ with the least violation of constraints.

III. Main Results

In this section, we recast the problem of collaboratively fulfilling spatial long-term constraints in MAS as an unconstrained distributed optimization problem. We first introduce and differentiate two graph types that model agents' coupling constraints (tasks) and their communication capabilities. Next, we reformulate the problem into an unconstrained centralized optimization framework and propose a distributed strategy to approximate its solution.

A. Task Dependency and Communication Graphs

Inspired by [7], we introduce the task dependency graph for the MAS. As shown in (2), each agent's constraints (tasks) may depend on the positions of other agents in the system. To capture this, we define a *directed* graph $\mathcal{G}^\psi(\mathcal{V}, \mathcal{E}^\psi)$, referred to as the multi-agent *task dependency graph*, where $\mathcal{E}^\psi \subseteq \mathcal{V} \times \mathcal{V}$ represents the set of directed edges. An edge $(i, j) \in \mathcal{E}^\psi$ exists if any constraint function of agent i in (2) depends on agent j's position $\boldsymbol{x}_j$. A self-edge (i, i) exists if agent i has a constraint that depends solely on $\boldsymbol{x}_i$, i.e., $\psi_{i,k}(\boldsymbol{x}_i) > 0$. We define $\mathcal{N}_i^\psi = \{j \in \mathcal{V} \mid (i, j) \in \mathcal{E}^\psi\}$ as the set of agent i's (out) neighbors in $\mathcal{G}^\psi$. The presence of an edge (i, j) indicates that agent i must satisfy at least one constraint relative to its (out) neighbor agent j, but not vice versa. A set $\mathcal{V}^M \subseteq \mathcal{V}$ is called a *maximal dependency cluster* of $\mathcal{G}^\psi$ if for all $i, j \in \mathcal{V}^M$, i and j are connected[1], and for all $q \in \mathcal{V} \setminus \mathcal{V}^M$, $i \in \mathcal{V}^M$, i and q are not connected. Thus, there are no task dependencies between different maximal dependency clusters.

We define $\mathcal{G}^c(\mathcal{V}, \mathcal{E}^c)$ as the *communication graph* of the agents, where $\mathcal{E}^c \subseteq \mathcal{V} \times \mathcal{V}$ is the set of undirected edges, consisting of unordered pairs $(i, j) \in \mathcal{E}^c$ for all $i, j \in \mathcal{V}$, indicating communication links between agents i and j. Additionally, $\mathcal{N}_i^c = \{j \in \mathcal{V} \mid (i, j) \in \mathcal{E}^c\}$ denotes the set of agent i's neighbors in the undirected communication graph $\mathcal{G}^c$. A communication link between agents i and j signifies they can exchange local information. For each maximal dependency cluster in $\mathcal{G}^\psi(\mathcal{V}, \mathcal{E}^\psi)$, we associate a communication subgraph $\mathcal{G}_i^c(\mathcal{V}_i^M, \mathcal{E}_i^c)$, for $i = 1, \ldots, m_{dc}$, where $m_{dc} \le N$ represents the number of clusters, and $\mathcal{V}_i^M \subseteq \mathcal{V}$ and $\mathcal{E}_i^c \subseteq \mathcal{E}^c$.

Assumption 1: The communication subgraphs $\mathcal{G}_i^c(\mathcal{V}_i^M, \mathcal{E}_i^c)$, $i = 1, \ldots, m_{dc}$, corresponding to maximal dependency clusters are undirected and connected.

Remark 1: Assumption 1 does not require neighboring agents in the task dependency graph $\mathcal{G}^\psi$ to be neighbors in the communication graph $\mathcal{G}^c$, or vice versa.

Since the spatial constraints (tasks) within each maximal dependency cluster are independent of others, coordination of agents in each cluster can be handled separately. Hence, without loss of generality, we assume the following.

Assumption 2: The task dependency graph $\mathcal{G}^\psi(\mathcal{V}, \mathcal{E}^\psi)$ is composed of one maximal dependency cluster.

B. Reformulating the Problem as a Centralized Optimization

Inspired by the method in [19], we consolidate each agent's constraints into a single constraint as follows:

$$\bar{\alpha}_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i}) > 0, \quad i = 1, \ldots, N, \quad (3)$$

where $\bar{\alpha}_i : \mathbb{R}^d \times \mathbb{R}^{n_i d} \to \mathbb{R}$ is given by:

$$\bar{\alpha}_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i}) \coloneqq \min\{\psi_{i,1}(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i}), \ldots, \psi_{i,m_i}(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})\}. \quad (4)$$

Clearly, if $\bar{\alpha}_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i}) \le 0$, then at least one of agent i's constraints is violated. We call $\bar{\alpha}_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})$ the *consolidated constraint (task) function* of agent i.

We can now merge the consolidated constraints of all N agents into a single global constraint, representing satisfaction of the spatial constraints for the entire MAS as follows:

$$\bar{\beta}(\boldsymbol{x}) \coloneqq \min\{\bar{\alpha}_1(\boldsymbol{x}_1, \boldsymbol{x}_{\mathcal{I}_1}), \ldots, \bar{\alpha}_N(\boldsymbol{x}_N, \boldsymbol{x}_{\mathcal{I}_N})\} > 0. \quad (5)$$

Note that when $\bar{\beta}(\boldsymbol{x}) \le 0$, at least one agent fails to meet one or more of its constraints. We refer to $\bar{\beta}(\boldsymbol{x})$ as the *global constraint (task) function of the MAS*. Both functions in (4) and (5) are continuous, yet typically nonsmooth.

If $\bar{\beta}(\cdot)$ has compact level sets (see Lemma 1), then since it is continuous, [20, Proposition 2.10] guarantees that $\bar{\beta}(\cdot)$ possesses at least one global maximizer. Thus, we can define:

$$\bar{\beta}^* \coloneqq \max_{\boldsymbol{x}} \bar{\beta}(\boldsymbol{x}), \quad (6)$$

and

$$\boldsymbol{x}^* = \arg\max \bar{\beta}(\boldsymbol{x}), \quad (7)$$

where $\boldsymbol{x}^*$ represents the optimal configuration of agents, maximizing the MAS's global constraint function. Here, $\bar{\beta}(\boldsymbol{x})$ serves as the objective function in (7), with its optimal value indicating how well the spatial constraints are satisfied. If $\bar{\beta}^* = \bar{\beta}(\boldsymbol{x}^*) > 0$, all agents' constraints are *collectively feasible*, and $\boldsymbol{x}^*$ satisfies all constraints. If $\bar{\beta}^* = \bar{\beta}(\boldsymbol{x}^*) \le 0$, at least one agent fails to meet one or more of its constraints,

[1]Here, i and j are considered connected if there is an undirected path along the directed edges in $\mathcal{E}^\psi$ connecting nodes i and j.

making $\boldsymbol{x}^*$ the *least-violating multi-agent formation* that maximizes $\bar{\beta}(\cdot)$. Thus, solving (7) promotes cooperative behavior among agents to effectively satisfy both individual and coupled long-term constraints in the MAS. Note that solving the optimization problem (7) requires information from all agents, leading to a centralized solution.

Assumption 3: At least one of the MAS constraint functions $\psi_{i,k}(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i}), k = 1, \dots, m_i, i = 1, \dots, N$, approaches $-\infty$ along any trajectory in $\mathbb{R}^{Nd}$ on which $\|\boldsymbol{x}\| \to +\infty$

Remark 2: Assumption 3 is not restrictive in practice and typically holds for well-posed multi-agent spatial constraints. It can always be satisfied by introducing an auxiliary individual constraint for each agent, $\psi_{i,\text{aux}}(\boldsymbol{x}_i) := c_{\text{aux}} - \|\boldsymbol{x}_i\|^2 > 0, i = 1, \dots, N$, where $c_{\text{aux}} > 0$ is a sufficiently large constant. This constraint defines a large ball around the origin in the MAS operating space, encompassing all other spatial constraints of agent i without interfering with them. Moreover, One can show that adding such auxiliary constraints can ensure strict convexity of the proposed global MAS objective function in this article (see Lemma3).

As the following lemma shows, Assumption 3 is necessary and sufficient for the level sets of $\bar{\beta}(\boldsymbol{x})$ to be compact.

Lemma 1: Function $-\bar{\beta}(\boldsymbol{x})$ (and thus $\bar{\beta}(\boldsymbol{x})$) has compact level sets if and only if Assumption 3 holds.

Proof: Refer to [18]. ■

In the next subsection, we discuss how optimization problem (7) can be approximately solved in a distributed way, allowing agents to determine their optimal positions $\boldsymbol{x}_i^*, i = 1, \dots, N$, using only local information.

C. Multi-Agent Coordination via Distributed Optimization

We propose a DO scheme to approximate the centralized nonsmooth problem in (7). The key is leveraging the Log-Sum-Exp (LSE) function, which delivers a smooth under-approximation of the $\min$ operators in (4) and (5).

For each agent, we define:

$$\alpha_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i}) := -\frac{1}{\nu_\alpha} \ln\Big(\sum_{k=1}^{m_i} e^{-\nu_\alpha\, \psi_{i,k}(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})}\Big), \tag{8}$$

for which $\alpha_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i}) \le \bar{\alpha}_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i}) \le \alpha_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i}) + \frac{1}{\nu_\alpha}\ln(m_i)$ holds. Note that, $\alpha_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})$ provides a smooth under-approximation of agent i's consolidated constraint function $\bar{\alpha}_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})$ in (4), and $\nu_\alpha > 0$ is a tuning parameter. As ν_α increases, the approximation improves, i.e., $\alpha_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i}) \to \bar{\alpha}_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})$ as $\nu_\alpha \to \infty$. Moreover, $\alpha_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i}) > 0$ guarantees that $\bar{\alpha}_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i}) > 0$. Thus, if $\alpha_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i}) > 0$, then $\boldsymbol{x}_i$ and $\boldsymbol{x}_{\mathcal{I}_i}$ satisfy agent i's constraints. We now define:

$$\bar{\beta}_{\text{ua}}(\boldsymbol{x}) := \min\{\alpha_1(\boldsymbol{x}_1, \boldsymbol{x}_{\mathcal{I}_1}), \dots, \alpha_N(\boldsymbol{x}_N, \boldsymbol{x}_{\mathcal{I}_N})\} \tag{9}$$

for which $\bar{\beta}_{\text{ua}}(\boldsymbol{x}) \le \bar{\beta}(\boldsymbol{x}) \le \min\{\alpha_1(\boldsymbol{x}_1, \boldsymbol{x}_{\mathcal{I}_1}) + \frac{1}{\nu_\alpha}\ln(m_1), \dots, \alpha_N(\boldsymbol{x}_N, \boldsymbol{x}_{\mathcal{I}_N}) + \frac{1}{\nu_\alpha}\ln(m_N)\} \le \bar{\beta}_{\text{ua}}(\boldsymbol{x}) + \frac{1}{\nu_\alpha}\ln(\bar{m})$ holds, where $\bar{m} := \max\{m_1, \dots, m_N\}$. To introduce a smooth under-approximation of (5), we replace the $\min$ operator in (9) with the LSE function, yielding:

$$\beta(\boldsymbol{x}) := -\frac{1}{\nu_\beta} \ln\Big(\sum_{i=1}^{N} e^{-\nu_\beta \alpha_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})}\Big), \tag{10}$$

for which $\beta(\boldsymbol{x}) \le \bar{\beta}_{\text{ua}}(\boldsymbol{x}) \le \beta(\boldsymbol{x}) + \frac{1}{\nu_\beta}\ln(N)$ holds, where $\nu_\beta > 0$ is a tuning parameter such that $\beta(\boldsymbol{x}) \to \bar{\beta}_{\text{ua}}(\boldsymbol{x})$ as $\nu_\beta \to \infty$. From the above inequalities, we have:

$$\beta(\boldsymbol{x}) \le \bar{\beta}(\boldsymbol{x}) \le \beta(\boldsymbol{x}) + \frac{1}{\nu_\beta}\ln(N) + \frac{1}{\nu_\alpha}\ln(\bar{m}). \tag{11}$$

Thus, $\beta(\boldsymbol{x})$ in (10) serves as a smooth under-approximation of the MAS's global constraint function $\bar{\beta}(\boldsymbol{x})$ in (5), and its accuracy can be improved by tuning $\nu_\beta > 0$ and $\nu_\alpha > 0$.

From (11) and Lemma 1 one can conclude that $-\beta(\boldsymbol{x})$ is radially unbounded and thus $\beta(\boldsymbol{x})$ has compact level curves. As a result, in lieu of [20, Proposition 2.10], we define:

$$\beta^* := \max_{\boldsymbol{x}} \beta(\boldsymbol{x}), \tag{12}$$

where $\beta^* \le \bar{\beta}^* \le \beta^* + \frac{1}{\nu_\beta}\ln(N) + \frac{1}{\nu_\alpha}\ln(\bar{m})$. It is evident that $\beta^* > 0$ ensures the feasibility of MAS constraints. Conversely, $\beta^* \le -\frac{1}{\nu_\beta}\ln(N) - \frac{1}{\nu_\alpha}\ln(\bar{m})$ is sufficient to indicate the infeasibility of MAS constraints.

From (12), we can verify that the centralized optimization problem in (6) can be approximately solved by maximizing $\beta(\boldsymbol{x})$. Consequently, the optimal agent positions (formation) for collaboratively satisfying the spatial multi-agent constraints can be approximated by solving:

$$\hat{\boldsymbol{x}}^* := \arg\max \beta(\boldsymbol{x}), \tag{13}$$

where $\hat{\boldsymbol{x}}^*$ is the approximated optimal multi-agent formation such that $\|\boldsymbol{x}^* - \hat{\boldsymbol{x}}^*\| \le \epsilon$. From (12), it is guaranteed that $\epsilon \to 0$ as $\nu_\alpha, \nu_\beta \to +\infty$, since $\beta^* \to \bar{\beta}^*$. Although smaller values of ν_α and ν_β increase the gap between β^* and $\bar{\beta}^*$, this does not necessarily imply that $\epsilon \ge 0$ will grow. In fact, ϵ may remain small even for low values of ν_α and ν_β.

Apart from ensuring smoothness in (12) and (13), $\beta(\boldsymbol{x})$ enables a distributed solution to (13). Since $\ln(\cdot)$ is strictly increasing and $\sum_{i=1}^{N} e^{-\nu_\beta \alpha_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})}$ is positive, we have $\hat{\boldsymbol{x}}^* = \arg\max -\frac{1}{\nu_\beta}\ln\big(\sum_{i=1}^{N} e^{-\nu_\beta \alpha_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})}\big)$, and thus:

$$\hat{\boldsymbol{x}}^* = \arg\min \sum_{i=1}^{N} e^{-\nu_\beta \alpha_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})}. \tag{14}$$

Now define

$$h_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i}) := \sum_{k=1}^{m_i} e^{-\nu_\alpha \psi_{i,k}(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})}, \quad i = 1, \dots, N. \tag{15}$$

Substituting (8) into (14) and using (15) yields $\hat{\boldsymbol{x}}^* = \arg\min \sum_{i=1}^{N} h_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})^{\frac{\nu_\beta}{\nu_\alpha}}$. Consequently, defining

$$f_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i}) := h_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})^{\frac{\nu_\beta}{\nu_\alpha}}, \quad i = 1, \dots, N, \tag{16}$$

leads to

$$f(\boldsymbol{x}) := \sum_{i=1}^{N} f_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i}), \tag{17}$$

representing the global objective function of the MAS, which is composed of the sum of agents' local objective functions $f_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})$. As a result, $\hat{\boldsymbol{x}}^*$ is the minimizer of both (13) and the following optimization problem:

$$\min_{\boldsymbol{x}} f(\boldsymbol{x}) = \sum_{i=1}^{N} f_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i}), \tag{18}$$

which aligns with DO setups [9], [21].

Note that each agent's objective function $f_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})$ depends solely on its own constraint functions $\psi_{i,k}(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})$, for $k = 1, \dots, m_i$. As a result, based on the constraints in (2), a private local objective function, defined in (16), can be formulated for each agent. By solving the smooth unconstrained DO problem (18), agents can collaboratively reach a formation that meets both individual and coupled constraints in MAS as much as possible. In particular, the feasibility of the MAS constraints under the multi-agent formation $\hat{\boldsymbol{x}}^*$ from the above optimization problem is determined by checking whether $\bar{\beta}(\hat{\boldsymbol{x}}^*) > 0$ (feasible) or $\bar{\beta}(\hat{\boldsymbol{x}}^*) \leq 0$ (infeasible).

Remark 3: Each agent's consolidated constraint function $\bar{\alpha}_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})$ in (4), or its smooth under-approximation $\alpha_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})$ in (8), can be interpreted as a local benefit function. By maximizing it, each agent seeks to satisfy its individual and coupled constraints as much as possible in an egoistic manner. Since each agent's benefit function depends on the positions of other agents, this egoistic optimization can be analyzed through noncooperative game theory [22], potentially leading to Nash equilibria. However, this approach does not always ensure the satisfaction of feasible multi-agent spatial constraints. The DO problem in (18) resolves this by collaboratively maximizing the smallest $\bar{\alpha}_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})$ across the MAS, as shown in (5) and (6).

D. Properties of the Multi-Agent Objective Function (17)

To identify suitable algorithms for solving the DO problem (18), it is crucial to examine the properties of the global multi-agent objective function $f(\boldsymbol{x})$. The following lemma provides a sufficient condition for the log-convexity of $f(\boldsymbol{x})$ in (17), which implies its convexity.

Lemma 2: If all agents' constraint functions in (2), i.e., $\psi_{i,k}(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})$, $k = 1, \dots, m_i, i = 1, \dots, N$ are concave, then the global objective function $f(\boldsymbol{x})$ in (17) is log-convex.

Proof: See [18]. ∎

The convexity of $f(\boldsymbol{x})$ in (17) is crucial for many DO algorithms, but identifying conditions for strict convexity, which guarantees a unique minimizer, is equally important. This is addressed in the following lemma.

Lemma 3: Let all $\psi_{i,k}(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})$ in (2) be concave functions. If at least one of the constraint functions $\psi_{i,k}(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})$, $k = 1, \dots, m_i$,, for each agent is strictly concave, then the global objective function $f(\boldsymbol{x})$ in (17) is strictly log-convex.

Proof: The proof is available in [18]. ∎

The closed-form gradient of agent i's local objective function $f_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})$ defined in (16) with respect to $\boldsymbol{x}$ is derived as follows (function arguments omitted for simplicity):

$$\nabla f_i = \frac{\partial f_i}{\partial \boldsymbol{x}}^\top = -\nu_\beta \, h_i^{(\frac{\nu_\beta}{\nu_\alpha}-1)} \sum_{k=1}^{m_i} \frac{\partial \psi_{i,k}}{\partial \boldsymbol{x}}^\top e^{-\nu_\alpha \psi_{i,k}}, \tag{19}$$

for all $i = 1, \dots, N$. Finally, it is not difficult to conclude that $f(\boldsymbol{x})$ and $\nabla f(\boldsymbol{x}) = \sum_{i=1}^{N} \nabla f_i$ are locally Lipschitz.

E. Distributed Optimization Algorithm and Control Design

First, note that, with a slight abuse of notation, we write $f_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})$ in (18) as $f_i(\boldsymbol{x})$. Let $\boldsymbol{x}_j^{(i)}$ denote agent i's local estimate of $\boldsymbol{x}_j$ and assume, without loss of generality, that $\boldsymbol{x}_i^{(i)}(0) = \boldsymbol{x}_i(0)$ for all $i = 1, \dots, N$. Moreover, let $\boldsymbol{x}^{(i)} := \mathrm{col}(\boldsymbol{x}_j^{(i)}) \in \mathbb{R}^{Nd}$ represent agent i's local estimate of $\boldsymbol{x}$.

To solve the unconstrained optimization problem (18) in a distributed manner, one can reformulate it as an equivalent consensus-based optimization problem, as shown in [21, Lemma 3.1], as follows:

$$\text{minimize } \tilde{f}(\tilde{\boldsymbol{x}}) = \sum_{i=1}^{N} f_i(\boldsymbol{x}^{(i)}), \text{ subject to } \boldsymbol{L}\tilde{\boldsymbol{x}} = \boldsymbol{0}_{N^2 d}, \tag{20}$$

where $\boldsymbol{0}_{N^2 d} \in \mathbb{R}^{N^2 d}$ is the vector of zeros, $\tilde{\boldsymbol{x}} = \mathrm{col}(\boldsymbol{x}^{(i)})_{i=1,\dots,N} \in \mathbb{R}^{N^2 d}$ represents the stacked vector of all agents' estimates, and $\boldsymbol{L} := \mathcal{L} \otimes \mathbf{I}_{Nd} \in \mathbb{R}^{N^2 d \times N^2 d}$, in which $\mathcal{L} \in \mathbb{R}^{N \times N}$ is the Laplacian matrix [23] corresponding to the undirected, connected inter-agent communication graph $\mathcal{G}^c$ and $\otimes$ denotes the Kronecker product. Notice that $\boldsymbol{L}\tilde{\boldsymbol{x}} = \boldsymbol{0}_{N^2 d}$ holds when $\boldsymbol{x}^{(i)} = \boldsymbol{x}^{(j)}$ for all $i, j \in \mathcal{V}$. In other words, the optimal solution of (20) must satisfy $\tilde{\boldsymbol{x}}^* = \boldsymbol{1}_N \otimes \hat{\boldsymbol{x}}^*$ for all $\hat{\boldsymbol{x}}^* \in \mathbb{R}^{Nd}$, where $\boldsymbol{1}_N \in \mathbb{R}^N$ is the vector of ones.

For a strictly convex function $\tilde{f}(\tilde{\boldsymbol{x}})$, the solution to the consensus-based DO problem (20) can be derived using the continuous-time distributed algorithm proposed in [24]:

$$\dot{\boldsymbol{x}}^{(i)} = -k_1 \nabla f_i(\boldsymbol{x}^{(i)}) - k_2 \sum_{j \in \mathcal{N}_i^c} (\boldsymbol{x}^{(i)} - \boldsymbol{x}^{(j)}) - \boldsymbol{z}_i, \tag{21a}$$

$$\dot{\boldsymbol{z}}_i = k_1 k_2 \sum_{j \in \mathcal{N}_i^c} (\boldsymbol{x}^{(i)} - \boldsymbol{x}^{(j)}). \tag{21b}$$

with $\sum_{i=1}^{N} \boldsymbol{z}_i(0) = \boldsymbol{0}_{Nd}$, and positive gains $k_1, k_2 > 0$.

To implement the above distributed protocol, each agent must compute the gradient of its local objective and share its solution estimate $\boldsymbol{x}^{(i)}$ with neighbors in the communication graph $\mathcal{G}^c(\mathcal{V}, \mathcal{E}^c)$. By (21) and the initial condition $\boldsymbol{x}_i^{(i)}(0) = \boldsymbol{x}_i(0)$, each agent's controller is given by:

$$\boldsymbol{u}_i = (\mathbf{e}_N^i \otimes \mathbf{I}_d)^\top \dot{\boldsymbol{x}}^{(i)}, \tag{22}$$

where $\mathbf{e}_n^i := [0 \dots 1 \dots 0]^\top \in \mathbb{R}^n$.

Theorem 1: Consider the multi-agent system in (1) with continuously differentiable long-term spatial constraints in (2). Under Assumptions 1-3 and Lemma 3, for any $\boldsymbol{x}^{(i)}(0)$ with $\boldsymbol{x}_i^{(i)}(0) = \boldsymbol{x}_i(0)$ and $\boldsymbol{z}_i(0) = \boldsymbol{0}_{N^2 d}$, for all $i = 1, \dots, N$, the distributed control protocol (21) and (22) ensures that all agents asymptotically converge to their optimal positions, i.e., $\boldsymbol{x}(t) \to \hat{\boldsymbol{x}}^*$ as $t \to \infty$.

Proof: The proof follows from [24, Theorem 8]. ∎

Remark 4: Note that although Theorem 1 guarantees only the asymptotic convergence of agents' positions $\boldsymbol{x}(t)$ to their optimal positions $\hat{\boldsymbol{x}}^*$, the multi-agent constraints may be satisfied at a finite, though unknown, time once $\bar{\beta}(\boldsymbol{x}(t)) > 0$.

F. Avoiding Potential Numerical Issues in Implementing (21)

From (15) and (16), it is evident that each agent's local objective function is a sum of exponential terms involving $\psi_{i,k}$ functions. Specifically, in any open subset of $\mathbb{R}^{nN_i}$ where agent i's constraints are violated (i.e., $\psi_{i,k}(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i}) < 0$), the local objective function $f_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})$ grows exponentially towards infinity. This behavior also applies to ∇f_i, as seen in (19). The increase in both $f_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})$ and ∇f_i becomes more pronounced when $\frac{\nu_\beta}{\nu_\alpha} > 1$. Consequently, the global objective function $f(\boldsymbol{x})$ in (17) also grows exponentially in regions where the multi-agent constraints are violated, i.e., when $\bar{\beta}(\boldsymbol{x}) < 0$ in (5). Thus, while $f(\boldsymbol{x})$ and $f_i(\boldsymbol{x}_i, \boldsymbol{x}_{\mathcal{I}_i})$, $i = 1, \ldots, N$, are well-defined over the entire spaces $\mathbb{R}^{Nd}$ and $\mathbb{R}^{(n_i+1)d}$, with decrease of $\bar{\beta}(\boldsymbol{x})$, $f(\boldsymbol{x})$ behaves similar to a barrier function outside the bounded set $\Omega := \{\boldsymbol{x} \mid \bar{\beta}(\boldsymbol{x}) > 0\} \subset \mathbb{R}^{Nd}$. Therefore, if the initial values of $\boldsymbol{x}^{(i)}(0)$, $i = 1, \ldots, N$, in (20) are such that $\beta(\boldsymbol{x}^{(i)}(0)) \ll 0$, the proposed DO algorithm (21) may face numerical overflow at $t = 0$. To mitigate this, we propose treating the tunable parameter $\nu_\beta \in (0, +\infty)$ as a time-varying function. Specifically, based on the structure of (16), initializing $\nu_\beta(0)$ with a sufficiently small value (e.g., $\nu_\beta(0) = 0.01$) and gradually increasing it to some sufficiently large finite value $\nu_\beta^{\text{final}} > \nu_\beta(0)$ (e.g., $\nu_\beta^{\text{final}} > \ln(N)$) during the execution of (20) helps in preventing a potential numerical overflow at $t = 0$. Recall that even a small ν_β^{final} can still yield a good approximate solution $\hat{\boldsymbol{x}}^*$ to $\boldsymbol{x}^*$, as explained below (13).

IV. Simulation Results

Consider (1) with $N = 3$ and $d = 2$ under the DO-based control scheme (21) and (22). In all subsequent simulations, we initialize $\boldsymbol{z}_i(0) = \boldsymbol{0}_{Nd}$ for all $i = 1, \ldots, N$, with the initial positions $\boldsymbol{x}_i(0)$ and the values of $\boldsymbol{x}^{(i)}(0)$ randomly chosen. The random initialization of $\boldsymbol{x}^{(i)}(0)$ is constrained by $\boldsymbol{x}_i^{(i)}(0) = \boldsymbol{x}_i(0)$. Moreover, we set $k_1 = k_2 = 1$ in (21). As discussed in Subsection III-F, ν_β is treated as a slowly increasing parameter for implementing (21), where $\nu_\beta(t) = 0.022t + 0.01$ for $0 \le t \le T$ and $\nu_\beta(t) = \nu_\beta^{\text{nom}}$ for all $t > T$ with $\nu_\beta^{\text{nom}} = 5$ and $T := (\nu_\beta^{\text{nom}} - 0.01)/0.022$. Additionally, in (15) and (16), we set $\nu_\alpha = 5$. The agents' communication graph is a line graph with the undirected edge set $\mathcal{E}^c = \{(1,2), (2,3)\}$, for which $\mathcal{L} = [1, -1, 0; -1, 2, -1; 0, -1, 1]$.

Case A: *Feasible Spatial Constraints*

In this example, the long-term constraints of the agents, as specified in (2), are considered as follows. For agent 1: $\psi_{1,1}(\boldsymbol{x}_1) = 1 - \|\boldsymbol{x}_1 - [2, 0]^\top\|^2 > 0$. For agent 2: $\psi_{2,1}(\boldsymbol{x}_2, \boldsymbol{x}_1) = 3^2 - \|\boldsymbol{x}_2 - \boldsymbol{x}_1\|^2 > 0$, and $\psi_{2,2}(\boldsymbol{x}_2, \boldsymbol{x}_3) = 2^2 - \|\boldsymbol{x}_2 - \boldsymbol{x}_3\|^2 > 0$. For agent 3: $\psi_{3,1}(\boldsymbol{x}_3) = 1 - \|\boldsymbol{x}_3 - [-2, 0]^\top\|^2 > 0$. Fig.1(top-left) illustrates the agents' final positions after 300 seconds. Fig.(5) (top-right) shows the evolution of $\bar{\beta}(\boldsymbol{x})$ in (5) and $\beta(\boldsymbol{x})$ in (10), evaluated at agents' positions. The convergence of $\bar{\beta}(\boldsymbol{x})$ to a positive constant ($\bar{\beta}(\hat{\boldsymbol{x}}^*) \approx 1$) confirms that the multi-agent constraints are collectively feasible, with all agents' constraints being satisfied from approximately $t = 13.48$ as $\bar{\beta}(\boldsymbol{x})$ remains positive for all $t \ge 13.48$. In this example agents reach the approximate optimal configuration $\hat{\boldsymbol{x}}^* = [(\hat{\boldsymbol{x}}_1^*)^\top, (\hat{\boldsymbol{x}}_2^*)^\top, (\hat{\boldsymbol{x}}_3^*)^\top]^\top \in \mathbb{R}^6$, where $\hat{\boldsymbol{x}}_1^* = [1.99, 0.00]$, $\hat{\boldsymbol{x}}_2^* = [-0.61, 0.03]$, and $\hat{\boldsymbol{x}}_3^* = [-1.99, 0.00]$ after 300 seconds. Fig.1 (bottom-left) depicts the evolution of the entries of the vector $\tilde{\boldsymbol{x}} \in \mathbb{R}^{18}$ in (20), partitioned as agents estimation vectors $\boldsymbol{x}^{(i)} \in \mathbb{R}^6, i = 1, 2, 3$. Fig. 1 (bottom-right) presents $\boldsymbol{z}_i$ evolution for each agent. Each vector entry is shown in a different color, with corresponding entries across agents using the same color but a different line style.

Case B: *Tightly Feasible Spatial Constraints*

In this example, the long-term constraints of agents 1 and 3 are kept the same as in *Case A* while agent 2's constraints are modified as follows: $\psi_{2,1}(\boldsymbol{x}_2, \boldsymbol{x}_1) = (1.7)^2 - \|\boldsymbol{x}_2 - \boldsymbol{x}_1\|^2 > 0$, and $\psi_{2,2}(\boldsymbol{x}_2, \boldsymbol{x}_3) = (0.7)^2 - \|\boldsymbol{x}_2 - \boldsymbol{x}_3\|^2 > 0$, enforcing a tighter feasible space for agent 2 compared with *Case A*.

Fig. 2 presents the simulation results for this case. Although the optimal value of $\beta(\boldsymbol{x})$ is negative ($\beta(\hat{\boldsymbol{x}}^*) \approx -0.1$), its maximizer $\hat{\boldsymbol{x}}^*$ yields a positive value for $\bar{\beta}(\boldsymbol{x})$ ($\bar{\beta}(\hat{\boldsymbol{x}}^*) \approx 0.1$), confirming that the multi-agent constraints are collectively feasible. Moreover, all agents' constraints are satisfied for $t \ge 62.21$. In this simulation, the agents reached the approximate optimal configuration: $\hat{\boldsymbol{x}}_1^* = [1.10, 0.00]$, $\hat{\boldsymbol{x}}_2^* = [-0.49, 0.00]$, and $\hat{\boldsymbol{x}}_3^* = [-1.10, 0.00]$ after 300 seconds. Notably, agents 1 and 3 approach the boundary of their constraints, deviating from their target positions to assist agent 2 in satisfying its coupled constraints. Recall that agents 1 and 3 do not have access to agent 2's local objective function nor its constraints and this collaborative behavior emerges from solving the optimization problem (18) in a distributed manner, solely by sharing local estimates $\boldsymbol{x}^{(i)}, i = 1, 2, 3$, via the communication network. Hence, this approach is useful in practice, as it allows agents to collaborate without knowing each other's spatial constraints and preserves privacy.

Case C: *Infeasible Spatial Constraints*

In this example, agents 1 and 3 constraints are the same as *Case A* and agent 2's constraints are further shrunk to $\psi_{2,1}(\boldsymbol{x}_2, \boldsymbol{x}_1) = (1.4)^2 - \|\boldsymbol{x}_2 - \boldsymbol{x}_1\|^2 > 0$, and $\psi_{2,2}(\boldsymbol{x}_2, \boldsymbol{x}_3) = (0.4)^2 - \|\boldsymbol{x}_2 - \boldsymbol{x}_1\|^2 > 0$, leading to an infeasible scenario. The simulation results are shown in Fig. 3. As depicted in Fig. 3 (right), the maximum value of $\beta(\boldsymbol{x})$ is negative ($\beta(\hat{\boldsymbol{x}}^*) \approx -0.35$), and its optimizer $\hat{\boldsymbol{x}}^*$ gives $\bar{\beta}(\hat{\boldsymbol{x}}^*) \approx -0.18$, indicating that the multi-agent constraints are infeasible. Despite this, the agents achieve a least-violating spatial configuration. The agents reached the approximate optimal configuration: $\hat{\boldsymbol{x}}_1^* = [0.98, 0.00]$, $\hat{\boldsymbol{x}}_2^* = [-0.98, 0.00]$, and $\hat{\boldsymbol{x}}_3^* = [-0.41, 0.00]$, with none of the agents fully satisfying their constraints. Although agents 1 and 3 slightly violated their constraints, their adjustments contributed to minimizing overall violations of the constraints at the group level.

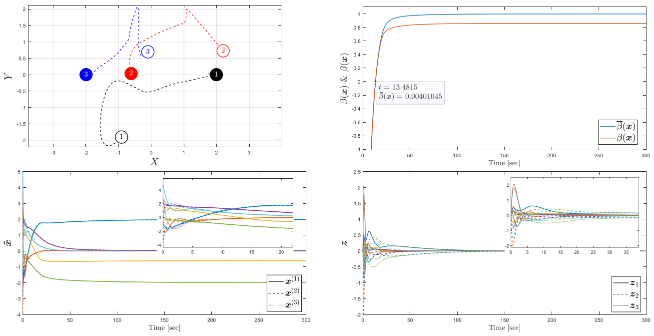

Fig. 1. Simulation results under multi-agent spatial constraints of Case A.

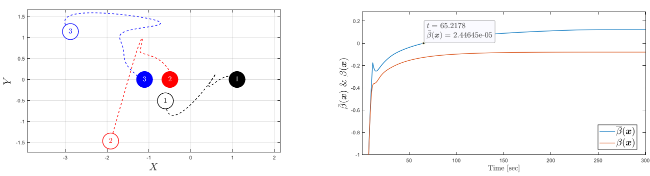

Fig. 2. Simulation results under multi-agent spatial constraints of Case B.

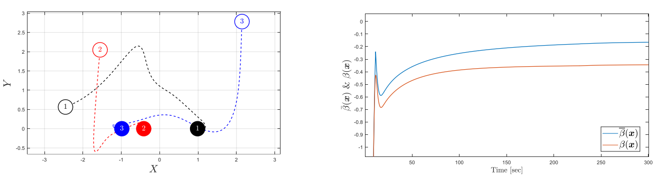

Fig. 3. Simulation results under multi-agent spatial constraints of Case C.

V. Conclusion

In this work, we addressed long-term spatial constraints in multi-agent systems, where agents not only satisfy their own constraints but also assist others by forming a desired configuration collaboratively. We first formulated the problem as a centralized optimization, introducing an objective function whose positive values indicate constraint satisfaction, with higher values signifying better fulfillment. To design distributed control protocols, we derived an alternative objective function expressed as the sum of local functions, each dependent only on the agent's own constraints, This enabled us to propose a distributed optimization scheme approximating the centralized solution. We also established conditions for the convexity and strict convexity of the global objective function. Finally, using a continuous-time distributed optimization algorithm, we developed a control protocol for single integrator agents. We expect that extending our method will help tackle emerging challenges in multi-robot systems with spatial constraints, including collaborative coordination under spatiotemporal specifications and formation control.

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