Adaptive Cooperative Manipulation with Rolling Contacts

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***Abstract*— In this paper we present a novel adaptive cooperative manipulation controller for multiple mobile robots with rolling contacts. Our approach exploits rolling effects of passive end-effectors and does not require force/torque sensing. Moreover, the proposed scheme is robust to uncertain dynamics of the object and agents including object center of mass, inertia, weight, and Coriolis terms. In addition, we present a novel closed-form internal force controller that guarantees no slip throughout the manipulation task. The adaptive controller design ensures boundedness of the estimated model parameters in predefined sets. Numerical simulations validate the effectiveness of the proposed approach.**

I. INTRODUCTION

Recent technological advancements have led to the concept of automated manufacturing where the ability to transport objects/packages autonomously is key to the production process. One popular approach to perform object transport is via cooperative manipulation, that is, transport/manipulation of an object by using multiple mobile manipulators.

Existing methods in cooperative manipulation aim to track a desired object reference via hybrid force/position and impedance/adaptive control schemes [1]–[4]. However, those methods rely on the assumption that each agent is *rigidly* fixed to the object, allowing it to apply any force/torque at the contact point. The rigidity assumption is highly restrictive as it only applies to objects on which a rigid grasp can be formed. Furthermore, many existing approaches are dependent on force/torque sensors mounted on each robot, which can be expensive or difficult to equip appropriately. In this work we remove the rigidity assumption and relax the dependency on tactile sensors to contact location sensors only. We propose exploiting natural rolling of a passive end-effector to accomplish the cooperative manipulation objective. Non-rigid/rolling contacts increase the number of objects that can be grasped, increase the workspace of the system, and allow for modular manipulation scenarios in which robots can be swapped in/out to adjust the grasp online.

Rolling contacts complicate the problem as each contact may only apply a force that respects friction cone constraints to prevent slip, instead of an arbitrary wrench associated with rigid contacts [5]. Early robotic grasping approaches required exact knowledge of the agent's dynamics [5], [6]. Other recent techniques are robust to model uncertainties, but neglect rolling effects or dynamics [7]–[9], while other more sensor-deprived approaches assume the object is weightless [10], [11]. The approach from [12] assumes a priori bounded states, which does not apply to the mobile manipulators considered here. Adaptive control schemes that have also been developed require force *and* contact location sensing, and assume boundedness of the uncertain parameter estimates [13], [14], or are limited to set-point (constant reference) manipulation [15]. Thus, there is no robust cooperative manipulation approach that ensures stability to a reference trajectory with non-rigid, rolling contacts and no force/torque sensing. Furthermore, for the collaborative manipulation proposed here, it is critical to ensure the object does not slip. This is neglected by most of the aforementioned approaches, which assume either rigid grasps or simply no slip without guarantees. Methods of ensuring slip prevention are developed typically by solving an optimization problem online [7], [16], [17]. However [7], [17] neglect the dynamics of the system, which may perturb the system and cause slip. The approach in [16] uses a conservative bound on the dynamics, which overcompensates the amount of force required to hold the object. Finally, most related works (e.g., [1], [3], [4], [6], [11]) consider accurate knowledge of the object center of mass, which can be difficult to obtain in practice, especially in cases of complicated object shapes.

In this paper, we present a novel adaptive cooperative manipulation control scheme for rolling contact points that does not use force/torque sensing. The proposed controller ensures asymptotic stability to a reference trajectory and is robust to uncertain dynamic model parameters including object/robot center of mass, inertia, and weight. Furthermore, we propose a novel internal force controller that ensures no slip during the manipulation motion without neglecting nor overcompensating for the system dynamics. Numerical simulations are used to validate the proposed approach.

Notation: $\boldsymbol{v}^{\mathcal{E}}$ indicates that the vector $\boldsymbol{v}$ is written with respect to a frame $\mathcal{E}$, and if there is no explicit frame defined, $\boldsymbol{v}$ is written with respect to the inertial frame, $\mathcal{P}$. The operator $(\cdot)\times$ denotes the skew-symmetric matrix representation of the cross-product. $SO(3)$ denotes the special orthogonal group of dimension 3, and S^n is the unit $(n-1)$-dimensional sphere. The $r \times r$ identity matrix is denoted by I_r and the r-dimensional vector of zeros by $\mathbf{0}_r$. The terms $\succeq, \preceq$ denote element-wise vector inequalities. The null space of a matrix B is denoted $\mathfrak{N}(B)$, and the interior of a set $\mathscr{A}$ is $\text{Int}(\mathscr{A})$.

II. PROBLEM FORMULATION

Consider $N \in \mathbb{N}$ robotic agents, consisting of a holonomic moving base and a robotic arm, grasping a rigid object in 3D space. Let their generalized joint space variables and respective derivatives be $\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i \in \mathbb{R}^{n_i}$ with $n_i \geq 3, \forall i \in$

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$\mathscr{N} := \{1, ..., N\}$. Here $\boldsymbol{q}_i$ consists of the degrees of freedom of the robotic arm as well as the moving base. The overall joint configuration is then $\boldsymbol{q} := [\boldsymbol{q}_1^T, ..., \boldsymbol{q}_N^T]^T$, $\dot{\boldsymbol{q}} := [\dot{\boldsymbol{q}}_1^T, ..., \dot{\boldsymbol{q}}_N^T]^T \in \mathbb{R}^n$ with $n := \sum_{i \in \mathscr{N}} n_i$. Each agent has a smooth, convex "fingertip" (i.e. passive end-effector) of high stiffness that is in contact with an object via a smooth contact surface. Let the inertial frame be denoted by $\mathcal{P}$, and a fingertip frame, $\mathcal{F}_i$, fixed at the point $\boldsymbol{p}_{f_i} \in \mathbb{R}^3$ on each agent end-effector. The translational and rotational velocities of $\mathcal{F}_i$ with respect to $\mathcal{P}$ are denoted by $\boldsymbol{v}_{f_i}, \boldsymbol{\omega}_{f_i} \in \mathbb{R}^3$, respectively. The rotation matrix from $\mathcal{F}_i$ to $\mathcal{P}$ is $R_{pf_i} := R_{pf_i}(\boldsymbol{q}_i) \in SO(3)$. The contact frame, $\mathcal{C}_i$, is located at the contact point, $\boldsymbol{p}_{c_i} \in \mathbb{R}^3$ and defined as a Gauss frame [18] where one of the axes is defined orthonormal to the contact plane. The vector from $\mathcal{F}_i$ to $\mathcal{C}_i$ is $\boldsymbol{p}_{fc_i} := \boldsymbol{p}_{c_i} - \boldsymbol{p}_{f_i} \in \mathbb{R}^3$. A visual representation of the contact geometry for the ith agent is shown in Figure 1.

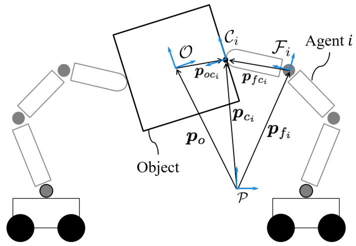

Fig. 1: Two robotic agents in contact with an object.

The dynamics of agent i in the grasp is defined by [18]:

$$M_i \ddot{\boldsymbol{q}}_i + C_i \dot{\boldsymbol{q}}_i + \boldsymbol{g}_i = -J_{h_i}^T \boldsymbol{f}_{c_i} + \boldsymbol{u}_i \tag{1}$$

where $M_i := M_i(\boldsymbol{q}_i) \in \mathbb{R}^{n_i \times n_i}$ is the positive definite inertia matrix, $C_i := C_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i) \in \mathbb{R}^{n_i \times n_i}$ is the Coriolis/centrifugal matrix, $\boldsymbol{g}_i := \boldsymbol{g}_i(\boldsymbol{q}_i)$ is the gravity torque, $\boldsymbol{f}_{c_i} \in \mathbb{R}^3$ is the contact force, $\boldsymbol{u}_i \in \mathbb{R}^{n_i}$ is the joint torque control input, and $J_{h_i} := J_{h_i}(\boldsymbol{q}_i, \boldsymbol{p}_{fc_i}) := \begin{bmatrix} I_3 & -(\boldsymbol{p}_{fc_i})\times \end{bmatrix} J_{s_i}(\boldsymbol{q}_i) \in \mathbb{R}^{3 \times n_i}$ is the agent Jacobian matrix, where $J_{s_i}(\boldsymbol{q}_i) \in \mathbb{R}^{6 \times n_i}$ is the manipulator Jacobian [18]. The full hand Jacobian matrix is $J_h := \text{diag}\{[J_{h_i}]_{i \in \mathscr{N}}\} \in \mathbb{R}^{3N \times n}$. We emphasize that the dynamical parameters (masses, moments of inertia) appearing in M_i, C_i, $\boldsymbol{g}_i$ are considered to be *unknown*, $\forall i \in \mathscr{N}$. The dynamics (1) can be written in vector form as

$$M\ddot{\boldsymbol{q}} + C\dot{\boldsymbol{q}} + \boldsymbol{g} = -J_h^T \boldsymbol{f}_c + \boldsymbol{u}, \tag{2}$$

where $M := M(\boldsymbol{q}) := \text{diag}\{[M_i]_{i \in \mathscr{N}}\}$, $C := C(\boldsymbol{q}, \dot{\boldsymbol{q}}) := \text{diag}\{[C_i]_{i \in \mathscr{N}}\} \in \mathbb{R}^{n \times n}$, and $\boldsymbol{g} := \boldsymbol{g}(\boldsymbol{q}) := [\boldsymbol{g}_1^T, \ldots, \boldsymbol{g}_N^T]^T$, $\boldsymbol{f}_c := [\boldsymbol{f}_{c_1}^T, \ldots, \boldsymbol{f}_{c_N}^T]^T$, $\boldsymbol{u} := [\boldsymbol{u}_1^T, \ldots, \boldsymbol{u}_N^T]^T \in \mathbb{R}^n$.

In contrast to the related literature, we assume here tracking of a traceable point $\boldsymbol{p}_o$ on the object surface (that can be tracked by common sensors) instead of the center of mass, whose information is considered unknown. Let $\mathcal{O}$ be a reference frame fixed at $\boldsymbol{p}_o$, Let also $R_{po} \in SO(3)$ be the respective rotation matrix, which maps from $\mathcal{O}$ to $\mathcal{P}$. Let $\boldsymbol{x}_o := [\boldsymbol{p}_o^T, \boldsymbol{\eta}_o^T]^T \in \mathbb{M} := \mathbb{R}^3 \times \mathbb{T}$, $\boldsymbol{v}_o := [\dot{\boldsymbol{p}}_o^T, \boldsymbol{\omega}_o^T]^T \in \mathbb{R}^6$ denote the pose and generalized velocity of the object frame, with $\boldsymbol{\eta}_o \in \mathbb{T}$ an orientation vector and $\mathbb{T}$ the associated domain. The position vector from $\mathcal{O}$ to the respective contact point is $\boldsymbol{p}_{oc_i} := \boldsymbol{p}_{c_i} - \boldsymbol{p}_o \in \mathbb{R}^3$.

We denote the actual object center of mass location with $\bar{\boldsymbol{p}}_o \in \mathbb{R}^3$, and without loss of generality we align the object body frame with $\mathcal{O}$ so that $\boldsymbol{\eta}_o = \bar{\boldsymbol{\eta}}_o$ and $\bar{\boldsymbol{\omega}}_o = \boldsymbol{\omega}_o$. Thus we let $\bar{\boldsymbol{x}}_o := [\bar{\boldsymbol{p}}_o^T, \boldsymbol{\eta}_o^T]^T \in \mathbb{M}$ be the object pose with respect to the inertial frame $\mathcal{P}$ and $\bar{\boldsymbol{v}}_o := [\dot{\bar{\boldsymbol{p}}}_o^T, \boldsymbol{\omega}_o^T]^T \in \mathbb{R}^6$. Let $\bar{\boldsymbol{p}}_{oc_i} := \boldsymbol{p}_{c_i} - \bar{\boldsymbol{p}}_o$ with $\boldsymbol{p}_{oc} := [\boldsymbol{p}_{oc_1}^T, ..., \boldsymbol{p}_{oc_N}^T]^T \in \mathbb{R}^{3N}$. The conventional object dynamics with respect to the object center of mass are given by the Newton-Euler formulation:

$$\bar{M}_o \dot{\bar{\boldsymbol{v}}}_o + \bar{C}_o \bar{\boldsymbol{v}}_o + \bar{\boldsymbol{g}}_o = \bar{G} \boldsymbol{f}_c \tag{3}$$

where $\bar{M}_o := \bar{M}_o(\bar{\boldsymbol{\eta}}_o) \in \mathbb{R}^{6 \times 6}$ is the object inertia matrix, $\bar{C}_o := \bar{C}_o(\bar{\boldsymbol{\eta}}_o, \bar{\boldsymbol{\omega}}_o) \in \mathbb{R}^{6 \times 6}$ is the object Coriolis and centrifugal matrices, $\bar{G} := \bar{G}(\bar{\boldsymbol{p}}_{oc}) \in \mathbb{R}^{6 \times 3N}$ is the grasp map, and $\bar{\boldsymbol{g}}_o \in \mathbb{R}^6$ is the gravity acting on the object.

The grasp map, $\bar{G}$, maps the concatenated contact force, $\boldsymbol{f}_c \in \mathbb{R}^{3N}$, to the net wrench acting on the object *center of mass* and is defined by $\bar{G} := [\bar{G}_1, ..., \bar{G}_N]$ where $\bar{G}_i := \bar{G}_i(\bar{\boldsymbol{p}}_{oc_i}) := [I_3, -(\bar{\boldsymbol{p}}_{oc_i})\times]^\top \in \mathbb{R}^{6 \times 3}$. We note that $\bar{G}$ is the conventional grasp map commonly used in grasping [18].

To perform the rigid body transformation, let $J_a := J_a(\bar{\boldsymbol{\eta}}_o) \in \mathbb{R}^{6 \times 6}$ be defined as:

$$J_a(\bar{\boldsymbol{\eta}}_o) := \begin{bmatrix} I_3 & (R_{po}\boldsymbol{p}_{\bar{o}o}^{\mathcal{O}})\times \\ \boldsymbol{0}_3 & I_3 \end{bmatrix} \tag{4}$$

where $\boldsymbol{p}_{\bar{o}o}^{\mathcal{O}} := \boldsymbol{p}_o^{\mathcal{O}} - \bar{\boldsymbol{p}}_o^{\mathcal{O}}$, such that $\bar{\boldsymbol{v}}_o$ and $\boldsymbol{v}_o$ can be related via $\bar{\boldsymbol{v}}_o = J_a \boldsymbol{v}_o$, which is derived by differentiating $\bar{\boldsymbol{p}}_o = \boldsymbol{p}_o + R_{po}(\bar{\boldsymbol{p}}_o^{\mathcal{O}} - \boldsymbol{p}_o^{\mathcal{O}})$. Note that $\boldsymbol{p}_{\bar{o}o}^{\mathcal{O}}$ is constant.

Substitution of $\bar{\boldsymbol{v}}_o = J_a \boldsymbol{v}_o$ and left multiplication by J_a^T in (3) yields the adjusted dynamics with respect to $\boldsymbol{p}_o$:

$$M_o \dot{\boldsymbol{v}}_o + C_o \boldsymbol{v}_o + \boldsymbol{g}_o = G \boldsymbol{f}_c, \tag{5}$$

where $M_o := M_o(\bar{\boldsymbol{\eta}}_o) := J_a^T \bar{M}_o J_a \in \mathbb{R}^{6 \times 6}$, $C_o := C_o(\bar{\boldsymbol{\eta}}_o, \boldsymbol{\omega}_o) := J_a^T(\bar{M}_o \dot{J}_a + \bar{C}_o J_a) \in \mathbb{R}^{6 \times 6}$, $\boldsymbol{g}_o := \boldsymbol{g}_o(\bar{\boldsymbol{\eta}}_o) := J_a^T \bar{\boldsymbol{g}}_o \in \mathbb{R}^6$, and $G := J_a^T \bar{G} \in \mathbb{R}^{6 \times 3N}$, for which it holds $G = [G_1, \ldots, G_N]$, with $G_i = J_a^T \bar{G}_i = [I_3, -(\boldsymbol{p}_{oc_i}\times)]^\top$, where we have used $R_{po}\bar{\boldsymbol{p}}_{\bar{o}o}^{\mathcal{O}} = \boldsymbol{p}_{\bar{o}o}$. Note that $G = G(\boldsymbol{p}_{oc})$, i.e., G is *not* dependent on $\bar{\boldsymbol{p}}_o$. Note also by the relation $\bar{\boldsymbol{p}}_o = \boldsymbol{p}_o - R_{po}\boldsymbol{p}_{\bar{o}o}^{\mathcal{O}}$, that M_o, C_o, $\boldsymbol{g}_o$ are functions of $\bar{\boldsymbol{\eta}}_o = \boldsymbol{\eta}_o$, $\bar{\boldsymbol{\omega}}_o = \boldsymbol{\omega}_o$ with dependency on the constant but unknown term $\boldsymbol{p}_{\bar{o}o}^{\mathcal{O}}$. Similarly to the agents, the object parameters appearing in the terms M_o, C_o, $\boldsymbol{g}_o$ are considered to be *unknown*.

Regarding the object orientation, we use the unit quaternion choice $\boldsymbol{\eta}_o := [\phi_o, \boldsymbol{\epsilon}_o^T]^T \in \mathbb{T} := \mathsf{S}^3$, where $\phi_o \in [-1, 1]$ and $\boldsymbol{\epsilon}_o \in \mathbb{R}^3$ are the scalar and vector part, respectively, satisfying $\phi_o^2 + \boldsymbol{\epsilon}_o^T \boldsymbol{\epsilon}_o = 1$. Moreover, it holds that [4]

$$\dot{\boldsymbol{\eta}}_o = \frac{1}{2} E_\eta(\boldsymbol{\eta}_o)\boldsymbol{\omega}_o \Rightarrow \boldsymbol{\omega}_o = 2E_\eta(\boldsymbol{\eta}_o)^T \dot{\boldsymbol{\eta}}_o, \tag{6}$$

where $E_\eta : \mathsf{S}^3 \to \mathbb{R}^{4 \times 3}$ is the matrix

$$E_\eta(\boldsymbol{\eta}) := \begin{bmatrix} -\boldsymbol{\epsilon}^T \\ \phi I_3 - (\boldsymbol{\epsilon})\times \end{bmatrix}, \quad \forall \boldsymbol{\eta} = [\phi, \boldsymbol{\epsilon}^T]^T \in \mathsf{S}^3.$$

The more practical consideration of rolling contacts, as opposed to a rigid grasp, requires no slip to occur between the agents and object by ensuring that each contact force remains inside the friction cone defined by:

$$\mathscr{F}_{c_i} := \{\boldsymbol{f}_{c_i}^{\mathcal{C}_i} \in \mathbb{R}^3 : f_{n_i}\mu \geq \sqrt{f_{x_i}^2 + f_{y_i}^2}\} \tag{7}$$

where $\boldsymbol{f}_{c_i}^{\mathcal{C}_i} = (f_{x_i}, f_{y_i}, f_{n_i})$ is the contact force at i written in frame $\mathcal{C}_i$ with tangential force components $f_{x_i}, f_{y_i} \in \mathbb{R}$ and normal force component $f_{n_i} \in \mathbb{R}$, and $\mu \in \mathbb{R}_{>0}$ is the friction coefficient. The full friction cone is the Cartesian product of all the friction cones: $\mathscr{F}_c := \mathscr{F}_{c_1} \times ... \times \mathscr{F}_{c_n}$.

When the contact points do not slip, the grasp relation $J_h \dot{\boldsymbol{q}} = \bar{G}^T \bar{\boldsymbol{v}}_o$ holds [6], which becomes

$$\boldsymbol{v}_c = J_h \dot{\boldsymbol{q}} = G^T \boldsymbol{v}_o, \tag{8}$$

where $\boldsymbol{v}_c := [\boldsymbol{v}_{c_1}^T, \dots, \boldsymbol{v}_{c_N}^T]^T \in \mathbb{R}^{3n}$ is the vector of contact velocities. The following assumptions are made for the grasp:

Assumption 1. *The grasp consists of $N \geq 3$ agents with non-collinear contact points and $\mathfrak{N}(G) \bigcap \mathrm{Int}(\mathscr{F}_c) \neq \emptyset$.*

Assumption 2. *The matrix $J_h(\boldsymbol{q})$ is non-singular, and the contact points do not exceed the fingertip surface.*

Remark 1. *Note that $N \geq 3$ agents with non-collinear contact points ensures G is full row rank [18]. The condition $\mathfrak{N}(G) \bigcap \mathrm{Int}(\mathscr{F}_c) \neq \emptyset$ is the standard force-closure condition [6]. Force-closure depends on the initial grasp, and can be ensured by existing high-level grasp planning methods [19]. Moreover, by incorporating optimization techniques, as e.g. in [20], or internal motions of redundant agents ($n_i > 3$), we can enforce prevention of excessive rolling of the contacts and guarantee avoidance of singular configurations and thus relax Assumption 2.*

We also assume that the contact vectors $\boldsymbol{p}_{fc_i}^{\mathcal{F}_i}$ and $\dot{\boldsymbol{p}}_{fc_i}^{\mathcal{F}_i}$ are measured accurately online, $\forall i \in \mathscr{N}$ (which can be achieved by sensors or forward simulation of the contact dynamics).

Let now a desired pose trajectory, $\boldsymbol{p}_\mathrm{d} : \mathbb{R}_{\geq 0} \to \mathbb{R}^3$, $\boldsymbol{\eta}_\mathrm{d} := [\phi_\mathrm{d}, \epsilon_\mathrm{d}^T]^T : \mathbb{R}_{\geq 0} \to \mathsf{S}^3$, to be tracked by $\boldsymbol{x}_o$. To that end, we define the position error $\boldsymbol{e}_p := \boldsymbol{p}_o - \boldsymbol{p}_\mathrm{d}$ as well as the quaternion product $\boldsymbol{e}_\eta := \boldsymbol{\eta}_\mathrm{d} \otimes \boldsymbol{\eta}_o^+$, as an orientation error metric [4], where $\boldsymbol{\eta}^+ := [\phi, -\boldsymbol{\epsilon}^T]^T$ denotes the quaternion conjugate. The aim is then to regulate $\boldsymbol{e}_\eta$ to $[\pm 1, \boldsymbol{0}_3^T]^T$ [4]. Moreover, we aim at ensuring that the fingers are always in contact with the object and slipping is avoided. Formally, the problem is defined as follows.

Problem 1. *Given a desired bounded, smooth object pose trajectory defined by $\boldsymbol{p}_d : \mathbb{R}_{\geq 0} \to \mathbb{R}^3$, $\boldsymbol{\eta}_d : \mathbb{R}_{\geq 0} \to \mathsf{S}^3$, with bounded first and second derivatives, determine a control law $\boldsymbol{u}$ in* (2) *such that the following conditions hold:*

1) $\lim_{t\to\infty} \big(\boldsymbol{e}_p(t), \boldsymbol{e}_\eta(t)\big) = \big(\boldsymbol{0}_3, [\pm 1, \boldsymbol{0}_3^T]^T\big)$
2) $\boldsymbol{f}_{c_i}^{C_i}(t) \in \mathscr{F}_{c_i}, \forall t > 0,\ i \in \mathscr{N}$.

III. PROPOSED CONTROL SCHEME

Without loss of generality, we assume that $n_i = 3$, $\forall i \in \mathscr{N}$, i.e., the agents are not redundant. The proposed solution can be trivially extended to redundant cases, e.g., by following the analysis of [18, Chapter 6]. By combining the agent and object dynamics (2), (5) as well as (8), we can obtain the coupled dynamics

$$\tilde{M} \dot{\boldsymbol{v}}_o + \tilde{C} \boldsymbol{v}_o + \tilde{\boldsymbol{g}} = G J_h^{-T} \boldsymbol{u}, \tag{9}$$

where $\tilde{M} := \tilde{M}(\tilde{\boldsymbol{x}}) := M_o + G J_h^{-T} M J_h^{-1} G^T$, $\tilde{C} := \tilde{C}(\tilde{\boldsymbol{x}}, \dot{\tilde{\boldsymbol{x}}}) := C_o + G J_h^{-T}(C J_h^{-1} G^T + M \frac{d}{dt}(J_h^{-1} G^T))$, $\tilde{\boldsymbol{g}} := \tilde{\boldsymbol{g}}(\tilde{\boldsymbol{x}}) := \boldsymbol{g}_o + G J_h^{-T} \boldsymbol{g}$, and $\tilde{\boldsymbol{x}} := [\boldsymbol{\eta}_o^T, \boldsymbol{q}^T, \boldsymbol{p}_{fc}^T, \boldsymbol{p}_{oc}^T]^T \in \mathbb{T} \times \mathbb{R}^{n+6N}$. By using Lemma 1 of [4], we can prove the following, which states useful properties of (9):

Lemma 1. *[4] The matrix $\tilde{M}$, is symmetric and positive-definite, and $\dot{\tilde{M}} - 2\tilde{C}$ is skew-symmetric.*

Next, the left-hand side of the object dynamics is linearly parameterized with respect to the dynamic parameters as:

$$M_o(\boldsymbol{\eta}_o)\dot{\boldsymbol{v}}_o + C_o(\boldsymbol{\eta}_o, \boldsymbol{\omega}_o)\boldsymbol{v}_o + \boldsymbol{g}_o = Y_o(\boldsymbol{\eta}_o, \boldsymbol{\omega}_o, \boldsymbol{v}_o, \dot{\boldsymbol{v}}_o)\boldsymbol{\nu}_o,$$

where $\boldsymbol{\nu}_o \in \mathbb{R}^{l_o}$, $l_o \in \mathbb{N}$, is a vector containing unknown object dynamic parameters as well as the term $\boldsymbol{p}_{\bar{o}o}^{\mathcal{O}}$, introduced in (4), and $Y_o : \mathbb{T} \times \mathbb{R}^{18} \to \mathbb{R}^{6\times l_o}$ is a known regressor matrix. Similarly, the part of (9) that concerns the robotic agents can be linearly parameterized as:

$$M J_h^{-1} G^T \dot{\boldsymbol{v}}_o + \left(M \frac{\partial}{\partial t}(J_h^{-1} G^T) + C J_h^{-1} G^T \right) \boldsymbol{v}_o + \boldsymbol{g} = Y(\tilde{\boldsymbol{x}}, \dot{\tilde{\boldsymbol{x}}}, \boldsymbol{v}_o, \dot{\boldsymbol{v}}_o)\boldsymbol{\nu},$$

where $\boldsymbol{\nu} \in \mathbb{R}^l, l \in \mathbb{N}$, is a vector of unknown dynamic parameters of the agents, and $Y : \mathbb{T} \times \mathbb{R}^{2n+15N} \to \mathbb{R}^{6N\times l}$ is the respective known regressor matrix.

Therefore, the left-hand side of the coupled dynamics (9) can be written as

$$\tilde{M}\dot{\boldsymbol{v}}_o + \tilde{C}\boldsymbol{v}_o + \tilde{\boldsymbol{g}} = Y_o(\boldsymbol{\eta}_o, \boldsymbol{\omega}_o, \boldsymbol{v}_o, \dot{\boldsymbol{v}}_o)\boldsymbol{\nu}_o + G J_h^{-T} Y(\tilde{\boldsymbol{x}}, \dot{\tilde{\boldsymbol{x}}}, \boldsymbol{v}_o, \dot{\boldsymbol{v}}_o)\boldsymbol{\nu} \tag{10}$$

Let now $\hat{\boldsymbol{\nu}} \in \mathbb{R}^l$, $\hat{\boldsymbol{\nu}}_o \in \mathbb{R}^{l_o}$, be the estimates of $\boldsymbol{\nu}$ and $\boldsymbol{\nu}_o$, respectively, by the agents, and the respective errors $\boldsymbol{e}_\nu := \hat{\boldsymbol{\nu}} - \boldsymbol{\nu}$, and $\boldsymbol{e}_{\nu_o} := \hat{\boldsymbol{\nu}}_o - \boldsymbol{\nu}_o$.

Regarding the pose errors, as described in Problem 1, these are $\boldsymbol{e}_p = \boldsymbol{p}_o - \boldsymbol{p}_\mathrm{d}$ and $\boldsymbol{e}_\eta = \boldsymbol{\eta}_\mathrm{d} \otimes \boldsymbol{\eta}_o^+$, which can be shown to satisfy [4]:

$$\boldsymbol{e}_\eta = \begin{bmatrix} e_\phi \\ \boldsymbol{e}_\epsilon \end{bmatrix} := \begin{bmatrix} \phi_o\phi_\mathrm{d} + \boldsymbol{\epsilon}_o^T\boldsymbol{\epsilon}_\mathrm{d} \\ \phi_o\boldsymbol{\epsilon}_\mathrm{d} - \phi_\mathrm{d}\boldsymbol{\epsilon}_o + (\boldsymbol{\epsilon}_o)\times\ \boldsymbol{\epsilon}_\mathrm{d} \end{bmatrix} \tag{11a}$$

$$\dot{\boldsymbol{e}}_\eta = \begin{bmatrix} \dot{e}_\phi \\ \dot{\boldsymbol{e}}_\epsilon \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\boldsymbol{e}_\epsilon^T\boldsymbol{e}_\omega \\ -\frac{1}{2}(e_\phi I_3 + (\boldsymbol{e}_\epsilon)\times)\boldsymbol{e}_\omega - (\boldsymbol{e}_\epsilon)\times\ \dot{\boldsymbol{\omega}}_\mathrm{d} \end{bmatrix}, \tag{11b}$$

where $\boldsymbol{e}_\omega := \boldsymbol{\omega}_o - \boldsymbol{\omega}_\mathrm{d} \in \mathbb{R}^3$ and $\boldsymbol{\omega}_\mathrm{d} = 2E(\boldsymbol{\eta}_\mathrm{d})\dot{\boldsymbol{\eta}}_\mathrm{d}$, similarly to (6). Note that, except for $(\boldsymbol{e}_p, \boldsymbol{e}_\eta, \boldsymbol{e}_\omega) = (\boldsymbol{0}_3, [\pm 1, \boldsymbol{0}_3^T]^T, \boldsymbol{0}_3)$ (the desired equilibirum), the point $(\boldsymbol{e}_p, \boldsymbol{e}_\eta, \boldsymbol{e}_\omega) = (\boldsymbol{0}_3, [0, \tilde{\boldsymbol{e}}_\epsilon^T]^T, \boldsymbol{0}_3)$ is also an equilibrium of (11), for any $\tilde{\boldsymbol{e}}_\epsilon \in \mathsf{S}^2$. The latter represents an undesired local minimum of the dynamics (11) and stems from topological obstructions of the orientation space [21]. Hence, a continuous controller cannot achieve *global* stabilization of $(\boldsymbol{e}_p, \boldsymbol{e}_\eta, \boldsymbol{e}_\omega)$ to the desired equilibrium point. As will be shown later, however, we will achieve almost global stabilization, i.e., from all initial conditions other than the aforementioned undesired local minimum configurations that satisfy $e_\phi(0) = 0$, which form a lower-dimensional manifold.

We provide next the proposed control protocol. First, we design the reference velocity signal $\boldsymbol{v}_f \in \mathbb{R}^6$ and the associated velocity error $\boldsymbol{e}_v$ as

$$\boldsymbol{v}_f := \boldsymbol{v}_\mathrm{d} - K\boldsymbol{e} \tag{12a}$$

$$\boldsymbol{e}_v := \boldsymbol{v}_o - \boldsymbol{v}_f, \tag{12b}$$

where $K := \text{diag}\{k_p I_3, k_\eta I_3\} \in \mathbb{R}^3$ is a positive definite matrix, with k_p, k_η positive constants, $\boldsymbol{e} := [\boldsymbol{e}_p^T, -\frac{\boldsymbol{e}_\epsilon^T}{e_\phi^3}]^T$, and $\boldsymbol{v}_\text{d} := [\dot{\boldsymbol{p}}_\text{d}^T, \boldsymbol{\omega}_\text{d}^T]^T$. As will be shown later, $e_\phi(0) \neq 0 \Rightarrow e_\phi(t) \neq 0$, $\forall t \geq 0$, thus (12a) is well defined.

We design now the control protocol as

$$u = Y_r \hat{\boldsymbol{\nu}} + J_h^T (G^* \boldsymbol{f}_\text{d} + \boldsymbol{f}_\text{int}), \tag{13}$$

where G^* is the Moore-Penrose pseudoinverse of G, $\boldsymbol{f}_\text{d} := Y_{o_r} \hat{\boldsymbol{\nu}}_o - \boldsymbol{e} - K_v \boldsymbol{e}_v$ with $K_v \in \mathbb{R}^6$ a positive definite gain matrix, $Y_r := Y(\tilde{\boldsymbol{x}}, \dot{\tilde{\boldsymbol{x}}}, \boldsymbol{v}_f, \dot{\boldsymbol{v}}_f)$, $Y_{o_r} := Y_o(\boldsymbol{\eta}_o, \boldsymbol{\omega}_o, \boldsymbol{v}_f, \dot{\boldsymbol{v}}_f)$, and $\boldsymbol{f}_\text{int}$ is a term in the nullspace of G to prevent contact slip, which will be designed later. Moreover, we design

$$\dot{\hat{\boldsymbol{\nu}}} = \text{Proj}(\hat{\boldsymbol{\nu}}, -\Gamma Y_r^T J_h^{-1} G^T \boldsymbol{e}_v), \tag{14a}$$

$$\dot{\hat{\boldsymbol{\nu}}}_o = \text{Proj}(\hat{\boldsymbol{\nu}}_o, -\Gamma_o Y_{o_r}^T \boldsymbol{e}_v), \tag{14b}$$

with $\Gamma \in \mathbb{R}^{l \times l}, \Gamma_o \in \mathbb{R}^{l_o \times l_o}$ positive definite constant matrices, and Proj() the projection operator, satisfying [22]:

$$(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})^T (W^{-1} \text{Proj}(\hat{\boldsymbol{\theta}}, W\boldsymbol{z}) - \boldsymbol{z}) \leq \boldsymbol{0}_{l_z}, \tag{15}$$

for any symmetric positive definite $W \in \mathbb{R}^{l_z \times l_z}$, and $\forall \boldsymbol{\theta}, \hat{\boldsymbol{\theta}}, \boldsymbol{z} \in \mathbb{R}^{l_z}$, for some $l_z \in \mathbb{N}$. Moreover, by appropriately choosing the initial conditions of the estimates $\hat{\boldsymbol{\nu}}(0)$, $\hat{\boldsymbol{\nu}}(0)$, we guarantee via the projection operator that $\hat{\boldsymbol{\nu}}(t)$, $\hat{\boldsymbol{\nu}}_o(t)$ will stay uniformly bounded in predefined sets defined by finite constants $\bar{\hat{\boldsymbol{\nu}}}$, $\bar{\hat{\boldsymbol{\nu}}}_o$, i.e., $\|\hat{\boldsymbol{\nu}}(t)\| \leq \bar{\hat{\boldsymbol{\nu}}}$, $\|\hat{\boldsymbol{\nu}}_o(t)\| \leq \bar{\hat{\boldsymbol{\nu}}}_o$, $\forall t \geq 0$. Hence, we can bound the respective errors as

$$\|\boldsymbol{e}_\nu(t)\| \leq \bar{e}_\nu := \bar{\hat{\boldsymbol{\nu}}} + \|\boldsymbol{\nu}\| \tag{16a}$$

$$\|\boldsymbol{e}_{\nu_o}(t)\| \leq \bar{e}_{\nu_o} := \bar{\hat{\boldsymbol{\nu}}}_o + \|\boldsymbol{\nu}_o\|, \tag{16b}$$

for finite constants $\bar{e}_\nu, \bar{e}_{\nu_o}$. More details can be found in [22].

We design next the internal force component $\boldsymbol{f}_\text{int}$ to guarantee slip prevention. Slip is addressed by ensuring the contact forces remain inside the friction cone as specified in (7). The friction cone can be approximated as a pyramid with $n_s \in \mathbb{N}_{>3}$ sides, which results in a linear constraint with respect to the contact forces [16], [23]: $\Lambda_i(\mu) R_{pc_i}^T \boldsymbol{f}_{c_i} \succeq 0, \forall i \in \mathscr{N}$, or in vector form,

$$\Lambda(\mu) R_{pc}^T \boldsymbol{f}_c \succeq 0, \tag{17}$$

where $R_{pc} := \text{diag}\{[R_{pc_i}]_{i \in \mathscr{N}}\}$, with $R_{pc_i} \in SO(3)$ the rotation matrices mapping the contact frames $\mathcal{C}_i$ to the inertial frame $\mathcal{P}$, $\Lambda(\mu) := \text{diag}\{[\Lambda_i(\mu)]_{i \in \mathscr{N}}\}$, with $\Lambda_i(\mu) \in \mathbb{R}^{n_s \times 3}$ define the pyramid matrices, and $\mu \in \mathbb{R}_{>0}$ is the friction coefficient; $\boldsymbol{f}_\text{int}$ must satisfy (17) (i.e., $\Lambda(\mu) R_{pc}^T \boldsymbol{f}_\text{int} \succeq 0$), as well as $G\boldsymbol{f}_\text{int} = 0$, and have a positive local normal component to the contact plane. To enforce these conditions we design[1] $\boldsymbol{f}_{\text{int}_i} = f'_\text{int} R_{pc_i} \boldsymbol{\ell}_i$, where $\boldsymbol{\ell}_i := [\ell_{i_x}, \ell_{i_y}, \ell_{i_z}]^T$ is the internal force direction in the contact frame $\mathcal{C}_i$, $i \in \mathscr{N}$, and $f'_\text{int} \in \mathbb{R}_{>0}$ is a gain parameter to be designed. Without loss of generality let ℓ_{i_z} be aligned with the normal direction of the contact frame such that $\ell_{i_z} > 0$, $i \in \mathscr{N}$, ensures that only pushing forces are applied at each contact. Satisfaction of the aforementioned conditions is done by solving the following convex quadratic program to define the internal force controller $\boldsymbol{f}_\text{int} = f'_\text{int} R_{pc} \boldsymbol{\ell}^*$, with

$$\boldsymbol{\ell}^* = \text{argmin}_{\boldsymbol{\ell}} \sum_{i \in \mathscr{N}} \|\boldsymbol{\ell}\|^2 \tag{18a}$$

$$\text{s. t.} \begin{cases} G R_{pc} \boldsymbol{\ell} = \boldsymbol{0}_6, \\ \ell_{i_z} > 0, & \forall i \in \mathscr{N}, \\ \Lambda_i(\mu) \boldsymbol{\ell}_i \succ \boldsymbol{0}_{n_s}, & \forall i \in \mathscr{N}, \end{cases} \tag{18b}$$

where $\boldsymbol{\ell} := [\boldsymbol{\ell}_1^T, \ldots, \boldsymbol{\ell}_N^T]^T$. Since the contact points form a force-closure configuration, (18) has a feasible solution.

Finally, to satisfy (17), $\boldsymbol{f}_\text{int}$ must apply sufficient force inside the friction cone to reject perturbations that will arise during the manipulation motion that can push the contact force outside of the friction cone. Rejection of these perturbations is performed by designing the gain f'_int as follows. For simplicity we define the terms $\boldsymbol{k} = \Lambda(\mu) R_{pc}^T G^* \boldsymbol{f}_\text{d}, \boldsymbol{l} = \Lambda(\mu) \boldsymbol{\ell}$, and we denote by k_j and l_j the jth element of $\boldsymbol{k}$ and $\boldsymbol{l}$ respectively for $j \in \{1, \ldots, Nn_s\}$.

Noting that $\Lambda(\mu)\boldsymbol{\ell} \succ 0$ from (18), we define the *decreasing* function $\kappa : \mathbb{R} \to \mathbb{R}_{\geq 0}$ as $\kappa(x) = -x$, if $x \leq -1$, $\kappa(x) = q(x)$, if $-1 \leq x \leq 0$, and $\kappa(x) = 0$, if $x \geq 0$, where $q(x) \geq 0$, $\forall x \in [-1, 0]$, is an appropriate polynomial that ensures continuous differentiability of κ, for instance $q(x) = x^3 + 2x^2$. Then one can verify that $\kappa(x) + 1 \geq -x$, $\forall x \in \mathbb{R}$. We now design the magnitude scaling as

$$f'_\text{int} = \frac{\kappa(\min_j\{k_j\}) + 1 + \varepsilon}{\min_j\{l_j\}}, \tag{19}$$

where $\varepsilon \in \mathbb{R}_{>0}$ is a tuning gain. The intuition behind (19) is to upper bound elements of the control and the system dynamics to prevent either from pushing the contact force outside of the friction cone. The term $\kappa(\min_j\{k_j\}) + 1$ cancels out any effects from $\boldsymbol{f}_d$. The term ε handles the bounded uncertainties, and it does not have to be an upper bound of all the dynamic terms, as [16], hence reducing the amount of applied squeezing force. The stability and slip prevention guarantees of the proposed controller are presented in the following theorem.

Theorem 1. *Consider N robotic agents in contact with an object, described by the dynamics* (2), (5), *and suppose Assumptions 1 and 2 hold. Let the desired object pose* ($\boldsymbol{p}_\text{d}$, $\boldsymbol{\eta}_\text{d}$) $: \mathbb{R}_{\geq 0} \to \mathbb{R}^3 \times \mathsf{S}^3$ *be bounded with bounded first and second derivatives. Moreover, assume that $e_\phi(0) \neq 0$ and $\boldsymbol{f}_{c_i}^{C_i}(0) \in \mathscr{F}_{c_i}$, $\forall i \in \mathscr{N}$. Then, the control protocol* (12a)-(19) *guarantees that $\lim_{t \to \infty} \big(\boldsymbol{e}_p(t), \boldsymbol{e}_\eta(t)\big) = \big(\boldsymbol{0}_3, [\pm 1, \boldsymbol{0}_3^T]^T\big)$, as well as boundedness of all closed-loop signals. Moreover, by choosing a sufficiently large ε in* (19), *it holds that $\boldsymbol{f}_c^{C_i}(t) \in \mathscr{F}_{c_i}, \forall t > 0$, $i \in \mathscr{N}$.*

Proof. Consider the stack vector state $\boldsymbol{\chi} := [\boldsymbol{e}_p^T, \boldsymbol{e}_\epsilon^T, \boldsymbol{e}_v^T, \boldsymbol{e}_\nu^T, \boldsymbol{e}_{\nu_o}^T]^T \in \mathscr{X} := \mathbb{R}^{12+\ell+\ell_o}$. Next, note by (2), (3), and (8) that, when $\boldsymbol{f}_{c_i}^{C_i}(\boldsymbol{\chi}) \in \mathscr{F}_{c_i}$, each $\boldsymbol{f}_{c_i}^{C_i}$ can be written as a function of the stack state, i.e., $\boldsymbol{f}_{c_i}^{C_i} = \boldsymbol{f}_{c_i}^{C_i}(\boldsymbol{\chi})$, $\forall i \in \mathscr{N}$. Consider also the set

$$\mathscr{U} := \{\boldsymbol{\chi} \in \mathscr{X} : \|\boldsymbol{e}_\epsilon\| < \bar{e}_\epsilon, \|\boldsymbol{e}_p\| < \bar{e}_p, \|\boldsymbol{e}_v\| < \bar{e}_v,$$
$$\|\boldsymbol{e}_\nu\| < \tilde{e}_\nu, \|\boldsymbol{e}_{\nu_o}\| < \tilde{e}_{\nu_o}, \boldsymbol{f}_{c_i}^{C_i}(\boldsymbol{\chi}) \in \text{Int}(\mathscr{F}_{c_i}), \forall i \in \mathscr{N}\},$$

[1]We use the notation $\boldsymbol{f}_\text{int} = [\boldsymbol{f}_{\text{int}_1}^T, \ldots, \boldsymbol{f}_{\text{int}_N}^T]^T$.

for some positive constants $\bar{e}_\epsilon > 1$, $\bar{e}_v$, $\bar{e}_p$ satisfying $\|\boldsymbol{e}_v(0)\| < \bar{e}_v$, $\|\boldsymbol{e}_p(0)\| < \bar{e}_p$, and $\tilde{e}_\nu, \tilde{e}_{\nu_o}$ larger than $\bar{e}_\nu, \bar{e}_{\nu_o}$, respectively, which were introduced in (16). Note that $\boldsymbol{\chi}(0) \in \mathscr{U}$. Next, by using (13) and (14), one obtains the closed-loop dynamics $\dot{\boldsymbol{\chi}} = \boldsymbol{h}_\chi(\boldsymbol{\chi}, t)$, where $\boldsymbol{h}_\chi : \mathscr{X} \times \mathbb{R}_{\geq 0} \to \mathscr{X}$ is a function that is continuous in t and locally Lipschitz in $\boldsymbol{\chi}$. Hence, there exists a positive time constant $\tau > 0$ and a unique solution $\boldsymbol{\chi} : [0, \tau) \to \mathscr{U}$, i.e., defined for $[0, \tau)$ and satisfying $\boldsymbol{\chi}(t) \in \mathscr{U}$, $\forall t \in [0, \tau)$. Hence, slip is prevented and the dynamics (9) are well-defined, for $t \in [0, \tau)$.

Let now the Lyapunov function

$$V := \frac{1}{2}\boldsymbol{e}_p^T\boldsymbol{e}_p + \frac{2}{e_\phi^2} + \frac{1}{2}\boldsymbol{e}_v^T\tilde{M}\boldsymbol{e}_v + \frac{1}{2}\boldsymbol{e}_\nu^T\Gamma^{-1}\boldsymbol{e}_\nu + \frac{1}{2}\boldsymbol{e}_{\nu_o}^T\Gamma_o^{-1}\boldsymbol{e}_{\nu_o}.$$

Since $e_\phi(0) \neq 0$, it holds that $V(0) \leq \bar{V}_0$ for a finite positive $\bar{V}_0$. Differentiation of V, use of the skew symmetry of $\dot{\tilde{M}} - 2\tilde{C}$, $\boldsymbol{v}_o = \boldsymbol{e}_v + \boldsymbol{v}_f$, (10), substitution of the control law (13) and adaptation law (14) and invoking the projection operator property (15) yields $\dot{V} \leq -\boldsymbol{e}^T K \boldsymbol{e} - \boldsymbol{e}_v^T K_v \boldsymbol{e}_v$. Thus $\dot{V}$ is negative semi-definite, and V is bounded in a compact set as $V(t) \leq V(0)$, $\forall t \in [0, \tau)$. In addition, $e_\phi(t) \neq 0$, $\forall t \in [0, \tau)$. Hence, the terms $\boldsymbol{e}_p(t)$, $\boldsymbol{e}_\epsilon(t)$, $e_\phi(t)$ are bounded in a compact set defined by $V(0)$ and not dependent on τ, $\forall t \in [0, \tau)$. Therefore, since $\boldsymbol{p}_\text{d}(t)$ and $\boldsymbol{\eta}_\text{d}(t)$ are bounded and have bounded derivatives and in view of Assumption 2 and properties of Euler-Lagrange systems, one can easily conclude the boundedness of $\boldsymbol{p}_o(t)$, $\boldsymbol{\eta}_o(t)$ $\boldsymbol{v}_o(t)$, $\boldsymbol{v}_f(t)$, $\dot{\boldsymbol{v}}_f(t)$, $\tilde{\boldsymbol{x}}, \dot{\tilde{\boldsymbol{x}}}$, $Y(\tilde{\boldsymbol{x}}, \dot{\tilde{\boldsymbol{x}}}, \boldsymbol{v}_o, \dot{\boldsymbol{v}}_o)$, Y_r, $Y_o(\boldsymbol{\eta}_o, \boldsymbol{\omega}_o, \boldsymbol{e}_v, \dot{\boldsymbol{e}}_v)$, Y_{o_r} in compact sets that are independent of τ, $\forall t \in [0, \tau)$. We prove next the slip prevention using the design of the internal force component $\boldsymbol{f}_\text{int}$. By using (2), (5) and (8), one obtains the following expression for the interaction forces $\boldsymbol{f}_c = B^{-1}(J_h M^{-1}[\boldsymbol{u} - \boldsymbol{g} - \left(CJ_h^{-1}G^T + M\frac{d}{dt}(J_h^{-1}G^T)\right)\boldsymbol{v}_o] + G^T M_o^{-1}(C_o\boldsymbol{v}_o + \boldsymbol{g}_o))$ where $B := J_h M^{-1} J_h^T + G^T M_o^{-1} G$, which, by replacing $\boldsymbol{u}$, using $\boldsymbol{v}_f = \boldsymbol{e}_v + \boldsymbol{v}_o$ and (10), adding and subtracting $B^{-1}G^T M_o^{-1}\boldsymbol{f}_\text{d} = B^{-1}G^T M_o^{-1} G G^* \boldsymbol{f}_{o,d}$ and adding $B^{-1}G^T M_o^{-1} G \boldsymbol{f}_\text{int} = \boldsymbol{0}_{3N}$, becomes

$$\boldsymbol{f}_c = G^*\boldsymbol{f}_\text{d} + \boldsymbol{f}_\text{int} + \boldsymbol{h} \tag{20}$$

where $\boldsymbol{h} := B^{-1}J_h M^{-1}(g - Y(\tilde{\boldsymbol{x}}, \dot{\tilde{\boldsymbol{x}}}, \boldsymbol{e}_v, \dot{\boldsymbol{e}}_v)\nu + Y_r\boldsymbol{e}_\nu) + B^{-1}G^T M_o^{-1}(\boldsymbol{e} + K_v\boldsymbol{e}_v + Y_o(\boldsymbol{e}_o, \boldsymbol{\omega}_o, \boldsymbol{e}_v, \dot{\boldsymbol{e}}_v)\nu_o - Y_{o_r}\boldsymbol{e}_{\nu_o} - g_o)$.

By combining the aforementioned expression with (17), one obtains the following condition for slip prevention:

$$\Lambda(\mu) R_{pc}^T \boldsymbol{f}_\text{int} \succeq -\Lambda(\mu) R_{pc}^T G^* \boldsymbol{f}_\text{d} - \Lambda(\mu) R_{pc}^T \boldsymbol{h}. \tag{21}$$

Note that due to the aforementioned Lyapunov analysis, as well as the adaptation laws (14) through the projection operator, $\boldsymbol{e}$, $\boldsymbol{e}_v(t)$, $\dot{\boldsymbol{e}}_v(t)$, $\boldsymbol{e}_\nu(t)$, $\boldsymbol{e}_{\nu_o}(t)$ are bounded in compact set independent of τ, $\forall t \in [0, \tau)$. By combining this with the aforementioned analysis, we conclude that that $\boldsymbol{h}$ is bounded for all $\forall t \in [0, \tau)$ in a compact set, independent of τ. Hence, by denoting ε_h the maximum bound of the elements of $\pm\Lambda(\mu) R_{pc}^T \boldsymbol{h}$ and using the designed internal force component $\boldsymbol{f}_\text{int} = f'_\text{int} R_{pc} \boldsymbol{\ell}$, a sufficient condition for (21) to hold is for the jth element to satisfy $l_j f'_\text{int} \geq -k_j + \varepsilon_h$, $\forall j \in \{1, \ldots, Nn_s\}$. By substituting (19), the left side satisfies $l_j \min\{l_j\}^{-1}\kappa(\min_j\{k_j\}) + 1 + \varepsilon \geq \kappa(\min_j\{k_j\}) + 1 + \varepsilon \geq -k_j + \varepsilon$, where we use $\kappa(x) \geq 0$, $\kappa(x) + 1 \geq -x$, $\forall x \in \mathbb{R}$, and $\kappa(\min_j(k_j)) > \kappa(k_j)$, $\forall j \in \{1, \ldots, Nn_s\}$, since $\kappa()$ is decreasing. Hence, by choosing a large enough ε we guarantee $\varepsilon \geq \varepsilon_h$ and hence contact slip is avoided $\forall t \in [0, \tau)$. In fact, the internal forces analysis above and the fact that Λ defines pyramid constraints imply that $\boldsymbol{f}_{c_i}^{\mathcal{C}_i} \in \bar{\mathscr{F}}_{c_i}$, where $\bar{\mathscr{F}}_{c_i}$ is a compact subset of $\text{Int}(\mathscr{F}_{c_i})$, $\forall i \in \mathscr{N}$. Therefore, since $\boldsymbol{e}_\nu$ and $\boldsymbol{e}_{\nu_o}$ are uniformly bounded through the projection operator by $\bar{e}_\nu$ and $\bar{e}_{\nu_o}$, respectively, by choosing large enough $\bar{e}_p$ and $\bar{e}_v$ in the definition of $\mathscr{U}$, $\chi(t)$ belongs to a compact subset $\bar{\mathscr{U}}$ of $\mathscr{U}$, $\forall t \in [0, \tau)$. Thus we achieve forward completeness and $\tau = \infty$. Note, finally, that $\boldsymbol{u}(t)$, as designed in (13), is bounded, $\forall t \geq 0$. Therefore, one can conclude that $\dot{\boldsymbol{e}}_v(t)$ and thus $\ddot{\boldsymbol{q}}(t)$ is bounded, $\forall t \geq 0$. Hence, it follows that $\ddot{V}(t)$ is also bounded, $\forall t \geq 0$. Thus by invoking Barbalat's lemma, it follows that $\lim_{t\to\infty} \dot{V}(t) = 0$ and so $\lim_{t\to\infty} \boldsymbol{e}(t) \to \boldsymbol{0}_6$ and $\lim_{t\to\infty} \boldsymbol{e}_v(t) \to \boldsymbol{0}_6$. This implies that $\lim_{t\to\infty} \boldsymbol{e}_\epsilon(t) \to \boldsymbol{0}_3$, which, given that $\boldsymbol{e}_\eta$ is a unit quaternion and $e_\phi(t) \neq 0$, $\forall t \geq 0$, ensures asymptotic stability of the pose error as $\lim_{t\to\infty}(\boldsymbol{e}_p(t), \boldsymbol{e}_\eta(t)) = (\boldsymbol{0}_3, [\text{sgn}(e_\phi(0)), \boldsymbol{0}_3^T]^T)$.

□

Remark 2. *Note that the bound ε_h of $\boldsymbol{h}$ in (20) can be computed a priori. In practice, the terms ν, ν_o can be known a priori up to a certain accuracy, leading thus to respective bounds. Hence, one can compute upper bounds for $V(0)$ and hence for $\boldsymbol{e}$, $\boldsymbol{e}_v$, $\boldsymbol{e}_\nu$, and $\boldsymbol{e}_{\nu_o}$. Since the structure of the dynamic terms is known, this can also lead to a bound of the terms B^{-1}, M^{-1}, M_o, $Y(\cdot)$, $Y_o(\cdot)$, Y_r, and Y_{o_r} that appear in $\boldsymbol{h}$. Hence, tuning of ε to overcome ε_h can be done off-line.*

IV. SIMULATION RESULTS

We implement here the proposed control scheme on three 6 DOF mobile manipulators consisting of a 3 DOF, 3 kg base (X-Y translation, rotation about Z) and a 3DOF manipulator with 3 identical links of length 0.3 m and mass of 0.5 kg each. The objective is to transport a 2 kg box along the desired reference trajectory defined by $\boldsymbol{p}_\text{d}(t) := [0.1\sin(0.125t), 0.1\sin(0.125t), 0.1\sin(0.125t)]^T$, $\boldsymbol{\eta}_\text{d}(t) := [\cos(0.1\sin(.125t)), 0, 0, \sin(0.1\sin(0.125t))]^T$. The control gains used are: $k_p = 1$, $k_\eta = .5$, $K_v = \text{diag}[5, 5, 5, 2, 2, 2]$, $\varepsilon = 0.1$, $\Gamma_o = 0.5I_{lo\times lo}$, $\Gamma = 0.5I_{l\times l}$. We consider 30% error in all uncertain parameter (including the object center of mass), and the projection operator enforces the following bounds on the uncertain terms: $\bar{\tilde{\nu}} = 2.25$, $\bar{\tilde{\nu}}_o = 1.5$. The simulation results are depicted in Figs. 2-4 for 50 seconds. More specifically, Figs. 2 shows the resulting error trajectories of the object-agent system, which satisfy $\lim_{t\to\infty} \boldsymbol{e}_p(t) = \boldsymbol{0}_3$, $\lim_{t\to\infty} \boldsymbol{e}_\varepsilon(t) = \boldsymbol{0}_3$, and $\lim_{t\to\infty} \boldsymbol{e}_\phi(t) = \text{sgn}(\boldsymbol{e}_\phi(0)) = 1$ in the presence of rolling effects. Fig. 3 illustrates the boundedness of the uncertain parameters, $\hat{\boldsymbol{\nu}}, \hat{\boldsymbol{\nu}}_o$ that is enforced by the proposed control scheme. Fig. 4 shows the required friction, $\mu_{ri} := \frac{\sqrt{f_{x_i}^2 + f_{y_i}^2}}{f_{n_i}}$, which denotes the minimum friction coefficient necessary to prevent slip throughout the motion [16], as well as the control inputs. If the required

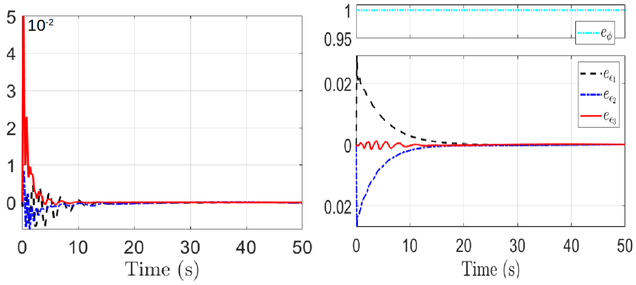

Fig. 2: The evolution $\boldsymbol{e}_p(t)$, $e_\phi(t)$, $\boldsymbol{e}_\epsilon(t)$, $\forall t \in [0, 50]$.

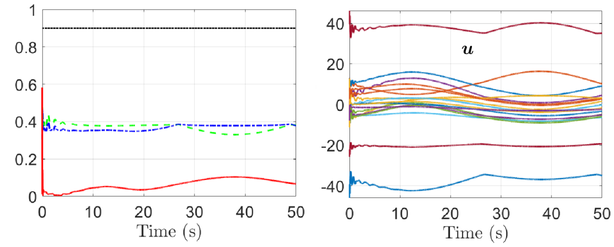

Fig. 4: The required friction to prevent slip for the three agents (left) with the black dashed line representing $\mu = 0.9$, and the resulting inputs $\boldsymbol{u}(t)$ of the agents (right), $\forall t \in [0, 50]$.

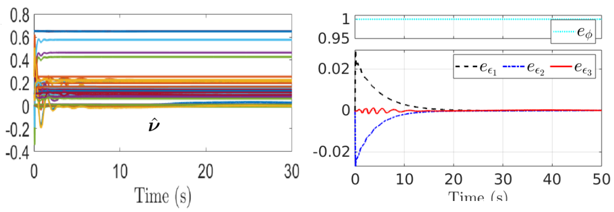

Fig. 3: The evolution of $\hat{\boldsymbol{\nu}}(t)$ and $\hat{\boldsymbol{\nu}}_o(t)$, $\forall t \in [0, 50]$.

friction surpasses the true coefficient, then the contact point will slip and the grasp is compromised. As shown in Fig. 4, however, the required friction for each contact is below the true coefficient of $\mu = 0.9$, and hence slip is prevented as guaranteed by the proposed method.

V. CONCLUSION AND FUTURE WORK

This paper presented a control scheme for cooperative manipulation with rolling contacts, robust to center of mass location and dynamic uncertainties. A novel internal force scheme was introduced for contact slip prevention. Future directions will address grasp reconfiguration as well as decentralization of the control protocol.

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