Splitting and merging control of multiple platoons with Signal Temporal Logic*

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***Abstract*— Coordination of multiple platoons involves the design of vehicles' trajectories ensuring splitting and merging of different platoons in a safe and energy-optimal manner. In this work, we consider the *split-merge-maintain* problem between any pair of platoons in which a number of vehicles temporally split from the rest, allowing the vehicles of another platoon to merge, and move with the preceding and following vehicles as a single, large platoon. Here, the splitting, merging and distance-maintaining tasks are expressed in Signal Temporal Logic (STL) and a control barrier function (CBF) is introduced to encode the STL constraints. The control inputs of the vehicles are, then, found as a solution to a computationally efficient, convex, quadratic program. The effectiveness of the proposed method is verified in simulation.**

I. Introduction

Nowadays, multi-vehicle coordination has received increased attention as it is considered a promising solution towards reducing traffic congestion and fuel emission in urban environments and highways. Among different driving scenarios, existing work has focused on vehicle platooning, in which vehicles moving in the same lane are expected to maintain a desired distance from their preceding vehicle, and move at a constant velocity while communicating with a limited number of neighboring vehicles. In the majority of the works, consensus protocols are designed for longitudinal control under limited inter-vehicle communication as in [1]–[3] and stability is established often under communication delays and/or varying communication topologies. In [4]–[7] the splitting and/or merging problem of single vehicles was studied and distributed or centralized control schemes were proposed for the design of safe trajectories ensuring these tasks. A detailed review on merging control strategies for connected and automated vehicles (CAVs) can be found in [8].

Contrary to single-platoon approaches, coordination of multiple platoons has received considerably less attention in literature [9]–[15]. Formation control of multiple platoons has been studied in [10], [11] for single and double integrator dynamics with the problem being formulated as a group consensus control problem. In [12] a four layer hierarchical approach is considered and longitudinal and lateral controllers are proposed ensuring consensus of vehicles in the same string and among different strings of vehicles. In these methods actuation limitations are not considered and the requirement of vehicles' positive velocity along their corresponding lane is ignored. Merging and splitting control of platoons has been considered in [13], where a two-layer hierarchical is proposed. In the first layer vehicles are organized in a virtual formation according to a scheduling policy enabling the splitting and merging of the platoons. Then, in the second layer a longitudinal controller is designed to follow the virtual reference of the formation and a potential function based lateral controller is proposed for ensuring safe lane changing. [14] studies the multi-platoon merging problem at curvilinear roads, accounting for input saturation and acceleration limitations while [15] introduces a two-stage distributed model predictive control scheme for merging of two adjacent platoons. In the first stage a space-making DMPC controller is introduced to temporally increase the distance among two given vehicles and then, a second DMPC controller is introduced to ensure smooth lane changing and fitting to the target platoon.

In all the aforementioned approaches, lane-changing, splitting and merging tasks are considered to be spatial tasks while the time required for their execution is either considered as a soft constraint or totally ignored. Recently, a formal language called Signal Temporal Logic (STL) was introduced in [16] to express complex, time-constrained tasks as the ones discussed above. Contrary to other logics, STL is evaluated over continuous signals and is equipped with a robustness metric, defined as for example in [17], [18] expressing the degree of satisfaction or violation of a task. In the majority of existing work, planning under STL tasks is formulated as an MILP problem where the dynamics and predicate functions are linear as for example in [19]–[22]. [23]–[26] consider the design of feedback controllers for nonlinear input affine systems in which the STL constraints are encoded using control barrier functions (CBFs). Finally, a first step towards combining the benefits of longer horizon problems with control barrier functions was made in [27].

In this work we consider the split-merge-maintain problem for systems of platoons that are subject to input constraints. In the considered scenario a pair of platoons, initially moving at different lanes is asked to eventually merge in one of the lanes and move as a single, large platoon. To achieve this, the first platoon needs to open space to allow the second platoon to merge and eventually minimize the distance with the vehicles of the "hosting" platoon resuming the platooning mode. These complex tasks are expressed in Signal Temporal Logic and encoded using control barrier functions. The desired velocity of the vehicles is, then, found as a solution to a CBF-QP problem that ensures minimum violation of

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the STL formula, when the satisfaction of the latter is not possible due to actuation limitations or tight time-constraints.

The remainder of this paper is organized as follows: Section II includes the problem preliminaries, Section III introduces the problem formulation, Section IV demonstrates the efficacy of the proposed controller in simulation and Section V summarizes the main results of this work and discusses some ideas for future research.

II. Preliminaries

True and false are denoted by $\top, \bot$ respectively. Scalars and vectors are denoted by non-bold and bold letters respectively. The partial derivative of a function $\mathfrak{b}(\mathbf{x}, t)$ evaluated at $(\mathbf{x}', t')$ with respect to t and $\mathbf{x}$ is abbreviated by $\frac{\partial \mathfrak{b}(\mathbf{x}',t')}{\partial t} = \frac{\partial \mathfrak{b}(\mathbf{x},t)}{\partial t}\Big|_{\substack{\mathbf{x}=\mathbf{x}'\\ t=t'}}$ and $\frac{\partial \mathfrak{b}(\mathbf{x}',t')}{\partial \mathbf{x}} = \frac{\partial \mathfrak{b}(\mathbf{x},t)}{\partial \mathbf{x}}\Big|_{\substack{\mathbf{x}=\mathbf{x}'\\ t=t'}}$ respectively. The latter is considered to be a row vector. An extended class $\mathcal{K}$ function $\alpha : \mathbb{R} \to \mathbb{R}_{\geq 0}$ is a locally Lipschitz continuous and strictly increasing function with $\alpha(0) = 0$. $A \otimes B$ denotes the Kronecker product of the matrices $A \in \mathbb{R}_n, B \in \mathbb{R}_m$. Given a set $Q \subset \mathbb{R}$, the set 2^Q denotes its power set. The Cartesian product of the sets $U_1, \dots, U_n$ is denoted by $U = \prod_{i=1}^n U_i$.

A. Signal Temporal Logic

Signal Temporal Logic (STL) determines whether a predicate μ is true or false based on the value of a continuously differentiable function $h : \mathbb{R}^n \to \mathbb{R}$ as follows:

$$\mu = \begin{cases} \top, & h(\mathbf{x}) \geq 0 \\ \bot, & h(\mathbf{x}) < 0. \end{cases} \tag{1}$$

The basic STL formulas are given by the grammar:

$$\phi := \top \mid \mu \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid \mathcal{G}_{[a,b]}\phi \mid \mathcal{F}_{[a,b]}\phi \mid \phi_1\ \mathcal{U}_{[a,b]}\ \phi_2,$$

where ϕ_1, ϕ_2 are STL formulas and $\mathcal{G}_{[a,b]}$, $\mathcal{F}_{[a,b]}$, $\mathcal{U}_{[a,b]}$ is the always, eventually and until operator defined over the interval $[a, b]$ with $0 \leq a \leq b$. Let $\mathbf{x} \models \phi$ denote the satisfaction of the formula ϕ by a signal $\mathbf{x} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$. The formula ϕ is satisfiable if $\exists\ \mathbf{x} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ such that $\mathbf{x} \models \phi$. The STL semantics for a signal $\mathbf{x} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ are recursively defined according to [16, Sec. 2.2]. STL is equipped with a robustness metric determining how robustly an STL formula ϕ is satisfied at time t. Given a continuous signal $\mathbf{x} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, the STL robust semantics [17] are recursively defined as follows: $\rho^{\mu}(\mathbf{x}, t) = h(\mathbf{x}(t))$, $\rho^{\neg\phi}(\mathbf{x}, t) = -\rho^{\phi}(\mathbf{x}, t)$, $\rho^{\phi_1 \wedge \phi_2}(\mathbf{x}, t) = \min(\rho^{\phi_1}(\mathbf{x}, t), \rho^{\phi_2}(\mathbf{x}, t))$, $\rho^{\phi_1\ \mathcal{U}_{[a,b]}\ \phi_2}(\mathbf{x}, t) = \max_{t_1 \in [t+a,t+b]} \min(\rho^{\phi_2}(\mathbf{x}, t_1), \min_{t_2 \in [t,t_1]} \rho^{\phi_1}(\mathbf{x}, t_2))$, $\rho^{\mathcal{F}_{[a,b]}\phi}(\mathbf{x}, t) = \max_{t_1 \in [t+a,t+b]} \rho^{\phi}(\mathbf{x}, t_1)$, $\rho^{\mathcal{G}_{[a,b]}\phi}(\mathbf{x}, t) = \min_{t_1 \in [t+a,t+b]} \rho^{\phi}(\mathbf{x}, t_1)$. Finally, $\mathbf{x} \models \phi$ holds, if $\rho^{\phi}(\mathbf{x}, 0) > 0$.

B. Control Barrier Functions for STL satisfaction

In this section we summarize the basic steps towards designing a control barrier function (CBF) for STL satisfaction as described in [24], [25]. Consider the STL fragment:

$$\psi = \top \mid \mu \mid \neg\mu, \tag{2a}$$

$$\bar{\varphi} = \mathcal{G}_{[a,b]}\psi \mid \mathcal{F}_{[a,b]}\psi \mid \psi_1\ \mathcal{U}_{[a,b]}\ \psi_2, \tag{2b}$$

$$\phi' = \bigwedge_{l=1}^{n_\phi} \bar{\varphi}_l, \tag{2c}$$

where ψ_1, ψ_2 are STL formulas of the form (2a), $\bar{\varphi}_l$, $l = 1, \dots, n_\phi$ are STL formulas of the form (2b), $n_\phi \geq 1$ and $0 \leq a \leq b < \infty$. By definition of the STL semantics, it is sufficient to ensure the satisfaction of a formula ϕ, defined as a conjunction of eventually and always formulas φ_i as follows:

$$\phi = \bigwedge_{i \in \mathcal{I}} \varphi_i, \tag{3}$$

where $\mathcal{I} = \mathcal{I}^{\mathcal{F}} \cup \mathcal{I}^{\mathcal{G}}$, $\mathcal{I}^{\mathcal{F}} = \{i \in \mathcal{I} : \varphi_i = \mathcal{F}_{[a_i,b_i]}\psi_i\}$ and $\mathcal{I}^{\mathcal{G}} = \{i \in \mathcal{I} : \varphi_i = \mathcal{G}_{[a_i,b_i]}\psi_i\}$. For each subformula $\varphi_i, i \in \mathcal{I}$, let $\mathfrak{b}_i : \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ be a function defined as:

$$\mathfrak{b}_i(\mathbf{x}, t) = -\gamma_i(t) + h_i(\mathbf{x}),$$

where $h_i : \mathbb{R}^n \to \mathbb{R}$ is the predicate function corresponding to φ_i, assumed to be continuously differentiable and $\gamma_i : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is a function describing a desired temporal behavior of the system that ensures satisfaction of φ_i with a minimum robustness r. As in [25], the performance functions $\gamma_i(t)$ are defined as piecewise linear functions as follows:

$$\gamma_i(t) = \begin{cases} \frac{\gamma_{i,\infty} - \gamma_{i,0}}{t_i^*} t + \gamma_{i,0}, & \text{if } t < t_i^* \\ \gamma_{i,\infty}, & \text{if } t \geq t_i^* \end{cases}, \tag{4}$$

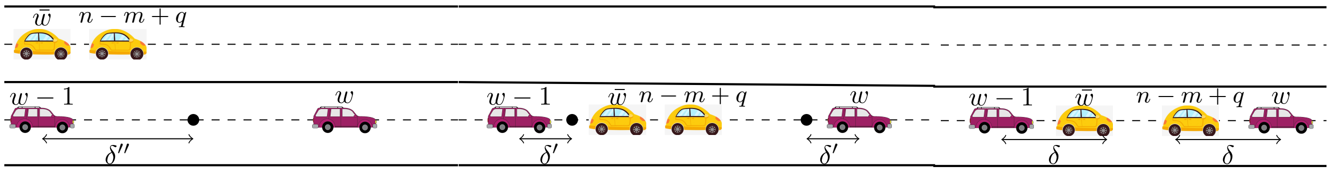
where $\gamma_{i,0}, \gamma_{i,\infty}, t_i^*$ are parameters depending on the robustness value r and chosen as in [25] to satisfy $\mathfrak{b}_i(\mathbf{x}(0), 0) \geq 0$ and $\mathfrak{b}_i(\mathbf{x}(t), t) \geq 0$ or equivalently $h_i(\mathbf{x}(t)) \geq r$, for every $t \geq t_i^*$. Based on the functions $\mathfrak{b}_i(\mathbf{x}, t)$, the CBF function $\mathfrak{b} : \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}$ corresponding to ϕ is defined as:

$$\mathfrak{b}(\mathbf{x}, t) = -\frac{1}{\eta} \ln\left(\sum_{i \in \mathcal{I}} o_i(t) \exp\left(-\eta \mathfrak{b}_i(\mathbf{x}, t)\right)\right), \tag{5}$$

where $\eta > 0$ is a parameter to be tuned, $o_i : \mathbb{R}_{\geq 0} \to \{0, 1\}$ is an integer valued function with $o_i(t) = 1$, for $t \in T^i$ and $o_i(t) = 0$, otherwise, where $T^i = [0, b_i)$, if $i \in \mathcal{I}^{\mathcal{F}} \cup \{i' \in \mathcal{I}^{\mathcal{G}} : a_{i'} = 0\}$, or $T^i = [0, a_i) \cup (a_i, b_i)$ if $i \in \{i' \in \mathcal{I}^{\mathcal{G}} : a_{i'} \neq 0\}$. Observe that due to the deactivation policy the proposed barrier function is differentiable only at $\mathbb{R}^n \times (\mathbb{R}_{\geq 0} \backslash \Sigma)$, where $\Sigma \subseteq \{a_i, b_i : i \in \mathcal{I}\}$. For this particular choice of $\mathfrak{b}(\mathbf{x}, t)$ it can be shown [24] that $\mathfrak{b}(\mathbf{x}, t) \leq \min_{i \in \mathcal{A}(t)} \mathfrak{b}_i(\mathbf{x}, t)$, where $\mathcal{A}(t) = \{i \in \mathcal{I} : o_i(t) = 1\}$. Therefore, if there exists $\mathbf{x} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ such that $\mathfrak{b}(\mathbf{x}(t), t) \geq 0$ for every $t \geq 0$, then $\mathfrak{b}_i(\mathbf{x}(t), t) \geq 0$, for every $i \in \mathcal{A}(t)$ and the set $\mathcal{C}(t)$ is rendered forward invariant, where $\mathcal{C}(t) = \{\mathbf{x} \in \mathbb{R}^n : \mathfrak{b}(\mathbf{x}, t) \geq 0\}$.

C. Graph Theory

An *undirected* graph G is defined as a pair $G = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{1, \dots, n\} \subset \mathbb{N}$ is a finite set of nodes and $\mathcal{E} \subseteq \{(w, w') \in \mathcal{N} \times \mathcal{N} : w \neq w'\}$. The *adjacency matrix* A of G is the $n \times n$ symmetric matrix whose elements $\bar{a}_{ww'}, w, w' \in \mathcal{N}$ are defined as follows: $\bar{a}_{ww'} = 1$, if $(w, w') \in \mathcal{E}$ and $\bar{a}_{ww'} = 0$, otherwise. The degree $\bar{d}_w$ of a node w is the cardinality of N_w, i.e., $\bar{d}_w = \sum_{w' \in N_w} \bar{a}_{ww'}$, where $N_w = \{w' \in \mathcal{N} : (w, w') \in \mathcal{E}\}$ denotes the nodes



(a) Splitting Task (b) Merging Task (c) Maintain Distance

Fig. 1: Split-Merge-Maintain Task for a pair of platoons.

that can communicate with w. Then, the *degree matrix* Δ of G is defined as the diagonal matrix $n \times n$, containing the degrees of the nodes of G on the diagonal, i.e., $\Delta = \text{diag}(\bar{d}_1, \ldots, \bar{d}_n)$. Based on these matrices, we can define the *Laplacian* matrix $\mathcal{L}$ of G as $\mathcal{L} = \Delta - A$. A *path* is a sequence of edges connecting two distinct vertices. A graph is *connected*, if there exists a path between any pair of vertices. A *component* of G [28] is a subset of G, associated with a minimal partitioning of the vertex set, such that each partition is connected. A connected graph has a single *connected component*.

III. Problem Formulation

Consider n vehicles that are initially divided in m platoon systems. The leading vehicle of each platoon is called the *leader* of the platoon and the remaining vehicles are called *followers*. Leaders are able to communicate with their neighboring followers and/or other leaders. On the other hand, followers are only able to communicate with neighboring vehicles of the same platoon. Consider the communication graph $G = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is the set of agents and $\mathcal{E}$ is the set of edges, where $\bar{e} = (w, w') \in \mathcal{E}$ iff vehicles w, w' communicate. The graph G is assumed to be static and undirected. Note that we do not pose any assumption on the connectivity of the whole graph although we assume that the graph has m connected components, i.e., each platoon is assumed to be connected. Let $\mathbf{x}_w = \begin{bmatrix} x_w^1 & x_w^2 \end{bmatrix}^T$ denote the position of the w-th vehicle, where $w \in \mathcal{N} = \{1, \ldots, n\}$. The position of each vehicle evolves according to the following equation:

$$\dot{\mathbf{x}}_w = -\sum_{w' \in N_w} (\mathbf{x}_w - \mathbf{x}_{w'} - \mathbf{x}_{ww'}^{des}) + \mathbf{u}_w, \tag{6}$$

where $\mathbf{u}_w \in U_w \subseteq \mathbb{R}^2$ is the control input of the w-th vehicle, N_w includes the vehicles with which w can communicate and $\mathbf{x}_{ww'}^{des} = \mathbf{x}_w^{des} - \mathbf{x}_{w'}^{des}$ is the desired relative position between agent w and agent w', which is defined as the difference between the absolute desired positions $\mathbf{x}_w^{des}, \mathbf{x}_{w'}^{des} \in \mathbb{R}^2$ and is constant. Here, only $\mathbf{x}_{ww'}^{des}$ needs to be known by the vehicles, hence extra communication or unnecessary reference frame changes are avoided. Each vehicle is assumed to have limited actuation capabilities, i.e., $U_w \subset \mathbb{R}^2$ is a bounded, convex set with $\mathbf{0} \in U_w$ for every $w \in \mathcal{N}$. Here, for simplicity we will consider box constraints. Thus, $U_w, w \in \mathcal{N}$ is defined as follows:

$$U_w = \prod_{k=1}^{2} [-u_{w,\max}^k, u_{w,\max}^k], \tag{7}$$

where $\mathbf{u}_w = \begin{bmatrix} u_w^1 & u_w^2 \end{bmatrix}^T$ and $0 < u_{w,\max}^k < \infty$ for every $w \in \mathcal{N}$ and $k \in \{1, 2\}$. Stacking the dynamics of the individual vehicles defined in (6), we may write the dynamics of the multi-vehicle system as follows:

$$\dot{\mathbf{x}} = -(\mathcal{L} \otimes I_2)\mathbf{x} + \mathbf{d} + \mathbf{u}, \tag{8}$$

where $\mathbf{x} = \begin{bmatrix} \mathbf{x}_1^T & \ldots & \mathbf{x}_n^T \end{bmatrix}^T \in \mathbb{R}^{2n}$, $\mathbf{u} = \begin{bmatrix} \mathbf{u}_1^T & \ldots \mathbf{u}_n^T \end{bmatrix}^T \in U \subseteq \mathbb{R}^{2n}$ is the state and input of the centralized system respectively, $U = \prod_{w \in \mathcal{N}} U_w$, $\mathcal{L} \in \mathbb{R}_{n \times n}$ is the Laplacian matrix of the graph G, and $\mathbf{d} \in \mathbb{R}^{2n}$ is a vector with elements $\tilde{d}_w, w \in \mathcal{N}$ defined as $\tilde{d}_w = \sum_{w' \in N_w} \mathbf{x}_{ww'}^{des}$. To simplify notation, let $\mathbf{x} = \begin{bmatrix} \mathbf{x}_F^T & \mathbf{x}_L^T \end{bmatrix}^T$, where $\mathbf{x}_F \in \mathbb{R}^{2(n-m)}$ and $\mathbf{x}_L \in \mathbb{R}^{2m}$ denote the states of the followers and leaders of the platoons, respectively.

In this work we will consider the problem of splitting and merging of platoons while ensuring that vehicles move close together as a platoon at all other times. This task is formally described by the STL formula ϕ, defined as:

$$\phi = \bigwedge_{l \in L} \bigwedge_{j \in J} \varphi_j^l, \tag{9}$$

where $L \subset 2^Q$ is the index set of the split-merge-maintain tasks between any pair of platoons (q, q') with $q, q' \in Q = \{1, \ldots, m\}$ and $J = \{1, 2, 3\}$ is the set of the indices of the STL formulas describing the split, merge or platoon-maintaining task between the l-th pair of platoons (q, q').

Here, the formulas $\varphi_j^l, l \in L, j \in J$ are defined as follows:

$$\varphi_1^l = \mathcal{G}_{[a_l, c_l]} \left(x_w^1 - x_{w-1}^1 \geq \delta'' \right), \tag{10a}$$

$$\begin{aligned}\varphi_2^l = \mathcal{G}_{[b_l, e_l]} \big(&(x_{\bar{w}}^1 - x_{w-1}^1 \geq \delta') \wedge (x_{n-m+q}^1 - x_w^1 \leq -\delta') \\ &\wedge (\|x_{n-m+q}^2 - y_{q'}\|_2 \leq \epsilon) \big),\end{aligned} \tag{10b}$$

$$\varphi_3^l = \mathcal{G}_{[d_l, f_l]} \big((x_{\bar{w}}^1 - x_{w-1}^1 \leq \delta) \wedge (x_{n-m+q}^1 - x_w^1 \geq -\delta) \big), \tag{10c}$$

where $0 \leq a_l \leq b_l \leq c_l \leq d_l \leq e_l \leq f_l$, $\epsilon \leq \frac{\chi}{2}$, $\chi > 0$ is the width of each lane, $\delta'' > 0$ and $\delta \geq \delta' > 0$. Here, $w, w-1$ are two consecutive vehicles of the q'-th platoon, $\bar{w}$ is the last follower of the q-th platoon and $y_{q'} \in \mathbb{R}$ denotes the x^2 coordinate of the center line of the lane at which the q'-th platoon moves. Intuitively, φ_1^l requires vehicles $w, w-1$ to increase their inter-distance along the horizontal axis, and hence temporarily split the q'-th platoon into two smaller platoons until time instant c_l. In (10b) the last follower of the q-th platoon should move to the lane of the q'-th platoon overtaking the $(w-1)$-th vehicle while the leader should follow the w-th vehicle. Additionally, the leader of the q-th platoon should always stay close to the center line of the new lane between time instants b_l and e_l. Finally, in (10c) the vehicles should move towards minimizing their relative distance, and thus recovering the platooning mode. Specifically, in the time interval $[d_l, f_l]$ the vehicles $\bar{w}, w$ should not be more than δ meters ahead of the vehicle $w-1$ and $n-m+q$ respectively. An illustration of the *split-merge-maintain* task is shown in Figure 1.

Observe that the STL formulas, defined by (10a)-(10c), belong to the STL fragment considered in [24] with the predicate functions of the corresponding always formulas being concave on $\mathbf{x}$. Hence, the STL constraints can be encoded using a control barrier function $\mathfrak{b}(\mathbf{x}, t)$ defined as in Section II-B.

Based on the above we may define the QP problem considered in this work as follows:

$$\min_{(\mathbf{u},\epsilon)\in U\times\mathbb{R}_{\geq 0}} \|\mathbf{u}-\mathbf{u}_{nom}\|_2^2+\epsilon^2, \tag{11}$$

subject to:

$$\frac{\partial\mathfrak{b}(\mathbf{x},t)}{\partial\mathbf{x}}(-(\mathcal{L}\otimes I_2)\mathbf{x}+\mathbf{d}+\mathbf{u})+\frac{\partial\mathfrak{b}(\mathbf{x},t)}{\partial t}\geq-\alpha(\mathfrak{b}(\mathbf{x},t))-\epsilon, \tag{11a}$$

$$C(-(\mathcal{L}\otimes I_2)\mathbf{x}+\mathbf{d}+\mathbf{u})>\mathbf{0} \tag{11b}$$

where $C\in\mathbb{R}_{n\times 2n}$ is a matrix with elements $c_{sp}=1$ if $p=2(s-1)+1$, and $c_{sp}=0$ otherwise, and $u_{nom}:\mathbb{R}^{2n}\times\mathbb{R}_{\geq 0}\to U$ is a nominal controller that is assumed to be continuous in $\mathbf{x}$ and piewecise continuous in t. Here, the nominal controller $\mathbf{u}_{nom}(\mathbf{x},t)$ is an offline designed controller that may introduce a desired action plan for the vehicles ensuring the split-merge-maintain task [13] but possibly not with the desired robustness r or not necessarily within the desired time intervals. Assuming that at time t' $\mathbf{x}(t')\in\mathcal{C}(t')$ holds, (11a) ensures that $\lim_{t\to t'^+}\mathbf{x}(t)\in\lim_{t\to t'^+}\mathcal{C}(t)$, if $\epsilon^*=0$, or otherwise that the designed, optimal controller $\mathbf{u}^*(\mathbf{x},t')$ will violate the specification minimally. Constraint (11b), relevant to platooning applications, is a novel constraint introduced in this work to ensure that the vehicles move forward along the horizontal axis and hence, avoid sudden stopping in the middle of the road reducing the possibility of accidents and/or traffic delays. By definition of the matrix C, (11b) is equivalent to $\dot{\mathbf{x}}_w^1>0$ for every $w\in\mathcal{N}$ or to the following constraint:

$$\mathbf{u}_w^1>\sum_{w'\in N_w}(\mathbf{x}_w^1-\mathbf{x}_{w'}^1-\mathbf{x}_{ww'}^{1,des}),\quad w\in\mathcal{N}, \tag{12}$$

where $\mathbf{x}_{ww'}^{1,des}\in\mathbb{R}$ is the desired relative position between vehicles w, w' along the horizontal axis. Based on the above we can now state the main result of this work as follows:

Theorem 1. *Consider the system* (8) *and the STL formula* ϕ *defined by* (9) *and* (10a)-(10c). *Assume that* $\mathbf{x}(0)\in\mathcal{C}(0)$ *and* $\mathcal{C}(t)\neq\emptyset$, *for every* $t\geq 0$. *For a given, linear, extended class* $\mathcal{K}$ *function* $\alpha:\mathbb{R}\to\mathbb{R}_{\geq 0}$ *and an open, bounded set* $\mathcal{D}\subset\mathbb{R}^{2n}$ *such that* $\mathcal{C}(t)\subset\mathcal{D}$ *is true for every* $t\geq 0$, *the following hold:*

(i) Assume that (11) *is feasible at every* $(\mathbf{x},t)\in\mathcal{D}\times(\mathbb{R}_{\geq 0}\backslash\Sigma)$ *and let* $(\mathbf{u}^*(\mathbf{x},t),\epsilon^*(t))\in U\times\mathbb{R}_{\geq 0}$ *denote its optimal solution at* $(\mathbf{x},t)\in\mathcal{D}\times(\mathbb{R}_{\geq 0}\backslash\Sigma)$. *If* $\epsilon^*(t)=0$ *for every* $t\in\mathbb{R}_{\geq 0}\backslash\Sigma$, *then* $\rho^\phi(\mathbf{x},0)\geq r>0$.

(ii) If $\mathbf{u}_{nom}:\mathbb{R}^{2n}\times\mathbb{R}_{\geq 0}\to U$ *is a nominal control input that is continuous in* $\mathbf{x}$ *and piecewise continuous in* t *and satisfies* $\mathbf{u}_{nom}(\mathbf{x},t)\in K(\mathbf{x},t)$, *for every* $(\mathbf{x},t)\in\mathcal{D}\times\mathbb{R}_{\geq 0}\backslash\Sigma$, *where* $K(\mathbf{x},t)\subseteq U$ *is defined as:*

$$K(\mathbf{x},t)=\big\{\mathbf{u}\in U:\dot{\mathfrak{b}}(\mathbf{x},t)\geq-\alpha(\mathfrak{b}(\mathbf{x},t)),\\ C(-(\mathcal{L}\otimes I_2)\mathbf{x}+\mathbf{d}+\mathbf{u})>0\big\},$$

with $\dot{\mathfrak{b}}(\mathbf{x},t)=\frac{\partial\mathfrak{b}(\mathbf{x},t)}{\partial\mathbf{x}}(-(\mathcal{L}\otimes I_2)\mathbf{x}+\mathbf{d}+\mathbf{u})+\frac{\partial\mathfrak{b}(\mathbf{x},t)}{\partial t}$, *then, the system* (8) *with* $\mathbf{u}=\mathbf{u}_{nom}(\mathbf{x},t)$ *ensures that* $\rho^\phi(\mathbf{x},0)\geq r>0$.

(iii) Let $\epsilon_{wc}=\sup_{\mathcal{D}\times(\mathbb{R}_{\geq 0}\backslash\Sigma)}\epsilon^*(t)$, *where* $\epsilon^*(t)$ *is the optimal solution of* (11) *for* $t\in\mathbb{R}_{\geq 0}\backslash\Sigma$. *The set:*

$$\mathcal{C}_{wc}(t)=\{\mathbf{x}\in\mathbb{R}^{2n}:\mathfrak{b}(\mathbf{x},t)\geq\alpha^{-1}(-\epsilon_{wc})\}$$

is forward invariant if $\mathcal{C}_{wc}(t)\subset\mathcal{D}$ *for every* $t\geq 0$.

Proof. The proof of (i) and (ii) follows similar arguments to [23, Th. 1] and (iii) follows the same reasoning as [23, Th. 2]. ■

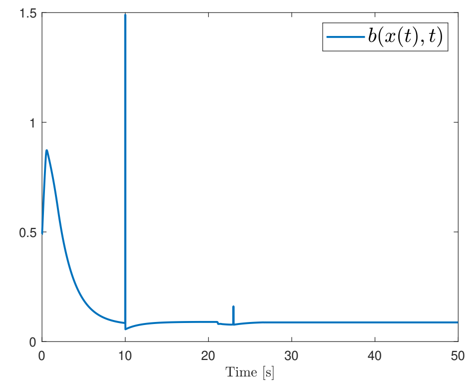


Fig. 2: Evolution of $\mathfrak{b}(\mathbf{x}(t),t)$ over time.

IV. Numerical Example

In this example we consider 5 vehicles and 2 platoons. The first platoon consists of vehicles $\{1,4\}$ and the second of $\{2,3,5\}$, where vehicles 4 and 5 are the platoon leaders of platoon 1 and 2, respectively. Here, G consists of 2 connected components with each connected component corresponding to a platoon of vehicles. Specifically, the Laplacian of the graph $\mathcal{L}$ and the vector $\mathbf{d}\in\mathbb{R}^{10}$ are given as follows:

$$\mathcal{L}=\begin{bmatrix}1&0&0&-1&0\\0&2&-1&0&-1\\0&-1&1&0&0\\-1&0&0&1&0\\0&-1&0&0&1\end{bmatrix},$$

$$\mathbf{d}^T=\begin{bmatrix}-1.5&0&-0.2&0&-1&0&1.5&0&1.2&0\end{bmatrix}.$$

Additionally, the bounds of the intervals of satisfaction of the formulas introduced in (10a)-(10c) are chosen to be

$(a, b, c, d, e, f) = (10, 23, 23, 30, 50, 50)$. The width of the road is chosen to be $\chi = 1$ and $\begin{bmatrix} y_q & y_{q'} \end{bmatrix} = \begin{bmatrix} \chi & 0 \end{bmatrix}$. Here, we choose $\delta'' = 6$, $\delta = 3.5$, $\epsilon = \sqrt{0.1}$ and $\delta' = 2.5$. The maximum allowable input on both directions is $u^k_{w,\max} = 10$, for every $w \in \mathcal{N}$, $k \in \{1, 2\}$. In order to ensure that vehicle 1 will always follow vehicle 4 (the leader), especially during the lane changing task, in addition to the STL task defined in (9) we consider the following task:

$$\varphi_4 = \mathcal{G}_{[0,50]}(x^1_4 - x^1_1 \geq 1),$$

and thus enforce the satisfaction of the formula $\phi' = \phi \wedge \varphi_4$. We design a barrier function for ϕ' with $r = 0.009$ and set $\alpha = 0.5$ and $\eta = 40$. The parameters of the performance functions are chosen as follows: $\boldsymbol{\gamma}_0 = \begin{bmatrix} -5 & -5 & -20 & -6.5 & -20 & -10 & -20 \end{bmatrix}$, $\boldsymbol{\gamma}_\infty = \begin{bmatrix} 0.01 & 0.011 & 0.01 & 0.01 & 0.01 & 0.01 & 0.01 \end{bmatrix}$ and $\mathbf{t}^* = \begin{bmatrix} 10 & 23 & 23 & 23 & 30 & 30 & 0 \end{bmatrix}$, where the v-th element of $\boldsymbol{\gamma}_0, \boldsymbol{\gamma}_\infty, \mathbf{t}^*$ corresponds to the predicate function $h_{j\kappa}(\mathbf{x})$ where j is the index of φ_j (we omit the superscript l for simplicity) and κ is the order of the predicate in the conjunction, e.g., $h_{22}(\mathbf{x}) = -\delta' - x^1_{n-m+q} + x^1_w$. The initial condition of the system is chosen to be $\mathbf{x}^T = \begin{bmatrix} -2.5 & 1 & -2 & 0 & -3 & 0 & -1 & 1 & 0 & 0 \end{bmatrix}$. Here, we consider $u^k_{w,nom}(\mathbf{x}, t) = 0$ for every $w \in \{1, 2, 3\}$, $k \in \{1, 2\}$ and $(\mathbf{x}, t) \in \mathbb{R}^{10} \times \mathbb{R}_{\geq 0}$. For vehicles $w \in \{4, 5\}$, we set $u^2_{w,nom}(\mathbf{x}, t) = 0$ and $u^1_{w,nom}(\mathbf{x}, t) = 0.1$, for every $(\mathbf{x}, t) \in \mathbb{R}^{10} \times \mathbb{R}_{\geq 0}$. Intuitively, the nominal controller enforces a constant, desired speed on the longitudinal axis for the leaders, while the followers are expected to move under the formation protocol, i.e., towards maintaining a desired relative position with respect to their neighbors. Observe that the considered nominal controller does not ensure the satisfaction of the split-merge-maintain task, but rather enforces vehicles to move across their current lanes. Despite this choice of nominal control, as shown in Figure 2, the proposed controller guarantees the satisfaction of the STL formula ϕ'. Note that the satisfaction or violation of the formula is determined by the sign of the barrier function $\mathfrak{b}(\mathbf{x}, t)$ at every time t. If $\mathfrak{b}(\mathbf{x}, t) \geq 0$ for every time t, then the formula is satisfied. In Figure 2 the evolution of the barrier function with respect to time is shown for the closed loop trajectory. The sudden spikes at times 10 and 23 sec are a result of the deactivation policy, discussed in Section II-B. Observe further that $\inf_{t\in[0,50]} \mathfrak{b}(\mathbf{x}(t), t) \geq 0.0559$. By the first statement of Theorem 1, this implies that $\rho^{\phi'}(\mathbf{x}, 0) \geq 0.009$, i.e., the split-merge-maintain task is satisfied with a minimum robustness 0.009. The satisfaction of the task can be also verified by Figure 3. As shown there, the leader of the lower platoon accelerates from 0 to 10sec from which time and until 23sec maintains the desired distance of 6m with its first follower. After the lane changing, from 27 sec onward the vehicles start moving at a constant distance from each other. Observe that the distance between vehicles 4,5 (the leaders) and 1,2 eventually becomes constant and equal to 2.673 and 2.686 meters respectively with both values belonging to the desired interval $(\delta', \delta) = (2.5, 3.5)$ m. Snapshots of the vehicles' actions, especially during the lane changing task, are shown in Figure 4. It worth noticing that despite the fact that collision avoidance is not enforced as a hard constraint in this scenario, it is ensured throughout the duration of the task. All computations were performed on an Intel Core i7-8665U with 16GB RAM using MATLAB. The QP problem was solved with `quadprog` at a frequency of 100Hz.

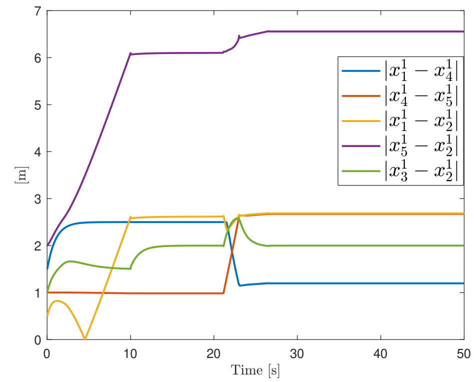


Fig. 3: Vehicles' distance along the x-axis as a function of time.

V. Conclusions-Future Work

In this work, we consider the multi-platoon coordination problem and express a usual split-merge-maintain scenario in Signal Temporal Logic. The STL constraints are encoded in continuous-time using existing time-varying control barrier functions (CBFs) and a QP problem is designed that ensures the least-violation of the STL formula and forward movement of the vehicles. Future work will improve the proposed approach by ensuring STL satisfaction and an acceptable vehicle behavior e.g., with respect to human comfort. The problem would also benefit from a decentralized, non-cooperative approach that can be ensured by decomposing the global STL formula to local ones as for example in [29].

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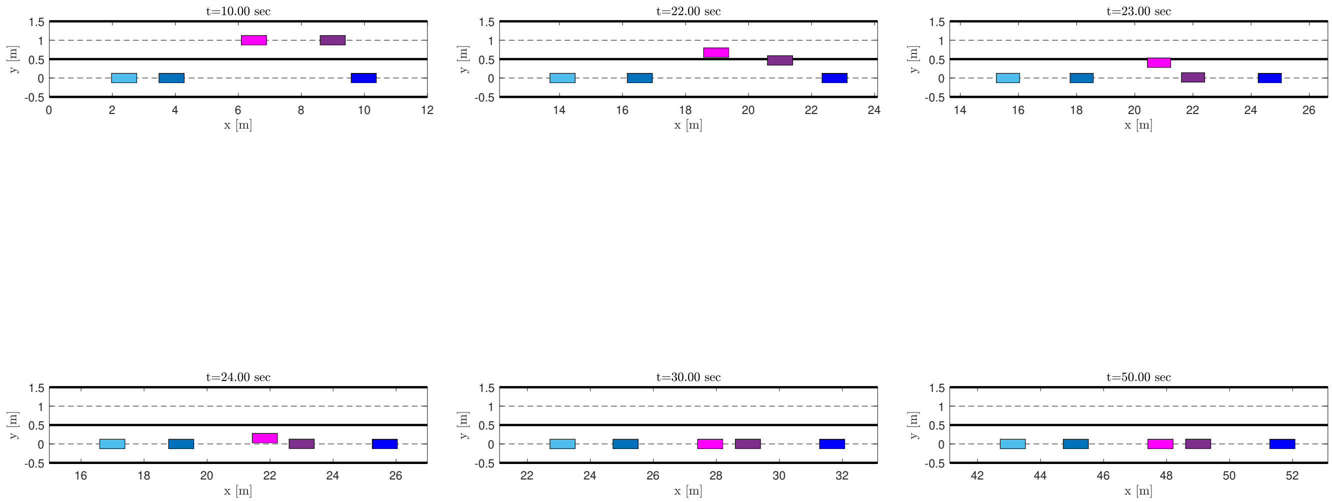

Fig. 4: Snapshots of the vehicles' actions towards satisfying the split-merge-maintain task. The purple and the blue rectangle represents the leader of the upper and lower platoon respectively.

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