Distributed cooperative control and collision avoidance for multiple kinematic agents

Dimos V. Dimarogonas and Kostas J. Kyriakopoulos

Abstract— **This paper contains two main contributions: (i) a provably correct distributed control strategy for collision avoidance and convergence of multiple holonomic agents to a desired feasible formation configuration and (ii) a connection between formation infeasibility and flocking behavior in holonomic kinematic multi-agent systems. In particular, it is shown that when inter-agent formation objectives cannot occur simultaneously in the state-space then, under certain assumptions, the agents velocity vectors and orientations converge to a common value at steady state, under the same control strategy that would lead to a feasible formation. Convergence guarantees are provided in both cases using tools from algebraic graph theory and Lyapunov analysis.**

I. Introduction

Multi-agent Navigation is a field that has recently gained increasing attention both in the robotics and the control communities, due to the need for autonomous control of more than one mobile robotic agents in the same workspace. While most efforts in the past had focused on centralized planning, specific real-world applications have lead researchers throughout the globe to turn their attention to decentralized concepts. The motivation for this work comes from many application domains one of the most important of which is the field of micro robotics ([12],[7]), where a team of a potentially large number of autonomous micro robots must cooperate in the sub micron level.

Among the various specifications that the control design aims at achieving in the case of a multi-agent team, formation convergence and achievement of flocking behavior are two objectives that have been pursued extensively in the last few years. The main feature of formation control is the cooperative nature of the equilibria of the system. Agents must converge to a desired configuration encoded by the inter-agent relative positions. Many feedback control schemes that achieve formation stabilization to a desired formation in a distributed manner have been proposed in literature, see for example [22],[11],[10] for some recent results. Of particular interest is also the so-called consensus problem, in which agents must converge to the same point in the state space ([13],[17],[16], [2],[9],[19],[18]). On the other hand, flocking behavior involves convergence of the velocity vectors and orientations of the agents to a common value at steady state; contributions include [8], [21],[15],[20].

In this paper, the problem of formation control is considered. The main feature of formation control is the cooperative nature of the equilibria of the system. Agents must converge to a desired configuration encoded by the inter-agent relative positions. Inspired by our previous work ([3],[4]) involving decentralized navigation and collision avoidance of multi-agent systems to non-cooperative equilibria (i.e. each agent had a specific goal configuration not related to the goal positions of the others) in this paper we propose a methodology that handles the problem of formation control satisfying at the same time, the collision avoidance objective in a distributed manner.

A further issue we also deal with in this paper, is the case where the desired formation in infeasible. That is the case when inter-agent objectives cannot occur simultaneously in the state space. By decoupling the two objectives (collision avoidance and formation convergence) it can be shown that under certain assumptions formation infeasibility forces the agents velocity vectors to a common value at steady state.

This paper contains similar results with our previous work on sphere world topologies ([5]). The main distinction between the point world and the sphere world topologies is that collision avoidance can be treated in a totally distributed manner in the first case. In the sphere world topology of [5], the decentralization level of the collision avoidance objective was limited by the fact that agents had to have knowledge of the exact number of agents in the group. In this paper, we exploit the point world topology to treat the problem in a totally distributed manner.

The rest of the paper is organized as follows: section II presents the system definition and problem statement. Section III presents the proposed control scheme. The stability analysis is provided in section IV. Section V contains a result relating formation infeasibility and flocking behavior. In section VI simulation results are presented for a number of non-trivial multi agent navigational tasks. Section VII summarizes the conclusions and indicates our current research.

II. System and Problem Definition

Consider a system of N point agents operating in the same workspace $W \subset \mathbb{R}^2$. Let $q_i \in \mathbb{R}^2$ denote the position of agent i. The configuration space is spanned by $q = [q_1, \ldots, q_N]^T$. The motion of each agent is described by the single integrator:

$$\dot{q}_i = u_i, i \in \mathcal{N} = [1, \ldots, N] \tag{1}$$

where u_i denotes the velocity (control input) for each agent.

Each agents' objective is to converge to a desired relative configuration with respect to a certain subset of the rest of the team, in a manner that will lead the whole team to a

The authors are with the Control Systems Lab, Department of Mechanical Engineering, National Technical University of Athens, 9 Heroon Polytechniou Street, Zografou 15780, Greece ddimar,kkyria@mail.ntua.gr

desired formation. Specifically, each agent is assigned with a specific subset N_i of the rest of the team, called agent i's *communication set* with which it can communicate in order to achieve the desired formation. Specifically, agent i has knowledge of the relative positions of agents belonging to N_i. Following the literature on formation control [14],[21], the desired formation can be encoded in terms of a *formation graph*:

Definition 1: The *formation graph* $G = \{Q, E, C\}$ is an undirected graph that consists of (i) a set of vertices $Q = \{1, ..., N\}$ indexed by the team members, (ii) a set of edges, $E = \{(i,j) \in Q \times Q\}$ containing pairs of nodes that represent inter-agent formation specifications, therefore for $(i,j) \in E \Rightarrow j \in N_i$ and (iii) a set of labels $C = \{c_{ij}\}$, where $(i,j) \in E$, that specify the desired inter-agent relative positions $q_i - q_j = c_{ij} \in \mathbb{R}^2$ in the desired formation configuration.

The objective of each agent i is to be stabilized in a desired relative position c_{ij} with respect to each member j of N_i, avoiding at the same time collisions. Collision avoidance is meant in the sense that the point agents are not found at the same point in the state space at each time instant. The collision avoidance procedure is distributed in the sense that each agent has to have only local knowledge of the agents that are very close at each time instant. We assume that each agent has sense of agents (apart from the ones belonging to its communication set) that are found within a circle of radius d around the agent. This circle is called the *sensing zone* of each agent i. The subset of $\mathcal{N}$ including the agents that belong to the sensing zone of i at each time instant is denoted by M_i. Hence

$$M_i = \{j \in \mathcal{N}, j \neq i : \|q_i - q_j\| \leq d\}$$

Hence each agent requires knowledge of the states of agents belonging to the sets N_i, M_i at each time instant.

We assume that the formation graph is undirected, in the sense that $i \in N_j \Leftrightarrow j \in N_i, \forall i, j \in \mathcal{N}, i \neq j$. It is obvious that $(i,j) \in E$ iff $i \in N_j \Leftrightarrow j \in N_i$. We also assume that there are no conflicting interagent objectives, in the sense that $c_{ij} = -c_{ji}, \forall i, j \in \mathcal{N}, i \neq j$.

III. Control Strategy

Let us define the formation objective for agent i

$$\gamma_i \triangleq \frac{1}{2} \sum_{j \in N_i} \|q_i - q_j - c_{ij}\|^2 \tag{2}$$

Let us also define V_{ij} as a potential field to deal with the collision avoidance between agents i and $j \in M_i$. We require that V_{ij} has the following properties:

1) V_{ij} is a function of the square norm of the Euclidean distance between agents i, j, i.e.

$$V_{ij} = V_{ij}\left(\underbrace{\|q_i - q_j\|^2}_{\beta_{ij}}\right) = V_{ij}\left(\beta_{ij}\right)$$

2) $V_{ij} \to \infty$ whenever $\beta_{ij} \to 0$.
3) It is everywhere continuously differentiable.
4) $\frac{\partial V_{ij}}{\partial q_i} = 0$ and $V_{ij} = 0$ whenever $\|q_i - q_j\| > d$.

It is straightforward to see that if the potential field satisfies these requirements, then agent i needs to have only knowledge of the states of agents within M_i at each time instant to fulfil the collision avoidance objective. The fourth requirement also guarantees that

$$\sum_{j \in M_i} \frac{\partial V_{ij}}{\partial q_i} = \sum_j \frac{\partial V_{ij}}{\partial q_i}$$

The gradient and the partial derivative of V_{ij} are computed by $\nabla V_{ij} = 2\rho_{ij} D_{ij} q$ and $\frac{\partial V_{ij}}{\partial q_i} = 2\rho_{ij}\left(D_{ij}\right)_i q$ where

$$\rho_{ij} \triangleq \frac{\partial V_{ij}}{\partial \beta_{ij}}$$

and the matrices D_{ij},$(D_{ij})_i$, for $i < j$, can be shown to be given by

$$D_{ij} = \left[\begin{array}{ccccc} & & O_{(i-1)\times N} & & \\ O_{1\times(i-1)} & 1 & O_{1\times(j-i-1)} & -1 & O_{1\times(N-j)} \\ & & O_{(j-i-1)\times N} & & \\ O_{1\times(i-1)} & -1 & O_{1\times(j-i-1)} & 1 & O_{1\times(N-j)} \\ & & O_{(N-j)\times N} & & \end{array}\right] \otimes I_2$$

and

$$\left(D_{ij}\right)_i = \left[\begin{array}{ccccc} O_{1\times(i-1)} & 1 & O_{1\times(j-i-1)} & -1 & O_{1\times(N-j)} \end{array}\right] \otimes I_2$$

The definition of the matrices D_{ij},$(D_{ij})_i$, for $i > j$ is straightforward.

There are many alternatives for the construction of the potential field V_{ij}. In this paper, we use the following definition of V_{ij}:

$$V_{ij}(\beta_{ij}) = \begin{cases} a/\beta_{ij}, \beta_{ij} < c \\ h\left(\beta_{ij} - d^2\right)^2, c \leq \beta_{ij} < d^2 \\ 0, \beta_{ij} \geq d^2 \end{cases}$$

This definition obviously fulfills requirements 1,2,4. The parameters a, c, h are chosen so that V_{ij} is everywhere continuously differentiable. Using simple calculus, it is easily derived that the parameters a, c, d, h should satisfy the conditions $d^2 = 3c, a = 4hc^3$. These conditions guarantee that $V_{ij}(\beta_{ij})$ is continuously differentiable at the points $\beta_{ij} = c$ and $\beta_{ij} = d^2$. Hence, the design parameters a, h can be chosen so that the sensing radius d of the agent can be chosen arbitrarily small.

Figure 1 shows a plot of the function V_{ij} with respect to β_{ij} for $h = 100$ and $d^2 = 0.001$.

This definition of V_{ij} guarantees that the potential field has the following important symmetry property:

$$\rho_{ij} = \rho_{ji}, \forall i, j \in \mathcal{N}, i \neq j$$

The last property is crucial in the stability analysis of the proposed control scheme, as will be shown in the analysis that follows.

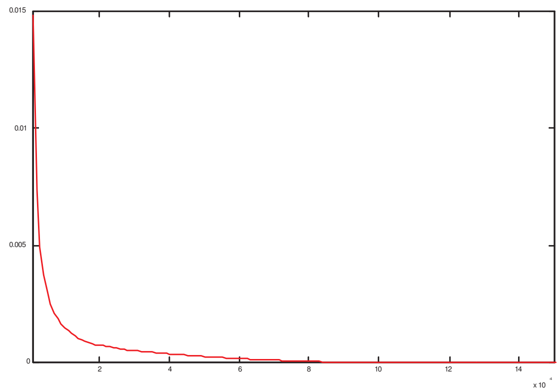

Fig. 1. The function V_{ij} for $h = 100$ and $d^2 = 0.001$.

We propose the following control law for each agent i:

$$u_i = -\sum_{j \in M_i} \frac{\partial V_{ij}}{\partial q_i} - \frac{\partial \gamma_i}{\partial q_i} \tag{3}$$

The stability analysis of the system (1) under the control law (3) is contained in the next sections.

IV. Stability Analysis

A. Tools from Algebraic Graph Theory

In this subsection we review some tools from algebraic graph theory that we shall use in the stability analysis of the next sections. The following can be found in any standard textbook on algebraic graph theory(e.g. [1], [6]).

For an undirected graph G with n vertices the *adjacency matrix* $A = A(G) = (a_{ij})$ is the $n \times n$ matrix given by

$$a_{ij} = \begin{cases} 1, \text{if } (i,j) \in E \\ 0, \text{otherwise} \end{cases}$$

If there is an edge connecting two vertices i, j, i.e. $(i,j) \in E$, then i, j are called *adjacent*. A *path* of length r from a vertex i to a vertex j is a sequence of $r+1$ distinct vertices starting with i and ending with j such that consecutive vertices are adjacent. If there is a path between any two vertices of the graph G, then G is called *connected* (otherwise it is called *disconnected*). The *degree* d_i of vertex i is defined as the number of its neighboring vertices, i.e.

$$d_i = \{\#j : (i,j) \in E\}$$

Let Δ be the $n \times n$ diagonal matrix of d_i's. The (combinatorial) *Laplacian* of G is the symmetric positive semidefinite matrix $\mathcal{L} = \Delta - A$. The Laplacian captures many interesting topological properties of the graph. Of particular interest in our case is the fact that for a connected graph, the Laplacian has a single zero eigenvalue and the corresponding eigenvector is the vector of ones, $\overrightarrow{\mathbf{1}}$.

The last property has lead to the result regarding the connection between formation non-feasibility and flocking behavior discussed in section V. The next paragraphs of this section contain the stability analysis of the formation scheme.

B. Stability of a feasible formation

In the following we derive a sufficient condition that guarantees that the proposed control design of the previous section drives the agents to the desired formation configuration in the case of formation feasibility. The function

$$V = \sum_i \left(\gamma_i + \sum_{j \neq i} V_{ij} \right)$$

is used as a candidate Lyapunov function for the multi-agent system. Differentiating V with respect to time we get

$$\dot{V} = (\nabla V)^T \cdot \dot{q}$$

Differentiating γ_i with respect to q_i we have

$$\frac{\partial \gamma_i}{\partial q_i} = \sum_{j \in N_i} (q_i - q_j - c_{ij}) = \sum_{j \in N_i} (q_i - q_j) + c_{ii}$$

where $c_{ii} = -\sum_{j \in N_i} c_{ij}$. We can then compute

$$\begin{bmatrix} -\frac{\partial \gamma_1}{\partial q_1} \\ \vdots \\ -\frac{\partial \gamma_N}{\partial q_N} \end{bmatrix} = -(Lq + c_l)$$

where $c_l \triangleq [c_{11}, \ldots, c_{NN}]^T$, $L = \mathcal{L} \otimes I_2$, $\mathcal{L}$ is the Laplacian of the formation graph and $\otimes$ denotes the matrix Kronecker product, as usual. After simple calculations we also get

$$\sum_i \nabla \gamma_i = 2(Lq + c_l) \tag{4}$$

Furthermore,

$$\sum_i \sum_{j \neq i} \nabla V_{ij} = 2 \left(\sum_i \sum_{j \neq i} \rho_{ij} D_{ij} \right) q = 2(R_1 \otimes I_2) q$$

where the matrix R_1 can be computed by

$$(R_1)_{ij} = \begin{cases} \sum_{j \neq i} \rho_{ij} + \sum_{j \neq i} \rho_{ji}, i = j \\ -\rho_{ij} - \rho_{ji}, i \neq j \end{cases}$$

The last equation has been derived based on the form of the D_{ij} matrices.

The gradient of the candidate Lyapunov function is now given by

$$\nabla V = 2(z + (R_1 \otimes I_2) q)$$

where $z \triangleq Lq + c_l = (\mathcal{L} \otimes I_2) q + c_l$. We also have

$$\dot{q} = \begin{bmatrix} -\frac{\partial \gamma_1}{\partial q_1} \\ \vdots \\ -\frac{\partial \gamma_N}{\partial q_N} \end{bmatrix} + \begin{bmatrix} -\sum_{j \in M_1} \frac{\partial V_{1j}}{\partial q_1} \\ \vdots \\ -\sum_{j \in M_N} \frac{\partial V_{Nj}}{\partial q_N} \end{bmatrix}$$

The second element is calculated by

$$\begin{bmatrix} -\sum_{j \in M_1} \frac{\partial V_{1j}}{\partial q_1} \\ \vdots \\ -\sum_{j \in M_N} \frac{\partial V_{Nj}}{\partial q_N} \end{bmatrix} = \begin{bmatrix} -\sum_{j \neq 1} \frac{\partial V_{1j}}{\partial q_1} \\ \vdots \\ -\sum_{j \neq N} \frac{\partial V_{Nj}}{\partial q_N} \end{bmatrix} = -2(R_2 \otimes I_2) q$$

The elements of the matrix R_2 are computed based on the form of the D_{ij} matrix and are given by

$$(R_2)_{ij} = \left\{ \begin{array}{l} \sum\limits_{j \neq i} \rho_{ij}, i = j \\ -\rho_{ij}, i \neq j \end{array} \right.$$

Hence

$$\dot{q} = -z - 2\left(R_2 \otimes I_2\right) q$$

Using now the symmetry of the potential functions we get

$$\rho_{ij} = \rho_{ji} \Rightarrow R_1 = 2R_2$$

so that

$$\begin{array}{l} \dot{V} = \left(\nabla V\right)^T \cdot \dot{q} = \\ = -2\left(z + \left(R_1 \otimes I_2\right) q\right)^T \left(z + 2\left(R_2 \otimes I_2\right) q\right) \\ \stackrel{R_1 = 2R_2}{\Rightarrow} \dot{V} = -2\left\|\left(z + 2\left(R_2 \otimes I_2\right) q\right)\right\|^2 \leq 0 \end{array} \tag{5}$$

The first result of this section establishes collision avoidance between the team members. This is established in the following Lemma:

Lemma 1: Assume that the multi-agent system is driven by the control law (3) and let $\phi\left(t, q(0)\right)$ denote the trajectory of the closed loop system at time $t \geq 0$ starting from an initial condition $q(0)$ at $t = 0$. Define the collision free set $\mathcal{I}\left(q\right) = \left\{q \middle| \left\|q_i - q_j\right\| > 0, \forall i, j \in \mathcal{N}, i \neq j\right\}$. Then the set $\mathcal{I}\left(q\right)$ is invariant for the trajectories of the closed loop system.

Proof: For every initial condition $q(0) \in \mathcal{I}(q)$, the time derivative of V remains non-positive for all $t \geq 0$, by virtue of (5). Hence $V\left(\phi\left(t, q(0)\right)\right) \leq V\left(\phi\left(0, q(0)\right)\right) = V\left(q(0)\right) < \infty$ for all $t \geq 0$. Since $V \to \infty$ if and only if $\left\|q_i - q_j\right\| \to 0$ for at least one pair $i, j \in \mathcal{N}$, we conclude that $\phi\left(t, q(0)\right) \in \mathcal{I}\left(q\right)$, for all $t \geq 0$. $\diamondsuit$

Lemma 1 guarantees collision avoidance. The next Theorem refers to the formation convergence objective:

Theorem 1: Assume that the multi-agent system is driven by the control law (3) and let $\phi\left(t, q(0)\right)$ denote the trajectory of the closed loop system at time $t \geq 0$ starting from an initial condition $q(0)$ at $t = 0$. Assume that the following hold:

1) The formation graph is connected.
2) The condition

$$\text{rank}\left(\begin{array}{cc} \left(2R_2 + \mathcal{L}\right) \otimes I_2 & c_l \end{array}\right) > \text{rank}\left(\left(2R_2 + \mathcal{L}\right) \otimes I_2\right)$$

holds for all q such that

$$\begin{array}{l} \exists t \geq 0 : \phi\left(t, q(0)\right) = q \in C \triangleq \\ \triangleq \left\{q \in W \middle| \exists i, j, i \neq j : \left\|q_i - q_j\right\| \leq d\right\} \end{array}$$

3) The formation configuration is feasible, in the sense that

$$\exists q \in W \backslash C : \left(\mathcal{L} \otimes I_2\right) q + c_l = 0$$

Then, under the control law (3) the state of the system converges to the desired formation configuration.

Proof: The last assumption of the theorem implies that the algebraic equation $z + 2\left(R_2 \otimes I_2\right) q = 0$ does not have a solution whenever the matrix R_2 is not identically zero for all q that belong to the trajectory of the closed loop system. Hence

$$\dot{V} = -2\left\|\left(z + 2\left(R_2 \otimes I_2\right) q\right)\right\|^2 \leq 0$$

with equality holding only when $\rho_{ij} = 0 \forall i, j \Rightarrow R_2 = 0$. Furthermore, the level sets of V define compact sets with respect to the agents' relative positions. Specifically, for all $(i, j) \in E$ we have

$$\begin{array}{l} V \leq c \Rightarrow \gamma_i \leq c \Rightarrow \frac{1}{2}\left\|q_i - q_j - c_{ij}\right\|^2 \leq c \Rightarrow \\ \left\|q_i - q_j - c_{ij}\right\| \leq \sqrt{2c} \Rightarrow \left|\left\|q_i - q_j\right\| - \left\|c_{ij}\right\|\right| \leq \sqrt{2c} \Rightarrow \\ \Rightarrow -\sqrt{2c} + \left\|c_{ij}\right\| \leq \left\|q_i - q_j\right\| \leq \sqrt{2c} + \left\|c_{ij}\right\| \Rightarrow \\ \Rightarrow 0 \leq \left\|q_i - q_j\right\| \leq \sqrt{2c} + c_{\max} \end{array}$$

where $c_{\max} \triangleq \max\limits_{(i,j) \in E} \left\|c_{ij}\right\|$. Connectivity of the formation graph ensures that the maximum length of a path connecting two vertices of the graph is at most $N - 1$. Hence $0 \leq \left\|q_i - q_j\right\| \leq \left(\sqrt{2c} + c_{\max}\right)\left(N - 1\right), \ \forall i, j \in \mathcal{N}$.

Application of LaSalle's invariance principle ensures the convergence of the system to the largest invariant subset of the set $S = \left\{q : Lq + c_l = 0\right\}$.

For all $i \in \mathcal{N}$, let c_i denote the configuration of agent i in a desired formation configuration with respect to the global coordinate frame. It is then obvious that $c_{ij} = c_i - c_j \forall (i, j) \in E$ for all possible desired final formations. Define $q_i - q_j - c_{ij} = q_i - q_j - \left(c_i - c_j\right) = \tilde{q}_i - \tilde{q}_j$. Then we have $Lq + c_l = 0 \Rightarrow L\tilde{q} = 0 \Rightarrow \mathcal{L}\tilde{x} = \mathcal{L}\tilde{y} = 0$ where $\tilde{x}, \tilde{y}$ the stack vectors of $\tilde{q}$ in the x, y directions. The fact that the formation graph is connected implies that the Laplacian has a simple zero eigenvalue with corresponding eigenvector the vector of ones, $\overrightarrow{\mathbf{1}}$. This guarantees that both $\tilde{x}, \tilde{y}$ are eigenvectors of $\mathcal{L}$ belonging to span$\{\overrightarrow{\mathbf{1}}\}$. Therefore all $\tilde{q}_i$ are equal to a common vector value c. Hence $\tilde{q}_i = c \ \forall i \Rightarrow q_i - q_j = c_{ij} \forall i, j, j \in N_i$. We conclude that the agents converge to the desired relative configuration. $\diamondsuit$

V. Formation infeasibility results in flocking behavior

The key assumption behind the stability analysis of the previous section is *formation feasibility*, namely that there exists a configuration $q \in W \backslash C$ such that $Lq + c_l = 0$. But what happens when there does *not* exist such a configuration in the state space? The answer is contained in the next theorem:

Theorem 2: Assume that the first two assumptions of Theorem 1 hold. Under these assumptions, the system reaches a configuration in which all agents have the same velocities and orientations even if the formation feasibility assumption of this theorem does not hold.

Proof : Equation $\dot{V} = -2\left\|\left(z + 2\left(R_2 \otimes I_2\right) q\right)\right\|^2$ guarantees that the system converges to the set $W \backslash C$ at steady state. Hence, at steady state system kinematics are given by:

$$\dot{q} = -\left(Lq + c_l\right) \tag{6}$$

Differentiating equation(6) wrt time we get

$$\dot{q} = -\left(Lq + c_l\right) \Rightarrow \ddot{q} = -L\dot{q} \tag{7}$$

Using $W = \frac{1}{2}\left\|\dot{q}\right\|^2$ as a candidate Lyapunov function for the differential equation (7) and taking its time derivative we have

$$W = \frac{1}{2}\left\|\dot{q}\right\|^2 \Rightarrow \dot{W} = \dot{q}^T\ddot{q} = -\dot{q}^T L\dot{q} \leq 0$$

LaSalle's Invariance Principle guarantees that the state of the system converges to the largest invariant subset of the set $S = \left\{\dot{q}|\dot{W} = 0\right\}$. Since $\ddot{q} = -L\dot{q}$ we necessarily have $\ddot{q} = 0$ inside S. Hence agent velocities converge to a constant value. Using the notation v_x, v_y for the N-dimensional stack vectors of the components of the agents' velocities in the x, y directions at steady state, we have

$$\dot{W} = 0 \Rightarrow \dot{q}^T\left(\mathcal{L} \otimes I_2\right)\dot{q} = 0 \Rightarrow v_x^T\mathcal{L}v_x + v_y^T\mathcal{L}v_y = 0$$

at steady state. This implies that both v_x, v_y are eigenvectors of $\mathcal{L}$ corresponding to the zero eigenvalue, meaning that v_x, v_y belong to span$\{\overrightarrow{\mathbf{1}}\}$, which ensures that all agent velocity vectors will have the same components at steady state, and will therefore be equal. $\diamondsuit$

This simple result shows that formation non-feasibility is directly related to a phenomenon with many similarities to what is known as flocking behavior in multi-agent systems. Please note that the final common velocities in the result of theorem 2 are not necessarily equal to zero.

VI. Simulations

To verify the results of the previous paragraphs we provide two computer simulations.

The first simulation in Figure 2 involves convergence to a feasible formation configuration. Specifically, we implement a line formation of four holonomic agents with communication sets given by

$$N_1 = \{2, 3, 4\}, N_2 = \{1\}, N_3 = \{1\}, N_4 = \{1\}$$

It is easily verified that the corresponding communication graph is connected. The four agents aim to converge to a line formation and the desired inter-agent relative positions are chosen accordingly. Screenshots I-V show the evolution in time of the multi-agent team. In screenshot I, A-i denotes the initial position of agent i. In the last screenshot, the agents converge to the desired line formation configuration. A collision avoidance maneuver between agents 1 and 3 occurs in screenshot IV. The values of the parameters in this simulation are: $d^2 = 1e-5, h = 1, a = (4/27)1e-15$.

The second simulation in Figure 3 involves four agents and a non-feasible formation configuration. The values of the parameters in this simulation are the same as previously while the desired inter-agent distances have been slightly perturbed in order to achieve formation infeasibility. Hence the third assumption of Theorem 1 may no longer be valid. The formation configuration may be rendered infeasible in this way. Theorem 2 however, guarantees that the system will reach a configuration where all agents will have the same velocities. Screenshots I-V of Figure 3 show the evolution in time and achievement of velocity alignment for the multi-agent system. The last screenshot shows that the velocities of the four agents converge to a common value.

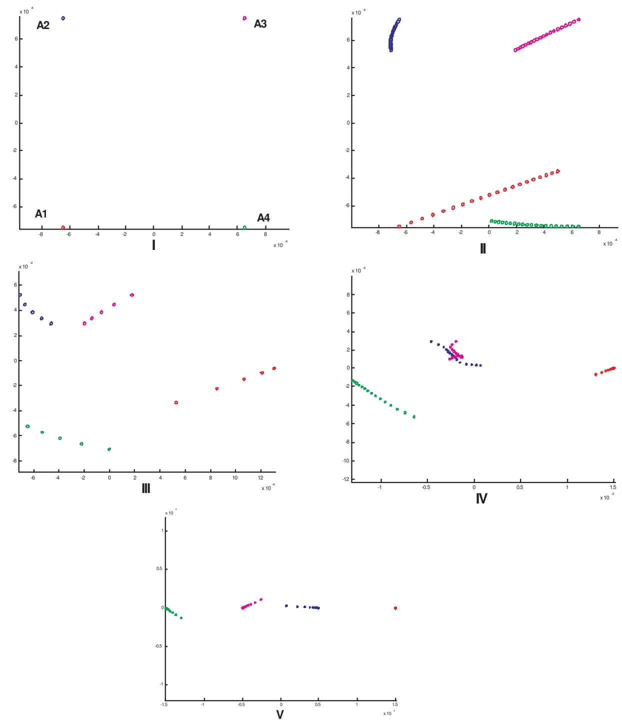

Fig. 2. Four agents converge to a line formation

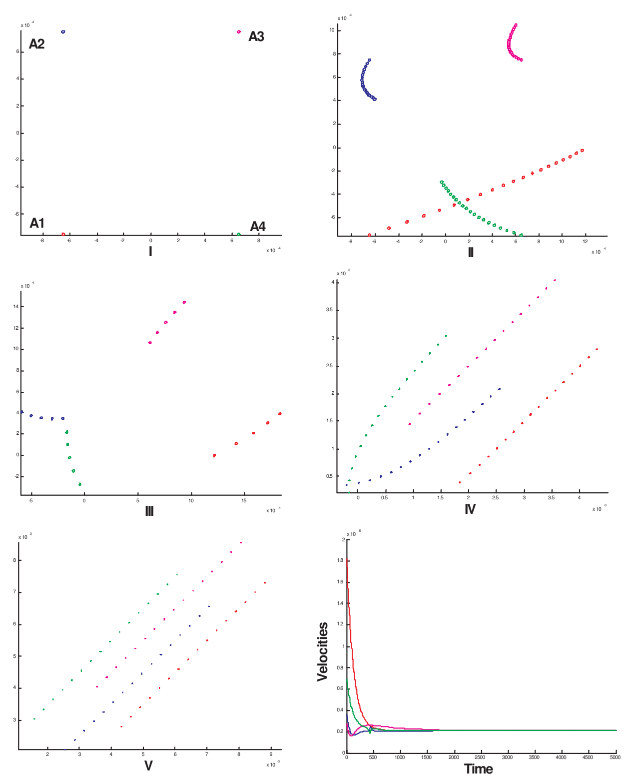

Fig. 3. Formation infeasibility results in flocking behavior for the multi-agent system.

VII. Conclusions

A provably correct distributed feedback control strategy that achieves convergence of a multi-agent system to a desired formation configuration avoiding at the same time collisions has been proposed. The collision avoidance and formation convergence objectives are treated in a decoupled manner. The symmetry of the potential field that ensures collision avoidance is used in the stability analysis of the system. When inter-agent objectives that specify the desired formation cannot occur simultaneously in the state space the desired formation is infeasible. It has been shown that under certain assumptions, formation infeasibility forces the agents velocity vectors to a common value at steady state. This provides a connection between formation infeasibility and flocking behavior for the multi-agent system.

Current research involves extending the current results to more general motion models, including three-dimensional models and general nonlinear dynamics. Another direction of research is to take into account directed graphs and switching communication topology.

VIII. Acknowledgements

The authors want to acknowledge the contribution of the European Commission through contract I-SWARM (IST-2004-507006).

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