Decentralized Control of Multi-Agent Systems Under Acyclic Spatio-Temporal Task Dependencies

Gregorio Marchesini, Siyuan Liu, Lars Lindemann and Dimos V. Dimarogonas

***Abstract*— We introduce a novel distributed sampled-data control method tailored for heterogeneous multi-agent systems under a global spatio-temporal task with acyclic dependencies. Specifically, we consider the global task as a conjunction of independent and collaborative tasks, defined over the absolute and relative states of agent pairs. Task dependencies in this form are then represented by a task graph, which we assume to be acyclic. From the given task graph, we provide an algorithmic approach to define a distributed sampled-data controller prioritizing the fulfilment of collaborative tasks as the primary objective, while fulfilling independent tasks unless they conflict with collaborative ones. Moreover, communication maintenance among collaborating agents is seamlessly enforced within the proposed control framework. A numerical simulation is provided to showcase the potential of our control framework.**

I. Introduction

Over the past decade, Control Barrier Functions (CBFs) have emerged as versatile control solutions in the realm of cyber-physical systems. Indeed, CBF-like constraints have enabled the seamless integration of multiple control objectives into optimal control frameworks, such as model predictive control and quadratic programming-based controllers, as well as analytical feedback control laws [1]–[5].

This work is concerned with the development of a novel distributed sampled-data controller applicable to heterogeneous multi-agent systems subject to communication and spatio-temporal constraints expressed as Signal Temporal Logic (STL) specifications. For multi-agent systems, [6]–[10] have developed distributed/decentralized control laws meeting several objectives, such as formation control and stabilization, as well as connectivity and collision avoidance constraints. Nevertheless, the interplay between connectivity, safety, and spatio-temporal task fulfilment requires further investigation. In [7], [8], the proposed CBF-enabled distributed control law requires global knowledge of the entire system state, although the control inputs are computed locally. Hence, the framework is only applicable to systems with all-to-all communication. In [6], a novel decentralized control law merging stabilization and safety objectives into Lyapunov-like barrier functions for controlling a swarm of differential drive robots was proposed. In their approach, agents are assumed to be connected to a central broadcasting agent from which goal destinations are dispatched, while collisions are resolved by a local avoidance scheme. The works in [9], [10] offer a decentralized formation control approach with input limitations and collision/connectivity constraints satisfied through suitable CBFs; however, more general objectives, like spatio-temporal tasks, are not considered therein. At the same time, none of the previous approaches is designed to safely handle control inputs expressed in a zero-order hold fashion, as typically applied from real embedded controllers.

Given these previous results, we propose a quadratic programming-based decentralized control framework that incorporates STL specification (tasks), as well as communication constraints in the form of CBF-like constraints. Building upon [11]–[13], we provide a definition of sampled-data control barrier functions, which are applied to encode the control objectives and provide continuous-time guarantees over their satisfaction. Furthermore, following our previous work [14], we assume a global STL task assigned to the MAS can be decomposed as a conjunction of independent and collaborative tasks defined over the absolute and relative state of couples of agents in the system, respectively, from which an acyclic task graph is derived. The established acyclic task graph is then applied to define a leader-follower structure over each edge of the graph, such that leaders are responsible for the satisfaction of collaborative tasks over their assigned edge, while followers reduce their maximum control authority to favour the satisfaction of the former task. In summary, the main contribution of this work is the development of a novel sampled-data decentralized control framework that guarantees continuous-time satisfaction of spatio-temporal tasks with acyclic dependencies expressed as STL specifications, as well as communication constraints. The presentation is organised as follows: Section II summarizes preliminaries and problem formulation. In Section III we define sampled-data CBFs and how STL tasks can be encoded in this framework. In Section IV, our decentralized control scheme is presented, while Sections V-VI provide numerical simulations and conclusions.

Notation: Bold letters indicate vectors while capital letters indicate matrices. Vectors are considered to be column vectors. The notation $|\mathcal{A}|$ indicates the cardinality of the set $\mathcal{A}$. For a given scalar $\gamma \in \mathbb{R}$, let $\gamma\mathcal{A} := \{\gamma a | a \in \mathcal{A}\}$. Let $\oplus$ and $\ominus$ indicate the Minkowski sum and difference of two sets. Let $blk(A_1, \dots A_n)$ represent the block diagonal matrix with blocks given by the matrices $A_1, \dots A_n$. We use the notation

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$\partial_x := \frac{\partial}{\partial x}$ to identify the partial derivative w.r.t to x. The set $\mathbb{R}_+$ denotes the non-negative real numbers while the set $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$ is the set of natural numbers including zero.

II. Preliminaries and problem formulation

Let $\mathcal{V} = \{1, \dots N\}$ be the set of indices assigned to each agent in a multi-agent system and let each agent be governed by an input-affine dynamics of the form:

$$\dot{\boldsymbol{x}}_i = f_i(\boldsymbol{x}_i) + g_i(\boldsymbol{x}_i)\boldsymbol{u}_i, \tag{1}$$

with state $\boldsymbol{x}_i \in \mathbb{R}^n \subset \mathbb{X}_i$ and input vector $\boldsymbol{u}_i \in \mathbb{R}^{m_i} \subset \mathbb{U}_i$, where m_i is the input dimension for agent i. Let $\mathbb{X}_i$ be compact and $\mathbb{U}_i$ be compact and convex. Moreover, let the functions $f_i : \mathbb{R}^n \to \mathbb{R}^n$, $g_i : \mathbb{R}^n \to \mathbb{R}^{n \times m_i}$ be locally Lipschitz continuous over $\mathbb{X}_i$. The global MAS dynamics is then compactly written as

$$\dot{\boldsymbol{x}} = \bar{f}(\boldsymbol{x}) + \bar{g}(\boldsymbol{x})\boldsymbol{u} \tag{2}$$

where $\boldsymbol{x} := [\boldsymbol{x}_1^T, \dots \boldsymbol{x}_N^T]^T, \boldsymbol{u} := [\boldsymbol{u}_1^T, \dots \boldsymbol{u}_N^T]^T, \bar{f}(\boldsymbol{x}) := [f_1^T, \dots f_N^T]^T$ and $\bar{g}(\boldsymbol{x}) := blk(g_1, \dots g_N)$. Let $\boldsymbol{e}_{ij} := \boldsymbol{x}_i - \boldsymbol{x}_j \in \mathbb{X}_{ij}$ represent the relative state vector for each $i, j \in \mathcal{V}$ where $\mathbb{X}_{ij} := \mathbb{X}_i \ominus \mathbb{X}_j$. Also let $\boldsymbol{p}_i = S\boldsymbol{x}_i$ represent the position of each agent i for a given selection matrix[1] S and let the relative position vector $\boldsymbol{p}_{ij} := \boldsymbol{p}_i - \boldsymbol{p}_j$ and the communication radius $r_c > 0$ such that it is assumed that at time t agents i and j can collaborate toward the satisfaction of a collaborative task (defined in Sec. II-A) if $\|\boldsymbol{p}_{ij}\| \leq r_c$.

A. Signal Temporal Logic and Task Graph

Signal Temporal Logic (STL) is a predicate logic applied to formally define spatio-temporal behaviours for continuous time signals. Let the boolean-valued predicates $\mu(h(\boldsymbol{x})) := \begin{cases} \top & \text{if } h(\boldsymbol{x}) \geq 0 \\ \bot & \text{if } h(\boldsymbol{x}) < 0, \end{cases}$ where $h : \mathbb{R}^{n \cdot N} \to \mathbb{R}$ is a real-valued *predicate function* encoding some spatial constraints for the system. In the following, we consider predicate functions of the form $h_i := h_i(\boldsymbol{x}_i)$ or $h_{ij} := h_{ij}(\boldsymbol{e}_{ij})$ for $i, j \in \mathcal{V}$. We refer to the former as *independent* predicate function and the latter as *collaborative* predicate function.

Assumption 1: Individual/Collaborative predicate functions h_i and h_{ij} $\forall i, j \in \mathcal{V}$ are concave functions.

Typical examples of functions that fit into our framework are 1) communication maintenance $h_{ij}(\boldsymbol{e}_{ij}) = r_c^2 - \|\boldsymbol{p}_{ij}\|^2$ 2) relative position formation $h_{ij}(\boldsymbol{e}_{ij}) = r^2 - (\boldsymbol{p}_{ij} - \boldsymbol{c}_{ij})^T P(\boldsymbol{p}_{ij} - \boldsymbol{c}_{ij})$ with formation vector $\boldsymbol{c}_{ij} \in \mathbb{R}^n$, $r > 0$ and positive definite matrix $P \in \mathbb{R}^{n \times n}$, 3) polyhedral formation $h_{ij}(\boldsymbol{e}_{ij}) = -log\big(\sum_{i=1}^{p} exp(\boldsymbol{a}_i^T \boldsymbol{e}_{ij} - b_i)\big)$ where $\boldsymbol{a}_i \in \mathbb{R}^n$ and $b_i \in \mathbb{R}$ define the p hyperplanes $\{\boldsymbol{x} \in \mathbb{R}^n | \boldsymbol{a}_i^T \boldsymbol{x} - b_i \geq 0\}$ for some $p \geq 1$, 4) Go-to-goal specifications like $h(\boldsymbol{x}_i) = \epsilon^2 - \|\boldsymbol{p}_i - \boldsymbol{t}_i\|^2$ for some $\epsilon \geq 0$ with goal position $\boldsymbol{t}_i \in \mathbb{R}^3$. We consider the following Signal Temporal Logic grammar

$$\varphi_i := F_{[a,b]}\mu_i | G_{[a,b]}\mu_i, \tag{3a}$$

$$\varphi_{ij} := F_{[a,b]}\mu_{ij} | G_{[a,b]}\mu_{ij}, \tag{3b}$$

$$\phi_{ij} := \wedge_k \varphi_{ij}^k, \; \phi_i := \wedge_k \varphi_i^k, \tag{3c}$$

where $\mu_{ij} := \mu(h_{ij}(\boldsymbol{e}_{ij}))$ and $\mu_i := \mu(h_i(\boldsymbol{x}_i))$. The operators G and F are the *always* and *eventually* operators. For a given STL task ϕ, the notation $\boldsymbol{x}(t) \models \phi$ indicates that the state trajectory $\boldsymbol{x}(t)$ satisfies ϕ. All the conditions under which $\boldsymbol{x}(t)$ satisfies ϕ (the STL semantics) are given in [15], [16] and are not revised here due to space limitations. Although, recall that $\boldsymbol{x}(t) \models G_{[a,b]}\mu \Leftrightarrow \mu(h(\boldsymbol{x}(t))) := \top, \forall t \in [a, b]$ and $\boldsymbol{x}(t) \models F_{[a,b]}\mu \Leftrightarrow \exists \tau \in [a, b] : \mu(h(\boldsymbol{x}(\tau))) := \top$. Consistently with previous nomenclature, we refer to tasks of type (3a) and (3b) as individual and collaborative tasks respectively.

Collaborative and independent tasks from fragment (3) naturally induce the definition of an undirected *task graph* $\mathcal{G}_\psi(\mathcal{V}, \mathcal{E}_\psi)$ where $\mathcal{E}_\psi \subset \mathcal{V} \times \mathcal{V}$ with $(i, j) \in \mathcal{E}_\psi$ if there exists a collaborative task ϕ_{ij} as per (3b) among i and j. Due to the limited communication radius r_c, we also introduce the time-varying undirected communication graph $\mathcal{G}_c(\mathcal{V}, \mathcal{E}_c(t))$ where $(i, j) \in \mathcal{E}_c(t)$ if $\|\boldsymbol{p}_{ij}(t)\| \leq r_c$ and let $\mathcal{N}_\psi(i) = \{j \in \mathcal{V} | (i, j) \in \mathcal{E}_\psi\}$, $\mathcal{N}_c(i) = \{j \in \mathcal{V} | (i, j) \in \mathcal{E}_c(t)\}$ be the task and communication neighbours for agent i. The global task ψ assigned to the MAS is then compactly written as

$$\psi = \psi_{ind} \wedge \psi_{col} \tag{4}$$

where $\psi_{ind} := \wedge_{i \in \mathcal{V}} \phi_i, \quad \psi_{col} := \wedge_{i \in \mathcal{V}} (\wedge_{j \in \mathcal{N}_\psi(i)} \phi_{ij})$.

Assumption 2: The task graph $\mathcal{G}_\psi$ is acyclic.

While Assmp. 2 can be considered restrictive as it imposes the task dependencies to follow a tree structure, the authors recently proposed a formula rewrite decomposition for tasks in fragment (3) such that cyclic task graphs can be reformulated into acyclic ones under mild assumption [14].

B. Sampled-Data Control

Consider a sampling interval $\delta t > 0$, initial time t^0, sampling instants $t^k := t^0 + k\delta t$ for $k \in \mathbb{N}_0$ and let the shorthand notation $a^k := a(t^k)$ for any general quantity a (scalar or vectorial). We consider each agent to be subject to a piece-wise constant (p.w.c) input $\boldsymbol{u}_i : \mathbb{R}_+ \to \mathbb{U}$ such that

$$\boldsymbol{u}_i(t) = \boldsymbol{u}_i^k \in \mathbb{U}_i, \quad \forall t \in [t^k, t^{k+1}), \forall i \in \mathcal{V}. \tag{5}$$

Furthermore, let $\mathcal{U}_i^{\delta t}$ be the set of p.w.c inputs $\boldsymbol{u}_i(t)$ satisfying (5) for agent i and $\mathcal{X}_i$ as the set of state trajectories $\boldsymbol{x}_i : \mathbb{R}^+ \to \mathbb{X}_i$ satisfying (1) under $\boldsymbol{u}_i(t) \in \mathcal{U}_i^{\delta t}$.

Definition 1: Consider (2) and sampling interval $\delta t > 0$. The reachable set from state $\boldsymbol{x}^k$ is defined as $\mathcal{R}(\boldsymbol{x}^k, \delta t) := \{\boldsymbol{x} \in \mathbb{X} | \boldsymbol{x} = \int_{t^k}^{t} \bar{f}(\boldsymbol{x}(t)) + \bar{g}(\boldsymbol{x}(t))\boldsymbol{u}^k \, dt, \quad \boldsymbol{x}(t^k) = \boldsymbol{x}^k, \forall \boldsymbol{u}^k \in \mathbb{U}, \forall t \in [t^k, t^k + \delta t)\}$.

We hereafter consider that it is possible to compute the reachable set $\mathcal{R}(\boldsymbol{x}^k, \delta t)$ for a given initial state $\boldsymbol{x}^k$, while an upper bound of the actual reachable set is sufficient to preserve the soundness of our proposed approach.

C. Problem Formulation

The following problem formulation is given.

Problem 1: Given a time-varying communication graph $\mathcal{G}_c(t)$, and acyclic task graph $\mathcal{G}_\psi$ associated with the global task $\psi := \psi_{col} \wedge \psi_{ind}$ as per (4), define a decentralized p.w.c

[1] A selection matrix applies to select $m \leq n$ unique elements from a vector of dimensions n such that $S \in \mathbb{R}^{m \times n}$ with $S[i, j] \in \{0, 1\}$; $\sum_{j=1}^{n} S[i, j] = 1, \forall i = 1, \dots m$ and $\sum_{i=1}^{m} S[i, j] = 1, \forall j = 1, \dots n$.

control law $\boldsymbol{u}_i(t) \in \mathcal{U}_i^{\delta t}$ as per (5) such that 1) $\boldsymbol{x}(t) \models \psi$ and 2) $\mathcal{G}_\psi \subseteq \mathcal{G}_c(t),\ \forall t \in \mathbb{R}_+$ if $\mathcal{G}_\psi \subseteq \mathcal{G}_c(t^0)$.

By the term decentralized we intend that the control input $\boldsymbol{u}_i(t)$ at time t for agent i is computed based on the state of the neighbouring agents $j \in \mathcal{N}_c(i)$ rather than the global state $\boldsymbol{x}$ of the system. Hence, the communication maintenance objective 2) in Problem 1 is a precondition for the satisfaction of objective 1). In Theorem 1 our proposed feedback control law satisfying objective 1) under the assumption that $\mathcal{G}_\psi \subseteq \mathcal{G}_c(t)$ for all $t \in \mathbb{R}_+$ is provided. On the other hand, Corollary 1 clarifies how the same control solution handles the enforcement of the communication objective $\mathcal{G}_\psi \subseteq \mathcal{G}_c(t),\ \forall t \in \mathbb{R}_+$, by including communication tasks in conjunction with ψ_{col}.

III. Sampled-data time-varying CBFs

Sampled-data Control Barrier Functions (sdCBF) are considered as suitable framework to encode STL tasks for our distributed control approach. Let the scalar-valued function $b : \mathcal{D} \times \mathbb{R}_+ \to \mathbb{R}$ with $\mathcal{D} \subset \mathbb{R}^{n \cdot N}$ being open and with corresponding level set $\mathcal{C}(t)$ defined as

$$\mathcal{C}(t) = \{\boldsymbol{x} \in \mathcal{D} |\ b(\boldsymbol{x}, t) \geq 0\}. \tag{6}$$

The following definitions are given

Definition 2: A scalar function $b : \mathcal{D} \times \mathbb{R}_+ \to \mathbb{R}$ is *sd-differentiable* if it is differentiable over $\mathcal{D}$, piece-wise differentiable over $\mathbb{R}_+$ with discontinuities over a discrete set $S = \{\tau_0, \dots \tau_q\}$ for some $q \geq 0$ and such that $\partial_{\boldsymbol{x}} b(\boldsymbol{x}, t)$, $\partial_t b(\boldsymbol{x}, t)$ are Lipschitz continuous over $\mathcal{D} \times [\tau_i, \tau_{i+1}), \forall i = 0, \dots q-1$. Moreover, 1) the left limit $\lim_{t^- \to \tau_i} \mathcal{C}(t) \subseteq \mathcal{C}(\tau_i),\ \forall \tau_i \in S$, 2) $\tau_i = t^k$ for some $k \in \mathbb{N}_0$.

In Def. 2, Lipschitz continuity of the gradients $\partial_{\boldsymbol{x}} b(\boldsymbol{x}, t)$ implies Lipschitz continuity of the Lie derivative $L_{\bar{f}} b(\boldsymbol{x}, t)$ and $L_{\bar{g}} b(\boldsymbol{x}, t)$. At the same time condition 1) intuitively imposes that $\mathcal{C}(t)$ is at least locally expanding at points of time discontinuity in S. Eventually condition 2) enforces times of discontinuities to correspond to sampling instants.

Definition 3: The set $\mathcal{C}(t)$ is *forward invariant* over the time interval $[t^0, \tau]$ under $\boldsymbol{u}(t) \in \prod_i \mathcal{U}_i^{\delta t}$ if there exists a unique solution $\boldsymbol{x}(t) \in \prod_i \mathcal{X}_i$ to (2), such that $\boldsymbol{x}(t^0) \in \mathcal{C}(t^0) \Rightarrow \boldsymbol{x}(t) \in \mathcal{C}(t),\ \forall t \in [t^0, \tau]$.

Next, our definition of sampled-data Control Barrier Function is given, which differs from [11], [12] as only piece-wise differentiability over time is assumed in our settings.

Definition 4: Consider a sampling interval $\delta t > 0$, system (2) and the sd-differentiable function $b : \mathcal{D} \times \mathbb{R}_+ \to \mathbb{R}$ where $\mathcal{C}(t)$ in (6) is compact and such that $\mathcal{C}(t) \subset \mathcal{D},\ \forall t \geq 0$. Then $b(\boldsymbol{x}, t)$ is a sampled-data Control Barrier Function (sdCBF) if $\bar{g}(\boldsymbol{x}) \partial_{\boldsymbol{x}} b(\boldsymbol{x}, t) = \boldsymbol{0} \Leftrightarrow \partial_{\boldsymbol{x}} b(\boldsymbol{x}, t) = \boldsymbol{0}$ and there exists a *margin function* $\zeta : \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}$ defined as

$$\zeta(\boldsymbol{x}, t) = \lambda b(\boldsymbol{x}, t) + \partial_t b(\boldsymbol{x}, t) + \nu(\boldsymbol{x}, t), \tag{7}$$

where $\lambda \in \mathbb{R}_+$ and $\nu(\boldsymbol{x}, t) \leq \bar{\nu}(\boldsymbol{x}, t),\ \forall (\boldsymbol{x}, t) \in \mathcal{D} \times \mathbb{R}_+$ with[2]

$$\begin{aligned} \bar{\nu}(\boldsymbol{x}, t) = &\min_{\bar{\boldsymbol{x}} \in \mathcal{R}(\boldsymbol{x}, \delta t), \tau \in [t^k, t^{k+1}), \boldsymbol{u} \in \mathbb{U}} \\ &L_{\bar{f}} b(\boldsymbol{x}, t) + L_{\bar{g}} b(\boldsymbol{x}, t)\boldsymbol{u} - (L_{\bar{f}} b(\bar{\boldsymbol{x}}, \tau) + L_{\bar{g}} b(\bar{\boldsymbol{x}}, \tau)\boldsymbol{u}) + \\ &\partial_t b(\boldsymbol{x}, t) - \partial_t b(\bar{\boldsymbol{x}}, \tau) + \lambda b(\boldsymbol{x}, t) - \lambda b(\bar{\boldsymbol{x}}, \tau)); \end{aligned} \tag{8}$$

such that there exist a p.w.c input $\boldsymbol{u}(t) \in \prod_i \mathcal{U}_i^{\delta t}$ as per (5) satisfying the condition

$$L_{\bar{f}} b(\boldsymbol{x}^k, t^k) + L_{\bar{g}} b(\boldsymbol{x}^k, t^k)\boldsymbol{u}^k \geq -\zeta(\boldsymbol{x}^k, t^k) \tag{9}$$

for all $\boldsymbol{x}^k \in \mathcal{D}$ and t^k with $k \in \mathbb{N}_0$.

The term $\bar{\nu}(\boldsymbol{x}, t)$ intuitively represents the minimum local negative variation of the classical CBF constraint $L_{\bar{f}} b(\boldsymbol{x}, t) + L_{\bar{g}} b(\boldsymbol{x}, t)\boldsymbol{u} + \partial_t b(\boldsymbol{x}, t) + \lambda b(\boldsymbol{x}, t) \geq 0$ at $(\boldsymbol{x}, t)$, which defines the local effect of sampling over the constraint satisfaction. Note that $\bar{\nu}(\boldsymbol{x}, t)$ exists and is bounded since $L_{\bar{f}} b(\boldsymbol{x}, t), L_{\bar{g}} b(\boldsymbol{x}, t), \lambda b(\boldsymbol{x}, t), \partial_t b(\boldsymbol{x}, t)$ are Lipschitz over the intervals $\mathcal{D} \times [t^k, t^{k+1})$. The following lemma inspired by [12, Thm.2] justifies the definition of an sdCBF.

Lemma 1: Assume $b(\boldsymbol{x}, t)$ is an sdCBF as per Def. 4 for a sampling interval $\delta t > 0$ and let $\boldsymbol{x}^0 = \boldsymbol{x}(t^0) \in \mathcal{C}(t^0)$. If (2) is subject to a p.w.c input trajectory $\boldsymbol{u}(t) \in \prod_i \mathcal{U}_i^{\delta t}$ such that condition (9) is satisfied for all $\boldsymbol{x}^k \in \mathcal{D}$ and t^k with $k \in \mathbb{N}_0$, then $\mathcal{C}(t)$ is forward invariant over $[t^0, \infty]$.

Proof: Provided in [17, Lemma 1] ■

A. sdCBF for STL tasks

Given a general STL formula ϕ from fragment (3) such that the predicates h_i and h_{ij} respect Assmp. 1, then the authors in [1] define a three steps procedure to construct a function $b^\phi(\boldsymbol{x}, t)$ with associated set $\mathcal{C}^\phi(t)$ as per (6) such that forward invariance of $\mathcal{C}^\phi(t)$ implies $\boldsymbol{x}(t) \models \phi$ [1, Sec. A]. The resulting $b^\phi(\boldsymbol{x}, t)$ is an sd-differentiable function as per Def. 2 assuming the predicate functions are differentiable and the time intervals $[a, b]$ for the temporal operators of fragment (3) are such that $a = t^{k_a}, b = t^{k_b}$ for some $k_a, k_b \in \mathbb{N}_0$. Thus, for each task ϕ_i or ϕ_{ij}, an sd-differentiable functions $b_i^\phi(\boldsymbol{x}_i, t)$ or $b_{ij}^\phi(\boldsymbol{e}_{ij}, t)$, respectively, can be constructed as per steps A,B,C in [1, Sec. A] such that $b_i^\phi(\boldsymbol{x}_i, t)$ and $b_{ij}^\phi(\boldsymbol{e}_{ij}, t)$ are concave in their first argument. The next proposition relates the satisfaction of a task ϕ with the forward invariance of the level set $\mathcal{C}^\phi(t)$.

Proposition 1: Consider a task ϕ from (3) and let $b^\phi(\boldsymbol{x}, t)$ be sd-differentiable and constructed according to [1, Sec. A] with corresponding level set $\mathcal{C}^\phi(t)$ as per (6). If $\mathcal{C}^\phi(t)$ is forward invariant over $[t^0, \infty]$, then $\boldsymbol{x}(t) \models \phi$.

Proof: See [7, Cor. 1] and [1, Thm. 1] ■

The following assumption is considered.

Assumption 3: Consider ϕ from (3), the associated function $b^\phi(\boldsymbol{x}, t)$ from Prop. 1. Furthermore, let $\mathcal{B}^\phi(t) := \{\boldsymbol{x} \in \mathcal{D} | \partial_{\boldsymbol{x}} b(\boldsymbol{x}, t) = \boldsymbol{0}\}$ and $\zeta^\phi(\boldsymbol{x}, t) = \lambda b^\phi(\boldsymbol{x}, t) + \partial_t b^\phi(\boldsymbol{x}, t) + \nu^\phi(\boldsymbol{x}, t)$ from (7). Then $\lambda \in \mathbb{R}_+$ can be chosen such that $\zeta^\phi(\boldsymbol{x}, t) > \chi,\ \forall \boldsymbol{x} \in \mathcal{B}^\phi(t)$ for some $\chi > 0$.

[2]For a scalar function $b(\boldsymbol{x}, t) : \mathbb{R}^{n \cdot N} \to \mathbb{R}$ the Lie derivative of b along $\bar{f}$, $\bar{g}$, f_i and g_i is given as $L_{\bar{f}} b(\boldsymbol{x}, t) := \partial_{\boldsymbol{x}} b(\boldsymbol{x}, t) \bar{f}(\boldsymbol{x})$, $L_{\bar{g}} b(\boldsymbol{x}, t) := \partial_{\boldsymbol{x}} b(\boldsymbol{x}, t) \bar{g}(\boldsymbol{x})$, $L_{f_i} b(\boldsymbol{x}, t) := \partial_{\boldsymbol{x}_i} b(\boldsymbol{x}, t) f_i(\boldsymbol{x}_i)$, $L_{g_i} b(\boldsymbol{x}, t) := \partial_{\boldsymbol{x}_i} b(\boldsymbol{x}) g_i(\boldsymbol{x})$.

From the definition of sdCBF in Def. 4 we have $L_{\bar{g}}b^{\phi}(\boldsymbol{x},t) = \mathbf{0} \Leftrightarrow \partial_{\boldsymbol{x}}b^{\phi}(\boldsymbol{x},t) = \mathbf{0} \Rightarrow \boldsymbol{x} \in \mathcal{B}^{\phi}(t)$. Thus Assmp. 3 allows for the satisfaction of the sdCBF constraint (9) when $\boldsymbol{x}(t) \in \mathcal{B}^{\phi}(t)$. Since by construction $b^{\phi}(\boldsymbol{x},t)$ is concave then $\mathcal{B}^{\phi}(t)$ corresponds to the set of maximisers of $b^{\phi}(\boldsymbol{x},t)$ at time t such that $b^{\phi}(\boldsymbol{x},t) = b^{\phi}_{max}(t) = max_{\boldsymbol{x}\in\mathcal{D}}\, b(\boldsymbol{x},t)\ \forall \boldsymbol{x} \in \mathcal{B}^{\phi}(t)$. Thus, Assmp. 3 implies choosing a sufficiently large $\lambda \in \mathbb{R}_+$ such that $\lambda b^{\phi}_{max}(t) + \partial_t b(\boldsymbol{x},t) + \nu(\boldsymbol{x},t) > \chi\ \forall \boldsymbol{x} \in \mathcal{B}^{\phi}(t)\, \forall t \in \mathbb{R}_+$.

IV. Decentralized Control Solution

In this section, we define our control solution for Problem 1. The presentation is ordered as follows: In Sec. IV-A we present the global decentralized control scheme applied to satisfy objective 1) in Problem 1, assuming $\mathcal{G}_{\psi} \subseteq \mathcal{G}_c(t)$, for all t. Communication maintenance is then addressed in Corollary 1. For ease of presentation, we often adopt the shorthand notations $b^{\phi}_{ij}(t^k) := b^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k)$ and $b^{\phi}_{i}(t^k) := b^{\phi}_{i}(\boldsymbol{x}^k_{i}, t^k)$ when clear from the context.

A. Decentralized Control for STL satisfaction

Consider a given task ϕ_{ij} involving the relative state $\boldsymbol{e}_{ij}$ among agent i and j, with associated sdCBF $b^{\phi}_{ij}(\boldsymbol{e}_{ij},t)$. By expanding (9) it follows that $L_{\bar{f}}b^{\phi}_{ij}(\boldsymbol{e}_{ij},t) + L_{\bar{g}}b^{\phi}_{ij}(\boldsymbol{e}_{ij},t)\boldsymbol{u} = L_{f_i}b^{\phi}_{ij}(\boldsymbol{e}_{ij},t) + L_{g_i}b^{\phi}_{ij}(\boldsymbol{e}_{ij},t)\boldsymbol{u}_i + L_{f_j}b^{\phi}_{ij}(\boldsymbol{e}_{ij},t) + L_{g_j}b^{\phi}_{ij}(\boldsymbol{e}_{ij},t)\boldsymbol{u}_j$, such that, for agent i, we define the *worst impact* and *best impact* over the satisfaction of ϕ_{ij}, respectively as

$$ {}^{i}\underline{\epsilon}^{\phi}_{ij}(t^k) = \min_{\boldsymbol{u}_i \in \gamma^k_i \mathbb{U}_i} L_{f_i}b^{\phi}_{ij}(t^k) + L_{g_i}b^{\phi}_{ij}(t^k)\boldsymbol{u}_i \qquad (10a)$$

$$ {}^{i}\bar{\epsilon}^{\phi}_{ij}(t^k) = \max_{\boldsymbol{u}_i \in \gamma^k_i \mathbb{U}_i} L_{f_i}b^{\phi}_{ij}(t^k) + L_{g_i}b^{\phi}_{ij}(t^k)\boldsymbol{u}_i, \qquad (10b)$$

where $\gamma^k_i \in (0,1],\ \forall i \in \mathcal{V}_i, \forall k \in \mathbb{N}_0$, is denoted as the *control reduction factor* which applies to shrinks the control set $\mathbb{U}_i$ to $\gamma^k_i\mathbb{U}_i$. The purpose of γ^k_i is to bound the value of the best and worst impacts in (10) to enforce the satisfaction of the sdCBF (9) over $b^{\phi}_{ij}(\boldsymbol{e}_{ij},t)$ and thus satisfy task ϕ_{ij} as clarified in Sec. IV-C.

Furthermore, a set of tokens $T_i = \{q_{ij} | \forall j \in \mathcal{N}_{\psi}(i)\}$ is introduced such that $q_{ij} \in \{l, f\}$ where l stands for *leader* and f stands for *follower*. The purpose of the tokens q_{ij} is to define an edge leader for each $(i,j) \in \mathcal{E}_{\psi}$ such that the following rules are defined $q_{ij} = l \Leftrightarrow q_{ji} = f$, $q_{ij} = l \Rightarrow q_{ik} \neq l\ \forall k \in \mathcal{N}_{\psi}(i) \setminus \{j\}$ for each $(i,j) \in \mathcal{E}_{\psi}$. With these rules, either i or j is a leader of the edge $(i,j) \in \mathcal{E}_{\psi}$, while each $i \in \mathcal{V}$ is the leader of at most one edge. Moreover, let $\mathcal{L}_{\psi}(i) := \{j \in \mathcal{N}_{\psi}(i) | q_{ij} = f\}$ be the leader set for agent i, corresponding to all the agents j for which i is a follower of $(i,j) \in \mathcal{E}_{\psi}$.

Theorem 1: Consider (2), a task graph $\mathcal{G}_{\psi} \subseteq \mathcal{G}_c(t)$, $\forall t \in \mathbb{R}_+$ such that tasks $\phi_{ij},\ \forall (i,j) \in \mathcal{E}_{\psi}$ and $\phi_i,\ \forall i \in \mathcal{V}$ are associated with $b^{\phi}_{ij}(\boldsymbol{e}_{ij},t)$, $b^{\phi}_{i}(\boldsymbol{x}_{i},t)$, respectively, as per [1, Sec. A]. Furthermore, let $\zeta^{\phi}_{ij}(\boldsymbol{e}_{ij},t)$, $\zeta^{\phi}_{i}(\boldsymbol{x}_{i},t)$ as per (7) and respecting Assmp. 3. Let each agent i be subject to the p.w.c input $\boldsymbol{u}_i(t) \in \mathcal{U}^{\delta t}_i$ such that $\boldsymbol{u}^k_i$ minimizes

$$\min_{\boldsymbol{u}_i, s_i, s_{ij}} \boldsymbol{u}^T_i\boldsymbol{u}_i + c_i\, s_i + \sum_{j\in\mathcal{L}_{\psi}(i)} c_{ij}\, s_{ij} \qquad (11a)$$

$$L_{f_i}b^{\phi}_{ij}(t^k) + L_{g_i}b^{\phi}_{ij}(t^k)\boldsymbol{u}_i \geq -\zeta^{\phi}_{ij}(t^k) - {}^{j}\underline{\epsilon}^{\phi}_{ij}(t^k),\ q_{ij} = l \qquad (11b)$$

$$L_{f_i}b^{\phi}_{ij}(t^k) + L_{g_i}b^{\phi}_{ij}(t^k)\boldsymbol{u}_i \geq -\frac{1}{2}\zeta^{\phi}_{ij}(t^k) - s_{ij},\ \forall j \in \mathcal{L}_{\psi}(i) \qquad (11c)$$

$$L_{f_i}b^{\phi}_{i}(t^k) + L_{g_i}b^{\phi}_{i}(t^k)\boldsymbol{u}_i \geq -\zeta^{\phi}_{i}(t^k) - s_i, \qquad (11d)$$

$$s_i \geq 0,\ s_{ij} \geq 0\ \forall j \in \mathcal{L}_{\psi}(i),\ \boldsymbol{u}_i \in \gamma^k_i\mathbb{U}_i, \qquad (11e)$$

where s_i, s_{ij} are slack variables with corresponding positive cost coefficients $c_i, c_{ij} \in \mathbb{R}_+$, $\gamma^k_i \in (0,1]$, $\forall i \in \mathcal{V}$, $\forall k \in \mathbb{N}_0$ and ${}^{j}\underline{\epsilon}^{\phi}_{ij}(t^k)$ is the worst impact from the follower j over task ϕ_{ij} as per (10a). If (11) is feasible for each t^k with $k \in \mathbb{N}_0$ then the solution $\boldsymbol{x}(t)$ to (2) under $\boldsymbol{u}(t) = [\boldsymbol{u}_i(t)^T]^T_{i\in\mathcal{V}}$ is such that $\boldsymbol{x}(t) \models \psi_{col}$. Moreover, if $s_i = 0$ for all t^k, then $\boldsymbol{x}(t) \models \psi_{col} \wedge \psi_{ind}$.

Proof: We first prove $\boldsymbol{x}(t) \models \psi_{col}$. Consider agent i with collaborative task ϕ_{ij} and sd-differentiable function $b^{\phi}_{ij}(\boldsymbol{e}_{ij},t)$ with $\zeta^{\phi}_{ij}(\boldsymbol{e}_{ij},t)$ as per (7) and $\mathcal{C}^{\phi}_{ij}(t)$ as per (6). Further let $q_{ij} = l$ and $q_{ji} = f$ (i is leader of $(i,j) \in \mathcal{E}_{\psi}$). Writing the left hand side of the sdCBF condition (9) over $b^{\phi}_{ij}(\boldsymbol{e}_{ij},t)$ gives $L_{f_i}b^{\phi}_{ij}(t^k) + L_{g_i}b^{\phi}_{ij}(t^k)\boldsymbol{u}_i + L_{f_j}b^{\phi}_{ij}(t^k) + L_{g_j}b^{\phi}_{ij}(t^k)\boldsymbol{u}_j \underset{(11b)}{\geq} -\zeta^{\phi}_{ij}(t^k) - {}^{j}\underline{\epsilon}^{\phi}_{ij}(t^k) + L_{f_j}b^{\phi}_{ij}(t^k) + L_{g_j}b^{\phi}_{ij}(t^k)\boldsymbol{u}_j \underset{(10a)}{\geq} -\zeta^{\phi}_{ij}(t^k)$, which corresponds to the satisfaction of the sdCBF constraint (9). Note that at the singularity points for which $L_{g_i}b^{\phi}_{ij}(\boldsymbol{e}_{ij},t) = \mathbf{0}$ we have from the chain rule that $\partial_{\boldsymbol{x}_i}b^{\phi}_{ij}(\boldsymbol{e}_{ij},t) = -\partial_{\boldsymbol{x}_j}b^{\phi}_{ij}(\boldsymbol{e}_{ij},t)$ and thus by Def. 4 we have $L_{g_i}b^{\phi}_{ij}(\boldsymbol{e}_{ij},t) = \mathbf{0} \Leftrightarrow L_{g_j}b^{\phi}_{ij}(\boldsymbol{e}_{ij},t) = \mathbf{0} \Rightarrow \partial_{\boldsymbol{x}}b^{\phi}_{ij}(\boldsymbol{e}_{ij},t) = \mathbf{0} \Rightarrow {}^{j}\underline{\epsilon}^{\phi}_{ij}(t^k) = 0$. Moreover, by Assmp. 3 on ζ^{ϕ}_{ij} we have that (11b) is satisfied independently of $\boldsymbol{u}^k_i$ or $\boldsymbol{u}^k_j$ in this situation. Letting $\mathcal{C}^{\phi}_{ij}(t)$ be the level set of b^{ϕ}_{ij} as per Prop. 6, then from Lemma 1 we have $\boldsymbol{x}(t) \in \mathcal{C}^{\phi}_{ij}(t)$, $\forall t \in \mathbb{R}_+$ and thus $\boldsymbol{x}(t) \models \phi_{ij}$ by Prop. 1. Since the argument can be repeated for every edge $(i,j) \in \mathcal{E}_{\psi}$ then $\boldsymbol{x}(t) \models \psi_{col}$. Turning to the satisfaction of ψ_{ind}, consider independent tasks ϕ_i with associated $b^{\phi}_{i}(\boldsymbol{x}_i,t)$, $\zeta^{\phi}_{i}(\boldsymbol{x}_i,t)$ and level set $\mathcal{C}^{\phi}_{i}(t)$ as per (6). If $s_i = 0$, $\forall i \in \mathcal{V}$, for all t^k, then constraint (11d) corresponds to the sdCBF condition (9) and thus $\boldsymbol{x}_i(t) \in \mathcal{C}^{\phi}_{i}(t)$, $\forall t \in \mathbb{R}_+$. From Prop. 1 we then have $\boldsymbol{x}(t) \models \phi_i$, $\forall i \in \mathcal{V} \Rightarrow \boldsymbol{x}(t) \models \psi_{ind}$. ∎

It is highlighted that constraint (11b) represents the sdCBF constraint (9) over $b^{\phi}_{ij}(\boldsymbol{e}_{ij},t)$, where the follower j is assumed to select the input $\boldsymbol{u}_j \in \gamma^k_j\mathbb{U}_j$ associated with the worst impact as per (10a). The control law (11) differs from previous controllers [6]–[8] as follows: **1)** Unlike [7], [8], agent i requires only local knowledge of state $\boldsymbol{x}_i$ and $\boldsymbol{e}_{ij}, \forall j \in \mathcal{N}_{\psi}(i)$. **2)** In contrast to [6], [7], and akin to [8], each agent is subject to $|\mathcal{N}_{\psi}(i)|$ constraints from collaborative tasks ϕ_{ij} plus one constraint from independent task ϕ_i. However, unlike [8], all constraints (11c)-(11d) are satisfied only up to a slack variable, while only (11b) is strictly

Algorithm 1 Control Reduction Computation (Agent i)

Compute ${}^{i}\Upsilon^{\phi}_{ij}(t^k)$ for each $j \in \mathcal{N}_\psi(i)$ and $\nu^{\phi}_{i}(t^k)$
if i in $\mathcal{F}_\psi$ **then**
  Set $\gamma_i^k = 1$
  Compute $\bar{\boldsymbol{u}}_i^k$ (14) and best impact ${}^{i}\bar{\epsilon}^{\phi}_{ij}(t^k)$ (10b).
else
  while Any $j \in \mathcal{L}_\psi(i)$ has not yet fixed γ_j^k **do**
    for $j \in \mathcal{L}_\psi(i)$ **do**
      if j has fixed γ_j^k **then**
        Receive ${}^{j}\bar{\epsilon}^{\phi}_{ij}(t^k)$ (10b)
        Receive ${}^{j}\Upsilon^{\phi}_{ij}(t^k)$ (13)
        Compute $\nu^{\phi}_{ij}(t^k)$ (12) and $\zeta^{\phi}_{ij}(t^k)$ (8)
        Compute $\underline{\boldsymbol{u}}_i^k$ (14) and $\tilde{\gamma}_{ij}^k$ (15)
  Fix $\gamma_i^k = \min_{j \in \mathcal{L}_\psi(i)} \tilde{\gamma}_{ij}$

satisfied, corresponding to the unique edge $(i,j) \in \mathcal{E}_\psi$ for which i is the leader ($q_{ij} = l$). **3)** Unlike [6]–[9], the control law (11) is implemented in a sampled-data fashion which is suitable for embedded-systems implementation.

In Theorem 1 it is assumed that $\mathcal{G}_\psi \subseteq \mathcal{G}_c(t)$. Nevertheless, a communication task can be explicitly given over each edge by imposing $\varphi^{com}_{ij} = G_{[0,\infty]} h^{com}_{ij}$ where $h^{com}_{ij} = r_c^2 - \|\boldsymbol{p}_{ij}\|^2$, which respects Assumption 1.

Corollary 1: Consider (2) and task graph $\mathcal{G}_\psi \subseteq \mathcal{G}_c(t^0)$, such that the conjunction of tasks ϕ_{ij} includes a communication task $\varphi^{com}_{ij} = G_{[0,\infty]} h^{com}_{ij}$ with $h^{com}_{ij} = r_c^2 - \|\boldsymbol{p}_{ij}\|^2$ as per (3b). If each agent i is subject to a p.w.c input $\boldsymbol{u}_i(t) \in \mathcal{U}_i^{\delta t}$ as per Thm. 1, then $\boldsymbol{x}(t) \models \psi_{col} \Rightarrow \boldsymbol{x}(t) \models \varphi^{com}_{ij}$, $\forall (i,j) \in \mathcal{E}_\psi$ and $\mathcal{G}_\psi \subseteq \mathcal{G}_c(t)$, $\forall t \in \mathbb{R}_+$.

B. Margin function computation

In (11) the margin functions $\zeta^{\phi}_{ij}(\boldsymbol{e}_{ij}, t) = \lambda b^{\phi}(\boldsymbol{e}^k_{ij}, t^k) + \partial_t b^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k) + \nu^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k)$ for all $j \in \mathcal{N}_\psi(i)$ and $\zeta^{\phi}_{i}(\boldsymbol{x}_i, t) = \lambda b^{\phi}_i(\boldsymbol{x}^k_i, t^k) + \partial_t b^{\phi}_i(\boldsymbol{x}^k_i, t^k) + \nu^{\phi}_i(\boldsymbol{x}^k_i, t^k)$ as per (7) for some (possibly different) $\lambda \in \mathbb{R}_+$ must be computed at each t^k. Particularly, $\nu^{\phi}_i(\boldsymbol{x}^k_i, t^k)$ and $\nu^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k)$ must satisfy $\nu^{\phi}_i(\boldsymbol{x}^k_i, t^k) \leq \bar{\nu}^{\phi}_i(\boldsymbol{x}^k_i, t^k)$ and $\nu^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k) \leq \bar{\nu}^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k)$ with $\bar{\nu}^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k)$ and $\bar{\nu}^{\phi}_i(\boldsymbol{x}^k_i, t^k)$ being according to (8). To reduce conservativism, it is beneficial to compute values $\nu^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k)$, $\nu^{\phi}_i(\boldsymbol{x}^k_i, t^k)$ as close as possible to their optimal values $\bar{\nu}^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k)$, $\bar{\nu}^{\phi}_i(\boldsymbol{x}^k_i, t^k)$, recalling that this computation is accomplished online. Regarding $\nu^{\phi}_i(\boldsymbol{x}^k_i, t^k)$, it is possible to directly compute $\nu^{\phi}_i(\boldsymbol{x}^k_i, t^k) = \bar{\nu}^{\phi}_i(\boldsymbol{x}^k_i, t^k)$ by solving for (8) online at each time t^k since each agent i is aware of $\boldsymbol{x}^k_i$ and $\mathcal{R}_i(\boldsymbol{x}^k_i, \delta t)$, where $\mathcal{R}_i$ is the reachable set for agent i as per (1). On the other hand, computing $\bar{\nu}^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k)$ online requires solving the minimization program from (8) as:

$$
\begin{aligned}
&\bar{\nu}^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k) = \min\big\{ L_{f_i} b^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k) - L_{f_i} b^{\phi}_{ij}(\bar{\boldsymbol{e}}_{ij}, \tau) + \\
&(L_{g_i} b^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k) - L_{g_i} b^{\phi}_{ij}(\bar{\boldsymbol{e}}_{ij}, \tau))\boldsymbol{u}_i + L_{f_j} b^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k) - \\
&L_{f_j} b^{\phi}_{ij}(\bar{\boldsymbol{e}}_{ij}, \tau) + (L_{g_j} b^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k) - L_{g_j} b^{\phi}_{ij}(\bar{\boldsymbol{e}}_{ij}, \tau))\boldsymbol{u}_j + \\
&\partial_t b^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k) - \partial_t b^{\phi}_{ij}(\bar{\boldsymbol{e}}_{ij}, \tau) + \lambda b^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k) - \lambda b^{\phi}_{ij}(\bar{\boldsymbol{e}}_{ij}, \tau)\big\} \\
&s.t: \ \bar{\boldsymbol{e}}_{ij} \in \mathcal{R}_{ij}(\boldsymbol{e}_{ij}, \delta t), \tau \in [t^k, t^{k+1}), \boldsymbol{u}_i \in \mathbb{U}_i, \boldsymbol{u}_j \in \mathbb{U}_j;
\end{aligned}
$$

where $\mathcal{R}_{ij}(\boldsymbol{e}^k_{ij}, \delta t) := \mathcal{R}_i(\boldsymbol{x}^k_i, \delta t) \ominus \mathcal{R}_j(\boldsymbol{x}^k_j, \delta t)$. The set $\mathcal{R}_{ij}(\boldsymbol{e}^k_{ij}, \delta t)$ represents the reachable set of the relative state between i and j within a sampling interval δt whose computation requires knowledge of both the dynamics of i and j. Since we assume that agents do not share detailed knowledge of each other's dynamics, we only let $\mathcal{R}_i(\boldsymbol{x}^k_i, \delta t)$ and $\mathcal{R}_j(\boldsymbol{x}^k_j, \delta t)$ to be shared through communication among i and j at each sampling time t^k. Hence we define $\nu^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k)$ as

$$
\nu^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k) = {}^{i}\Upsilon^{\phi}_{ij}(t^k) + {}^{j}\Upsilon^{\phi}_{ij}(t^k) \tag{12}
$$

where

$$
\begin{aligned}
&{}^{i}\Upsilon^{\phi}_{ij}(t^k) := \min_{\boldsymbol{u}_i, \boldsymbol{e}_{ij}, \tau} \big\{ L_{f_i} b^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k) - L_{f_i} b^{\phi}_{ij}(\bar{\boldsymbol{e}}_{ij}, \tau) + \\
&(L_{g_i} b^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k) - L_{g_i} b^{\phi}_{ij}(\bar{\boldsymbol{e}}_{ij}, \tau))\boldsymbol{u}_i + \big(\lambda b^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k) - \\
&\lambda b^{\phi}_{ij}(\bar{\boldsymbol{e}}_{ij}, \tau) + \partial_t b^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k) - \partial_t b^{\phi}_{ij}(\bar{\boldsymbol{e}}_{ij}, \tau)\big)/2\big\} \\
&s.t: \ \bar{\boldsymbol{e}}_{ij} \in \mathcal{R}_{ij}(\boldsymbol{e}^k_{ij}, \delta t), \ \tau \in [t^k, t^{k+1}), \ \boldsymbol{u}_i \in \mathbb{U}_i;
\end{aligned} \tag{13a}
$$

$$
\begin{aligned}
&{}^{j}\Upsilon^{\phi}_{ij}(t^k) := \min_{\boldsymbol{u}_j, \bar{\boldsymbol{e}}_{ij}, \tau} \big\{ L_{f_j} b^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k) - L_{f_j} b^{\phi}_{ij}(\bar{\boldsymbol{e}}_{ij}, \tau) + \\
&(L_{g_j} b^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k) - L_{g_j} b^{\phi}_{ij}(\bar{\boldsymbol{e}}_{ij}, \tau))\boldsymbol{u}_j + \big(\lambda b^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k) - \\
&\lambda b^{\phi}_{ij}(\bar{\boldsymbol{e}}_{ij}, \tau) + \partial_t b^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k) - \partial_t b^{\phi}_{ij}(\bar{\boldsymbol{e}}_{ij}, \tau)\big)/2\big\} \\
&st: \ \bar{\boldsymbol{e}}_{ij} \in \mathcal{R}_{ij}(\boldsymbol{e}^k_{ij}, \delta t), \ \tau \in [t^k, t^{k+1}), \ \boldsymbol{u}_j \in \mathbb{U}_j,
\end{aligned} \tag{13b}
$$

and we note that $\nu^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k) \leq \bar{\nu}^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k)$[3]. Thus, for each task ϕ_{ij}, agent i computes ${}^{i}\Upsilon^{k}_{ij}$ while it receives the value of ${}^{j}\Upsilon^{k}_{ij}$ at each t^k from all $j \in \mathcal{N}_\psi(i)$ such that $\nu^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k)$ are computed by direct summation as per (12)

C. Control reduction factor computation

At last the control reduction factors $\gamma_i^k, \forall i \in \mathcal{V}$ and the worst impact ${}^{j}\underline{\epsilon}^{\phi}_{ij}(t^k)$ as per (10a) must be computed to implement (11). In this section, we clarify, how these terms are computed online.

Initially, consider a single task ϕ_{ij} with corresponding sdCBF $b^{\phi}_{ij}(\boldsymbol{e}_{ij}, t)$ and margin function $\zeta^{\phi}_{ij}(\boldsymbol{e}_{ij}, t)$ as per (7). Furthermore, let i be the leader of the edge $(i,j) \in \mathcal{E}_\psi$ and assume at time t^k that the control reduction factor γ_i^k for the edge leader i is fixed. We next derive an analytical approach to compute γ_j^k for the follower agent j such that no matter how $\boldsymbol{u}_j \in \gamma_j^k \mathbb{U}_j$ is chosen at time t^k, there exist $\boldsymbol{u}_i^k \in \gamma_i^k \mathbb{U}_i$ from the leader such that (11b) is satisfied. Let $V^{\phi}_{ij}(t^k) = \max_{\boldsymbol{u}_i \in \gamma_i^k \mathbb{U}_i} \min_{\boldsymbol{u}_j \in \gamma_j^k \mathbb{U}_j} L_{f_i} b^{\phi}_{ij}(t^k) + L_{g_i} b^{\phi}_{ij}(t^k)\boldsymbol{u}_i + L_{f_j} b^{\phi}_{ij}(t^k) + L_{g_j} b^{\phi}_{ij}(t^k)\boldsymbol{u}_j + \zeta^{\phi}_{ij}(t^k)$, where $V^{\phi}_{ij}(t^k)$ intuitively represents the value function of a two-player game over the sdCBF condition (9) for task ϕ_{ij}, with agent i and j being the maximizing (leader) and minimizing (follower) player, respectively. If γ_j^k is designed such that $V^{\phi}_{ij}(t^k) \geq 0$ at each time t^k, then there exists $\boldsymbol{u}_i \in \gamma_i^k \mathbb{U}_i$ from the leader such that constraint (11b) is satisfied leading to the satisfaction of ϕ_{ij}. The following result holds.

[3]Summing the objectives in (13), the original objective in (8) is found. The inequality is then justified since the minimum of the sum is greater than the sum of the minimums

Proposition 2: Consider the minimization $\min_{x\in\mathcal{X}} f(x)$ where $\mathcal{X}$ is convex and $f(\alpha x) = \alpha f(x)$ with $\alpha > 0$. If $x^* \in \text{argmax}_{x\in\mathcal{X}}(f(x))$, then $\alpha x^* \in \text{argmax}_{x_\alpha \in \alpha\mathcal{X}}(f(x))$.

Since $\boldsymbol{u}_i$ and $\boldsymbol{u}_j$ enter linearly in $V^{\phi}_{ij}(t^k)$, we have from Prop. 2 and (10b)-(10a) that $V^{\phi}_{ij}(t^k) = L_{f_i} b^{\phi}_{ij}(t^k) + L_{g_i} b^{\phi}_{ij}(t^k)\bar{\boldsymbol{u}}^k_i \gamma^k_i + L_{f_j} b^{\phi}_{ij}(t^k) + L_{g_j} b^{\phi}_{ij}(t^k)\underline{\boldsymbol{u}}^k_j \gamma^k_j + \zeta^{\phi}_{ij}(t^k) = {}^i\bar{\epsilon}^{\phi}_{ij}(t^k) + {}^j\underline{\epsilon}^{\phi}_{ij}(t^k) + \zeta^{\phi}_{ij}(t^k)$ where

$$\underline{\boldsymbol{u}}^k_j = \underset{\boldsymbol{u}_j\in\mathbb{U}_j}{\text{argmin}} L_{g_j} b^{\phi}_{ij}(t^k)\boldsymbol{u}_j \,; \bar{\boldsymbol{u}}^k_i = \underset{\boldsymbol{u}_i\in\mathbb{U}_i}{\text{argmax}} L_{g_i} b^{\phi}_{ij}(t^k)\boldsymbol{u}_i. \quad (14)$$

Hence, after agent i and j independently compute $\bar{\boldsymbol{u}}^k_i$, $\underline{\boldsymbol{u}}^k_j$ by solving (14), an analytical expression of $V^{\phi}_{ij}(t^k)$ as a function of γ^k_i and γ^k_j is derived.

Assumption 4: Consider task ϕ_{ij} such that i is the leader of $(i, j) \in \mathcal{E}_\psi$. Then it holds ${}^i\bar{\epsilon}^{\phi}_{ij}(t^k) + \zeta^{\phi}_{ij}(t^k) + L_{f_i} b^{\phi}_{ij}(t^k) \geq 0$ for all t^k for system (2) under the control law (11), where ${}^i\bar{\epsilon}^{\phi}_{ij}(t^k)$ is best impact i as per (10b).

Assumption 4 is introduced to ensure that the leader agent i is always able to enforce $V^{\phi}_{ij}(t^k) \geq 0$ (and thus satisfy the sdCBF condition (9)) if the follower agent j chooses $\gamma^k_j = 0$ at time t^k (and thus $\boldsymbol{u}^k_j = \boldsymbol{0}$). At this point, once the follower agent j computes $\zeta^{\phi}_{ij}(t^k)$ (Sec IV-B) and the best impact ${}^i\bar{\epsilon}^{\phi}_{ij}(t^k)$ is sent from the leader i to j as per (10b), then imposing $V^{\phi}_{ij}(t^k) \geq 0$ implies $\gamma^k_j \leq \frac{-({}^i\bar{\epsilon}^{\phi}_{ij}(t^k)+\zeta^{\phi}_{ij}(t^k)+L_{f_i}b^{\phi}_{ij}(t^k))}{L_{g_j}b^{\phi}_{ij}(t^k)\underline{\boldsymbol{u}}^k_j}$ if $L_{g_j} b^{\phi}_{ij}(t^k)\underline{\boldsymbol{u}}^k_j < 0$ or $\gamma^k_j \geq \frac{-({}^i\bar{\epsilon}^{\phi}_{ij}(t^k)+\zeta^{\phi}_{ij}(t^k)+L_{f_i}b^{\phi}_{ij}(t^k))}{L_{g_j}b^{\phi}_{ij}(t^k)\underline{\boldsymbol{u}}^k_j}$ if $L_{g_j} b^{\phi}_{ij}(t^k)\underline{\boldsymbol{u}}^k_j \geq 0$. By Assmp. 4 the numerator of the previous expressions is non-positive, so any $\gamma^k_j \in (0, 1]$ would satisfy the latter conditioning and $\gamma^k_j = 1$ is set arbitrarily in this case. On the other hand, the former condition is the only binding one. Since j is possibly follower for a number of tasks equal to $|\mathcal{L}_\psi(i)|$, we have that γ^k_j is computed as $\gamma^k_j = \min_{i\in\mathcal{L}_\psi(j)}\{\tilde{\gamma}^k_{ij}\}$ where

$$\tilde{\gamma}^k_{ij} = \begin{cases} 1 \quad \text{if} \quad L_{g_j} b^{\phi}_{ij}(t^k)\underline{\boldsymbol{u}}^k_j \geq 0 \\ \min\{1, \frac{-({}^i\bar{\epsilon}^{\phi}_{ij}(t^k)+\zeta^{\phi}_{ij}(t^k)+L_{f_i}b^{\phi}_{ij}(t^k))}{L_{g_j}b^{\phi}_{ij}(t^k)\underline{\boldsymbol{u}}^k_j}\} \text{ else,} \end{cases} \quad (15)$$

$\forall i \in \mathcal{L}_\psi(j)$. For the special case $L_{g_j} b^{\phi}_{ij}(t^k) = \boldsymbol{0}$, it is known from Assmp. 3 that $L_{g_j} b^{\phi}_{ij}(t^k) = \boldsymbol{0} \Leftrightarrow L_{g_i} b^{\phi}_{ij}(t^k) = \boldsymbol{0} \Rightarrow \partial_{\boldsymbol{x}} b^{\phi}_{ij}(t^k) = \boldsymbol{0}$ and in this case $V^{\phi}_{ij} = \zeta^{\phi}_{ij}(t^k) \geq \chi$ for some $\chi > 0$ so that the solution $\tilde{\gamma}^k_{ij} = 1$ in (15) is taken. After γ^k_j is computed, the worst impact ${}^j\underline{\epsilon}^{\phi}_{ij}(t^k)$ in (10a) is computed from j and sent to the leaders $i \in \mathcal{L}_\psi(j)$ so that constraint (11b) is computed for each leader.

Until now we have assumed that for a given agent j, the leaders $i \in \mathcal{L}_\psi(j)$ have fixed their values of γ^k_i and we showed how γ^k_j is computed such that an input $\boldsymbol{u}_i \in \gamma^j_i\mathbb{U}_i$ satisfying (11b) exists for each leader. The decentralized algorithm Alg. 1 provides a way for all agents $i \in \mathcal{V}$ to fix their value of γ^k_i sequentially at each t^k. Namely, Alg. 1 start with each agent first computing ${}^i\Upsilon^{\phi}_{ij}(t_k)$, as per (13), and $\nu^{\phi}_i(t^k)$, as per Sec. IV-B, (line 1). Then, if agent i is a leaf node in $\mathcal{F}_\psi$ the value of γ^k_i is set to 1 and the best impact ${}^i\bar{\epsilon}^{\phi}_{ij}(t^k)$ is computed for the unique task ϕ_{ij} as per (10b). Selecting $\gamma^k_i = 1$ for the leaf nodes is justified since leaf nodes do not have any leader. If instead $i \notin \mathcal{F}_\psi$, then i continuously checks its leader neighbours $j \in \mathcal{L}_\psi(i)$. When any leader $j \in \mathcal{L}_\psi(i)$ has fixed its γ^k_j then i requires ${}^j\bar{\epsilon}^{\phi}_{ij}(t^k)$ as per (10b) and ${}^j\Upsilon^{\phi}_{ij}(t^k)$ as per (13). With this information, $\nu^{\phi}_{ij}(t_k) = {}^i\Upsilon^{\phi}_{ij}(t^k) + {}^j\Upsilon^{\phi}_{ij}(t^k)$ (and thus ζ^{ϕ}_{ij}) is computed together with $\tilde{\gamma}^k_{ij}$ as per (15) (lines 8-12). When $\tilde{\gamma}^k_{ij}$ are available for all $j \in \mathcal{L}_\psi(i)$, the minimum $\tilde{\gamma}^k_{ij}$ is taken as γ^k_j (line 13).

V. Simulations

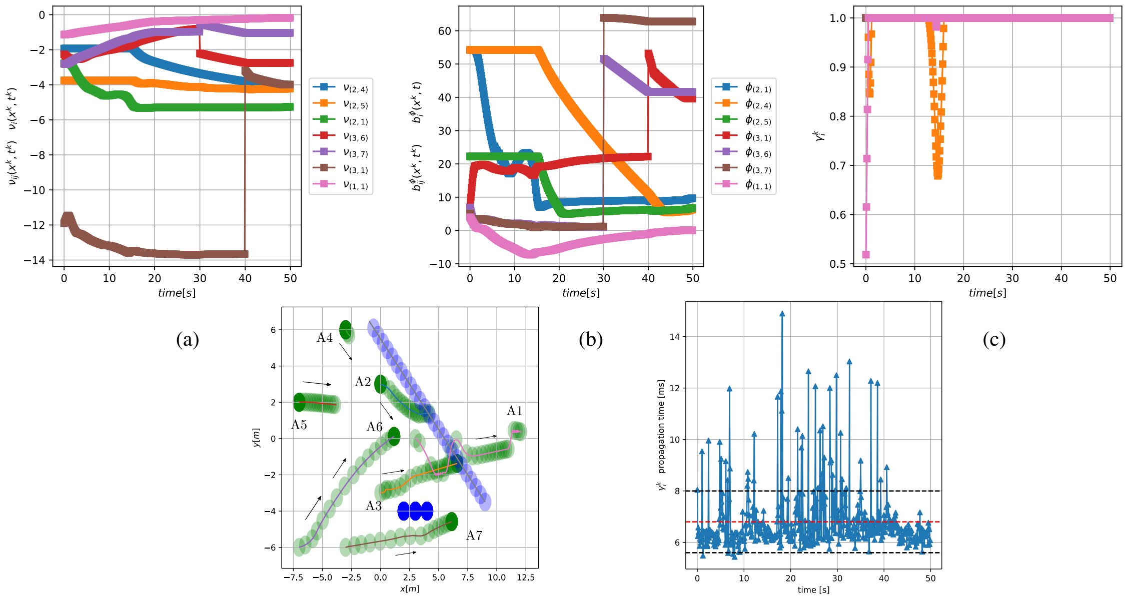
Consider a MAS composed of a single-integrator system $\dot{\boldsymbol{p}}_1 = \boldsymbol{u}_1 \in \mathbb{R}^2$ and 6 differential drive systems

$$\begin{bmatrix} \dot{\boldsymbol{p}}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} cos(\theta_i) & l\cdot sin(\theta_i) \\ -sin(\theta_i) & l\cdot cos(\theta_i) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix} \; \forall i \in 2, \ldots 7$$

where v_i and ω_i are linear and angular velocities respectively and $l > 0$ is a small *look-ahead* distance. We select a sampling interval $\delta t = 0.1s$ and communication radius $r_c = 8.5$. The control constraints are $\|\boldsymbol{u}_1\| \leq 0.9$ and $|v_i| \leq 0.6 \wedge |\omega_i| < 0.15$, $\forall i = 2, \ldots 7$. We then define the acyclic set of edges $\mathcal{E}_\psi = \{(1, 2), (1, 3), (2, 4), (2, 5), (3, 6), (3, 7), (1, 1)\}$ with collaborative formation tasks $\varphi_{12} = F_{[30,40]}(5 - \|\boldsymbol{p}_{12}\|), \varphi_{37} = F_{[20,30]}(2 - \|\boldsymbol{p}_{37} - [2.5, -2.5]^T\|), \varphi_{36} = G_{[20,30]}(2 - \|\boldsymbol{p}_{37} - [-2.5, 2.5]^T\|)$ and communication maintenance task φ^{comm}_{ij}, $\forall (i, j) \in \mathcal{E}_\psi$ as per Corollary 1. Moreover, let the independent task $\varphi_1 = G_{[20,50]}(3 - \|\boldsymbol{p}_1 - [15, 0]^T\|)$. A collision avoidance mechanism to avoid 3 static obstacles and one dynamic obstacle (blue circle in Fig. 1d) is considered in parallel to the control law (11). The simulation is run for 50 s on an Intel-Core i7-1265U with 32 GB of RAM, and the result is shown in Fig. 1d. The time evolution of the barrier functions $\boldsymbol{b}^{\phi}_{ij}(\boldsymbol{e}_{ij}, t)$ (Fig. 1b) shows that all the collaborative tasks are satisfied ($\boldsymbol{b}^{\phi}_{ij}(\boldsymbol{e}_{ij}, t) \geq 0 \; \forall (i, j) \in \mathcal{E}_\psi$) while the independent task for Agent 1 is unsatisfied with a delay of 20s with respect to the required time of satisfaction due to the prioritization of the collaborative tasks. The value of the functions $\nu^{\phi}_{ij}, \nu^{\phi}_i$ is shown in Fig. 1a, while Fig. 1e shows the computational time required to propagate control reduction factor γ^k_i from the leaf nodes $\{5, 6, 7, 4\}$ to the root agent 1 as per Alg. 1. This time includes the online computation of ν^{ϕ}_{ij},ν^{ϕ}_i as well as the computation of best/worst impacts $\bar{\epsilon}^{\phi}_{ij}$,$\underline{\epsilon}^{\phi}_{ij}$ at each t^k. Assuming zero communication time, the average propagation time is 6.9 msec with a standard deviation of 1 msec. In Fig. 1c the value of γ^k_i is shown at each t^k where for most of the agents, the value remains at $\gamma^k_i = 1$ while agents 1 and 3 have a minimum control reduction factor of 0.67 and 0.51 respectively at times $15.5s$ and $0s$.

VI. Conclusions

We showed a decentralised control approach for multi-agent systems subject to spatio-temporal and communication constraints exploring the notion of acyclic task graphs. Our framework is amenable to direct implementation into



(a) (b) (c)

(d) (e)

Fig. 1: (a) Margin functions $\boldsymbol{\nu}^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k)$, $\boldsymbol{\nu}^{\phi}_{i}(\boldsymbol{x}^k_{i}, t^k)$ for each barrier $b^{\phi}_{ij}(\boldsymbol{e}_{ij}, t), b^{\phi}_{i}(\boldsymbol{x}_{i}, t)$. (b) Barrier functions $b^{\phi}_{ij}(\boldsymbol{e}^k_{ij}, t^k), b^{\phi}_{i}(\boldsymbol{x}^k_{i}, t^k)$ associated with tasks ϕ_{ij}, ϕ_i. (c) Control reduction factor γ^k_i. (d) MAS simulation. The arrows represent directions of motion. Blue and green circles represent obstacles and agents respectively (e) Time required to compute γ^k_i sequentially for each $i \in \mathcal{V}$ at each k (Alg. 1).

sampled-data real embedded controllers with continuous time guarantees over task satisfaction. As future work, we explore the robustness and scalability of the proposed approach.

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