2022 IEEE 61st Conference on Decision and Control (CDC)
December 6-9, 2022. Cancún, Mexico

An Adaptive Cooperative Manipulation Control Framework for Multi-Agent Disturbance Rejection

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2022 IEEE 61st Conference on Decision and Control (CDC) | 978-1-6654-6761-2/22/$31.00 ©2022 IEEE | DOI: 10.1109/CDC51059.2022.9992478

***Abstract*— The success of a cooperative manipulation process depends on the level of disturbance rejection between the cooperating agents. However, this attribute may be jeopardized due to unexpected behaviors, such as joint saturation or internal collisions. This leads to deterioration in the performance of the manipulation task. In this paper, we present an adaptive distributed control framework that directly mitigates these internal disturbances, both in the joint (and task) spaces. With our approach, we show that including the manipulator-load coupling in the definition of the task error yields improved performance and robustness. To validate this statement, we provide stability guarantees and simulation results for two implementation cases.**

I. Introduction

Adaptive manipulation control dates back to [1]–[3], and has been used to perform single arm manipulation with limited knowledge of the environment, as in [4]. However, the application of adaptive control to cooperative manipulation is still limited [5]. Several works in this field have proposed a high-level controller that distributes contributions among agents [6], [7], while others have accounted for the deviation from their assigned contribution [8], [9].

Alternatively, different *leader-follower* approaches have been proposed in [10]–[12] to circumvent the need for communication between agents. However, these options do not account for abnormal behaviors by the cooperating partners; this is further exacerbated when the aberrant robot is the leader, propagating the undesired behavior to the rest of the agents. Similarly, distributed formulations have been employed for cooperative manipulation, including [13], where a distributed approach without adaptive control was applied, and [14], in which an unknown payload was manipulated with an optimization of the decoupled dynamics of both subsystems. Other relevant approaches include using distributed Bayesian learning to infer the load dynamics and internal manipulation coupling [15]; employing formation control for the cooperative manipulation task, such as in [16] and [17], which led to a distributed controller that requires only local measurements, and a solution that requires a model-based add-on to compensate for the internal disturbances,

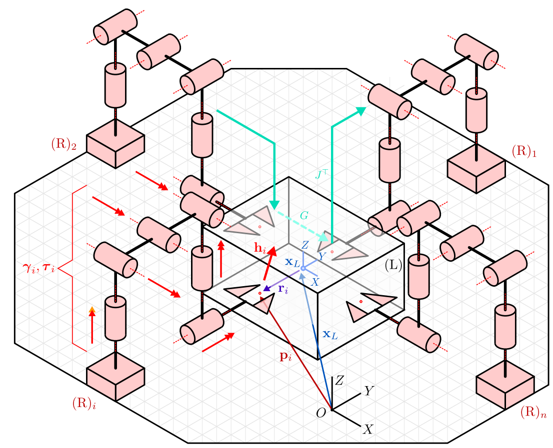

Fig. 1. Scheme of the cooperative manipulation system, with grasping setting and multi-agent disturbance transmission mechanism in aquamarine.

respectfully; finally, using decentralized sliding mode control for cooperative load transport, as in [18];

Returning to adaptive control, we must highlight that its application on multi-agent manipulation systems is generally done either in the joint space [3], [19] or in the task space [20]; but not both. In contrast, we present a control framework that not only applies adaptive control in both the joint space and task space, but also improves the robustness of the solution by accounting for an extra coupling criterion for the manipulators (see Fig. 1). In particular, we opt for a backstepping approach [21], [22] to simplify the controller design and introduce the aforementioned coupling criterion directly into the formulation. The main contributions of this paper can then be summarized as:

- **C1.** An adaptive backstepping framework that mitigates both joint- and task-space disturbances, considering the full dynamics of the load and the robot manipulators.
- **C2.** The analysis of two applications of this framework: one tracking the end-effector positions in concordance with the load trajectories, and the other minimizing the joint speeds as a load-manipulator coupling criterion.

The novelties of the these contributions compared with relevant works for the application are as follows. The work of [23] also proposed an adaptive approach for the collaborative manipulation task; but in that case, the adaptation is applied to the weight of the payload. In contrast, our framework can be distributed and only assumes prior knowledge of the inertial parameters of the object and its desired trajectory. In [24], a coupled dynamic formulation is used to obtain an adaptive controller to compensate for disturbances in the joint and task spaces. However, this approach tracks

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This work was supported by the Swedish Research Council (VR), the Swedish Foundation for Strategic Research (SSF), the Knut and Alice Wallenberg Foundation (KAW), the H2020 CANOPIES project, and the ERC LEAFHOUND project.

only the load pose, while we propose adding an extra coupling criterion for robustness. The approach in [25], in turn, also includes adaptation to uncertainties in the grasping setting and is formulated for any number of heterogeneous agents. However, this formulation involves calculating joint-space errors, despite their inherent limitations. Moreover, the optimization of the load-manipulator interaction is not fully explicit. Finally, in [26], the compensation for disturbances in the task and joint spaces is considered, but only for point masses with elastic grasping within a 1-D task-space and without theoretical guarantees.

The rest of the paper is organized as follows: in Section II, we introduce the cooperative manipulation problem and motivate the need for the proposed framework, which is detailed in Section III, including two particular cases. To validate these, in Section IV we provide realistic simulation results, highlighting the strengths and weaknesses of our approach. The manuscript is wrapped up with the conclusions in Section V.

Notation: $I_M \in \Re^{M\times M}$ and $0_{M\times N} \in \Re^{M\times N}$ denote the identity and zero matrices, respectively, with $0_M = 0_{M\times M}$, and $\mathbf{0}_M \in \Re^M$ the zero column vector; $S(\mathbf{v})$ is the skew-symmetric matrix for vector $\mathbf{v}$, and $|\mathbf{v}|_W^2 := \mathbf{v}^\top W \mathbf{v}$ its weighted norm square, where the subindex is omitted for the identity; A^+ the right-hand Moore-Penrose pseudoinverse of $A \in \Re^{M\times N}, M < N$, and $tr(A)$ stands for its trace. $\hat{\varpi}$ is an estimate of ϖ whose error reads $\tilde{\varpi} := \varpi - \hat{\varpi}$.

Acronyms: robot manipulator (RM), end-effector (EE), degrees of freedom (DoF).

II. System modeling

Let us consider the system in Fig. 1, in which a team of $n \geq 2$ RMs (R) –with $N \geq 3n$ DoF in total– carries a rigid body (L). The stacked joint-space configurations of (R) are $\boldsymbol{\gamma} \in \Re^N$; the derived stacked Cartesian positions $\mathbf{p} \in \Re^{3n}$; and the load pose $\mathbf{x}_L \in \Re^6$. Then, the kinematics of the overall system is given by

$$\mathbf{v}_L = \mathbb{T}\dot{\mathbf{x}}_L, \tag{1a}$$

$$\mathbf{v}_R = J\dot{\boldsymbol{\gamma}}, \tag{1b}$$

where $\mathbf{v}_L \in \Re^6$, $\mathbf{v}_R \in \Re^{6n}$ denote the operational-space velocities –i.e., including angular velocities– for (R) and (L), J is the geometric Jacobian for the EEs of (R); and

$$\mathbb{T} := \begin{pmatrix} I_3 & 0_3 \\ 0_3 & T \end{pmatrix}$$

T is the matrix that transforms a set of (L) angular rates into their equivalent angular velocity (see [27], Subsection II.A). Moreover, we define the EE position Jacobians as

$$\dot{\mathbf{p}} := J_p\dot{\boldsymbol{\gamma}}.$$

Subsequently, the dynamics of the (R)+(L) system is

$$M\dot{\mathbf{v}}_L + \mathbf{b}_L = G\mathbf{h}, \tag{2a}$$

$$B\ddot{\boldsymbol{\gamma}} + C\dot{\boldsymbol{\gamma}} + \mathbf{g}_R = \boldsymbol{\tau} - J^\top\mathbf{h}, \tag{2b}$$

with $M \in \Re^{6\times 6}$ –including the rotation inertia $\mathcal{I} \in \Re^{3\times 3}$– and $B \in \Re^{N\times M}$ the load and manipulators inertia matrices, respectively; $C \in \Re^{6\times 6}$ the Coriolis matrix for (R); $\mathbf{b}_L^\top := [(S(\boldsymbol{\omega}_L)\mathcal{I}\boldsymbol{\omega}_L)^\top + \mathbf{g}_L^\top, \mathbf{0}_3^\top]$, where $\boldsymbol{\omega}_L \in \Re^3$ are the angular speed of the load and $\mathbf{g}_L \in \Re^3$, $\mathbf{g}_R \in \Re^N$ account for gravity in both subsystems; $\boldsymbol{\tau} \in \Re^N$ the stacked joint torques, and $\mathbf{h} \in \Re^{6n}$ the stacked EE wrenches expressed in the inertial frame. The corresponding grasp matrix is defined by

$$G := \left(\begin{array}{cc|c|cc} I_3 & 0_3 & \multirow{2}{*}{$\cdots$} & I_3 & 0_3 \\ S(\mathbf{r}_1) & I_3 & & S(\mathbf{r}_n) & I_3 \end{array}\right) \in \Re^{6\times 6n},$$

where r_n is the vector from the load frame to the grasp location of the n-th robot [28], [29]. For simplicity, let us finally reformulate the dynamics of the (R)+(L) system in (2) in compact form

$$\mathbb{B}\dot{\mathbf{v}} + \mathbf{b} = P_c\mathbf{u}, \tag{3}$$

with $\mathbf{v}^\top := [\mathbf{v}_L^\top, \dot{\boldsymbol{\gamma}}^\top]$ the generalized velocities of the system, $\mathbf{b}^\top := [\mathbf{b}_L^\top, \mathbf{b}_R^\top]$ the stacked gravitational and Coriolis terms where $\mathbf{b}_R := C\dot{\boldsymbol{\gamma}} + \mathbf{g}_R$, $\mathbf{u} := [\mathbf{h}^\top, \boldsymbol{\tau}^\top]^\top \in \Re^{6n+N}$ the actuation vector and

$$\mathbb{B} := \begin{pmatrix} M & 0_{6\times N} \\ 0_{N\times 6} & B \end{pmatrix} \in \Re^{(6+N)\times(6+N)},$$

$$P_c := \begin{pmatrix} G & 0_{6\times N} \\ -J^\top & I_N \end{pmatrix} \in \Re^{(6+N)\times(6n+N)},$$

the compound system inertia and the matrix mapping from the actuation $\mathbf{u}$ to the system dynamics, respectively.

III. Adaptive Control Framework

There are several reasons why a cooperating agent might exhibit unexpected behaviors [9]. For instance, when a manipulator cannot meet the required end-effector wrench due to joint saturation [30], when an internal collision occurs, or if the joint torque sensors get uncalibrated [31]. These situations result in unexpected disturbances that the other agents have to mitigate to fulfill the cooperative manipulation task. This generally requires either measuring the differences with respect to a nominal behavior or characterizing the disturbances to propose a suitable indirect estimate [15], [32]. As these disturbances are hard to characterize and measuring the divergences from the undisturbed case requires enhanced sensor capabilities, we opt for a different approach.

In this work, we propose a control framework with a core controller obtained using a backstepping approach that, apart from tracking the load, also accounts for the manipulator dynamics. This already robust approach is then enhanced with an adaptive update law for a simple formulation of the uncertainties in *both* subsystems in (3) –as is [17]–, yielding

$$\mathbf{u} = \mathbf{u}_c + \mathbf{u}_d, \tag{4}$$

where $\mathbf{u}_c^\top := [\mathbf{h}_c^\top, \boldsymbol{\tau}_c^\top]$ are the control inputs to the system dynamics without accounting for uncertainties: $\mathbf{h}_c \in \Re^{6n}$ the nominal stacked wrenches and $\boldsymbol{\tau}_c \in \Re^N$ the nominal stacked torques; and $\mathbf{u}_d^\top := [\mathbf{d}^\top, \boldsymbol{\delta}^\top]$ are the terms to incorporate the uncertainties that we assume slow time-varying, i.e., $\dot{\mathbf{u}}_d \approx \mathbf{0}_{6n+N}$: $\mathbf{d} \in \Re^{6n}$ in the wrenches and $\boldsymbol{\delta} \in \Re^N$ in the joint-space. Throughout this manuscript, we refer to $\mathbf{d}$ as the load-space disturbance and $\boldsymbol{\delta}$ as the joint-space disturbance.

Remark 1: Due to the coupling induced by the Jacobian J and grasp matrix G, the projection of any time-varying behavior into the joint-space and load-space is closer to the slow time-varying condition. This makes adapting to $\mathbf{d}$ and $\boldsymbol{\delta}$ significantly more inclusive than just employing either of them. To illustrate this, consider a case in which a manipulator in (R) is under joint-space disturbances (Fig. 1). These would be first moderated in the same space, but not completely canceled. As the remaining disturbances are transmitted via the aforementioned matrices, our approach would also mitigate them after the coupling.

We formulate the general adaptive control framework using both the minimum norm joint torques and their derived contact wrenches, namely

$$\mathbf{u}_c = \Lambda\left(\Gamma\mathbf{e} + \boldsymbol{\varphi}\right) + P_c^{+}\mathbf{b} - P_d\hat{\mathbf{u}}_d, \tag{5}$$

with the adaptive update law for the uncertainties in (4),

$$\dot{\hat{\mathbf{u}}}_d = -\Gamma_d\Lambda_d^{\top}\Gamma_a\mathbf{e}. \tag{6}$$

From a descriptive perspective –later expanded and defined once we particularize the strategy for two application cases in the next section, with their associated matrix/vector sizes–, the parameters introduced in the formulation can be seen as

- $\mathbf{e}$ encapsulates the error metrics including proportional, derivative and integral actions for (L) and a (R)-(L) link through the manipulator dynamics, and is the main design option of the control framework;
- Λ is the matrix distributing the error tracking and feedforward action between the direct control input and the indirect contact wrenches and Λ_d reflects how the adaptive parameters should react to this error;
- Γ includes the control gains in the control core, with $\Gamma_d \in \Re^{(6n+N)\times(6n+N)}$ its positive definite counterpart for the adaptive module, and Γ_a scales how the error propagates into the adaptive estimation;
- $\boldsymbol{\varphi}$ accounts for the feedforward actions, i.e. terms directly including the reference speeds and accelerations, depending on the definition of the error metrics $\mathbf{e}$;
- P_c^{+} provides the solution of the under-determined system of equations in (3) for a $\mathbf{u}_c$ of lesser norm, namely
$$P_c^{+} = \begin{pmatrix} G^{+} & 0_{6n\times N} \\ J^{\top}G^{+} & I_N \end{pmatrix} \in \Re^{(6n+N)\times(6n+N)};$$
- P_d is the mapping from the estimated disturbances into the control actuation given by
$$P_d := \begin{pmatrix} G^{+}G & 0_{6n\times N} \\ -J^{\top}P_G & I_N \end{pmatrix} \in \Re^{(6n+N)\times(6n+N)}; \text{ and}$$
- $P_G := I_{6n} - G^{+}G$ is the projection matrix into the kernel of G.

Remark 2: It is worth discussing the impact of disturbances through P_d (above) in (5). While $\boldsymbol{\delta}$ is directly handled by $\boldsymbol{\tau}_c$, its equivalent for the task-space is distributed along the null- and column-spaces of G: i) the kernel contributions via P_G, i.e. internal forces that do not directly alter the load dynamics, are handled by the direct control input. In other words, part of the compensation of disturbances in the task space is achieved through internal stress on the object through P_G, as in [33]; ii) disturbances in the column-space are compensated indirectly via control wrenches. This implies that although the latter do not appear explicitly on the control input $\boldsymbol{\tau}_c$, they are indeed compensated.

Then, upon plugging the control law (5) and the uncertainty model in (4) into the compact dynamic formulation in (3), we obtain the closed-loop behavior of the system:

$$\dot{\mathbf{v}} = \mathbb{B}^{-1}P_c\left[\Lambda\left(\Gamma\mathbf{e} + \boldsymbol{\varphi}\right) + \tilde{\mathbf{u}}_d\right], \tag{7}$$

where $\tilde{\mathbf{u}}_d$ was the difference between the real disturbances and the estimation coming from the adaptive update law (6).

We now proceed to particularize the general framework above for two different sets of error metrics associated with two design concepts:

A. an approach in which both the load pose $\mathbf{x}_L$ and consistent (R) end-effector positions $\mathbf{p}$ are tracked simultaneously for robustness, similarly to [27]; but for the multi-agent disturbance scenario presented above.
B. an alternative solution in which only the load pose $\mathbf{x}_L$ is tracked; but we include the joint-space velocities of (R) in the Lyapunov candidate. This allows us to both link the (R) and (L) dynamics, thereby transforming the wrench control into a torque one, and softening the response of the controller.

Each of these particular cases is explored in a separate subsection below, highlighting their connection to (5) and (6), and detailing the values of their associated gains matrices. Please note that the general variables will be altered.

A. Consistent tracking of the load pose and EE positions

Let us define the task-space of the problem as in [27], i.e. $\mathbf{x} := [\mathbf{x}_L^{\top}, \mathbf{p}^{\top}]^{\top} \in \Re^{S}$, $S := 6 + 3n$ and $\dot{\mathbf{x}} = W\mathbf{v}$, with

$$W = \begin{pmatrix} \mathbb{T}^{-1} & 0_{6\times N} \\ 0_{3n\times 6} & J_p \end{pmatrix}.$$

In contrast to tracking only the load pose, this definition allows us to consistently track the load references in both the load and the manipulator dynamics (2), thus enhancing the robustness of this solution, as thoroughly discussed in [27]. Consequently, the kinodynamics of the compound (R)+(L) system (2) in error terms becomes

$$\dot{\mathbf{e}}_I = \mathbf{e}_P, \tag{8a}$$
$$\dot{\mathbf{e}}_P = \mathbf{e}_D, \tag{8b}$$
$$\dot{\mathbf{e}}_D = \boldsymbol{\varphi}(\ddot{\mathbf{x}}_r, \dot{\mathbf{x}}_r) + \dot{W}W^{+}\mathbf{e}_D - W\mathbb{B}^{-1}\left(P_c\mathbf{u} - \mathbf{b}\right), \tag{8c}$$

where $\mathbf{e}_P := \mathbf{x}_r - \mathbf{x}$ is the tracking error and $\mathbf{e}_I$ its implicit integral counterpart, $\varphi(\ddot{\mathbf{x}}_r, \dot{\mathbf{x}}_r) := \ddot{\mathbf{x}}_r - \dot{W}W^{+}\dot{\mathbf{x}}_r$. Then, we particularize the framework in (5) for this task.

Proposition 1: Consider the system (8) and the multi-agent disturbance model in (4) with its associated assumption of slow time-varying disturbances $\mathbf{u}_d$ and assume that we are operating away from any singularity of T. Then, under the control law (5) for the error metric $\mathbf{e} := [\mathbf{e}_I^{\top}, \mathbf{e}_P^{\top}, \mathbf{e}_D^{\top}]^{\top}$, and the particularization of the matrices in (5) given by (9), for $\Gamma_1, \Gamma_2, \Gamma_3 \in \Re^{S\times S}$ positive definite gain matrices,

$$\Lambda = \begin{pmatrix} G^{+}M\mathbb{T} & 0_{6n\times 3n} \\ J^{\top}G^{+}M\mathbb{T} & BJ_p^{+} \end{pmatrix},$$
$$\Lambda_d = \begin{pmatrix} \mathbb{T}^{-1}M^{-1}G & 0_{6\times N} \\ -J_pB^{-1}J^{\top} & J_pB^{-1} \end{pmatrix},$$

and the adaptive update laws (6), the closed-loop system (7) is asymptotically stabilized to zero, i.e., $\mathbf{e}\to\mathbf{0}$ as $t\to\infty$.

Proof: Let us use a backstepping approach and define $V_1 := |\mathbf{e}_I|^2/2 \geq 0$ and derive $\dot{V}_1 = \mathbf{e}_I^{\top}\mathbf{e}_P = -|\mathbf{e}_I|^2_{\Gamma_1} - \mathbf{z}_1^{\top}\mathbf{e}_I$, with $\mathbf{z}_1 := -\Gamma_1\mathbf{e}_I - \mathbf{e}_P$ the error of the first step. To cope with it, we define $V_2 := V_1 + |\mathbf{z}_1|^2/2 \geq 0$, and thus $\dot{V}_2 = -|\mathbf{e}_I|^2_{\Gamma_1} - \mathbf{z}_1^{\top}(\mathbf{e}_I + \Gamma_1\mathbf{e}_P + \mathbf{e}_D) = -|\mathbf{e}_I|^2_{\Gamma_1} - |\mathbf{z}_1|^2_{\Gamma_2} - \mathbf{z}_2^{\top}\mathbf{z}_1$, with $\mathbf{z}_2 := -\Gamma_2\mathbf{z}_1 + \mathbf{e}_I + \Gamma_1\mathbf{e}_P + \mathbf{e}_D$ the error of this second step. Analogously, we define $V_3 := V_2 + |\mathbf{z}_2|^2/2 \geq 0$ under the error dynamics in (8c), and then we have

$$\begin{aligned}\dot{V}_3 &= -|\mathbf{e}_I|^2_{\Gamma_1} - |\mathbf{z}_1|^2_{\Gamma_2} + \mathbf{z}_2^{\top}(\dot{\mathbf{z}}_2 - \mathbf{z}_1) \\ &= -|\mathbf{e}_I|^2_{\Gamma_1} - |\mathbf{z}_1|^2_{\Gamma_2} \\ &\quad + \mathbf{z}_2^{\top}\Big[\Gamma_1\mathbf{e}_I + (\Gamma_2\Gamma_1 + 2I_S)\,\mathbf{e}_P + (\Gamma_2 + \Gamma_1)\,\mathbf{e}_D \\ &\quad + \boldsymbol{\varphi} + \dot{W}W^{+}\mathbf{e}_D - W\mathbb{B}^{-1}(P_c\mathbf{u} - \mathbf{b})\Big].\end{aligned}$$

If we plug (5) for the proposed gains in (9) into the above –as in [27]–, we obtain that

$$\begin{aligned}\dot{V}_3 &= -|\mathbf{e}_I|^2_{\Gamma_1} - |\mathbf{z}_1|^2_{\Gamma_2} - |\mathbf{z}_2|^2_{\Gamma_3} - \mathbf{z}_2^{\top}W\mathbb{B}^{-1}P_c\tilde{\mathbf{u}}_d \\ &= -|\mathbf{e}_I|^2_{\Gamma_1} - |\mathbf{z}_1|^2_{\Gamma_2} - |\mathbf{z}_2|^2_{\Gamma_3} - \mathbf{e}^{\top}\Gamma_a^{\top}\Lambda_d\tilde{\mathbf{u}}_d.\end{aligned}$$

Finally, we propose the Lyapunov function to mitigate the uncertainties in (4): $V_A := V_3 + |\tilde{\mathbf{u}}_d|^2_{\Gamma_d^{-1}}/2 \geq 0$. Under the assumption of slow time-varying disturbances ($\dot{\mathbf{u}}_d \approx \mathbf{0}_{6n+N}$) and using the adaptive update law in (6), we obtain that

$$\begin{aligned}\dot{V}_A &= -|\mathbf{e}_I|^2_{\Gamma_1} - |\mathbf{z}_1|^2_{\Gamma_2} - |\mathbf{z}_2|^2_{\Gamma_3} + \tilde{\mathbf{u}}_d^{\top}\left(\Gamma_d^{-1}\dot{\hat{\mathbf{u}}}_d - \Lambda_d^{\top}\Gamma_a\mathbf{e}\right) \\ &= -|\mathbf{e}_I|^2_{\Gamma_1} - |\mathbf{z}_1|^2_{\Gamma_2} - |\mathbf{z}_2|^2_{\Gamma_3}.\end{aligned}$$

Since $V_A > 0$ and $\dot{V}_A \leq 0$, all the trajectories are bounded, and we can invoke LaSalle's Invariance Principle. To rule out all other conditions for $\dot{V}_A \leq 0$ different from the desired equilibrium, we analyze $\mathbf{z}_1$ and $\mathbf{z}_2$. From the definition of $\mathbf{z}_1 := -\Gamma_1\mathbf{e}_I - \mathbf{e}_P$, it follows that $\mathbf{e}_P = \mathbf{0}_S$. Analogously, as $\mathbf{z}_2 := -\Gamma_2\mathbf{z}_1 + \mathbf{e}_I + \Gamma_1\mathbf{e}_P + \mathbf{e}_D$, we obtain $\mathbf{e}_D = \mathbf{0}_S$ and therefore $\mathbf{e} = \mathbf{0}_{3S}$, thus concluding the proof. ■

B. Load pose tracking with softened joint-space response

In contrast to Subsection III-A, we define the task-space as $\mathbf{x} := \mathbf{x}_L \in \Re^6$, that is, only accounting for the load pose. As this traditional option does not yield a clear interconnection with the dynamics of the manipulator, it tends to be associated with an open-loop torque allocation based on (2b), (i.e., $\boldsymbol{\tau} = B\ddot{\boldsymbol{\gamma}} + C\dot{\boldsymbol{\gamma}} + \mathbf{g}_R + J^{\top}\mathbf{h}$). Here, nevertheless, we propose a different approach: adding the joint speeds of (R) to the proposed Lyapunov function so that this control allocation comes directly from the closed-loop formulation. For that purpose, we write the error kinodynamics of (2a) as

$$\dot{\mathbf{e}}_I = \mathbf{e}_P, \tag{10a}$$
$$\dot{\mathbf{e}}_P = \mathbf{e}_D, \tag{10b}$$
$$\dot{\mathbf{e}}_D = \boldsymbol{\varphi}_R(\ddot{\mathbf{x}}_r, \dot{\mathbf{x}}_r) - \mathbb{T}^{-1}\dot{\mathbb{T}}\mathbf{e}_D - \mathbb{T}^{-1}M^{-1}(G\mathbf{h} - \mathbf{b}_L), \tag{10c}$$

where $\mathbf{e}_P := \mathbf{x}_r - \mathbf{x}$ is the tracking error and $\mathbf{e}_I$ its implicit integral counterpart, while $\varphi_R(\ddot{\mathbf{x}}_r, \dot{\mathbf{x}}_r) := \ddot{\mathbf{x}}_r + \mathbb{T}^{-1}\dot{\mathbb{T}}\dot{\mathbf{x}}_r$.

Proposition 2: Consider the system (10) and the multi-agent disturbances slow time-varying disturbances $\mathbf{u}_d$ in (4) and assume that T is full-ranked. Then, under the control law (5) for $\mathbf{e} := [\mathbf{e}_I^{\top}, \mathbf{e}_P^{\top}, \mathbf{e}_D^{\top}, -\dot{\boldsymbol{\gamma}}]^{\top}$, $\boldsymbol{\varphi} = [\boldsymbol{\varphi}_R^{\top}, \mathbf{0}_N^{\top}]^{\top}$, the control gains in (11), where $\Gamma_4, \Gamma_5, \Gamma_6 \in \Re^{6\times 6}$, $\Gamma_7 \in \Re^{N\times N}$ are positive definite gain matrices; and with

$$\Lambda = \begin{pmatrix} G^{+}M\mathbb{T} & 0_{6n\times N} \\ J^{\top}G^{+}M\mathbb{T} & B \end{pmatrix},\ \Lambda_d = \begin{pmatrix} \mathbb{T}^{-1}M^{-1}G & 0_{6\times N} \\ -B^{-1}J^{\top} & B^{-1} \end{pmatrix},$$

and the adaptive update laws (6) for these matrices, the closed-loop system (7) is asymptotically stabilized to zero, i.e., $\mathbf{e}\to\mathbf{0}$ as $t\to\infty$.

Proof: Let us use a backstepping approach, define $V_1 := |\mathbf{e}_I|^2/2 \geq 0$ and derive $\dot{V}_1 = \mathbf{e}_I^{\top}\mathbf{e}_P = -|\mathbf{e}_I|^2_{\Gamma_4} - \mathbf{z}_1^{\top}\mathbf{e}_I$, $\mathbf{z}_1 := -\Gamma_4\mathbf{e}_I - \mathbf{e}_P$ the error of this first backstepping step. To cope with it, we define $V_2 := V_1 + |\mathbf{z}_1|^2/2 \geq 0$, and hence $\dot{V}_2 = -|\mathbf{e}_I|^2_{\Gamma_4} - \mathbf{z}_1^{\top}(\mathbf{e}_I + \Gamma_4\mathbf{e}_P + \mathbf{e}_D) = -|\mathbf{e}_I|^2_{\Gamma_4} - |\mathbf{z}_1|^2_{\Gamma_5} - \mathbf{z}_2^{\top}\mathbf{z}_1$, with $\mathbf{z}_2 := -\Gamma_5\mathbf{z}_1 + \mathbf{e}_I + \Gamma_4\mathbf{e}_P + \mathbf{e}_D$ the error of the second step. Instead of adding a standard third step, we opt for a Lyanpunov candidate that includes the coupling with the manipulator, namely $V_3 := V_2 + |\mathbf{z}_2|^2/2 + |\dot{\boldsymbol{\gamma}}|^2/2 \geq 0$. The derivative of such candidate under (10c) and (2b) is

$$\begin{aligned}\dot{V}_3 &= -|\mathbf{e}_I|^2_{\Gamma_4} - |\mathbf{z}_1|^2_{\Gamma_5} + \begin{bmatrix}\mathbf{z}_2 \\ -\dot{\boldsymbol{\gamma}}\end{bmatrix}^{\top}\begin{bmatrix}\dot{\mathbf{z}}_2 - \mathbf{z}_1 \\ -\ddot{\boldsymbol{\gamma}}\end{bmatrix} \\ &= -|\mathbf{e}_I|^2_{\Gamma_1} - |\mathbf{z}_1|^2_{\Gamma_5} + \begin{bmatrix}\mathbf{z}_2 \\ -\dot{\boldsymbol{\gamma}}\end{bmatrix}^{\top}\begin{bmatrix}\dot{\mathbf{e}}_D \\ -\ddot{\boldsymbol{\gamma}}\end{bmatrix} \\ &\quad + \begin{bmatrix}\mathbf{z}_2 \\ -\dot{\boldsymbol{\gamma}}\end{bmatrix}^{\top}\begin{bmatrix}\Gamma_4\mathbf{e}_I + (\Gamma_5\Gamma_4 + 2I_6)\,\mathbf{e}_P + (\Gamma_5 + \Gamma_4)\,\mathbf{e}_D \\ \mathbf{0}_N\end{bmatrix}.\end{aligned}$$

Using (2b) and (10c), and the notation in (3), we obtain

$$\begin{aligned}\dot{V}_3 &= -|\mathbf{e}_I|^2_{\Gamma_4} - |\mathbf{z}_1|^2_{\Gamma_5} \\ &\quad + \begin{bmatrix}\mathbf{z}_2 \\ -\dot{\boldsymbol{\gamma}}\end{bmatrix}^{\top}\left[\boldsymbol{\varphi} - \begin{pmatrix}(M\mathbb{T})^{-1} & 0_{6\times N} \\ 0_{N\times 6} & B^{-1}\end{pmatrix}(P_c\mathbf{u} - \mathbf{b})\right] \\ &\quad + \begin{bmatrix}\mathbf{z}_2 \\ -\dot{\boldsymbol{\gamma}}\end{bmatrix}^{\top}\begin{bmatrix}\Gamma_4\mathbf{e}_I + (\Gamma_5\Gamma_4 + 2I_6)\mathbf{e}_P + \left(\Gamma_5 + \Gamma_4 - \mathbb{T}^{-1}\dot{\mathbb{T}}\right)\mathbf{e}_D \\ \mathbf{0}_N\end{bmatrix}.\end{aligned}$$

If we then plug in (5) with the particularization above and the proposed gains in (11) –as in [27]–, the derivative of the Lyapunov candidate becomes

$$\begin{aligned}\dot{V}_3 &= -|\mathbf{e}_I|^2_{\Gamma_4} - |\mathbf{z}_1|^2_{\Gamma_5} - |\mathbf{z}_2|^2_{\Gamma_6} - |\dot{\boldsymbol{\gamma}}|^2_{\Gamma_7} \\ &\quad - \begin{bmatrix}\mathbf{z}_2 \\ -\dot{\boldsymbol{\gamma}}\end{bmatrix}^{\top}\begin{pmatrix}(M\mathbb{T})^{-1} & 0_{6\times N} \\ 0_{N\times 6} & B^{-1}\end{pmatrix}P_c\tilde{\mathbf{u}}_d \\ &= -|\mathbf{e}_I|^2_{\Gamma_4} - |\mathbf{z}_1|^2_{\Gamma_5} - |\mathbf{z}_2|^2_{\Gamma_6} - |\dot{\boldsymbol{\gamma}}|^2_{\Gamma_7} - \mathbf{e}^{\top}\Gamma_a^{\top}\Lambda_d\tilde{\mathbf{u}}_d.\end{aligned}$$

Finally, we mitigate the adaptive errors with the Lyapunov function $V_A := V_3 + |\tilde{\mathbf{u}}_d|^2_{\Gamma_d^{-1}}/2 \geq 0$. Under the assumption of slow time-varying disturbances ($\dot{\mathbf{u}}_d \approx \mathbf{0}_{6n+N}$) and

$$\Gamma=\begin{pmatrix}\Gamma_1+\Gamma_3\Gamma_2\Gamma_1+\Gamma_3 & \Gamma_2\Gamma_1+\Gamma_3\Gamma_1+\Gamma_3\Gamma_2+2I_S & \Gamma_1+\Gamma_2+\Gamma_3+\dot{W}W^{+}\end{pmatrix},\ \Gamma_a=\begin{pmatrix}\Gamma_2\Gamma_1+I_S & \Gamma_1+\Gamma_2 & I_S\end{pmatrix}. \tag{9}$$

$$\Gamma=\begin{pmatrix}\Gamma_4+\Gamma_6\Gamma_5\Gamma_4+\Gamma_6 & \Gamma_5\Gamma_4+\Gamma_6\Gamma_4+\Gamma_6\Gamma_5+2I_6 & \Gamma_4+\Gamma_5+\Gamma_6-\mathbb{T}^{-1}\dot{\mathbb{T}} & 0_{6\times N} \\ & 0_{N\times 18} & & \Gamma_7\end{pmatrix},\ \Gamma_a=\begin{pmatrix}\Gamma_5\Gamma_4+I_6 & \Gamma_4+\Gamma_5 & I_6 & 0_{6\times N} \\ & 0_{N\times 18} & & I_N\end{pmatrix}. \tag{11}$$

introducing the adaptive update law in (6), we obtain that

$$\begin{aligned}\dot{V}_A &= -\,|\mathbf{e}_I|^2_{\Gamma_4} - |\mathbf{z}_1|^2_{\Gamma_5} - |\mathbf{z}_2|^2_{\Gamma_6} - |\dot{\gamma}|^2_{\Gamma_7} + \tilde{\mathbf{u}}_d^\top\left(\Gamma_d^{-1}\dot{\hat{\mathbf{u}}}_d - \Lambda_d^\top\Gamma_a\mathbf{e}\right) \\ &= -\,|\mathbf{e}_I|^2_{\Gamma_4} - |\mathbf{z}_1|^2_{\Gamma_5} - |\mathbf{z}_2|^2_{\Gamma_6} - |\dot{\gamma}|^2_{\Gamma_7}\,.\end{aligned}$$

Since $V_A > 0$ and $\dot{V}_A \leq 0$, all the trajectories are bounded, and we can invoke LaSalle's Invariance Principle. As in the previous discussion, we rule out all solutions of $\dot{V}_A = 0$ different from $\mathbf{e} = \mathbf{0}_{18+N}$. Since $\mathbf{z}_1 := -\Gamma_4\mathbf{e}_I - \mathbf{e}_P$, we obtain $\mathbf{e}_P = \mathbf{0}_6$, and recalling $\mathbf{z}_2 := -\Gamma_5\mathbf{z}_1 + \mathbf{e}_I + \Gamma_4\mathbf{e}_P + \mathbf{e}_D$, it follows that $\mathbf{e}_D = \mathbf{0}_6$, thus concluding the proof. ■

IV. Validation

To validate the framework, we analyze the results of both application cases in simulation, using the MathWorks® MATLAB/Simulink multi-body Simscape tool. The proposed cooperative manipulation test bench is depicted in Fig. 2, and consists of two RMs with 6 DoF sharing a load – with a mass of 2.117 kg and inertia in the body frame of $[\mathcal{I}_x, \mathcal{I}_y, \mathcal{I}_z] = [0.0297, 0.0316, 0.0440]$ kg m^2– following a sinusoidal pose trajectory. It is important to note that this load is above the combined maximum loads of the manipulators, allowing us to obtain a conservative evaluation of the tracking performance. Moreover, we include a temporal torque saturation for one of the joints to evaluate the robustness of the framework against the unexpected internal behaviors mentioned above. Moving to the implementation, we include the well-known σ-modification [34] of the adaptive update in (6) to improve its robustness against drift. The gains associated with this and the control gains chosen for both cases are detailed in Table I, where $[\phi, \theta, \psi]$ denote the roll-pitch-yaw angles of the load.

Once the simulation settings are clear, let us discuss the results in Fig. 3. For the first application case in Subsection III-A, we can see that the base strategy –without the adaptive update law– already provides solid tracking capabilities. In particular, we wish to highlight that during the saturation disturbances ($t \in [4, 6]$s) this core approach just suffers limited oscillations in load position and attitude. These results are further improved when the adaptive update law is used. For example, the adaptive add-on reduces the oscillations in the load pose produced by the disturbances due to the change in $\hat{\mathbf{d}}, \hat{\boldsymbol{\delta}}$ around $t = 5$s, while maintaining the nominal behavior before and after them.

Regarding the results for the second case (Subsection III-B), we can see that the base controller is capable of tracking the load position but produces noticeable oscillations in the attitude from the start of the disturbances. Thanks to the adaptive update law, those are mitigated in the complete strategy, but without reaching the nominal behavior prior

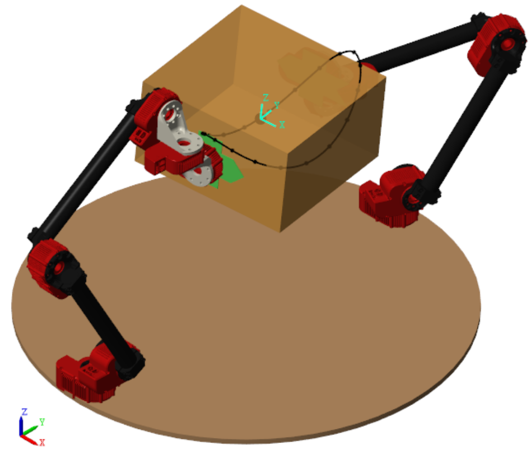

Fig. 2. Cooperative manipulation test bench system with two RMs.

TABLE I

Control gains and values for the σ-modification

		Subsection III-A			Subsection III-B			
Core (5)		Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6	Γ_7
(L)	Pos.	0.5	30.0	6.0	0.5	30.0	6.0	-
	ϕ	1.5	120.0	6.0	0.8	66.0	3.3	-
	θ	0.5	42.0	6.0	1.3	102.0	5.1	-
	ψ	0.6	160.0	0.8	0.3	72.0	0.4	-
(R)	p_{xy}	0.4	9.0	4.0	-	-	-	-
	p_z	0.6	18.0	4.0	-	-	-	-
	$\dot{\gamma}_{1\text{-}5}$	-	-	-	-	-	-	60.5
	$\dot{\gamma}_6$	-	-	-	-	-	-	300.0
Adaptive (6)		Γ_d	σ		Γ_d	σ		
(L)	d_{xyz}	0.01	0.6		0.01	3.0		
	$d_{\phi\theta\psi}$	0.01	2.4		0.01	3.0		
(R)	$\boldsymbol{\delta}$	0.12	1.4		0.02	150.0		

to them nor the tracking capabilities of the previous case. Nevertheless, we recall that the validation is performed under severe conditions, both in the load mass and in the joint torque saturation.

In general, we can infer that the proposed framework mitigates the disturbances produced by the joint saturation in one of the manipulators, which propagate from this to the other manipulator through the gasp map. As this involves coping with disturbances in both task and joint spaces, the adaptive update law employed in the framework has also proven useful in modulating such disturbances.

V. Conclusion

In this work, we present a control framework which extends previous collaborative manipulation approaches using contact wrenches with an interconnection criterion between

Fig. 3. Simulation results for particularizations detailed in Subsections III-A and III-B, with and without the adaptive update law in (6), including the saturation of the torque of the third DoF of $(\mathrm{R})_1$ for $t \in [4, 6]$s (whose effects on the tracking performance are highlighted in light orange).

the load and manipulator mechanics. This new paradigm leads to a joint torque solution that comes directly from the control design and provides enhanced disturbance rejection capabilities. Moreover, we focus on the problem of unexpected disturbances within the robot collaboration, for which we propose adding an adaptive update law to the base framework. This strategy is then particularized for two application cases: one extending the task-space with the consistent tracking of the end-effector positions of the manipulators, and another one with the joint speeds. The stability of the solution for these two cases is analyzed with Lyapunov methods, and their simulation results are used to validate the framework. Finally, these results are thoroughly discussed, with special emphasis on the robustness of the strategy and the safe response against the aforementioned internal disturbances. Future work will extend the number of manipulators beyond two, and hardware experiments will be used to validate our method. In addition, more quantitative comparisons between our work and others will be conducted.

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