Event-Triggered Tracking Control of Networked and Quantized Control Systems

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Abstract— **This paper studies the tracking control problem of networked and quantized control systems under both multiple networks and event-triggered mechanisms. Multiple networks are to connect the plant and reference system with decentralized controllers to guarantee their information transmission, whereas event-triggered mechanisms are to reduce the information transmission via multiple networks. In this paper, all networks are independent and asynchronous and have local event-triggered mechanisms, which are based on local measurements and determine whether the local measurements need to be transmitted. We first implement an emulation-based approach to develop a novel hybrid model for tracking control of networked and quantized control systems. Next, sufficient conditions are derived and decentralized event-triggered mechanisms are designed to ensure the tracking performance. Finally, a numerical example is given to illustrate the obtained results.**

I. Introduction

The introduction of wired/wireless networks to connect multiple smart devices leads to networked control systems (NCS), the area of which includes three activities [1]: control of networks; control over networks; and multi-agent systems. The presence of networks improves efficiency and flexibility of integrated applications, and reduces installation and maintenance time and costs [2], [3]. Smart devices are physically distributed and interconnected such that their communications are via different types of networks, which result in many issues like transmission delays, packet dropouts, quantization, etc. Therefore, the main challenge is how to design the control scheme to limit the effects of the network-induced issues and to achieve the desired performances while keeping the information transmission as minimal as possible. One suitable approach is periodic event-triggered control (PETC) [4]–[6], combining time-triggered control (TTC) [7], [8] and event-triggered control (ETC) [9], [10]. The PETC allows the triggering condition to be evaluated with a predefined sampling period to decide the information transition, and leads to a balance between TTC and ETC by avoiding the continuous evaluation of the triggering condition [4], [6].

Many existing results on NCS focus mainly on stability analysis and stabilization control, and both TTC and ETC/PETC have been addressed [3], [4], [11], [12]. However, tracking control, as a fundamental problem in control theory [13], [14], received less attention [15]–[17]. The main objective of the tracking control is to design controllers such that the plant can track the given reference trajectory as close as possible [18], [19]. In the tracking control, the controller consists of two parts [14]: the feedforward part to induce the reference trajectory for the plant, and the feedback part to drive the plant to converge to the reference trajectory. As opposed to the traditional tracking control, the main challenge of the tracking control of NCS is that only local/partial information is transmitted to the plant due to limited-capacity communication networks. In addition, the information transmission via networks may be a error source affecting the tracking performance [14]. Therefore, both network-induced errors and local interaction rules need to be considered simultaneously, and thus result in additional difficulties in the tracking performance analysis.

In this paper, we study the event-triggered tracking control problem for networked and quantized control systems (NQCS), where several issues caused by the network and quantization are included [20]. To this end, we implement an emulation-like approach [3], [13], [14], and develop a novel hybrid model using the formalism in [21] to address the event-triggered tracking control for NQCS, which is our first contribution. In particular, a general scenario is considered: multiple independent and asynchronous networks are applied to ensure the communication among different components. This scenario stems from many physical systems, where different communication channels are applied to connect sensors, controllers and actuators. Hence, this setting recovers the architectures in [13], [14] for NCS and [16], [17] for MAS as special cases. In this setting, a general hybrid model is developed to incorporate all issues caused by multiple networks and decentralized event-triggered mechanisms (ETMs). Our second contribution is to apply the Lyapunov-based approach to investigate the tracking performance. Specifically, motivated by multiple Lyapunov functions approach and under reasonable assumptions, the decentralized ETMs are designed to reduce the frequency of the information transmission, and the tradeoff between the maximally allowable sampling period (MASP) and the maximally allowable delay (MAD) is derived to guarantee the tracking performance.

II. Preliminaries

$\mathbb{R} := (-\infty, +\infty)$; $\mathbb{R}_{\geq 0} := [0, +\infty)$; $\mathbb{R}_{>0} := (0, +\infty)$; $\mathbb{N} := \{0, 1, 2, \ldots\}$; $\mathbb{N}_+ := \{1, 2, \ldots\}$. Given two sets $\mathcal{A}$ and

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$\mathcal{B}$, $\mathcal{B}\backslash\mathcal{A} := \{x : x \in \mathcal{B}, x \notin \mathcal{A}\}$. $|\cdot|$ denotes the Euclidean norm. Given two vectors $x, y \in \mathbb{R}^n$, $(x, y) := (x^{\mathrm{T}}, y^{\mathrm{T}})^{\mathrm{T}}$ for simplicity of notation, and $\langle x, y\rangle$ denotes the usual inner product. $\mathbb{E}$ denotes the vector with all components being 1, I denotes the identity matrix of appropriate dimension, and diag$\{A, B\}$ denotes the block diagonal matrix made of the matrices A and B. Given a function $f : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, $f(t^+) := \limsup_{s\to 0^+} f(t + s)$. A function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{K}$ if it is continuous, $\alpha(0) = 0$, and strictly increasing; it is of class $\mathcal{K}_\infty$ if it is of class $\mathcal{K}$ and unbounded. $\beta : \mathbb{R}^2_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{KL}$ if $\beta(s, t) \in \mathcal{K}$ for fixed $t \geq 0$ and $\beta(s, t)$ decreases to zero as $t \to 0$ for fixed $s \geq 0$. A function $\beta : \mathbb{R}^3_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{KLL}$ if $\beta(r, s, t) \in \mathcal{KL}$ for fixed $s \geq 0$ and $\beta(r, s, t) \in \mathcal{KL}$ for fixed $t \geq 0$.

Consider the hybrid system [21]:

$$\begin{cases} \dot{x} = F(x, w), \quad (x, w) \in C, \\ x^+ = G(x, w), \quad (x, w) \in D, \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the system state, $w \in \mathbb{R}^m$ is the external input, $F : C \to \mathbb{R}^n$ is the flow map, $G : D \to \mathbb{R}^m$ is the jump map, C is the flow set and D is the jump set. For the hybrid system (1), the following basic assumptions are presented: the sets $C, D \subset \mathbb{R}^n \times \mathbb{R}^m$ are closed; F is continuous on C; and G is continuous on D.

Definition 1 ([21]): The system (1) is *input-to-state stable (ISS)* from w to x, if there exist $\beta \in \mathcal{KLL}$ and $\gamma \in \mathcal{K}_\infty$ such that $|x(t, j)| \leq \beta(|x(0, 0)|, t, j) + \gamma(\|w\|_\infty)$ for all $(t, j) \in \mathrm{dom}x$.

III. Problem Formulation

Consider the following nonlinear system

$$\dot{x}_{\mathrm{p}} = f_{\mathrm{p}}(x_{\mathrm{p}}, u), \quad y_{\mathrm{p}} = g_{\mathrm{p}}(x_{\mathrm{p}}), \tag{2}$$

where $x_{\mathrm{p}} \in \mathbb{R}^{n_{\mathrm{p}}}$ is the system state, $u \in \mathbb{R}^{n_u}$ is the control input, and $y_{\mathrm{p}} \in \mathbb{R}^{n_y}$ is the plant output. Similar to [13]–[19], the reference system tracked by (2) is of the form:

$$\dot{x}_{\mathrm{r}} = f_{\mathrm{p}}(x_{\mathrm{r}}, u_{\mathrm{f}}), \quad y_{\mathrm{r}} = g_{\mathrm{p}}(x_{\mathrm{r}}). \tag{3}$$

where $x_{\mathrm{r}} \in \mathbb{R}^{n_{\mathrm{r}}}$ is the reference state ($n_{\mathrm{r}} = n_{\mathrm{p}}$), $u_{\mathrm{f}} \in \mathbb{R}^{n_u}$ is the feedforward control input, and $y_{\mathrm{r}} \in \mathbb{R}^{n_y}$ is the reference output. Assume that the reference system (3) has a unique solution for any initial condition and any input.

To track (3), the controller for (2) in the absence of the network and quantizer is $u = u_{\mathrm{c}} + u_{\mathrm{f}}$, where $u_{\mathrm{c}} \in \mathbb{R}^{n_u}$ is the feedback item from the following controller

$$\dot{x}_{\mathrm{c}} = f_{\mathrm{c}}(x_{\mathrm{c}}, y_{\mathrm{p}}, y_{\mathrm{r}}), \quad u_{\mathrm{c}} = g_{\mathrm{c}}(x_{\mathrm{c}}), \tag{4}$$

where $x_{\mathrm{c}} \in \mathbb{R}^{n_{\mathrm{c}}}$ is the state of the feedback controller; $u_{\mathrm{f}} \in \mathbb{R}^{n_u}$ is the feedforward item and is related to plant state and reference state. We assume that f_{p} and f_{c} are continuous; g_{p} and g_{c} are continuously differentiable.

Since the emulation-based approach is applied [3], [13], [14], the feedback controller (4) is assumed to be designed for the network-free and quantization-free case. Hence, our objective is to implement the designed controller over both ETMs and multiple networks and quantizers, and to ensure the tracking performance of the system (2)-(4) under reasonable assumptions and the designed decentralized ETMs.

A. Information Transmission over Multiple Networks

The information is sampled via the sensors, quantized and then determined (by the ETM to be designed) to be transmitted via the network. Since the sensors and actuators may be of different types, the connection among the plant, the reference and the controller may be via multiple networks (e.g., wired/wireless networks [2], [12]). Therefore, the information is transmitted via multiple networks.

Assumption 1: In the case that the ETM is implemented, all sensors and actuators are connected via $N \in \mathbb{N}_+$ independent and asynchronous networks.

For each network $i \in \mathcal{N} := \{1, \ldots, N\}$, the information to be transmitted is denoted by $z_i := (y^i_{\mathrm{p}}, y^i_{\mathrm{r}}, u^i_{\mathrm{f}}, u^i_{\mathrm{c}}) \in \mathbb{R}^{n^i_z}$ with $n^i_z := n^i_y + n^i_{\mathrm{r}} + n^i_{\mathrm{f}} + n^i_{\mathrm{c}}$. The dynamics of z_i is

$$\dot{z}_i = f^i_z(z_i, x_{\mathrm{p}}, x_{\mathrm{r}}, x^i_{\mathrm{c}}), \tag{5}$$

where f^i_z can be computed explicitly via (2)-(4). The dependence of $\dot{z}_i$ on x_{p} and x_{r} comes from the potential dependence of y^i_{p} (or y^i_{r}) on x_{p} (or x_{r}). Denote $z := (z_1, \ldots, z_N) \in \mathbb{R}^{n_z}$, $n_z := \sum_{i=1}^N n^i_z$, and $\dot{z} = f_z := (f^1_z, \ldots, f^N_z) \in \mathbb{R}^{n_z}$. Because of the band-limited capacity of each network and spatial locations of its sensors and actuators, all sensors and actuators of each network are grouped into $\ell_i \in \mathbb{N}_+$ nodes to access to the network [11]. Correspondingly, z_i is partitioned into ℓ_i parts. For the i-th network, its sampling time sequence is given by $\{t^i_j : i \in \mathcal{N}, j \in \mathbb{N}_+\}$, which is strictly increasing. At t^i_j, one and only one node is allowed to access to the i-th network, and this node is chosen by an time-scheduling protocol. For the i-th network, the sampling intervals are defined as $h^i_j := t^i_{j+1} - t^i_j$, where $i \in \mathcal{N}$ and $j \in \mathbb{N}_+$. Since it takes time to compute and transmit the information, there exist transmission delays $\tau^i_j \geq 0$ such that the information is received at the arrival times $r^i_j = t^i_j + \tau^i_j$.

Assumption 2: For the i-th network, $i \in \mathcal{N}$, there exist constants $T_i \geq \Delta_i \geq 0$ and $\varepsilon_i \in (0, T_i)$ such that $\varepsilon_i \leq h^i_j \leq T_i$ and $0 \leq \tau^i_j \leq \min\{\Delta_i, h^i_j\}$ hold for all $j \in \mathbb{N}_+$.

In Assumption 2, $T_i > 0$ is called the *maximally allowable sampling period (MASP)* for the i-th network, $\Delta_i \geq 0$ is called the *maximally allowable delay (MAD)*, and $\varepsilon_i > 0$ is the minimal interval of two successive transmissions.

The sampled information is quantized before being transmitted. For each network, each node $j \in \{1, \ldots, \ell_i\}$ has a quantizer. The quantizer is a piecewise continuous function $\bar{q}^i_j : \mathbb{R}^{n^i_j} \to \mathcal{Q}^i_j \subset \mathbb{R}^{n^i_j}$, where $\mathcal{Q}^i_j$ is finite. The following assumption is made for the quantizer; see also [22].

Assumption 3: For all $i \in \mathcal{N}$ and $j \in \{1, \ldots, \ell_i\}$, there exist $\mathfrak{m}^i_j > \mathfrak{n}^i_j > 0$ and $\mathfrak{n}_{0j} > 0$ such that for all $z^i_j \in \mathbb{R}^{n^i_j}$: i) $|z^i_j| \leq \mathfrak{m}^i_j \Rightarrow |\bar{q}^i_j(z^i_j) - z^i_j| \leq \mathfrak{n}^i_j$; ii) $|z^i_j| > \mathfrak{m}^i_j \Rightarrow |\bar{q}_j(z^i_j)| > \mathfrak{m}^i_j - \mathfrak{n}^i_j$; iii) $|z^i_j| \leq \mathfrak{n}_{0j} \Rightarrow \bar{q}^i_j(z^i_j) \equiv 0$.

In Assumption 3, $\epsilon^i_j := \bar{q}^i_j(z_j) - z^i_j$ is the quantization error. $\mathfrak{m}^i_j$ is the range of the j-th quantizer in i-th network, $\mathfrak{n}^i_j$ is the bound on the quantization error. Based on the quantizer $\bar{q}^i_j$ and Assumption 3, the applied quantizer is of the form:

$$q^i_j(\mu^i_j, z^i_j) = \mu^i_j \bar{q}^i_j(z^i_j/\mu^i_j), \quad j \in \{1, \ldots, \ell_i\}, \tag{6}$$

where $\mu^i_j > 0$ is a time-varying quantization parameter.

Assumption 4 ([13]): The initial state $(x_{\mathrm{p}0}, x_{\mathrm{r}0}, x_{\mathrm{c}0})$ is known *a priori* and bounded. The quantization parameter μ_j^i is such that $|z_j^i| \leq \mathfrak{m}_j^i \mu_j^i$ for all $j \in \{1, \dots, \ell_i\}$ and $i \in \mathcal{N}$.

Assumption 4 is to ensure that the quantizer does not saturate. Combining all quantizers in ℓ_i nodes yields the overall quantizer: $q_i(\mu_i, z_i) := (q_1^i(\mu_1^i, z_1^i), \dots, q_{\ell_i}^i(\mu_{\ell_i}^i, z_{\ell_i}^i))$, where $\mu_i := (\mu_1^i, \dots, \mu_{\ell_i}^i) \in \mathbb{R}^{\ell_i}$ is evolving as

$$\dot{\mu}_i(t) = 0, \quad t \in (r_j^i, r_{j+1}^i), \tag{7}$$

$$\mu_i(r_j^{i+}) = \Omega_i \mu_i(r_j^i), \quad \Omega_i := \mathrm{diag}\{\Omega_1^i, \dots, \Omega_{\ell_i}^i\}, \tag{8}$$

where $\Omega_j^i \in (0, 1]$. The quantized measurement is defined as $\bar{\mathbf{z}}_i = (\bar{y}_{\mathrm{p}}^i, \bar{y}_{\mathrm{r}}^i, \bar{y}_{\mathrm{c}}^i, \bar{y}_{\mathrm{f}}^i) := (q_i(\mu_i, y_{\mathrm{p}}^i), q_i(\mu_i, y_{\mathrm{r}}^i), q_i(\mu_i, y_{\mathrm{c}}^i), q_i(\mu_i, y_{\mathrm{f}}^i))$; the quantization error is defined as $\epsilon_i := (\epsilon_{\mathrm{p}}^i, \epsilon_{\mathrm{r}}^i, \epsilon_{\mathrm{c}}^i, \epsilon_{\mathrm{f}}^i) = (\bar{y}_{\mathrm{p}}^i - y_{\mathrm{p}}^i, \bar{y}_{\mathrm{r}}^i - y_{\mathrm{r}}^i, \bar{u}_{\mathrm{c}}^i - u_{\mathrm{c}}^i, \bar{u}_{\mathrm{f}}^i - u_{\mathrm{f}}^i)$.

To reduce the transmission frequency, a local ETM is implemented for each network. That is, at t_j^i, only when the event-triggered condition for the i-th network is satisfied can the quantized measurement be transmitted. Define $\hat{z} := (\hat{y}_{\mathrm{p}}, \hat{y}_{\mathrm{r}}, \hat{u}_{\mathrm{c}}, \hat{u}_{\mathrm{f}}) \in \mathbb{R}^{n_z}$ as the received measurement after the transmission, and the control input received by the plant is $\hat{u} := \hat{u}_{\mathrm{c}} + \hat{u}_{\mathrm{f}}$. The network-induced errors are $e_{\mathrm{p}} := \hat{y}_{\mathrm{p}} - y_{\mathrm{p}}$, $e_{\mathrm{r}} := \hat{y}_{\mathrm{r}} - y_{\mathrm{r}}$, $e_{\mathrm{c}} := \hat{u}_{\mathrm{c}} - u_{\mathrm{c}}$ and $e_{\mathrm{f}} := \hat{u}_{\mathrm{f}} - u_{\mathrm{f}}$. Let $\hat{z} = (\hat{z}_1, \dots, \hat{z}_N)$ and $\mathbf{e} := \hat{z} - z = (\mathbf{e}_1, \dots, \mathbf{e}_N) \in \mathbb{R}^{n_z}$.

In $[r_j^i, r_{j+1}^i]$, the received measurement $\hat{z}_i$ is assumed to be implemented via the zero-order hold (ZOH) mechanism, that is, $\dot{\hat{z}}_i(t) = 0$ for $t \in [r_j^i, r_{j+1}^i]$. Whether $\hat{z}_i$ is updated at r_j^i is based on the local ETM at t_j^i. Assume that the event-triggered condition for the i-th network is $\Gamma_i \geq 0$, where the function $\Gamma_i : \mathbb{R}_{\geq 0} \to \mathbb{R}$ will be designed explicitly in Subsection V-B. $\Gamma_i \geq 0$ implies the information transmission, and $\hat{z}_i$ is updated with the latest information. That is, $\hat{z}_i$ is updated by

$$\hat{z}_i(r_j^{i+}) = \begin{cases} \bar{\mathbf{z}}_i(r_j^i) + \mathbf{h}_z^i(\kappa_i(t_j^i), \mathbf{e}_i(t_j^i)), \ \Gamma_i(t_j^i) \geq 0, \\ \hat{z}_i(r_j^i), \ \Gamma_i(t_j^i) < 0, \end{cases} \tag{9}$$

where $\kappa_i : \mathbb{R}_{\geq 0} \to \mathbb{N}$ is a counter to record the number of the successful transmission events. That is, $\kappa_i(t_j^{i+}) = \kappa_i(t_j^i) + 1$ if $\Gamma_i(t_j^i) \geq 0$, and $\kappa_i(t_j^{i+}) = \kappa_i(t_j^i)$ otherwise. $\mathbf{h}_z^i \in \mathbb{R}^{n_z}$ is the update function and depends on the time-scheduling protocol. Denote $\mathbf{h}_z^i := (\mathbf{h}_{\mathrm{p}}^i, \mathbf{h}_{\mathrm{r}}^i, \mathbf{h}_{\mathrm{c}}^i, \mathbf{h}_{\mathrm{f}}^i)$ from the definition of $\hat{z}_i$. Furthermore, we can rewrite (9) as $\hat{z}_i(r_j^{i+}) = (1 - \Upsilon(\Gamma_i(t_j^i)))\hat{z}_i(r_j^i) + \Upsilon(\Gamma_i(t_j^i))[\bar{\mathbf{z}}_i(r_j^i) + \mathbf{h}_z^i(\kappa_i(t_j^i), \mathbf{e}_i(t_j^i))]$, where $\Upsilon : \mathbb{R} \to \{0, 1\}$ is defined as $\Upsilon(\Gamma_i) = 1$ if $\Gamma_i \geq 0$ and $\Upsilon(\Gamma_i) = 0$ otherwise. Hence , $\mathbf{e}_i$ is updated by $\mathbf{e}_i(r_j^{i+}) = \hat{z}_i(r_j^{i+}) - z_i(r_j^{i+}) = \mathbf{e}_i(r_j^i) + \Upsilon(\Gamma_i(t_j^i))[\mathfrak{h}_z^i(\kappa_i(t_j^i), \mathbf{x}_i(t_j^i), \mathbf{e}_i(t_j^i), \mu_i(t_j^i)) - \mathbf{e}_i(t_j^i)]$, where $\mathfrak{h}_z^i(\kappa_i, x_i, \mathbf{e}_i, \mu_i) = \epsilon_i + \mathbf{h}_z^i(\kappa_i, \mathbf{e}_i)$ and $\mathbf{x}_i = (x_{\mathrm{p}}^i, x_{\mathrm{r}}^i, x_{\mathrm{c}}^i, u_{\mathrm{f}}^i)$.

IV. Development of Hybrid Model

After the presentation of the information transmission, we construct the hybrid model for the event-triggered tracking control of NQCS in this section. Define the tracking error $\eta := x_{\mathrm{p}} - x_{\mathrm{r}} \in \mathbb{R}^{n_{\mathrm{p}}}$, and $e_{\mathrm{a}} := (e_\eta, e_{\mathrm{c}}) := (e_{\mathrm{p}} - e_{\mathrm{r}}, e_{\mathrm{c}}) \in \mathbb{R}^{n_{\mathrm{a}}}$ with $e_{\mathrm{p}}, e_{\mathrm{r}}, e_{\mathrm{c}}$ defined in Section III, where $n_{\mathrm{a}} = n_y + n_{\mathrm{c}}$. Combining all variables and analyses in Subsection III-A, we derive the following impulsive model:

$$\left.\begin{aligned} \dot{\eta} &= F_\eta(\delta, \eta, x_{\mathrm{c}}, x_{\mathrm{r}}, e_{\mathrm{a}}, e_{\mathrm{f}}, e_{\mathrm{r}}) \\ \dot{x}_{\mathrm{c}} &= F_{\mathrm{c}}(\delta, \eta, x_{\mathrm{c}}, x_{\mathrm{r}}, e_{\mathrm{a}}, e_{\mathrm{f}}, e_{\mathrm{r}}) \\ \dot{x}_{\mathrm{r}} &= F_{\mathrm{r}}(\delta, x_{\mathrm{r}}, e_{\mathrm{f}}), \quad \dot{\mu} = 0 \\ \dot{e}_{\mathrm{a}} &= G_{\mathrm{a}}(\delta, \eta, x_{\mathrm{c}}, x_{\mathrm{r}}, e_{\mathrm{a}}, e_{\mathrm{f}}, e_{\mathrm{r}}) \\ \dot{e}_{\mathrm{r}} &= G_{\mathrm{r}}(\delta, \eta, x_{\mathrm{c}}, x_{\mathrm{r}}, e_{\mathrm{a}}, e_{\mathrm{f}}, e_{\mathrm{r}}) \\ \dot{e}_{\mathrm{f}} &= G_{\mathrm{f}}(\delta, \eta, x_{\mathrm{c}}, x_{\mathrm{r}}, e_{\mathrm{a}}, e_{\mathrm{f}}, e_{\mathrm{r}}) \end{aligned}\right\} \quad t^i \in [r_j^i, r_{j+1}^i], \tag{10a}$$

$$\begin{aligned} &\mu_i(r_j^{i+}) = \Omega_i \mu_i(r_j^i), \quad \delta_i(r_j^{i+}) = \delta_i(r_j^i), \\ &e_{\mathrm{a}}^i(r_j^{i+}) = e_{\mathrm{a}}^i(r_j^i) + \Upsilon(\Gamma_i(t_j^i))[-e_{\mathrm{a}}^i(t_j^i) \\ &\qquad h_{\mathrm{a}}^i(\kappa_i(t_j^i), x_i(t_j^i), e_i(t_j^i), \mu_i(t_j^i))], \\ &e_{\mathrm{r}}^i(r_j^{i+}) = e_{\mathrm{r}}^i(r_j^i) + \Upsilon(\Gamma_i(t_j^i))[-e_{\mathrm{r}}^i(t_j^i) \\ &\qquad + h_{\mathrm{r}}^i(\kappa_i(t_j^i), x_i(t_j^i), e_i(t_j^i), \mu_i(t_j^i))], \\ &e_{\mathrm{f}}^i(r_j^{i+}) = e_{\mathrm{f}}^i(r_j^i) + \Upsilon(\Gamma_i(t_j^i))[-e_{\mathrm{f}}^i(t_j^i) \\ &\qquad + h_{\mathrm{f}}^i(\kappa_i(t_j^i), x_i(t_j^i), e_i(t_j^i), \mu_i(t_j^i))], \end{aligned} \tag{10b}$$

where $e_i := (e_{\mathrm{a}}^i, e_{\mathrm{r}}^i, e_{\mathrm{f}}^i)$, $h_{\mathrm{a}}^i = (\mathfrak{h}_{\mathrm{p}}^i - \mathfrak{h}_{\mathrm{r}}^i, \mathfrak{h}_{\mathrm{c}}^i), h_{\mathrm{r}}^i = \mathfrak{h}_{\mathrm{r}}^i$ and $h_{\mathrm{f}}^i = \mathfrak{h}_{\mathrm{f}}^i$. $\delta := (\delta_1, \dots, \delta_N) \in \mathbb{R}^N$, and $\delta_i \in \mathbb{R}_{\geq 0}$ is to model the 'continuous' time of the i-th network and depends on u_{f}^i and/or its differential [13], [14]. Now, our objective is to derive reasonable conditions and ETMs simultaneously to guarantee ISS of the system (10) from $(e_{\mathrm{r}}, e_{\mathrm{f}})$ to (η, e_{a}).

We further transform (10) into a hybrid model in the formalism of [21]. Define $x := (\eta, x_{\mathrm{c}}, x_{\mathrm{r}}) \in \mathbb{R}^{n_x}$ and $e := (e_{\mathrm{a}}, e_{\mathrm{r}}, e_{\mathrm{f}}) \in \mathbb{R}^{n_e}$ with $n_x = n_{\mathrm{p}} + n_{\mathrm{c}} + n_{\mathrm{r}}$ and $n_e = n_{\mathrm{a}} + n_y + n_u$. Define $m := (m_1, \dots, m_N) \in \mathbb{R}^{n_e}$ with $m_i := h_i(\kappa_i, e_i) - e_i \in \mathbb{R}^{n_e^i}$, where $e_i := (e_{\mathrm{a}}^i, e_{\mathrm{r}}^i, e_{\mathrm{f}}^i)$ and $h_i := (h_{\mathrm{a}}^i, h_{\mathrm{r}}^i, h_{\mathrm{f}}^i)$ are defined in (10). Let $\kappa := (\kappa_1, \dots, \kappa_N) \in \mathbb{R}^N$ with $\kappa_i \in \mathbb{N}$ defined in (9); $\mu := (\mu_1, \dots, \mu_N) \in \mathbb{R}^{\mathfrak{L}}$ with $\mathfrak{L} := \sum_{i \in \mathcal{N}} \ell_i$ and $\mu_i \in \mathbb{R}$ defined in (7)-(8); $\tau := (\tau_1, \dots, \tau_N) \in \mathbb{R}^N$ with $\tau_i \in \mathbb{R}$ defined as a timer to record both sampling intervals and transmission delays for the i-th network; $b := (b_1, \dots, b_N) \in \mathbb{R}^N$ with $b_i \in \{0, 1\}$ to show whether the next jump is from the sampling event or the update event. Denote $\mathfrak{X} := (x, e, \mu, m, \delta, \tau, \kappa, b) \in \mathscr{R} := \mathbb{R}^{n_x} \times \mathbb{R}^{n_e} \times \mathbb{R}^{\mathfrak{L}} \times \mathbb{R}^{n_e} \times \mathbb{R}^N \times \mathbb{R}^N \times \mathbb{R}^N \times \{0, 1\}^N$, and the hybrid model is given by

$$\dot{\mathfrak{X}} = F(\mathfrak{X}), \ \mathfrak{X} \in C, \qquad \mathfrak{X}^+ = G(\mathfrak{X}), \ \mathfrak{X} \in D, \tag{11}$$

where $C := \cup_{i=1}^N C_i$, $D := \cup_{i=1}^N (D_{1i} \cup D_{2i})$, $C_i := \{\mathfrak{X} \in \mathscr{R} : (b_i, \tau_i) \in (\{0\} \times \mathbf{T}_i) \cup (\{1\} \times \mathbf{\Delta}_i)\}$, $D_{1i} := \{\mathfrak{X} \in \mathscr{R} : (b_i, \tau_i) \in \{0\} \times [\varepsilon_i, T_i]\}$, $D_{2i} := \{\mathfrak{X} \in \mathscr{R} : (b_i, \tau_i) \in \{1\} \times \mathbf{\Delta}_i\}$, and $\mathbf{T}_i := [0, T_i], \mathbf{\Delta}_i := [0, \Delta_i]$. The mapping F in (11) is defined as

$$F(\mathfrak{X}) := (f(\delta, x, e), g(\delta, x, e), 0, 0, \mathbb{E}, \mathbb{E}, 0, 0), \tag{12}$$

where $f := (F_\eta, F_{\mathrm{r}}, F_{\mathrm{c}})$ and $g := (G_{\mathrm{a}}, G_{\mathrm{r}}, G_{\mathrm{f}})$ are derived from (10a). The mapping G in (11) is defined as

$$G(\mathfrak{X}) := \begin{cases} G_1(\mathfrak{X}), & \mathfrak{X} \in D_1, \\ G_2(\mathfrak{X}), & \mathfrak{X} \in D_2, \end{cases} \tag{13}$$

with $G_1(\mathfrak{X}) = \cup_{i=1}^N G_{1i}(\mathfrak{X}), D_1 = \cup_{i=1}^N D_{1i}, G_2(\mathfrak{X}) = \cup_{i=1}^N G_{2i}(\mathfrak{X}), D_2 = \cup_{i=1}^N D_{2i}$,

$$G_{1i}(\mathfrak{X}) := \begin{cases} \mathfrak{G}_{1i}(\mathfrak{X}), & \mathfrak{X} \in D_{1i}, \\ \varnothing, & \mathfrak{X} \notin D_{1i}, \end{cases} \tag{14}$$

$\mathfrak{G}_{1i}(\mathfrak{X}) = (x, e, \mu, \mathcal{M}_{1i}(x, e, m, \kappa), \delta, \Lambda_i\tau, \kappa + \Upsilon(\Gamma_i)(I - \Lambda_i)\mathbb{E}, b + (I - \Lambda_i)\mathbb{E})$;

$$G_{2i}(\mathfrak{X}) := \begin{cases} \mathfrak{G}_{2i}(\mathfrak{X}), & \mathfrak{X} \in D_{2i}, \\ \varnothing, & \mathfrak{X} \notin D_{2i}, \end{cases} \tag{15}$$

and $\mathfrak{G}_{2i}(\mathfrak{X}) = (x, \mathcal{E}_i(x, e, m, \kappa), \mathbf{\Omega}_i\mu, \mathcal{M}_{2i}(x, e, m, \kappa), \delta, \tau, \kappa, \Lambda_i b)$, where $\Lambda_i := \operatorname{diag}\{\Lambda_i^1, \ldots, \Lambda_i^N\}$ with $\Lambda_i^k = 0$ if $k = i \in \mathcal{N}$ and $\Lambda_i^k = 1$ otherwise; $\mathbf{\Omega}_i := \operatorname{diag}\{\mathbf{\Omega}_i^1, \ldots, \mathbf{\Omega}_i^N\} \in \mathbb{R}^{\mathfrak{L}\times\mathfrak{L}}$ with $\mathbf{\Omega}_i^k = \Omega_i$ if $k = i \in \mathcal{N}$ and $\mathbf{\Omega}_i^k = I$ otherwise;

$$\begin{aligned} \mathcal{M}_{1i}(x, e, m, \kappa) &:= \Phi_i m + (I - \Phi_i) M_{1i}(x, e, m, \kappa), \\ \mathcal{M}_{2i}(x, e, m, \kappa) &:= \Phi_i m + (I - \Phi_i) M_{2i}(e, m), \\ \mathcal{E}_i(x, e, m, \kappa) &:= \Phi_i m + \Upsilon(\Gamma_i)(I - \Phi_i) E_i(e, m). \end{aligned}$$

Here, $\Phi_i := \operatorname{diag}\{\Phi_i^1, \ldots, \Phi_i^N\} \in \mathbb{R}^{n_e \times n_e}, M_{1i} := (M_{1i}^1, \ldots, M_{1i}^N) \in \mathbb{R}^{n_e}, M_{2i} := (M_{2i}^1, \ldots, M_{2i}^N) \in \mathbb{R}^{n_e}$ and $E_i := (E_i^1, \ldots, E_i^N) \in \mathbb{R}^{n_e}$. If $k = i$, then $\Phi_i^k = 0, M_{1i}^k = (1 - \Upsilon(\Gamma_i)) m_i + \Upsilon(\Gamma_i)(h_i(\kappa_i, e_i) - e_i), M_{2i}^k = -e_i - m_i$ and $E_i^k = e_i + m_i$. If $k \neq i$, then $\Phi_i^k = I, M_{1i}^k = M_{2i}^k = E_i^k = 0$.

V. Main Results

In this section, we first present necessary assumptions, then design the event-triggered condition for each network, and finally establish the convergence of the tracking error.

A. Assumptions

To begin with, some assumptions are presented.

Assumption 5: There exist a function $W_i : \mathbb{R}^{n_e^i} \times \mathbb{R}^{\ell_i} \times \mathbb{R}^{n_e^i} \times \mathbb{N} \times \{0, 1\} \to \mathbb{R}_{\geq 0}$ which is locally Lipschitz in (e_i, μ_i, m_i) for all $\kappa_i \in \mathbb{N}, b_i \in \{0, 1\}$, $\alpha_{\jmath i} \in \mathcal{K}_\infty$, $\jmath \in \{1, \ldots, 6\}$, and $\lambda_i \in [0, 1)$ such that for all $(e_i, \mu_i, m_i, \kappa_i, b_i) \in \mathbb{R}^{n_e^i} \times \mathbb{R}^{\ell_i} \times \mathbb{R}^{n_e^i} \times \mathbb{N} \times \{0, 1\}$,

$$\begin{aligned} &\alpha_{1i}(|e_{\mathrm{a}}^i|) \leq W_i(e_i, \mu_i, m_i, \kappa_i, b_i) \leq \alpha_{2i}(|e_i|), && (16) \\ &W_i(e_i, \mu_i, h_i(\kappa_i, e_i) - e_i, \kappa_i + 1, 1) \\ &\quad \leq \lambda_i W_i(e_i, \mu_i, m_i, \kappa_i, 0) + \alpha_{3i}(|e_{\mathrm{f}}^i|) + \alpha_{4i}(|e_{\mathrm{r}}^i|), && (17) \\ &W_i(e_i + m_i, \Omega_i\mu_i, -e_i - m_i, \kappa_i, 0) \\ &\quad \leq W_i(e_i, \mu_i, m_i, \kappa_i, 1) + \alpha_{5i}(|e_{\mathrm{f}}^i|) + \alpha_{6i}(|e_{\mathrm{r}}^i|). && (18) \end{aligned}$$

Assumption 6: There exist a continuous function $H_{ib_i} : \mathbb{R}^{n_x} \times \mathbb{R}^{n_e} \to \mathbb{R}_{>0}$, $\sigma_{1ib_i}, \sigma_{2ib_i} \in \mathcal{K}_\infty$ and $L_{ib_i} \in [0, \infty)$ such that $\langle \frac{\partial W_i(e_i, \mu_i, m_i, \kappa_i, b_i)}{\partial e_i}, g_i(\delta, x, e) \rangle \leq H_{ib_i}(x, e) + L_{ib_i} W_i(e_i, \mu_i, m_i, \kappa_i, b_i) + \sigma_{1ib_i}(|e_{\mathrm{f}}^i|) + \sigma_{2ib_i}(|e_{\mathrm{r}}^i|)$ for all $(x, \kappa_i, b_i) \in \mathbb{R}^{n_x} \times \mathbb{N} \times \{0, 1\}$ and almost all $(e_i, \mu_i, m_i) \in \mathbb{R}^{n_e^i} \times \mathbb{R}^{\ell_i} \times \mathbb{R}^{n_e^i}$.

Assumptions 5-6 are on the e_i-subsystem. Assumption 5 is to estimate the jumps of W_i at the discrete-time instants. Specifically, (17) is for the successful transmission jumps (i.e., $\Gamma_i \geq 0$), and (18) is for the update jumps. Assumption 6 is to estimate the derivative of W_i in the continuous-time intervals, and the coupling is shown via the function H_{ib_i}. Since Assumptions 5-6 are applied to the e_i-subsystem, (17)-(18) hold with respect to e_{r}^i and e_{f}^i, which are parts of e_i and treated as the internal disturbances caused by the network. Similar conditions have been considered in existing works [3], [13], [14], where however only a common communication network and TTC are studied.

Assumption 7: There exist a locally Lipschitz function $V : \mathbb{R}^{n_x} \to \mathbb{R}_{\geq 0}$, $\alpha_{1V}, \alpha_{2V}, \zeta_{1ib_i}, \zeta_{2ib_i}, \zeta_{3ib_i}, \zeta_{4ib_i} \in \mathcal{K}_\infty$, and $\mu, \theta_{ib_i}, \gamma_{ib_i} > 0, \bar{L}_{ib_i} \in \mathbb{R}$ such that $\alpha_{1V}(|\eta|) \leq V(x) \leq \alpha_{2V}(|x|)$ for all $x \in \mathbb{R}^{n_x}$, and for all $(e_i, \mu_i, m_i, \kappa_i, b_i) \in \mathbb{R}^{n_e^i} \times \mathbb{R}^{\ell_i} \times \mathbb{R}^{n_e^i} \times \mathbb{N} \times \{0, 1\}$ and almost all $x \in \mathbb{R}^{n_x}$,

$$\begin{aligned} &\langle \nabla V(x), f(\delta, x, e) \rangle \leq -\mu V(x) - \sum_{i=1}^N \big[H_{ib_i}^2(x, e) \\ &\quad + (\gamma_{ib_i}^2 - \theta_{ib_i}) W_i^2(e_i, \mu_i, m_i, \kappa_i, b_i) - K_{ib_i}(x, e, \mu, m) \\ &\quad - \varphi_{ib_i}(z_i) + \zeta_{1ib_i}(|e_{\mathrm{f}}^i|) + \zeta_{2ib_i}(|e_{\mathrm{r}}^i|) \big], && (19) \\ &\langle \nabla \varphi_{ib_i}(z_i), f_z^i(\delta, x, e) \rangle \leq \bar{L}_{ib_i} \varphi_i(z_i) + K_{ib_i}(x, e, \mu, m) \\ &\quad + H_{ib_i}^2(x, e) + \zeta_{3ib_i}(|e_{\mathrm{f}}^i|) + \zeta_{4ib_i}(|e_{\mathrm{r}}^i|), && (20) \end{aligned}$$

where H_{ib_i} is defined in Assumption 6, $\varphi_{ib_i} : \mathbb{R}^{n_z^i} \to \mathbb{R}_{\geq 0}$ is a locally Lipschitz function with $\varphi_{ib_i}(0) = 0$, and $K_{ib_i} : \mathbb{R}^{n_x} \times \mathbb{R}^{n_e} \times \mathbb{R}^{\mathfrak{L}} \times \mathbb{R}^{n_e} \to \mathbb{R}_{\geq 0}$ is a continuous function.

Assumption 7 is on the x-subsystem, and imply that the η-subsystem satisfies the ISS-like property from $(\sum_{i=1}^N W_i, e_{\mathrm{f}}, e_{\mathrm{r}})$ to η. This assumption is reasonable due to the implementation of the emulation-based approach, where the controller is assumed to be known *a priori* to ensure the tracking performance robustly in the network-free case. Hence, in the presence of the network, $(\sum_{i=1}^N W_i, e_{\mathrm{f}}, e_{\mathrm{r}})$ is treated as the disturbance. Moreover, (20) provides the growth bound on the derivative of the function φ_{ib_i} on the flow. Finally, for the linear case, Assumptions 5-7 can be represented as a whole linear matrix inequality; see e.g. [23].

B. Decentralized Event-Triggered Mechanisms

With Assumption 5-7, the ETM is designed for each network, that is, the function Γ_i in (10) is defined as a mapping from $\mathbb{R}^{n_z^i} \times \mathbb{R}^{n_e^i} \times \mathbb{R}^{\ell_i} \times \mathbb{R}^{n_e^i} \times \mathbb{N} \times \{0, 1\}$ to $\mathbb{R}$:

$$\begin{aligned} \Gamma_i(z_i, e_i, \mu_i, m_i, \kappa_i, b_i) &:= -b_i \gamma_{ib_i} W_i^2(e_i, \mu_i, m_i, \kappa_i, b_i) \\ &\quad - (1 - b_i)\rho_i \bar{\lambda}_i \varphi_{ib_i}(z_i), \end{aligned} \tag{21}$$

where W_i is from in Assumption 5, φ_{ib_i} is from in Assumption 7, $\rho_i \geq 0$ is a design parameter with $\rho_i \in [0, \bar{\rho}_i)$, and

$$\bar{\lambda}_i := \max\left\{\lambda_i, (1 - \rho_i \bar{L}_{i0})^{-1} \rho_i \gamma_{i0}\right\}, \tag{22}$$

$$\bar{\rho}_i := \begin{cases} 1, & \bar{L}_{i0} \leq -\gamma_{i0}, \\ \min\left\{1, (\bar{L}_{i0} + \gamma_{i0})^{-1}\right\}, & \bar{L}_{i0} > -\gamma_{i0}, \end{cases} \tag{23}$$

with λ_i in Assumption 5 and $\gamma_{i0}, \bar{L}_{i0}$ in Assumption 7.

With the function (21), the event-triggered condition is $\Gamma_i \geq 0$, which is similar to those in [10], [23] for the ETC in different contexts. One difference between (21) and the existing ones lies in the local logical variable b_i, which leads to two cases in (21). Since the case $b_i = 1$ implies that the update event will occur at the arrival instant, the ETM is not needed and $\Gamma_i = -\gamma_{i1} W_i^2(e_i, \mu_i, m_i, \kappa_i, 1) < 0$, which thus implies that the ETM will not be implemented in this case. In contrast, for the case $b_i = 0$, the next event is the transmission event, and the ETM is implemented to determine whether the sampled measurement will be transmitted. Hence, $\Gamma_i = \gamma_{i0} W_i^2(e_i, \mu_i, m_i, \kappa_i, 0) - \rho_i \bar{\lambda}_i \varphi_{i0}(z_i) \geq 0$ will be verified in this case. As a result, the parameters in (22)-(23) only depend on the case $b_i = 0$, and all designed event-triggered conditions are consistent with the transmission setup and decentralized since only local information is involved in each event-triggered condition.

Finally, consider the following differential equation

$$\dot{\phi}_{ib_i} = -2L_{ib_i}\phi_{ib_i} - \gamma_{ib_i}((1+\varrho_{ib_i})\phi_{ib_i}^2 + 1), \tag{24}$$

where $i \in \mathcal{N}$, $L_{ib_i} \geq 0$ is given in Assumption 6, and $\gamma_{ib_i} > 0$ is given in Assumption 7. $\varrho_{ib_i} \in (0, \bar{\lambda}_i^{-2}\phi_{ib_i}^{-2}(0) - 1)$ and $\phi_{ib_i}(0) \in (1, \bar{\lambda}_i^{-1})$ with $\bar{\lambda}_i$ in (22). From Claim 1 in [14], the solutions to (24) are strictly decreasing as long as $\phi_{ib_i} \geq 0$.

C. Tracking Performance Analysis

Now we are ready to state the main result of this section.

Theorem 1: Consider the system (13) and let Assumptions 1-7 hold. If the MASP T_i and the MAD Δ_i satisfy

$$\gamma_{i0}\phi_{i0}(\tau_i) \geq (1+\varrho_{i1})\bar{\lambda}_i^2 \gamma_{i1}\phi_{i1}(0), \quad \tau_i \in \mathbf{T}_i, \tag{25a}$$

$$\gamma_{i1}\phi_{i1}(\tau_i) \geq (1+\varrho_{i0})\gamma_{i0}\phi_{i0}(\tau_i), \quad \tau_i \in \mathbf{\Delta}_i, \tag{25b}$$

where ϕ_{ib_i} is the solution to (24) with $\phi_{ib_i}(0), \phi_{ib_i}(T_i) > 0$, then the system (11) is ISS from $(e_\mathrm{r}, e_\mathrm{f})$ to (η, e_a).

Theorem 1 implies the convergence of the tracking error to a region around the origin, and the size of the convergence region depends on the network-induced error $(e_\mathrm{r}, e_\mathrm{f})$. If the feedforward control inputs are transmitted directly to the plant and reference system, then $e_\mathrm{f} = 0, \varphi_1 \equiv 0$, and thus the convergence region can be further smaller. Comparing with previous works [12]–[14], [18], [23] on NCS and [15], [16] on MAS, the event-triggered tracking control problem is studied here for NQCS under decentralized ETMs and network constraints. In particular, quantization effects and/or time delays are not considered in [12], [14]–[16], [18], [23], and the time-triggered tracking control is addressed in [13], [14]. Therefore, a unified model is developed here and the tracking performance is achieved via less communication.

VI. Numerical Example

Consider two connected single-link robot arms, whose dynamics are presented as $(i = 1, 2)$

$$\begin{aligned} \dot{\mathfrak{q}}_\mathrm{p}^{i1} &= \mathfrak{q}_\mathrm{p}^{i2}, \\ \dot{\mathfrak{q}}_\mathrm{p}^{i2} &= -a_i \sin \mathfrak{q}_\mathrm{p}^{i1} + \sum_{j=1}^{2} b_{ij}(\mathfrak{q}_\mathrm{p}^{1j} - \mathfrak{q}_\mathrm{p}^{2j}) + c_i u_i, \end{aligned} \tag{26}$$

where $\mathfrak{q}_\mathrm{p}^i := (\mathfrak{q}_\mathrm{p}^{i1}, \mathfrak{q}_\mathrm{p}^{i2}) \in \mathbb{R}^2$ with the configuration coordinate $\mathfrak{q}_\mathrm{p}^{i1}$ and the velocity $\mathfrak{q}_\mathrm{p}^{i2}$, both of which are measurable, $u_i \in \mathbb{R}$ is the input torque, and $a_i, c_i > 0, b_{ij} \in \mathbb{R}$ are certain constants. The reference system is given by

$$\begin{aligned} \dot{\mathfrak{q}}_\mathrm{r}^{i1} &= \mathfrak{q}_\mathrm{r}^{i2}, \\ \dot{\mathfrak{q}}_\mathrm{r}^{i2} &= -a_i \sin \mathfrak{q}_\mathrm{r}^{i1} + \sum_{j=1}^{2} b_{ij}(\mathfrak{q}_\mathrm{r}^{1j} - \mathfrak{q}_\mathrm{r}^{2j}) + c_i u_\mathrm{f}^i, \end{aligned} \tag{27}$$

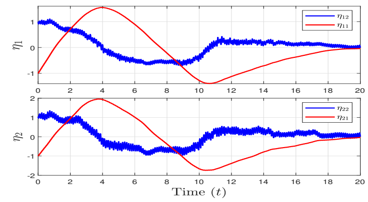

Fig. 1. Tracking errors under the RR protocol case and the ETM (28), where $T_1 = T_2 = 0.01$ and $\Delta_1 = \Delta_2 = 0.0015$.

where $\mathfrak{q}_\mathrm{r}^i := (\mathfrak{q}_\mathrm{r}^{i1}, \mathfrak{q}_\mathrm{r}^{i2}) \in \mathbb{R}^2$ are the measurable reference state, and $u_\mathrm{f}^i = 5\sin(5t)$ is the feedforward input. In the network-free case, the feedback controller is designed as $u_\mathrm{c}^i = -c_i^{-1}[a_i(\sin(\mathfrak{q}_\mathrm{p}^{i1}) - \sin(\mathfrak{q}_\mathrm{r}^{i1})) - (\mathfrak{q}_\mathrm{p}^{i1} - \mathfrak{q}_\mathrm{r}^{i1}) - (\mathfrak{q}_\mathrm{p}^{i2} - \mathfrak{q}_\mathrm{r}^{i2})]$ such that the tracking error is asymptotically stable.

We consider the case that the communication between the controller and the plant is via the ETMs and two communication networks and quantizers. The controller is applied via the ZOH devices and the networks has $\ell_i = 3$ nodes for $\mathfrak{q}_\mathrm{p}^{i1}$, $\mathfrak{q}_\mathrm{p}^{i2}$ and u_i, respectively. Set $\max_{i\in\{1,2\}, j\in\{1,2,3\}} \mathfrak{n}_j^i = 0.8$ and $\max_{i\in\{1,2\}, j\in\{1,2,3\}} \Omega_j^i = 0.6$. Hence, the feedback controller is $u_\mathrm{c}^i = -c_i^{-1}[a_i(\sin(\hat{\mathfrak{q}}_\mathrm{p}^{i1}) - \sin(\hat{\mathfrak{q}}_\mathrm{r}^{i1})) - (\hat{\mathfrak{q}}_\mathrm{p}^{i1} - \hat{\mathfrak{q}}_\mathrm{r}^{i1}) + (\hat{\mathfrak{q}}_\mathrm{p}^{i2} - \hat{\mathfrak{q}}_\mathrm{r}^{i2})]$. u_f^i is assumed to be transmitted to (27) directly, and $\hat{\mathfrak{q}}_\mathrm{r}^{i1}, \hat{\mathfrak{q}}_\mathrm{r}^{i2}$ are implemented in the ZOH fashion. That is, u_c^i knows but does not depend on $\mathfrak{q}_\mathrm{r}^{i1}, \mathfrak{q}_\mathrm{r}^{i2}$.

Let $D_i = \sqrt{3}\max\{1 + a_i, c_i\}$. From [13], we choose the appropriate Lyapunov function $W_i(e_i, \mu_i, m_i, \kappa_i, \tau_i, b_i)$. For instance, $W_i(e_i, \mu_i, m_i, \kappa_i, \tau_i, b_i) := \omega_i |e_\mathrm{a}^i| + |\mu_i|$ for the TOD protocol, where $\omega_i \in (0, (1 - \max_j \Omega_j^i)/\max_j \mathfrak{n}_j^i)$. $|\partial W(e_i, \mu_i, m_i, \kappa_i, \tau_i, b_i)/\partial e_i| \leq M_i$ with $M_i = \sqrt{\ell_i}$ for the RR protocol case and $M_i = 1$ for the TOD protocol case. Assumption 5 holds with $\lambda_i = \max\{\sqrt{(\ell_i - 1)/\ell_i}, \omega_i \mathfrak{m}_i \mathfrak{n}_j^i + \Omega_j^i\}$ and $\alpha_{3i} = \alpha_{4i} = \alpha_{5i} = \alpha_{6i} = 0$. Assumption 6 holds with $L_{i0} = M_i D_i, L_{i1} = M_i^2 D_i/\lambda_i$, $H_{i0}(x, e) = H_{i1}(x, e) = M_i(|\eta_{i2}| + |(b_{i1} - 1)\eta_{i1} + (b_{i2} - 1)\eta_{i2}| + |b_{i1}\eta_{(3-i)1}| + |b_{i2}\eta_{(3-i)2}|)$, $\sigma_{1i0}(v) = \sigma_{1i1}(v) = c_i M_i v$ and $\sigma_{2i0}(v) = \sigma_{2i1}(v) = 2a_i M_i v$ for $v \geq 0$.

To verify Assumption 7, define $V(\eta) := \sum_{i=1}^2 \phi_{i1}\eta_{i1}^2 + \phi_{i2}\eta_{i1}\eta_{i2} + \phi_{i3}\eta_{i2}^2$ with $\phi_{i1}, \phi_{i2}, \phi_{i3} \in \mathbb{R}$. Let $\hat{a}_i, \tilde{a}_i \in [-a_i, a_i]$ such that $a_i[\sin(\eta_{i1} + \mathfrak{q}_\mathrm{r}^{i1}) - \sin(\eta_{i1} + \mathfrak{q}_\mathrm{r}^{i1} + e_\eta^{i1} + e_\mathrm{r}^{i1})] = \hat{a}_i(e_\eta^{i1} + e_\mathrm{r}^{i1})$ and $a_i[\sin(\mathfrak{q}_\mathrm{r}^{i1}) - \sin(\mathfrak{q}_\mathrm{r}^{i1} + e_\mathrm{r}^{i1})] = \tilde{a}_i e_\mathrm{r}^{i1}$. Thus, $\langle \nabla V(\eta), F_\eta(\delta, x, e, \mu)\rangle \leq \sum_{i=1}^2 [-\phi_{i1}\eta_{i1}^2 + (2\phi_{i1} - 2\phi_{i3} - \phi_{i2})\eta_{i1}\eta_{i2} - (2\phi_{i3} - \phi_{i1})\eta_{i2}^2 + (\phi_{i2}\eta_{i1} + 2\phi_{i3}\eta_{i2})(b_{i1}(\eta_{11} - \eta_{21}) + b_{i2}(\eta_{12} - \eta_{22})) + 0.5(\varrho_{i0}^{-1} + \varrho_{i1}^{-1})(\phi_{i1}\eta_{i1} + 2\phi_{i3}\eta_{i2})^2 + 0.5\varrho_{i0}D_i|e_i|^2 + \varrho_{i1}(4a_i^2|e_\mathrm{r}^i|^2 + c_i^2|e_\mathrm{f}^i|^2)]$, where $\varrho_{i0}, \varrho_{i1} > 0$ are given in (24). If ϕ_1, ϕ_2, ϕ_3 are such that $-H_{ib_i}^2(x, e) - K_{ib_i}(x, e, \mu, m) - \varphi_{ib_i}(z_i) \geq -\phi_{i1}\eta_{i1}^2 + (2\phi_{i1} - 2\phi_{i3} - \phi_{i2})\eta_{i1}\eta_{i2} - (2\phi_{i3} - \phi_{i1})\eta_{i2}^2 + (\phi_{i1}\eta_{i1} + 2\phi_{i3}\eta_{i2})(b_{i1}(\eta_{11} - \eta_{21}) + b_{i2}(\eta_{12} - \eta_{22})) + 0.5(\varrho_{i0}^{-1} + \varrho_{i1}^{-1})(\phi_{i1}\eta_{i1} + 2\phi_{i3}\eta_{i2})^2$, then Assumption 7 holds with $\theta_{ib_i}(v) = \pi_i v^2, \gamma_{i0} = \sqrt{\pi_i + \varrho_{i0}D_i^2}, \gamma_{i1} = \sqrt{\pi_i + \varrho_{i1}M_i D_i^2/\lambda_i^2}, \zeta_{1ib_i}(v) = \varrho_{i1}a_i^2|v|^2$, $\zeta_{4ib_i}(v) = \varrho_{i1}a_i^2|v|^2$ and $\pi_i > 0$ is arbitrarily small.

Let $\phi_{11} = 8, \phi_{12} = 12, \phi_{13} = 6, \phi_{21} = 5, \phi_{22} = 7, \phi_{23} = 9, a_1 = 9.81 * 0.2, a_2 = 9.81 * 0.3, c_1 = 2, c_2 = 4, \pi_i = 0.005, \varrho_{i0} = 20$ and $\varrho_{i1} = \varrho_{i0}M_i/\lambda_1$. Thus, $L_{10} =$

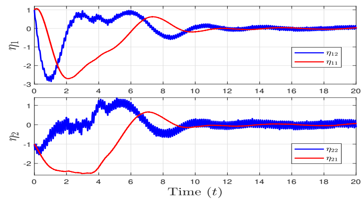

Fig. 2. Tracking errors under the TOD protocol case and the ETM (28), where $T_1 = T_2 = 0.014$ and $\Delta_1 = \Delta_2 = 0.0025$.

TABLE I

COMPARISON OF TRANSMISSION NUMBERS IN DIFFERENT CASES

Network	Event-triggering		Time-triggering
	RR case	TOD case	[13], [14]
Network 1	887	1606	2000
Network 2	1448	1838	2000

$8.8860, L_{11} = 18.8501, L_{20} = 12, L_{21} = 25.4558, \gamma_{10} = 22.9436, \gamma_{11} = 48.6706, \gamma_{20} = 30.9839, \gamma_{21} = 65.7267$ for the RR protocol case; $L_{10} = 5.1303, L_{11} = 10.8831, L_{20} = 6.9282, L_{21} = 14.6969, \gamma_{10} = 22.9436, \gamma_{11} = 28.1, \gamma_{20} = 30.9839, \gamma_{21} = 37.9473$ for the TOD protocol case. By the detailed computation, $\bar{\rho}_1 = 0.0501$ and $\bar{\rho}_2 = 0.0371$ for RR and TOD protocols. Hence, $\rho_i \in (0, \bar{\rho}_i)$, and the ETM is

$$\Gamma_i = -b_i\gamma_i|(e^i_\eta, e^i_{\mathrm{r}}, \mu_i)|^2 + (1-b_i)\rho_i\bar{\lambda}_i|\eta_i|^2 \geq 0. \quad (28)$$

Set $\phi_{10}(0) = \phi_{11}(0) = 1.1023$ and $\phi_{20}(0) = \phi_{21}(0) = 0.8816$ for the RR protocol case, and we have $T_1 = 0.0256, \Delta_1 = 0.0064, T_2 = 0.0161$, and $\Delta_2 = 0.0026$. Set $\phi_{10}(0) = \phi_{11}(0) = \phi_{20}(0) = \phi_{21}(0) = 1.0468$ for the TOD protocol case, and we have $T_1 = 0.0279, \Delta_1 = 0.00445, T_2 = 0.02115$, and $\Delta_2 = 0.0032$. To simplify the simulation, the transmission intervals and the transmission delays are constants. Under the ETM (28), Figs. 1-2 show the convergence and boundedness of tracking errors in RR and TOD protocol cases, respectively.

The numbers of information transmission in different cases are presented in Table I. We consider 23 units of time for the RR case and 32 units of time for the TOD case. The transmission numbers are the same (2000 times) in the time-triggered case [13], [14], whereas the transmission numbers are reduced to different extents in the event-triggered case. In particular, the transmission numbers of two networks in the RR case are less than these in the TOD case.

VII. CONCLUSIONS

We presented a Lyapunov-based emulation approach for the event-triggered tracking control problem of NQCS, where the information communication is via multiple asynchronous networks. To deal with this problem, we proposed a new hybrid model, and then established sufficient conditions and designed decentralized event-triggered mechanisms. The tradeoff between the MASP and the MAD was determined to guarantee the tracking performance. The effectiveness of the proposed approach was illustrated via a numerical example.

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