Barrier Function-based Model Predictive Control under Signal Temporal Logic Specifications*

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***Abstract*— In this work a continuous-time MPC scheme is presented for linear systems under Signal Temporal Logic (STL) specifications and input constraints. The satisfaction of the STL specifications is encoded by time-varying barrier functions and a least-violating control law is designed for cases when the satisfaction of the task with a given robustness value is not achieved, e.g., due to actuation limitations. The recursive feasibility of the proposed scheme is guaranteed when a time-varying terminal constraint is introduced. This constraint ensures a desired behavior for the system that guarantees the satisfaction of the task with pre-determined robustness.**

I. Introduction

Nowadays, an increasing interest has been shown in robotic-aided applications examples of which are coverage [1] and search and rescue missions [2]. These applications underline the need of autonomous systems capable of performing a variety of complex, time-constrained tasks in uncertain and dynamic environments. An important question arising in this context is how to efficiently encode such specifications for control. Signal Temporal Logic (STL) [3] has been proven an expressive language for describing complex tasks under strict deadlines. Contrary to Linear Temporal Logic (LTL), it offers robust semantics [4], [5] that allow the evaluation of the satisfaction of the task over a continuous-time signal. Hence, abstractions of the system dynamics, usually considered in LTL-planning [6]–[8] are avoided.

Existing methods for planning under STL specifications consider discrete-time systems and find plans as solutions to computationally demanding MILP problems [9]–[12]. Although suggestions towards reducing the computational load have been made [13], the complexity of the problem remains. This is partially related to the choice of the optimization horizon length which is often considered at least as large as the duration of the task [9], [11]. In addition to the computational complexity, [9]–[12] lack of feasibility guarantees. Authors in [9] ensure feasibility on a subset of the horizon by making use of past solutions while least violating solutions are proposed in [11] as a remedy when a feasible control law guaranteeing the satisfaction of the STL formula does not exist.

Another important limitation of the aforementioned methods is the lack of guarantees for the satisfaction of the task in continuous time. A solution to this problem is given in [12] with the design of a possibly non-convex problem. A different approach to STL planning is presented in [14] where the STL specifications are encoded using time-varying barrier functions and feedback control laws are designed for continuous-time, input-affine systems. Although computationally efficient, this method does not consider actuation limitations found for example, in every real mechanical system.

Aiming at overcoming the limitations of the aforementioned methods, we propose a continuous-time model predictive control scheme (MPC) for the satisfaction of a set of STL tasks by a linear system subject to state and input constraints. Although originally designed for linear systems, the proposed framework can also be applied to nonlinear, input-affine systems under some extra controllability assumptions. Motivated by [14], we encode the STL specifications using a time-varying barrier function and design a least-violating control law for the cases when the satisfaction of a task with a pre-determined robustness r_H is not possible due to input limitations. The recursive feasibility of the proposed scheme is guaranteed by a time-varying terminal constraint the satisfaction of which ensures the satisfaction of the task with a pre-determined robustness $r_F \leq r_H$. Contrary to existing approaches [9], [11] here the optimization horizon length can be chosen arbitrarily small and independent of the STL formula provided that the problem is initially feasible.

The remainder of the paper is as follows: Section II includes the preliminaries and problem formulation. Section III introduces the time-varying barrier functions and Section IV the proposed MPC scheme. Simulations are shown in Section V and conclusions are summarized in Section VI.

II. Preliminaries and Problem Formulation

True and false are denoted by $\top, \bot$ respectively. Scalars and vectors are denoted by non-bold and bold letters respectively. The partial derivative of a function $\mathfrak{b}(\mathbf{x}, t)$ evaluated at $(\mathbf{x}', t')$ with respect to t and $\mathbf{x}$ is abbreviated by $\frac{\partial \mathfrak{b}(\mathbf{x}',t')}{\partial t} = \frac{\partial \mathfrak{b}(\mathbf{x},t)}{\partial t}\Big|_{\substack{\mathbf{x}=\mathbf{x}' \\ t=t'}}$ and $\frac{\partial \mathfrak{b}(\mathbf{x}',t')}{\partial \mathbf{x}} = \frac{\partial \mathfrak{b}(\mathbf{x},t)}{\partial \mathbf{x}}\Big|_{\substack{\mathbf{x}=\mathbf{x}' \\ t=t'}}$ respectively. The latter is considered to be a row vector. An extended class $\mathcal{K}$ function $\alpha : \mathbb{R} \to \mathbb{R}_{\geq 0}$ is a locally Lipschitz continuous and strictly increasing function with $\alpha(0) = 0$. The function $\mathbf{u} : [t_1, t_2] \to \mathbb{R}^m$ has a property a.e. (almost everywhere) if the property holds everywhere in $[t_1, t_2]$ except from a set of points of measure zero. The weighted Euclidean norm

*This work was partially supported by the Wallenberg AI, Autonomous Systems and Software Program (WASP) funded by the Knut and Alice Wallenberg Foundation, the ERC CoG LEAFHOUND and the Swedish Research Council.

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of a vector $\boldsymbol{\zeta} \in \mathbb{R}^n$ is given by $\|\boldsymbol{\zeta}\|_Q = \sqrt{\boldsymbol{\zeta}^T Q \boldsymbol{\zeta}}$ where Q is a positive definite matrix of appropriate dimensions. The induced 2-norm of a rectangular matrix C is defined as: $\|C\| = \sigma_{\max}(C)$, where $\sigma_{\max}(C)$ is the maximum singular value of C. Given $a, b \in \mathbb{R}$, a divides b, denoted by $a|b$ if there exists an integer $k \neq 0$ such that $b = ka$.

Signal Temporal Logic (STL) determines whether a predicate μ is true or false by evaluating a continuously differentiable function $h : \mathbb{R}^n \to \mathbb{R}$ as follows:

$$\mu = \begin{cases} \top, & h(\mathbf{x}) \geq 0 \\ \bot, & h(\mathbf{x}) < 0 \end{cases}$$

for $\mathbf{x} \in \mathbb{R}^n$. The basic STL formulas are given by the grammar:

$$\phi := \top \mid \mu \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid \mathcal{G}_{[a,b]}\phi \mid \mathcal{F}_{[a,b]}\phi \mid \phi_1 \, \mathcal{U}_{[a,b]} \, \phi_2$$

where ϕ_1, ϕ_2 are STL formulas and $\mathcal{G}_{[a,b]}$, $\mathcal{F}_{[a,b]}$, $\mathcal{U}_{[a,b]}$ are the always, eventually and until operators defined over the interval $[a, b]$ with $0 \leq a \leq b < \infty$. Let $\mathbf{x} \models \phi$ denote the satisfaction of the formula ϕ by a signal $\mathbf{x} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$. The formula ϕ is satisfiable if $\exists \ \mathbf{x}' : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ such that $\mathbf{x}' \models \phi$. The STL semantics are defined in [3]. STL is equipped with robustness metrics determining how robustly an STL formula ϕ is satisfied at time t by a signal $\mathbf{x}$. These semantics are defined as follows [4], [5]: $\rho^{\mu}(\mathbf{x}, t) = h(\mathbf{x}(t))$, $\rho^{\neg\phi}(\mathbf{x}, t) = -\rho^{\phi}(\mathbf{x}, t)$, $\rho^{\phi_1 \wedge \phi_2}(\mathbf{x}, t) = \min(\rho^{\phi_1}(\mathbf{x}, t), \rho^{\phi_2}(\mathbf{x}, t))$, $\rho^{\phi_1 \, \mathcal{U}_{[a,b]} \, \phi_2}(\mathbf{x}, t) = \max_{t_1 \in [t+a, t+b]} \min(\rho^{\phi_2}(\mathbf{x}, t_1), \min_{t_2 \in [t, t_1]} \rho^{\phi_1}(\mathbf{x}, t_2))$, $\rho^{\mathcal{F}_{[a,b]}\phi}(\mathbf{x}, t) = \max_{t_1 \in [t+a, t+b]} \rho^{\phi}(\mathbf{x}, t_1)$, $\rho^{\mathcal{G}_{[a,b]}\phi}(\mathbf{x}, t) = \min_{t_1 \in [t+a, t+b]} \rho^{\phi}(\mathbf{x}, t_1)$. Finally, it should be noted that $\mathbf{x} \models \phi$ if $\rho^{\phi}(\mathbf{x}, 0) > 0$.

Consider the dynamical system:

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{u} \in \mathbb{R}^m$ are the state and input vectors of the system respectively, $A \in M_n(\mathbb{R})$, $B \in M_{n \times m}(\mathbb{R})$. Given a control signal $\mathbf{u} : [t_1, t_2] \to \mathbb{U}$ a solution $\mathbf{x} : [t_1, t_2] \to \mathbb{X}$ of (1) with $\mathbf{x}(t_1) = \mathbf{x}_1$ is an absolutely continuous function such that: $\mathbf{x}(t) = \mathbf{x}_1 + \int_{t_1}^{t} (A\mathbf{x}(\tau) + B\mathbf{u}(\tau)) d\tau$ holds a.e. in $[t_1, t_2]$.

Assumption 1. *The matrix B has full row rank ($n \leq m$).*

In this work the state and input of the system are constrained, i.e., $\mathbf{x} \in \mathbb{X}$, $\mathbf{u} \in \mathbb{U}$ where:

$$\mathbb{X} = \{\mathbf{x} \in \mathbb{R}^n : \ \|\mathbf{x}\| \leq d_x\} \subset \Omega \tag{2a}$$

$$\mathbb{U} = \{\mathbf{u} \in \mathbb{R}^m : \ \|\mathbf{u}\| \leq d_u\} \tag{2b}$$

with $d_x, \ d_u > 0$ and $\Omega \subseteq \mathbb{R}^n$ is an open, connected set.

Assumption 2. *Consider the dynamical system* (1) *subject to state and input constraints of the form $\mathbf{x} \in \mathbb{X}$, $\mathbf{u} \in \mathbb{U}$ where $\mathbb{X}, \mathbb{U}$ are defined as in* (2a)-(2b)*. Then, it holds: $d_u \sigma_{\min}(B) > \sigma_{\max}(A) d_x$, where $\sigma_{\min}(B), \sigma_{\max}(A)$ is the minimum and maximum singular value of B, A respectively.*

Assumption 2 ensures that the allowable control input is high enough to account for the worst case scenario, i.e., when the state of the system is about to leave the environment. This Assumption is necessary for guaranteeing the existence of a terminal controller as will be seen in Section IV.

A. Problem Formulation

In this work we consider an expressive fragment of STL defined as follows:

$$\psi := \top \mid \mu \mid \neg\mu \tag{3a}$$

$$\varphi := \mathcal{G}_{[a,b]}\psi \mid \mathcal{F}_{[a,b]}\psi \mid \psi_1 \, \mathcal{U}_{[a,b]} \, \psi_2 \tag{3b}$$

$$\phi'' := \bigwedge_{l=1}^{n_\phi} \varphi_l \tag{3c}$$

where $\psi_1, \ \psi_2$ are STL formulas of the form (3a), $\varphi_l, \ l = 1, \ldots, n_\phi$ are STL formulas of the form (3b), $n_\phi \geq 1$ and $0 \leq a \leq b < \infty$. Let $[a_l, b_l]$ be the time interval of the temporal operator of $\varphi_l, l \in \{1, \ldots, n_\phi\}$ in (3c). Consider the finite set of time instants $\{\tau_j\}_{j=0}^{J}$ with $\tau_0 = 0, \ \tau_j = j\Delta\tau, \ j = 1, \ldots, J$ where $\tau_J = \max_l b_l$ and $\Delta\tau > 0$ is a given constant. Observe that by definition of τ_J it holds that $\Delta\tau | \max_l b_l$ with $J = \frac{\max_l b_l}{\Delta\tau}$. Based on the above we may define the Problem considered in this paper as:

Problem 1. *Consider the dynamical system* (1) *subject to state and input constraints $\mathbf{x} \in \mathbb{X}$, $\mathbf{u} \in \mathbb{U}$ with $\mathbb{X}, \mathbb{U}$ defined by* (2a)-(2b)*. Given an STL formula ϕ'' as in* (3c)*, a positive prediction horizon length N and a sampling rate $\Delta\tau$ with $\Delta\tau | \max_l b_l$, design a control input $\mathbf{u}$ such that any solution $\mathbf{x} : [0, \tau_J] \to \mathbb{X}$ of* (1) *with initial condition $\mathbf{x}(0)$ guarantees $\rho^{\phi''}(\mathbf{x}, 0) \geq \bar{\rho}$ where $\bar{\rho}$ is maximized with respect to $\mathbf{u}$.*

III. Barrier Functions for Task Satisfaction

In this Section we introduce the control barrier functions (CBFs) the value of which determines the satisfaction or violation of a given STL formula. Here, we consider the formula $\mathcal{G}_{[a,b]} \ \psi_1 \wedge \mathcal{F}_{[b,b]} \ \psi_2$ the satisfaction of which implies the satisfaction of the until formula $\psi_1 \ \mathcal{U}_{[a,b]} \ \psi_2$ of (3b). Based on that, we design a CBF function evaluating the satisfaction of ϕ which is defined as follows:

$$\phi = \bigwedge_{i=1}^{p} \tilde{\varphi}_i \tag{4}$$

where $p = n_\phi + n_u$, n_ϕ is the total number of STL tasks in (3c) and n_u the number of until operators in (3c). The STL formula $\tilde{\varphi}_i$ has one of the following forms: $\mathcal{G}_{[a_i,b_i]}\psi, \ \mathcal{F}_{[a_i,b_i]}\psi, \ \mathcal{G}_{[a_i,b_i]}\psi_{1,z}, \ \mathcal{F}_{[b_i,b_i]}\psi_{2,z}$. The first two expressions account for the always and eventually formulas existing in (3c) while the last two are considered since the satisfaction of their conjunction implies the satisfaction of the z-th until formula of (3c). We denote the time interval associated with the temporal operator of $\tilde{\varphi}_i$ as $[\tilde{a}_i, \tilde{b}_i]$. For simplicity we will call $[\tilde{a}_i, \tilde{b}_i]$ as the time interval of satisfaction of $\tilde{\varphi}_i$.

Let $\mathfrak{b}_w(\mathbf{x}, t)$ with $\mathfrak{b}_w : \Omega \times [0, \infty) \to \mathbb{R}$, $w \in \{H, F\}$ be two differentiable functions on $\Omega \times (\sigma_w^k, \sigma_w^{k+1})$ where $\{\sigma_w^k\}_{k=0}^{\tilde{p}_w}$ is a finite set of points at which discontinuities with respect to t may occur. The set of points $\{\sigma_w^k\}$ is chosen such that $\sigma_w^0=0$, $\sigma_w^{\tilde{p}_w}=\infty$ and $\sigma_w^k \leq \sigma_w^{k+1},\ k=0\dots,\tilde{p}_w-2$ where $\tilde{p}_H \leq p+1+|\mathcal{I}_H^{\mathcal{G}}|,\ \tilde{p}_F \leq \tilde{p}_H+2,\ \mathcal{I}_H^{\mathcal{G}}=\{i\in\{1,\dots,p\}:\tilde{\varphi}_i=\mathcal{G}_{[\tilde{a}_i,\tilde{b}_i]}\psi_i\}$ is the set of indices of the always formulas in (4) and $|\mathcal{I}_H^{\mathcal{G}}|$ its cardinality. Based on $\mathfrak{b}_w,\ w\in\{H,F\}$ we may define the level set of $\mathfrak{b}_w(\mathbf{x},t)$ as:

$$\mathcal{C}_w(t)=\{\mathbf{x}\in\mathbb{X}|\ \mathfrak{b}_w(\mathbf{x},t)\geq 0\} \tag{5}$$

Let $N>0$ be the prediction horizon length.The function $\mathfrak{b}_H(\mathbf{x},t)$ is evaluated at each $t\in[\tau_j,\tau_j+N]$ for planning the agents' future actions towards the satisfaction of ϕ. The barrier function $\mathfrak{b}_F(\mathbf{x},t)$, called the *terminal barrier function* is designed for guaranteeing the recursive feasibility of the MPC scheme. This is achieved by introducing the constraint $\mathbf{x}(\tau_j+N)\in\mathcal{C}_F(\tau_j+N)$ in the MPC problem. In that way, we can ensure the satisfaction of $\phi'=\phi\wedge\tilde{\varphi}_{p+1}$ over time where:

$$\tilde{\varphi}_{p+1}=\mathcal{G}_{[0,\tilde{b}_{p+1}]}\psi_{p+1} \tag{6a}$$

$$\psi_{p+1}=\begin{cases}\top, & h_{p+1}(\mathbf{x})\geq 0\\ \bot, & h_{p+1}(\mathbf{x})<0\end{cases} \tag{6b}$$

$\tilde{b}_{p+1}=\tau_J+N$ and $h_{p+1}(\mathbf{x})=d_x^2-\|\mathbf{x}\|^2$. The task expressed by (6a)-(6b) is introduced for guaranteeing the satisfaction of the state constraints $\mathbf{x}\in\mathbb{X}$.

Let $\mathcal{I}_H=\{1,\dots,p\},\mathcal{I}_F=\mathcal{I}_H\cup\{p+1\}$ be the set of indices of the formulas associated with the design of $\mathfrak{b}_H(\mathbf{x},t),\mathfrak{b}_F(\mathbf{x},t)$ respectively. The barrier functions $\mathfrak{b}_w(\mathbf{x},t),w\in\{H,F\}$ are defined in two steps. In step A and for every $i\in\mathcal{I}_w,\ w\in\{H,F\}$ a desired, temporal behavior is introduced for the system guaranteeing the satisfaction of $\tilde{\varphi}_i$. Then, a barrier function $\mathfrak{b}_w^i(\mathbf{x},t)$ is defined as the error between the actual and desired temporal behavior of the system for every $\tilde{\varphi}_i$. At step B, the CBF $\mathfrak{b}_w(\mathbf{x},t),w\in\{H,F\}$ is defined as a function of the different $\mathfrak{b}_w^i(\mathbf{x},t),\ i\in\mathcal{I}_w$. By construction, a non-negative value of $\mathfrak{b}_w(\mathbf{x},t)$ at a time instant t implies $\mathfrak{b}_w^i(\mathbf{x},t)\geq 0$ for any $i\in\mathcal{I}_w$ contributing to $\mathfrak{b}_w(\mathbf{x},t)$. Hence, guaranteeing $\mathfrak{b}_w(\mathbf{x},t)\geq 0$ for every $t\geq 0$ ensures the satisfaction of $\phi,\ \phi'$ for $w=H,F$ respectively. Based on that, we may describe the two step procedure as follows:

a) Step A: The barrier function $\mathfrak{b}_w^i:\Omega\times[0,\infty)\to\mathbb{R}$ corresponding to the STL formula $\tilde{\varphi}_i,\ i\in\mathcal{I}_w,w\in\{H,F\}$ defined as:

$$\mathfrak{b}_w^i(\mathbf{x},t)=-\gamma_w^i(t)+h_i(\mathbf{x}) \tag{7}$$

where $h_i:\Omega\to\mathbb{R}$ is the predicate function of the predicate ψ_i associated with the temporal formula $\tilde{\varphi}_i$ and $\gamma_w^i(t),w\in\{H,F\}$ a designer-defined function describing a desired temporal behavior for the system. This behavior ensures the satisfaction of $\tilde{\varphi}_i$ with a maximum robustness r_w at a desired time instant, denoted by t_w^{*i}. The functions $h_i(\mathbf{x})$ are assumed to be continuously differentiable in Ω. Since $\mathbb{X}\subset\Omega$ is compact the restriction of h_i on $\mathbb{X}$ admits a maximum value. Let $h_i^{\max}=\max_{\mathbf{x}\in\mathbb{X}}h_i(\mathbf{x})$. As in [15], we consider $\gamma_w^i(t)$ functions of the form:

$$\gamma_w^i(t)=\begin{cases}\frac{\gamma_{w,\infty}^i-\gamma_{w,0}^i}{t_w^{*i}}t+\gamma_{w,0}^i, & \text{if } t<t_w^{*i}\\ \gamma_{w,\infty}^i, & \text{if } t\geq t_w^{*i}\end{cases} \tag{8}$$

where $\gamma_{w,0}^i,\gamma_{w,\infty}^i$ are parameters depending on the robustness value r_w and the time instant t_w^{*i} and satisfy:

$$\gamma_{w,0}^i\in(-\infty,h_i(\mathbf{x}(0))) \tag{9a}$$

$$\gamma_{w,\infty}^i\in(\max(r_w,\gamma_{w,0}^i),h_i^{\max}) \tag{9b}$$

$$t_w^{*i}=\begin{cases}\tilde{b}_i, & \text{if } \tilde{\varphi}_i=\mathcal{F}_{[\tilde{a}_i,\tilde{b}_i]}\ \psi_i\\ \tilde{a}_i, & \text{if } \tilde{\varphi}_i=\mathcal{G}_{[\tilde{a}_i,\tilde{b}_i]}\ \psi_i\end{cases} \tag{9c}$$

$$r_w\in\begin{cases}(0,h_i(\mathbf{x}(0))), & \text{if } t_w^{*i}=0\\ (0,h_i^{\max}), & \text{if } t_w^{*i}>0\end{cases} \tag{9d}$$

From (9a) $\gamma_{w,0}^i$ is chosen such that $\mathfrak{b}_w^i(\mathbf{x}(0),0)>0$ for every $w\in\{H,F\}$. In addition, due to (9b), for every time instant $t\geq t_w^{*i}$ we have that $\mathfrak{b}_w^i(\mathbf{x},t)\leq -r_w+h_i(\mathbf{x})$. Thus, $\mathfrak{b}_w^i(\mathbf{x},t)\geq 0$ implies $h_i(\mathbf{x})\geq r_w$ for all $t\geq t_w^{*i}$. Note that for every $i\in\mathcal{I}_w$ the function $\gamma_w^i(t)$ is piecewise differentiable.

b) Step B: The barrier functions $\mathfrak{b}_w^i(\mathbf{x},t),\ i\in\mathcal{I}_w,\ w\in\{H,F\}$ introduced above ensure the satisfaction of $\tilde{\varphi}_i$ when their value is non-negative for every $t\geq 0$. However, satisfying each $\tilde{\varphi}_i$ separately does not imply the satisfaction of ϕ or ϕ'. This motivates the design of a single barrier function $\mathfrak{b}_w(\mathbf{x},t),w\in\{H,F\}$ the value of which may determine whether ϕ,ϕ' is satisfied when $w=H,F$ respectively. This function is formally introduced for any $t\geq 0$ as follows:

$$\mathfrak{b}_w(\mathbf{x},t)=-\ln\left(\sum_{i\in\mathcal{I}_w}\exp\left(-\mathfrak{b}_w^i(\mathbf{x},t)\right)\right) \tag{10}$$

For this particular choice of $\mathfrak{b}_w(\mathbf{x},t)$ it is known [15, Eq. 2] that $\mathfrak{b}_w(\mathbf{x},t)\leq\min_{i\in\mathcal{I}_w}\mathfrak{b}_w^i(\mathbf{x},t)$ for any $w\in\{H,F\}$. Therefore, $\mathfrak{b}_w(\mathbf{x},t)\geq 0\Rightarrow\mathfrak{b}_w^i(\mathbf{x},t)\geq 0$ is always true for all $i\in\mathcal{I}_w,w\in\{H,F\}$ and $t\geq 0$. In this paper the barrier functions $\mathfrak{b}_w(\mathbf{x},t)$ are designed such that $r_H\geq r_F$ is true. To reduce conservatism when a large number of tasks is considered, authors in [14] propose removing $\mathfrak{b}_w^i(\mathbf{x},t)$ from $\mathfrak{b}_w(\mathbf{x},t),\ w\in\{H,F\}$ when the corresponding task is satisfied, i.e., at $t=\tilde{b}_i$. Additionally, based on [15] for any $i\in\mathcal{I}_w^{\mathcal{G}}$ the function $\mathfrak{b}_w^i(\mathbf{x},t)$ is also deactivated at $t=\tilde{a}_i$, where $\mathcal{I}_F^{\mathcal{G}}=\mathcal{I}_H^{\mathcal{G}}\cup\{p+1\}$. Let T_i denote the time interval at which $\mathfrak{b}_w^i(\mathbf{x},t)$ contributes to $\mathfrak{b}_w(\mathbf{x},t),\ w\in\{H,F\}$. Based on the above, $T_i=[0,\tilde{b}_i)$ if $\tilde{\varphi}_i=\mathcal{F}_{[\tilde{a}_i,\tilde{b}_i]}\ \psi_i$, $T_i=(0,\tilde{b}_i)$ if $\tilde{\varphi}_i=\mathcal{G}_{[0,\tilde{b}_i]}\ \psi_i$ or $T_i=[0,\tilde{a}_i)\cup(\tilde{a}_i,\tilde{b}_i)$ if $\tilde{\varphi}_i=\mathcal{G}_{[\tilde{a}_i,\tilde{b}_i]}\ \psi_i$. Furthermore, $i_w'=\arg\max_{i\in\mathcal{I}_w}\tilde{b}_i$ for any $w\in\{H,F\}$. The deactivation may be considered by making use of the integer functions $o_w^i(t)\in\{0,1\}$ defined as:

$$o_w^i(t)=\begin{cases}1, & t\in T_i\\ 0, & t\notin T_i\end{cases} \tag{11}$$

for any $t\geq 0$ and $i\neq i_w'$. When $i=i_w'$ we set $o_w^{i_w'}(t)=1$. The latter rule is introduced to avoid cases when the barrier function $\mathfrak{b}_w(\mathbf{x},t)$ becomes undefined, i.e., when $o_w^i(t)=$

$0, \forall i \in \mathcal{I}_w$. Based on the above, the modified barrier function $\mathfrak{b}_w(\mathbf{x}, t), t \geq 0,\ w \in \{H, F\}$ can be written as:

$$\mathfrak{b}_w(\mathbf{x}, t) = -\ln\left(\sum_{i \in \mathcal{I}_w} o_w^i(t) \exp\left(-\mathfrak{b}_w^i(\mathbf{x}, t)\right)\right) \tag{12}$$

IV. Control Approach

In this section we present a novel MPC scheme for the satisfaction of the STL formula ϕ under actuation and state constraints. Here, the optimization horizon can be chosen arbitrarily small as long as initial feasibility, a common assumption in MPC literature is assumed. The recursive feasibility of the proposed scheme is guaranteed by the satisfaction of a time-varying terminal constraint $\mathbf{x}(\tau_j + N) \in \mathcal{C}_F(\tau_j + N)$ where $\mathcal{C}_F(\tau_j + N)$ is defined by (5) with respect to the terminal barrier function $\mathfrak{b}_F(\mathbf{x}, t)$. Motivated by the work in [14], a terminal controller is also designed guaranteeing that $\mathbf{x}(t) \in \mathcal{C}_F(t)$ for any $t > \tau_j + N$ if $\mathbf{x}(\tau_j + N) \in \mathcal{C}_F(\tau_j + N)$ holds.

Definition 1. *The function $\mathfrak{b}_F(\mathbf{x}, t)$ is a valid control barrier function (vCBF) within each time interval $(\sigma_F^k, \sigma_F^{k+1})$, $k = 0, \ldots, \tilde{p}_F - 1$, if there exists an extended class $\mathcal{K}$ function α_F such that for all $(\mathbf{x}, t) \in \mathcal{C}_F(t) \times (\sigma_F^k, \sigma_F^{k+1})$, $k = 0, \ldots, \tilde{p}_F - 1$ it holds:*

$$\sup_{\mathbf{u} \in \mathbb{U}} \Big\{ \frac{\partial \mathfrak{b}_F(\mathbf{x}, t)}{\partial \mathbf{x}}(A\mathbf{x} + B\mathbf{u}) + \frac{\partial \mathfrak{b}_F(\mathbf{x}, t)}{\partial t} + \alpha_F(\mathfrak{b}_F(\mathbf{x}, t)) \Big\} \geq 0 \tag{13}$$

Assumption 3. *Consider the differentiable function $\mathfrak{b}_F(\mathbf{x}, t)$ on $\mathbb{X} \times (\sigma_F^k, \sigma_F^{k+1}), k = 0, \ldots, \tilde{p}_F - 1$ and let Assumption 2 hold. Consider further an extended class $\mathcal{K}$ function α_F and a given, positive constant δ_1 satisfying $\delta_1 > \frac{L_t + |\alpha_F(\chi)|}{d_u \sigma_{\min}(B) - \sigma_{\max}(A) d_x}$, where $L_t = \max_{i \in \mathcal{I}_F} \frac{d\gamma_F^i}{dt}|_{t=0}$ and $\chi < \inf_{(\mathbf{x}, t) \in \mathbb{X} \times [0, \tau_J + N]} \mathfrak{b}_F(\mathbf{x}, t)$. Then, for any $(\mathbf{x}, t) \in \mathbb{X} \times \big([0, \tau_J + N] \backslash \{\sigma_F^k\}_{k=0}^{\tilde{p}_F - 1}\big)$ with $\big\|\frac{\partial \mathfrak{b}_F(\mathbf{x}, t)}{\partial \mathbf{x}}\big\| \leq \delta_1$ it holds that: $\frac{\partial \mathfrak{b}_F(\mathbf{x}, t)}{\partial \mathbf{x}} A\mathbf{x} + \frac{\partial \mathfrak{b}_F(\mathbf{x}, t)}{\partial t} + \alpha_F(\mathfrak{b}_F(\mathbf{x}, t)) > 0$.*

Assumption 3 ensures that $\mathfrak{b}_F(\mathbf{x}, t)$ is non-negative when the control input $\mathbf{u} = \mathbf{0}$ is applied to (1) for any $(\mathbf{x}, t) \in \mathbb{X} \times \big([0, \tau_J + N] \backslash \{\sigma_F^k\}_{k=0}^{\tilde{p}_F - 1}\big)$ with $\big\|\frac{\partial \mathfrak{b}_F(\mathbf{x}, t)}{\partial \mathbf{x}}\big\| \leq \delta_1$. Notice that the value of δ_1 depends on the actuation capabilities of the system and the minimum STL performance requirements. A high value of δ_1 may induce conservatism on the design of $\mathfrak{b}_F(\mathbf{x}, t)$ when compared to, e.g., [15] not surprisingly given the input constraints present here.

Theorem 1. *Consider the system dynamics (1) subject to input constraints $\mathbf{u} \in \mathbb{U}$ with $\mathbb{U}$ defined as in (2b) and the STL formula $\phi' = \phi \wedge \tilde{\varphi}_{p+1}$ with $\phi, \tilde{\varphi}_{p+1}$ defined by (4) and (6a)-(6b) respectively. Let Assumptions 1-3 hold. Consider an extended class $\mathcal{K}$ function α_F and a control law $\bar{\mathbf{u}}(\mathbf{x}, t) := \bar{\mathbf{u}}$ with $\bar{\mathbf{u}}$ given by:*

$$\bar{\mathbf{u}} = \arg\min_{\mathbf{u} \in \mathbb{U}} \mathbf{u}^T \mathbf{u} \tag{14}$$

subject to:

$$\frac{\partial \mathfrak{b}_F(\mathbf{x}, t)}{\partial \mathbf{x}}(A\mathbf{x} + B\mathbf{u}) + \frac{\partial \mathfrak{b}_F(\mathbf{x}, t)}{\partial t} \geq -\alpha_F(\mathfrak{b}_F(\mathbf{x}, t)) \tag{14a}$$

Then, the function $\mathbf{x} : [0, \tau_J + N] \to \mathbb{X}$ satisfying (1) for $\bar{\mathbf{u}}$ a.e. guarantees $\rho^{\phi'}(\mathbf{x}, 0) \geq r_F > 0$ provided that $\mathbf{x}(0) \in \mathcal{C}_F(0)$.

Proof. We provide only a sketch of the proof here due to space limitations. The full proof will be presented in an upcoming journal submission. For $(\mathbf{x}, t) \in \mathbb{X} \times \big([0, \tau_J + N] \backslash \{\sigma_F^k\}_{k=0}^{\tilde{p}_F - 1}\big)$ with $\big\|\frac{\partial \mathfrak{b}_F(\mathbf{x}, t)}{\partial \mathbf{x}}\big\| > \delta_1$ it can be shown that $\mathbf{u}_{\text{feas}} = B^\dagger(-A\mathbf{x} + \mathbf{v}_{\text{feas}})$ is a feasible solution of (14) where $B^\dagger$ is the Moore-Penrose matrix of B and $\mathbf{v}_{\text{feas}} = (L_t - \alpha_F(\chi))\big\|\frac{\partial \mathfrak{b}_F(\mathbf{x}, t)}{\partial \mathbf{x}}\big\|^{-2} \frac{\partial \mathfrak{b}_F(\mathbf{x}, t)^T}{\partial \mathbf{x}}$. By [16, Prp 3.3.9] and due to the convexity of (14) we may conclude that the KKT conditions of (14) are necessary and sufficient. Finally, it can be shown that the proposed control law is continuous in $(\mathbf{x}, t)$. ■

The proposed MPC problem is solved at pre-determined, equidistant time instants τ_j based on the current state of the system $\mathbf{x}(\tau_j)$. The resulting control law is applied over a finite time interval $[\tau_j, \tau_{j+1})$ until the next state measurement $\mathbf{x}(\tau_{j+1})$ becomes available at τ_{j+1}. The aforementioned procedure is repeated for a finite number of times J with $J = \frac{\max_l b_l}{\Delta \tau}$. Given the actuation limitations of the agents satisfaction of ϕ might not be possible at all times as this decision might lead to excessive state and input costs. Therefore, we propose the relaxation of $\mathfrak{b}_H(\mathbf{x}, t) \geq 0$ by introducing a slack variable $\epsilon = \epsilon(t) \geq 0$ and imposing the constraint $\mathfrak{b}_H(\mathbf{x}, t) \geq -\epsilon$ for every $t \in [\tau_j, \tau_j + N)$. The relaxation factor ϵ is considered as an MPC variable and the goal is to minimize its value within $[\tau_j, \tau_j + N]$, $j = 0, \ldots, J$. The proposed framework can accommodate any differentiable cost function $L(\mathbf{u}, \mathbf{x}, \epsilon)$ such as the quadratic one, i.e., $L(\mathbf{u}, \mathbf{x}, \epsilon) = \|\mathbf{u}\|_{Q_u}^2 + \|\mathbf{x}\|_{Q_x}^2 + \|\epsilon\|_{Q_\epsilon}^2$. Based on the above, we may define the MPC problem as follows:

$$\min_{\mathbf{u}, \epsilon} \int_{\tau_j}^{\tau_j + N} L(\mathbf{u}, \mathbf{x}, \epsilon) dt \tag{15}$$

$$\text{s.t.} \qquad \dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}, \quad \text{a.e. } [\tau_j, \tau_j + N] \tag{15a}$$

$$\mathfrak{b}_H(\mathbf{x}, t) \geq -\epsilon, \quad t \in [\tau_j, \tau_j + N) \tag{15b}$$

$$\mathbf{x}(\tau_j) = \mathbf{x}_j \tag{15c}$$

$$\mathbf{x}(\tau_j + N) \in \mathcal{C}_F(\tau_j + N) \tag{15d}$$

$$\mathbf{x} \in \mathbb{X}, \quad t \in [\tau_j, \tau_j + N] \tag{15e}$$

$$\mathbf{u} \in \mathbb{U}, \quad t \in [\tau_j, \tau_j + N] \tag{15f}$$

$$\epsilon \in [0, \infty), \quad t \in [\tau_j, \tau_j + N] \tag{15g}$$

Equation (15a) defines the system dynamics while (15b) ensures that $\mathbf{x}(t)$ lies in the interior or at the closest possible distance from $\mathcal{C}_H(t)$. Constraints (15c), (15d) express the initial and terminal conditions of the states respectively. As discussed earlier (15d) ensures that the system behaves in a desired way towards the satisfaction of ϕ' with robustness r_F. Finally, (15e)-(15g) ensure that $\mathbf{x}, \mathbf{u}, \epsilon$ take values among the admissible.

For the optimal control problem (15) we make the following assumption on the regularity of $\mathbf{u}(t)$ on any time interval $[\tau_j, \tau_j + N],\ j = 0, \ldots, J$:

Assumption 4. *Any control input* $\mathbf{u} : [\tau_j, \tau_j + N] \to \mathbb{U}$*,* $j = 0, \ldots, J$ *satisfying* (15a)-(15g) *is continuous a.e. in* $[\tau_j, \tau_j + N]$.

Theorem 2. *Consider the system* (1) *and the STL formula* ϕ *defined by* (4)*. Let Assumptions 1-4 hold. Assume that the MPC problem* (15) *is feasible at* $\tau_0 = 0$*. Then,* (15) *is recursively feasible.*

Proof. Assume that (15) is feasible at $\tau_j,\ j \geq 1$. Let $\mathbf{u}_j : [\tau_j, \tau_j + N] \to \mathbb{U}$, $\epsilon_j : [\tau_j, \tau_j + N] \to [0, \infty)$ denote the control input and the violating factor respectively, found as a solution of (15) over the time interval $[\tau_j, \tau_j + N]$. Consider the candidate control input:

$$\mathbf{u}_{j+1}(t) = \begin{cases} \mathbf{u}_j(t), & t \in [\tau_{j+1}, \tau_j + N] \\ \bar{\mathbf{u}}(t), & t \in (\tau_j + N, \tau_{j+1} + N] \end{cases}$$

where $\bar{\mathbf{u}} : (\tau_j + N, \tau_{j+1} + N] \to \mathbb{U}$ is the optimal solution of (14). By Theorem 1, there always exists an admissible, control law $\bar{\mathbf{u}}(\bar{\mathbf{x}}(t), t)$ that is continuous in $\bar{\mathbf{x}}$, where $\bar{\mathbf{x}} : [\tau_j + N, \tau_{j+1} + N] \to \mathbb{X}$ is a solution of (1) for $\bar{\mathbf{u}}(\bar{\mathbf{x}}(t), t)$. The solution $\bar{\mathbf{x}}(t)$ is an absolutely continuous function in t. Hence, the feedback control law $\bar{\mathbf{u}}(\bar{\mathbf{x}}(t), t) = \bar{\mathbf{u}}(t)$ is continuous a.e. in $(\tau_j + N, \tau_{j+1} + N]$. Additionally, by feasibility of (15) at $[\tau_j, \tau_j + N]$ the feasible control $\mathbf{u}_j(t)$ satisfies Assumption 4. As a result the proposed control signal $\mathbf{u}_{j+1}(t)$ satisfies Assumption 4 guaranteeing the existence of solutions of (1) over the interval $[\tau_{j+1}, \tau_{j+1} + N]$. By Theorem 1 $\mathfrak{b}_F(\bar{\mathbf{x}}(t), t) \geq 0$ is true for any solution $\bar{\mathbf{x}}(t)$ of (1) for $\bar{\mathbf{u}}(t)$ and for all $t \in [\tau_j + N, \tau_{j+1} + N]$. Therefore, (15d)-(15e) is satisfied. In addition, since $\mathbf{u}_j(t), t \in [\tau_{j+1}, \tau_j + N]$ is a feasible input for the MPC at τ_j, any solution $\mathbf{x}_j$ of (1) for $\mathbf{u}_j$ satisfies the state constraints, i.e., $\mathbf{x}_j(t) \in \mathbb{X}$, $t \in [\tau_{j+1}, \tau_j + N]$. Note also that $\mathbf{u}_{j+1}(t) \in \mathbb{U}$, $t \in [\tau_{j+1}, \tau_{j+1} + N]$. Hence, (15e)-(15f) are satisfied. Let $\mathfrak{b}_H(\bar{\mathbf{x}}, t)$ be the value of the barrier function for $\bar{\mathbf{x}}(t),\ t \in (\tau_j + N, \tau_{j+1} + N]$. Consider the violation factor $\epsilon_{j+1}(t),\ t \in [\tau_{j+1}, \tau_{j+1} + N]$:

$$\epsilon_{j+1}(t) = \begin{cases} \epsilon_j(t), & t \in [\tau_{j+1}, \tau_j + N] \\ \bar{\epsilon}(t), & t \in (\tau_j + N, \tau_{j+1} + N] \end{cases}$$

where $\bar{\epsilon} := \bar{\epsilon}(t)$ is defined as:

$$\bar{\epsilon} = \begin{cases} 0, & \mathfrak{b}_H(\bar{\mathbf{x}}, t) \geq 0 \\ -\mathfrak{b}_H(\bar{\mathbf{x}}, t), & \mathfrak{b}_H(\bar{\mathbf{x}}, t) < 0 \end{cases}$$

By feasibility of (15) at τ_j it holds that: 1) $\epsilon_j(t) \geq 0$ is true for every $t \in [\tau_j, \tau_j + N]$ and 2) (15b) is satisfied for any solution $\mathbf{x}_j(t)$ of (1) when applying $\mathbf{u}_j(t)$ a.e. in $[\tau_j, \tau_j + N]$. By definition, $\bar{\epsilon} \geq 0$. For $\mathfrak{b}_H(\bar{\mathbf{x}}, t) \geq 0$, $\epsilon_{j+1} = 0$ implies the satisfaction of (15b). Finally, setting $\epsilon_{j+1} = -\mathfrak{b}_H(\bar{\mathbf{x}}, t)$ when $\mathfrak{b}_H(\bar{\mathbf{x}}, t) < 0$ results in $\epsilon_{j+1} + \mathfrak{b}_H(\bar{\mathbf{x}}, t) = 0$. Therefore, (15b) is satisfied as equality. Based on the analysis above the candidate solution $(\mathbf{u}_{j+1}, \epsilon_{j+1})$ satisfies the constraints (15a)-(15g). As a result, (15) is feasible over $[\tau_{j+1}, \tau_{j+1} + N]$. Since the above hold for any $j \geq 1$, (15) is recursively feasible. ■

Theorem 3. *Let the Assumptions of Theorem 2 hold. Let* $\mathbf{x}_{CL} : [0, \tau_J] \to \mathbb{X}$ *be a solution of* (1) *under the control law* $\kappa(\mathbf{x}(t)) = \begin{cases} \mathbf{u}_j(t), & t \in [\tau_j, \tau_{j+1}),\ j = 0, \ldots, J-1 \\ \mathbf{u}_J(t), & t = \tau_J \end{cases}$.
Then, $\rho^{\phi}(\mathbf{x}_{CL}, 0) \geq \min_{j=0,\ldots,J} \bar{\rho}_j$ *with* $\bar{\rho}_j \geq \inf_{t \in [\tau_j, \tau_j + N]} \mathfrak{b}_H(\mathbf{x}_{CL}(t), t) \geq -\epsilon_{wc}$, $j = 0, \ldots, J$ *where* $\epsilon_{wc} = \max_{j=0,\ldots,J} \epsilon_{wc}^j$*,* $\epsilon_{wc}^j = \sup_{t \in [\tau_j, \tau_j + N]} \epsilon(t)$ *and* $\bar{\rho}_j$ *is maximized over* $[\tau_j, \tau_j + N]$.

Proof. By Theorem 2, (15) is feasible for every $j = 0, \ldots, J$. This implies that $\kappa(\mathbf{x}(t))$ is always defined and due to Assumption 4 it is continuous a.e. in $[0, \tau_J]$. Hence, there always exists a solution $\mathbf{x}_{\mathrm{CL}} : [0, \tau_J] \to \mathbb{X}$ to (1) satisfying (15b) for every $t \in [0, \tau_J]$.

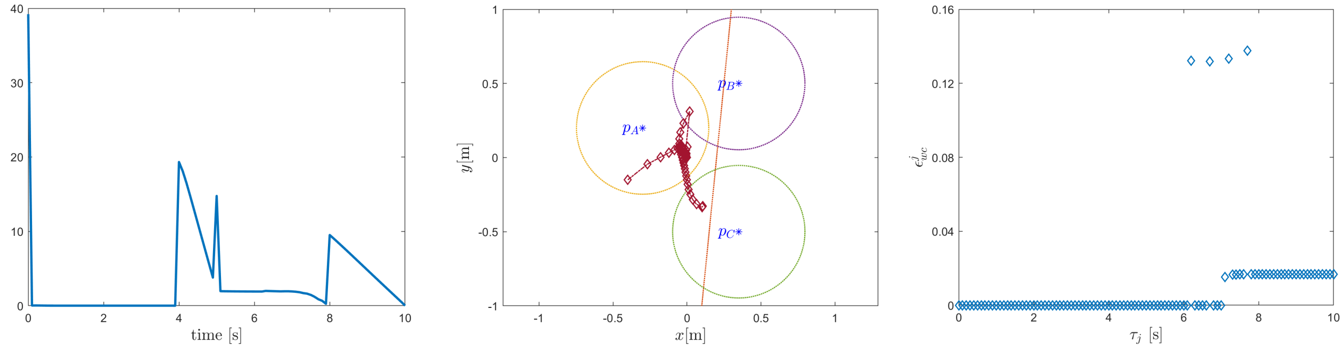
The formula ϕ, defined by (4) is a conjunction of always and eventually formulas $\tilde{\varphi}_i,\ i \in \mathcal{I}_H$. Hence, by definition of the robust semantics we have: $\rho^{\phi}(\mathbf{x}_{\mathrm{CL}}, 0) = \min_{i \in \mathcal{I}_H} \rho^{\tilde{\varphi}_i}(\mathbf{x}_{\mathrm{CL}}, 0)$. By construction, $\mathfrak{b}_H(\mathbf{x}, t) \leq \min_{i \in \mathcal{I}_H} \mathfrak{b}_H^i(\mathbf{x}, t)$. Hence, for $t \in T_i$ it holds that:

$$h_i(\mathbf{x}_{\mathrm{CL}}(t)) \geq \gamma_H^i(t) + \mathfrak{b}_H(\mathbf{x}_{\mathrm{CL}}(t), t) \tag{16}$$

By design of $\gamma_H^i(t)$ it holds that: $\gamma_H^i(t) \geq \gamma_H^i(t_H^{*i}) \geq r_H$ for every $t \in T_i$ where t_H^{*i} is the time instant at which the formula $\tilde{\varphi}_i$ is satisfied with robustness r_H and T_i is the time interval over which $\mathfrak{b}_H^i(\mathbf{x}, t)$ contributes to $\mathfrak{b}_H(\mathbf{x}, t)$. Equation (16) implies that $h_i(\mathbf{x}_{\mathrm{CL}}(t)) \geq r_H + \mathfrak{b}_H(\mathbf{x}_{\mathrm{CL}}(t), t) \geq r_H + \inf_{t' \in [\tau_j, \tau_j + N]} \mathfrak{b}_H(\mathbf{x}_{\mathrm{CL}}(t'), t'),\ t \in T_i$. Let $\bar{\rho}_j = r_H + \inf_{t' \in [\tau_j, \tau_j + N]} \mathfrak{b}_H(\mathbf{x}_{\mathrm{CL}}(t'), t')$. If $\tilde{\varphi}_i$ is an always formula it holds that: $\rho^{\tilde{\varphi}_i}(\mathbf{x}_{\mathrm{CL}}, 0) = \min_{t_1 \in [\tilde{a}_i, \tilde{b}_i]} h_i(\mathbf{x}_{\mathrm{CL}}(t_1)) \geq \bar{\rho}_j \geq \min_{j=0,\ldots,J} \bar{\rho}_j$. If the formula $\tilde{\varphi}_i$ is an eventually formula it holds that: $\rho^{\tilde{\varphi}_i}(\mathbf{x}_{\mathrm{CL}}, 0) = \max_{t_1 \in [\tilde{a}_i, \tilde{b}_i]} h_i(\mathbf{x}_{\mathrm{CL}}(t_1)) \geq \max_{t_1 \in [\tilde{a}_i, t_H^{*i}]} h_i(\mathbf{x}_{\mathrm{CL}}(t_1)) \geq \min_{j=0,\ldots,J} \bar{\rho}_j$. Considering the aforementioned results, we may conclude that $\rho^{\phi}(\mathbf{x}_{\mathrm{CL}}, 0) = \min_{i \in \mathcal{I}_H} \rho^{\tilde{\varphi}_i}(\mathbf{x}_{\mathrm{CL}}, 0) \geq \min_{j=0,\ldots,J} \bar{\rho}_j$. Note that $r_H > 0$ is true, and thus $\bar{\rho}_j \geq \inf_{t' \in [\tau_j, \tau_j + N]} \mathfrak{b}_H(\mathbf{x}_{\mathrm{CL}}(t'), t')$. By (15b) $\mathfrak{b}_H(\mathbf{x}_{\mathrm{CL}}(t'), t') \geq -\epsilon \geq -\epsilon_{\mathrm{wc}}^j$, $t' \in [\tau_j, \tau_j + N]$. Considering this result and for $\epsilon_{\mathrm{wc}} = \max_{j=0,\ldots,J} \epsilon_{\mathrm{wc}}^j$ we have that $\mathfrak{b}_H(\mathbf{x}_{\mathrm{CL}}(t'), t') \geq -\epsilon_{\mathrm{wc}}$. ■

V. Simulation Results

In this section we present a simulation scenario for a system with dynamics $\dot{\mathbf{x}} = -A\mathbf{x} + B\mathbf{u}$ where $\mathbf{x} \in \mathbb{R}^2$, $A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$. The initial condition of the system is $\mathbf{x}(0) = \begin{bmatrix} -0.4 & -0.15 \end{bmatrix}^T$. The system is subject to state and inputs constraints with $d_x = 1.5$ and $d_u = 9$. Next consider the formula $\phi'' = \bigwedge_{i=1}^{3} \varphi_i$ with the subformulas $\varphi_i, i = 1, \ldots, 3$ defined as: $\varphi_1 = \mathcal{G}_{[0,4]} \psi_1$, $\varphi_2 = \psi_2\, \mathcal{U}_{[5,8]} \psi_3$



(a) Barrier Function Evolution (b) State Trajectory (c) Violation Factor

Fig. 1: The evolution of the barrier function $\mathfrak{b}_H(\mathbf{x}_{\mathrm{CL}}(t), t)$, the closed loop trajectory $\mathbf{x}_{\mathrm{CL}}(t)$ and the maximum violation factor $\epsilon^j_{\mathrm{wc}}, j = 0, \ldots, J$ obtained by the proposed MPC scheme.

and $\varphi_3 = \mathcal{F}_{[8,10]}\psi_4$. The predicate functions corresponding to ψ_i, $i = 1, \ldots 4$ are defined as: $h_1(\mathbf{x}) = 0.2 - \|\mathbf{x} - p_A\|^2$, $h_2(\mathbf{x}) = \begin{bmatrix}-10 & 1\end{bmatrix}\mathbf{x} + 2$, $h_3(\mathbf{x}) = 0.2 - \|\mathbf{x} - p_B\|^2$, $h_4(\mathbf{x}) = 0.2 - \|\mathbf{x} - p_C\|^2$ where $p_A = \begin{bmatrix}-0.3 & 0.2\end{bmatrix}^T, p_B = \begin{bmatrix}0.35 & 0.5\end{bmatrix}^T, p_C = \begin{bmatrix}0.35 & -0.5\end{bmatrix}^T$. Based on ϕ'' the state of the system needs to stay close to p_A, approach p_B while respecting a safety requirement and finally reach p_C. The optimization horizon and sampling rate are chosen as $N = 3$ and $\Delta\tau = 0.1$ respectively. The robustness value corresponding to the barrier function $\mathfrak{b}_H(\mathbf{x}, t), \mathfrak{b}_F(\mathbf{x}, t)$ is $r_H = 0.1$ and $r_F = 0.01$ respectively. The closed loop trajectory of the system $\mathbf{x}_{\mathrm{CL}}(t)$ is shown in Figure 1b for $t \in [0, 10]$. Based on Theorem 3 and Figure 1c we have $\rho^\phi(\mathbf{x}, 0) \geq -0.1344$ which is a more conservative bound when compared to the results of Figure 1a from which we can deduce that $\rho^\phi(\mathbf{x}, 0) \geq 0.1$ since $\inf_{t\in[0,10]} \mathfrak{b}_H(\mathbf{x}_{\mathrm{CL}}(t), t) = 0.0026$. The jumps shown in Figure 1a at times $t = 0$, $t = 4$, $t = 5$ and $t = 8$ sec occur due to the deactivation policy described in section III. The simulations were performed in an Intel Core i7-8665U with 16GB RAM with the average computational time of the MPC being equal to 4.7375 sec.

VI. Conclusion

A model predictive control framework is proposed for a linear system under STL specifications. The satisfaction of the STL tasks is imposed within the MPC scheme using a time-varying barrier function, designed offline. The recursive feasibility of the proposed scheme is proven by introducing a time-varying terminal constraint that ensures a worst-case temporal behavior for the system guaranteeing the satisfaction of the task. Future work will extend the framework to multi-agent systems.

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