Decentralized Robust Control of Coupled Multi-Agent Systems under Local Signal Temporal Logic Tasks

Lars Lindemann and Dimos V. Dimarogonas

***Abstract*— Motivated by the recent interest in formal methods-based control of multi-agent systems, we adopt a bottom-up approach. Each agent is subject to a local signal temporal logic task that may depend on other agents' behavior. These dependencies pose control challenges since some of the tasks may be opposed to each other. We first develop a local continuous feedback control law and identify conditions under which this control law guarantees satisfaction of the local tasks. If these conditions do not hold, we propose to use the developed control law in combination with an online detection & repair scheme, expressed as a local hybrid system. After detection of a critical event, a three-stage procedure is initiated to resolve the problem. The theoretical results are illustrated in simulations.**

I. Introduction

Multi-agent systems under objectives such as consensus, formation control, and connectivity maintenance have been well studied [1]. The need for more complex objectives in robotic applications has led to formal methods-based control strategies where temporal logics are used to formulate high-level temporal tasks. Top-down approaches have been considered, as for instance in [2], by decomposing a global task into local ones. Top-down approaches are usually subject to high computational complexity. On the other hand, the works in [3], [4] favor a bottom-up approach, where local tasks are independently distributed to each agent. Here, however, feasibility of each local task does not imply feasibility of the conjunction of all local tasks. The works in [2]-[4] rely on automata-based verification techniques that discretize the physical environment and agent dynamics. We instead consider continuous-time dynamics without the need for discretizing neither environment nor agent dynamics in space or time. This paper extends our work in [5] to multi-agent systems and is, to the best of our knowledge, the first approach not making use of such discretization in this field.

We adopt to a bottom-up approach by considering local tasks formulated in signal temporal logic [6]. These tasks can depend on each other, i.e., also oppose each other. This makes the control of multi-agent systems under signal temporal logic tasks a challenge and the main research question in this paper. Signal temporal logic introduces the notion of space robustness [7], a robustness metric stating how robustly a signal satisfies a given task. In a first step, we identify conditions under which a continuous feedback control law, which is derived by combining space robustness and prescribed performance control [8], satisfies basic signal temporal logic tasks. If these conditions do not hold, an online detection & repair scheme is introduced by defining a local hybrid system [9] for each agent. Critical events will be detected and resolved in a three-stage procedure, gradually relaxing parameters such as robustness. One advantage of our decentralized approach is the low computational complexity due to the continuous feedback control laws. Furthermore, the team of agents is allowed to be heterogeneous with additional dynamic couplings among them. Robustness is considered with respect to disturbances and with respect to the signal temporal logic task. Multi-agent systems under signal temporal logic tasks have also been considered in [10] in a centralized approach, not investigating task dependencies, but with a special focus on communication.

Section II introduces preliminaries, while Section III states the problem definition. Section IV presents our solution to the stated problem, which is verified by simulations in Section V. Conclusions are given in Section VI.

II. Preliminaries

Scalars and column vectors are denoted by non-bold letters x and bold letters $\boldsymbol{x}$, respectively, while true and false are denoted by $\top$ and $\bot$; $\mathbb{R}$ are the real numbers, while $\mathbb{R}^n$ is the n-dimensional real vector space. The natural, non-negative, and positive real numbers are $\mathbb{N}$, $\mathbb{R}_{\geq 0}$, and $\mathbb{R}_{>0}$, respectively. For convenience, we abbreviate $\begin{bmatrix}\boldsymbol{x} & \boldsymbol{y}\end{bmatrix} := \begin{bmatrix}\boldsymbol{x}^T & \boldsymbol{y}^T\end{bmatrix}^T$.

A. Signal Temporal Logic (STL)

Signal temporal logic (STL) is a predicate logic and consists of predicates μ that are obtained after the evaluation of a predicate function $h : \mathbb{R}^n \to \mathbb{R}$ as $\mu := \begin{cases} \top \text{ if } h(\boldsymbol{x}) \geq 0 \\ \bot \text{ if } h(\boldsymbol{x}) < 0. \end{cases}$

With μ being a predicate, the STL syntax is given by

$$\phi \; ::= \; \top \mid \mu \mid \neg\phi \mid \phi \wedge \psi \mid \phi \, U_{[a,b]} \, \psi \; ,$$

where ϕ and ψ are STL formulas. The satisfaction relation $(\boldsymbol{x}, t) \models \phi$ indicates if the signal $\boldsymbol{x} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ satisfies ϕ at time t. Then, the STL semantics are inductively defined as $(\boldsymbol{x}, t) \models \mu$ if and only if $h(\boldsymbol{x}(t)) \geq 0$, $(\boldsymbol{x}, t) \models \neg\mu$ if and only if $\neg((\boldsymbol{x}, t) \models \mu)$, $(\boldsymbol{x}, t) \models \phi \wedge \psi$ if and only if $(\boldsymbol{x}, t) \models \phi \wedge (\boldsymbol{x}, t) \models \psi$, $(\boldsymbol{x}, t) \models \phi \, U_{[a,b]} \, \psi$ if and only if $\exists t_1 \in [t+a, t+b]$ s.t. $(\boldsymbol{x}, t_1) \models \psi \wedge \forall t_2 \in [t, t_1]$, $(\boldsymbol{x}, t_2) \models \phi$. Disjunction, eventually, and always-operator are derived as $\phi \vee \psi := \neg(\neg\phi \wedge \neg\psi)$, $F_{[a,b]}\phi := \top \, U_{[a,b]} \, \phi$, and $G_{[a,b]}\phi := \neg F_{[a,b]}\neg\phi$, respectively. Robust semantics, which are called

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The authors are with the Department of Automatic Control, School of Electrical Engineering, KTH Royal Institute of Technology, 100 44 Stockholm, Sweden. llindem@kth.se (L. Lindemann), dimos@kth.se (D.V. Dimarogonas)

space robustness $\rho^{\phi}(\boldsymbol{x},t)$ [7], are defined in Definition 1; $\rho^{\phi}(\boldsymbol{x},t)$ determines how robustly the signal $\boldsymbol{x}:\mathbb{R}_{\geq 0}\to\mathbb{R}^n$ satisfies ϕ at time t. It holds that $(\boldsymbol{x},t)\models\phi$ if $\rho^{\phi}(\boldsymbol{x},t)>0$.

Definition 1 (Space Robustness): The semantics of space robustness are inductively defined as [7, Definition 3]:

$$\begin{aligned}\rho^{\mu}(\boldsymbol{x},t) &:= h(\boldsymbol{x}(t))\\ \rho^{\neg\phi}(\boldsymbol{x},t) &:= -\rho^{\phi}(\boldsymbol{x},t)\\ \rho^{\phi\wedge\psi}(\boldsymbol{x},t) &:= \min\big(\rho^{\phi}(\boldsymbol{x},t),\rho^{\psi}(\boldsymbol{x},t)\big)\\ \rho^{F_{[a,b]}\phi}(\boldsymbol{x},t) &:= \max_{t_1\in[t+a,t+b]}\rho^{\phi}(\boldsymbol{x},t_1)\\ \rho^{G_{[a,b]}\phi}(\boldsymbol{x},t) &:= \min_{t_1\in[t+a,t+b]}\rho^{\phi}(\boldsymbol{x},t_1)\end{aligned}$$

We abuse the notation as $\rho^{\phi}(\boldsymbol{x}(t)) := \rho^{\phi}(\boldsymbol{x},t)$ if t is not explicitly contained in $\rho^{\phi}(\boldsymbol{x},t)$. For instance, $\rho^{\mu}(\boldsymbol{x}(t)) := \rho^{\mu}(\boldsymbol{x},t) := h(\boldsymbol{x}(t))$ since t is not an explicit parameter.

B. A Bottom-up Approach for Multi-Agent Systems

Consider a multi-agent system with M agents modeled by an undirected graph $\mathcal{G} := (\mathcal{V},\mathcal{E})$ [1]. The vertex and edge sets are $\mathcal{V} := \{v_1,\ldots,v_M\}$ and $\mathcal{E}\in\mathcal{V}\times\mathcal{V}$, respectively. Two agents $v_i, v_j\in\mathcal{V}$ can communicate if and only if there exists a path between v_i and v_j. A path is a sequence $v_i, v_{k_1},\ldots,v_{k_P},v_j$ such that $(v_i,v_{k_1}),\ldots,(v_{k_P},v_j)\in\mathcal{E}$.

Let $\boldsymbol{x}_i\in\mathbb{R}^n$, $\boldsymbol{u}_i\in\mathbb{R}^{m_i}$, and $\boldsymbol{w}_i\in\mathcal{W}_i$ be the state, input, and additive noise of agent v_i's dynamics with $\mathcal{W}_i\subset\mathbb{R}^n$ being a bounded set. Let $\boldsymbol{x} := \begin{bmatrix}\boldsymbol{x}_1 & \boldsymbol{x}_2 & \ldots & \boldsymbol{x}_M\end{bmatrix}$ be the stacked vector of all agents' states. Each agent v_i obeys

$$\dot{\boldsymbol{x}}_i = f_i(\boldsymbol{x}_i) + f_i^{\mathrm{c}}(\boldsymbol{x}) + g_i(\boldsymbol{x}_i)\boldsymbol{u}_i + \boldsymbol{w}_i, \qquad (1)$$

where $f_i^{\mathrm{c}}(\boldsymbol{x})$ describes preassumed dynamic couplings between agents. Also define $\boldsymbol{x}_i^{\mathrm{ext}} := \begin{bmatrix}\boldsymbol{x}_{j_1} & \ldots & \boldsymbol{x}_{j_{M-1}}\end{bmatrix}$ with $v_{j_1},\ldots,v_{j_{M-1}}\in\mathcal{V}\setminus\{v_i\}$, i.e., $\boldsymbol{x}_i^{\mathrm{ext}}$ is a stacked vector containing the states of all agents except of $\boldsymbol{x}_i$.

Assumption 1: The functions $f_i:\mathbb{R}^n\to\mathbb{R}^n$, $f_i^{\mathrm{c}}:\mathbb{R}^{nM}\to\mathbb{R}^n$, and $g_i:\mathbb{R}^n\to\mathbb{R}^{n\times m_i}$ are locally Lipschitz continuous, and $g_i(\boldsymbol{x}_i)g_i({\boldsymbol{x}_i}^T)$ is positive definite for all $\boldsymbol{x}_i\in\mathbb{R}^n$.

We now tailor the STL semantics to a multi-agent bottom-up approach. Each agent $v_i\in\mathcal{V}$ is subject to a local STL formula that is endowed with the subscript i, i.e., ϕ_i. Based on [4, Definition 3], local satisfaction of ϕ_i by the signal $\boldsymbol{x}_{\phi_i}:\mathbb{R}_{\geq 0}\to\mathbb{R}^{p_i}$ is defined in Definition 2. We will be more specific regarding $\boldsymbol{x}_{\phi_i}$ and p_i after Definition 4.

Definition 2 (Local Satisfaction): The signal $\boldsymbol{x}_{\phi_i}:\mathbb{R}_{\geq 0}\to\mathbb{R}^{p_i}$ locally satisfies ϕ_i if and only if $(\boldsymbol{x}_{\phi_i},0)\models\phi_i$.

Definition 3 (Local Feasibility): The formula ϕ_i is locally feasible if and only if $\exists\boldsymbol{x}_{\phi_i}:\mathbb{R}_{\geq 0}\to\mathbb{R}^{p_i}$ such that $\boldsymbol{x}_{\phi_i}$ locally satisfies ϕ_i.

Each local formula ϕ_i depends on agent v_i and may also depend on some other agents $v_j\in\mathcal{V}$. Consider $\boldsymbol{x}_j:\mathbb{R}_{\geq 0}\to\mathbb{R}^n$ to be the solution to (1) associated with agent v_j.

Definition 4 (Formula-Agent Dependency): If $\boldsymbol{x}_j(t)$ is not contained in $\boldsymbol{x}_{\phi_i}(t)$ for all $t\in\mathbb{R}_{\geq 0}$ and local satisfaction of ϕ_i, i.e., $(\boldsymbol{x}_{\phi_i},0)\models\phi_i$, can be evaluated, then ϕ_i does not depend on v_j. Otherwise, i.e., knowledge of $\boldsymbol{x}_j(t)$ is needed and hence $\boldsymbol{x}_j(t)$ is contained in $\boldsymbol{x}_{\phi_i}(t)$, then ϕ_i does depend on v_j and we say that agent v_j is participating in ϕ_i.

The set of participating agents in ϕ_i is $\mathcal{V}_{\phi_i} := \{v_{j_1},v_{j_2},\ldots,v_{j_{P(\phi_i)}}\}\subseteq\mathcal{V}$, where $P(\phi_i) := \sum_{j=1}^{|\mathcal{V}|}\chi_j(\phi_i)$ is a function indicating the total number of participating agents in ϕ_i with $\chi_j(\phi_i) := \begin{cases}1 & \text{if } \phi_i \text{ depends on } v_j\\ 0 & \text{otherwise.}\end{cases}$ Define $\boldsymbol{x}_{\phi_i}(t) := \begin{bmatrix}\boldsymbol{x}_{j_1}(t) & \ldots & \boldsymbol{x}_{j_{P(\phi_i)}}(t)\end{bmatrix}$ for all $t\in\mathbb{R}_{\geq 0}$ with $v_{j_1},\ldots,v_{j_{P(\phi_i)}}\in\mathcal{V}_{\phi_i}$, i.e., all agents participating in ϕ_i. It holds that $p_i := nP(\phi_i)$ so that $\boldsymbol{x}_{\phi_i}$ is completely defined.

We call ϕ_i a non-collaborative formula if and only if $P(\phi_i)=1$, i.e., ϕ_i does not depend on $v_j\in\mathcal{V}\setminus\{v_i\}$. Otherwise, i.e., $P(\phi_i)>1$, we call ϕ_i a collaborative formula. Global satisfaction of the set of formulas $\{\phi_1,\ldots,\phi_M\}$ by $\boldsymbol{x}:\mathbb{R}_{\geq 0}\to\mathbb{R}^{nM}$ is introduced in Definition 5.

Definition 5 (Global Satisfaction): The signal $\boldsymbol{x}:\mathbb{R}_{\geq 0}\to\mathbb{R}^{nM}$ globally satisfies $\{\phi_1,\ldots,\phi_M\}$ if and only if $\boldsymbol{x}_{\phi_i}$ locally satisfies ϕ_i for all agents $v_i\in\mathcal{V}$.

Definition 6 (Global Feasibility): The set of formulas $\{\phi_1,\ldots,\phi_M\}$ is globally feasible if and only if $\exists\boldsymbol{x}:\mathbb{R}_{\geq 0}\to\mathbb{R}^{nM}$ such that $\boldsymbol{x}$ globally satisfies $\{\phi_1,\ldots,\phi_M\}$.

Next, maximal dependency clusters are defined.

Definition 7 (Maximal Dependency Cluster): Consider the undirected dependency graph $\mathcal{G}_d := (\mathcal{V},\mathcal{E}_d)$ with $(v_i,v_j)\in\mathcal{E}_d\subseteq\mathcal{V}\times\mathcal{V}$ if and only if the formula ϕ_i depends on v_j; $\Xi\subseteq\mathcal{V}$ is a maximal dependency cluster if and only if $\forall v_i,v_j\in\Xi$ there is a path from v_i to v_j in $\mathcal{G}_d$ and $\nexists v_k\in\mathcal{V}\setminus\Xi$ such that there is a path from v_i to v_k in $\mathcal{G}_d$.

A multi-agent system under $\{\phi_1,\ldots,\phi_M\}$ induces $L\leq M$ maximal dependency clusters denoted by $\bar{\Xi} := \{\Xi_1,\ldots,\Xi_L\}$. These clusters are maximal, i.e., there are no formula-agent dependencies between different clusters.

Example 1: Consider the local tasks $\phi_1 := F_{[a_1,b_1]}(\|\boldsymbol{x}_1-\boldsymbol{x}_2\|\leq 1)$, $\phi_2 := F_{[a_2,b_2]}(\|\boldsymbol{x}_2\|\leq 1)$, and $\phi_3 := F_{[a_3,b_3]}(\|\boldsymbol{x}_3\|\leq 1)$. Then $\Xi_1=\{v_1,v_2\}$ and $\Xi_2=\{v_3\}$.

C. Hybrid Systems

Hybrid systems with external inputs have been modeled in [9] by considering hybrid inclusions as in Definition 8. The value of the state $\boldsymbol{z}_i$ after a jump is denoted by $\hat{\boldsymbol{z}}_i$. For a detailed review, the reader is referred to [9].

Definition 8: [9] A hybrid system is a tuple $\mathcal{H}_i := (C_i,F_i,D_i,G_i)$ where C_i, D_i, F_i, and G_i are the flow and jump set and the possibly set-valued flow and jump map, respectively. The continuous and discrete dynamics are

$$\begin{cases}\dot{\boldsymbol{z}}_i\in F_i(\boldsymbol{z}_i,\boldsymbol{u}_i^{\mathrm{int}},\boldsymbol{u}_i^{\mathrm{ext}}) & \text{for } (\boldsymbol{z}_i,\boldsymbol{u}_i^{\mathrm{int}},\boldsymbol{u}_i^{\mathrm{ext}})\in C_i\\ \hat{\boldsymbol{z}}_i\in G_i(\boldsymbol{z}_i,\boldsymbol{u}_i^{\mathrm{int}},\boldsymbol{u}_i^{\mathrm{ext}}) & \text{for } (\boldsymbol{z}_i,\boldsymbol{u}_i^{\mathrm{int}},\boldsymbol{u}_i^{\mathrm{ext}})\in D_i,\end{cases} \qquad (2)$$

where $\boldsymbol{z}_i\in\mathcal{Z}_i$ is a hybrid state with domain $\mathcal{Z}_i$, while $\boldsymbol{u}_i^{\mathrm{int}}\in\mathcal{U}_i^{\mathrm{int}}$ and $\boldsymbol{u}_i^{\mathrm{ext}}\in\mathcal{U}_i^{\mathrm{ext}}$. Furthermore, let $\mathfrak{H}_i := \mathcal{Z}_i\times\mathcal{U}_i^{\mathrm{int}}\times\mathcal{U}_i^{\mathrm{ext}}$.

III. Problem Statement

In this paper, the following STL fragment is considered:

$$\psi \;::=\; \top \mid \mu \mid \neg\mu \mid \psi_{(1)}\wedge\psi_{(2)} \qquad (3a)$$

$$\phi \;::=\; G_{[a,b]}\psi \mid F_{[a,b]}\psi \qquad (3b)$$

$$\theta^{\mathrm{s}_1} \;::=\; \bigwedge_{k=1}^{K}\phi_{(k)} \text{ with } b_{(k)}\leq a_{(k+1)} \qquad (3c)$$

$$\theta^{s_2} ::= F_{[c_{(1)},d_{(1)}]}\big(\psi_{(1)} \wedge F_{[c_{(2)},d_{(2)}]}(\psi_{(2)} \wedge \ldots)\big) \quad (3d)$$
$$\theta ::= \theta^{s_1} \mid \theta^{s_2}, \quad (3e)$$

where μ is a predicate and $\psi_{(1)}$, $\psi_{(2)}, \ldots$ are formulas of class ψ given in (3a), whereas $\phi_{(k)}$ with $k \in \{1, \ldots, K\}$ are formulas of class ϕ given in (3b) with corresponding time intervals $[a_{(k)}, b_{(k)}]$. Note the use of brackets, e.g. $\psi_{(1)}$, to distinguish from local formulas, e.g., ψ_1. For conjunctions of non-temporal formulas of class ψ, the robust semantics are approximated by a smooth function that preserves the property $(\boldsymbol{x}, 0) \models \psi$ if $\rho^{\psi}(\boldsymbol{x}) > 0$ as in [5].

Assumption 2: The robust semantics for a conjunction of q *non-temporal formulas* of class ψ given in (3a), i.e., $\rho^{\psi_{(1)}\wedge\ldots\wedge\psi_{(q)}}(\boldsymbol{x})$, are approximated by a smooth function as

$$\rho^{\psi_{(1)}\wedge\ldots\wedge\psi_{(q)}}(\boldsymbol{x}) \approx -\ln\Big(\sum_{i=1}^{q} \exp\big(-\rho^{\psi_{(i)}}(\boldsymbol{x})\big)\Big).$$

The objective in this paper is to consider local formulas of class ϕ given in (3b) that are independently distributed to each agent $v_i \in \mathcal{V}$. The proposed solution can then be extended to local formulas of class θ given in (3e) as instructed in [5]. Each agent $v_i \in \mathcal{V}$ is hence subject to a local formula ϕ_i of the form (3b). Let ψ_i correspond to ϕ_i.

Assumption 3: Each formula of class ψ given in (3a) that is contained in (3b) and associated with an agent v_i is: 1) s.t. $\rho^{\psi_i}(\boldsymbol{x}_{\phi_i})$ is concave and 2) well-posed in the sense that $(\boldsymbol{x}_{\phi_i}, 0) \models \psi_i$ implies $\|\boldsymbol{x}_{\phi_i}(0)\| \leq C < \infty$ for some $C \geq 0$.

Next, define the global optimum of $\rho^{\psi_i}(\boldsymbol{x}_{\phi_i})$ as $\rho_i^{\text{opt}} := \sup_{\boldsymbol{x}_{\phi_i} \in \mathbb{R}^{nP(\phi_i)}} \rho^{\psi_i}(\boldsymbol{x}_{\phi_i})$, which is straightforward to compute due to Assumption 2 and 3. Assumption 4 now guarantees that ϕ_i is locally feasible since $\rho_i^{\text{opt}} > 0$ implies that $\rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) > 0$ is possible.

Assumption 4: The optimum of $\rho^{\psi_i}(\boldsymbol{x}_{\phi_i})$ is s.t. $\rho_i^{\text{opt}} > 0$.

The goal is to derive a local control law $\boldsymbol{u}_i(\boldsymbol{x}_{\phi_i}, t)$ for each agent v_i such that $r_i \leq \rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) \leq \rho_i^{\max}$, where $r_i \in \mathbb{R}$ is a robustness measure, while $\rho_i^{\max} \in \mathbb{R}$ with $r_i < \rho_i^{\max}$ is a robustness delimiter. We look at each dependency cluster separately and then distinguish between two cases.

Problem 1: Assume that each agent v_i is subject to a local STL formula ϕ_i of the form (3b), hence inducing the maximal dependency clusters $\bar{\Xi} := \{\Xi_1, \ldots, \Xi_L\}$ with $L \leq M$. For each cluster Ξ_l with $l \in \{1, \ldots, L\}$, derive a control strategy as follows. *Case A)* Under the assumption that each agent $v_i, v_j \in \Xi_l$ is subject to the same formula, i.e., $\phi_i = \phi_j$, design a local feedback control law $\boldsymbol{u}_i(\boldsymbol{x}_{\phi_i}, t)$ such that $0 < r_i \leq \rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) \leq \rho_i^{\max}$ for all $v_i \in \Xi_l$, which means local satisfaction of ϕ_i. *Case B)* Otherwise, i.e., $\exists v_i, v_j \in \Xi_l$ such that $\phi_i \neq \phi_j$, each agent $v_i \in \Xi_l$ nevertheless initially applies the derived control law $\boldsymbol{u}_i(\boldsymbol{x}_{\phi_i}, t)$ for Case A. Design a local online detection & repair scheme for each agent $v_i \in \Xi_l$ such that $r_i \leq \rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) \leq \rho_i^{\max}$, where $r_i \in \mathbb{R}$, possibly negative, is maximized up to a precision of $\delta_i > 0$ with δ_i being a design parameter.

IV. Proposed Problem Solution

In the proposed solution to Case A in Problem 1 in Section IV-A, neither formula-agent dependencies nor dynamic couplings $f_i^c(\boldsymbol{x})$ pose difficulties. For Case B, both types of dependencies may lead to trajectories that do not satisfy the formulas. The proposed solution, introducing an online detection & repair scheme, is given in Section IV-B. We now first present the main idea of our work on single-agent systems [5], which is based on prescribed performance control [8] and now extended to multi-agent systems. For a thorough illustration, the reader is referred to [5]. Define the performance function γ_i for agent v_i in Definition 9 and the transformation function S in Definition 10.

Definition 9: The performance function $\gamma_i : \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$ is continuously differentiable, bounded, positive, non-increasing, and given by $\gamma_i(t) := (\gamma_i^0 - \gamma_i^\infty)\exp(-l_i t) + \gamma_i^\infty$ where $\gamma_i^0, \gamma_i^\infty \in \mathbb{R}_{>0}$ with $\gamma_i^0 \geq \gamma_i^\infty$ and $l_i \in \mathbb{R}_{\geq 0}$.

Definition 10: A transformation function $S : (-1, 0) \to \mathbb{R}$ is a strictly increasing function, hence injective and admitting an inverse. In particular, let $S(\xi) := \ln\Big(-\frac{\xi+1}{\xi}\Big)$.

Our objective, i.e., $r_i \leq \rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) \leq \rho_i^{\max}$, is achieved by prescribing a temporal behavior to $\rho^{\psi_i}(\boldsymbol{x}_{\phi_i}(t))$ through the design parameters γ_i and $\rho_i^{\max}$ as

$$-\gamma_i(t) + \rho_i^{\max} < \rho^{\psi_i}(\boldsymbol{x}_{\phi_i}(t)) < \rho_i^{\max}. \quad (4)$$

Note the use of $\rho^{\psi_i}(\boldsymbol{x}_{\phi_i}(t))$ and not $\rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0)$. When $\boldsymbol{x}_{\phi_i}$ is seen as a state, define $e_i(\boldsymbol{x}_{\phi_i}) := \rho^{\psi_i}(\boldsymbol{x}_{\phi_i}) - \rho_i^{\max}$,

$$\xi_i(\boldsymbol{x}_{\phi_i}, t) := \frac{e_i(\boldsymbol{x}_{\phi_i})}{\gamma_i(t)},$$

and $\epsilon_i(\boldsymbol{x}_{\phi_i}, t) := S\big(\xi_i(\boldsymbol{x}_{\phi_i}, t)\big)$. Now, (4) can be written as $-\infty < \epsilon_i(t) < \infty$. If $\epsilon_i(t)$ is bounded for all $t \geq 0$, then inequality (4) holds for all $t \geq 0$. The connection between $\rho^{\psi_i}(\boldsymbol{x}_{\phi_i}(t))$ in (4) and $\rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0)$ is made by γ_i, which needs to be chosen as explained in detail in [5] to obtain $0 < r_i \leq \rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) \leq \rho_i^{\max}$. Therefore, select the parameters

$$t_i^* \in \begin{cases} a_i & \text{if } \phi_i = G_{[a_i,b_i]}\psi_i \\ [a_i, b_i] & \text{if } \phi_i = F_{[a_i,b_i]}\psi_i, \end{cases} \quad (5)$$
$$\rho_i^{\max} \in \big(\max\big(0, \rho^{\psi_i}(\boldsymbol{x}_{\phi_i}(0))\big), \rho_i^{\text{opt}}\big) \quad (6)$$
$$r_i \in (0, \rho_i^{\max}) \quad (7)$$
$$\gamma_i^0 \in \begin{cases} (\rho_i^{\max} - \rho^{\psi_i}(\boldsymbol{x}_{\phi_i}(0)), \infty) & \text{if } t_i^* > 0 \\ (\rho_i^{\max} - \rho^{\psi_i}(\boldsymbol{x}_{\phi_i}(0)), \rho_i^{\max} - r_i] & \text{if } t_i^* = 0 \end{cases} \quad (8)$$
$$\gamma_i^\infty \in \Big(0, \min\big(\gamma_i^0, \rho_i^{\max} - r_i\big)\Big] \quad (9)$$
$$l_i \in \begin{cases} \mathbb{R}_{\geq 0} & \text{if } -\gamma_i^0 + \rho_i^{\max} \geq r_i \\ \frac{-\ln\left(\frac{r_i + \gamma_i^\infty - \rho_i^{\max}}{-(\gamma_i^0 - \gamma_i^\infty)}\right)}{t_i^*} & \text{if } -\gamma_i^0 + \rho_i^{\max} < r_i \end{cases} \quad (10)$$

where it has to hold that $\rho^{\psi_i}(\boldsymbol{x}_{\phi_i}(0)) > r_i$ if $t_i^* = 0$.

A. Global and Local Satisfaction Guarantees

Theorem 1 provides a global satisfaction guarantee if all clusters in $\bar{\Xi}$ satisfy the assumption of Case A in Problem 1.

Theorem 1: Let each agent $v_i \in \mathcal{V}$ be subject to ϕ_i as in (3b), hence inducing the maximal dependency clusters $\bar{\Xi} := \{\Xi_1, \ldots, \Xi_L\}$. Assume that for each $\Xi_l \in \bar{\Xi}$ it holds that: for all $v_i, v_j \in \Xi_l$ we have 1) v_i and v_j can communicate,

2) $\phi_i = \phi_j$, and 3) $t_i^* = t_j^*$, $\rho_i^{\max} = \rho_j^{\max}$, $r_i = r_j$, and $\gamma_i = \gamma_j$ are chosen as in (5)-(10). If for each agent $v_i \in \mathcal{V}$ Assumptions 1-4 hold and each agent v_i applies

$$\boldsymbol{u}_i(\boldsymbol{x}_{\phi_i}, t) := -\epsilon_i(\boldsymbol{x}_{\phi_i}, t) g_i(\boldsymbol{x}_i)^T \frac{\partial \rho^{\psi_i}(\boldsymbol{x}_{\phi_i})}{\partial \boldsymbol{x}_i}, \tag{11}$$

then it holds that $0 < r_i \leq \rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) \leq \rho_i^{\max}$ for all agents $v_i \in \mathcal{V}$, i.e., each agent v_i locally satisfies ϕ_i, which in turn guarantees global satisfaction of $\{\phi_1, \ldots, \phi_M\}$. All closed-loop signals are well-posed, i.e., continuous and bounded.

Proof: The proof can be found in [11]. ■

If $L = M$, i.e., each agent $v_i \in \mathcal{V}$ is subject to a non-collaborative formula ϕ_i, Theorem 1 trivially applies. For the next result, a stronger assumption on $f_i^{\mathrm{c}}(\boldsymbol{x})$ is needed.

Assumption 5: The function $f_i^{\mathrm{c}} : \mathbb{R}^{nM} \to \mathbb{R}^n$ is bounded.

Now consider a formula ϕ of the form (3b) and assume that each $v_i \in \mathcal{V}_\phi$ is subject to $\phi_i := \phi$. Then Theorem 2 guarantees satisfaction of ϕ if all agents $v_i \in \mathcal{V}_\phi$ collaborate.

Theorem 2: Let each agent $v_i \in \mathcal{V}$ satisfy Assumption 1 and 5. Consider a formula ϕ as in (3b) and let each agent $v_i \in \mathcal{V}_\phi$ be subject to $\phi_i := \phi$. Assume that for all $v_i, v_j \in \mathcal{V}_\phi$ it holds that: 1) v_i and v_j can communicate and 2) $t_i^* = t_j^*$, $\rho_i^{\max} = \rho_j^{\max}$, $r_i = r_j$, and $\gamma_i = \gamma_j$ are chosen as in (5)-(10). Assume further that all agents $v_k \in \mathcal{V} \setminus \mathcal{V}_\phi$ apply a control law $\boldsymbol{u}_k'$ such that $\boldsymbol{x}_k$ remains in a compact set Ω_k'. If for each agent $v_i \in \mathcal{V}_\phi$ Assumptions 2-4 hold and each $v_i \in \mathcal{V}_\phi$ applies (11), then $0 < r := r_i \leq \rho^\phi(\boldsymbol{x}_\phi, 0) \leq \rho_i^{\max} =: \rho^{\max}$, i.e., $(\boldsymbol{x}_\phi, 0) \models \phi$. All closed-loop signals are well-posed.

Proof: The proof can be found in [11]. ■

The assumption of $\boldsymbol{u}_k'$ is not restrictive and excludes finite escape time. For instance, if $\dot{\boldsymbol{x}}_k := f_k(\boldsymbol{x}_k)$ is asymptotically stable, then $\boldsymbol{u}_k'(\boldsymbol{x}_k) := -g_k(\boldsymbol{x}_k)^T \boldsymbol{x}_k$ keeps the state $\boldsymbol{x}_k$ in a compact set. If all agents $v_i \in \mathcal{V}_\phi$ apply the control law (11) under the conditions in Theorem 2 to satisfy ϕ, we refer to this as *collaborative control* in the remainder. Theorem 2 has further implications with respect to Case A in Problem 1. Consider again the induced maximal dependency clusters $\bar{\Xi} := \{\Xi_1, \ldots, \Xi_L\}$. Assume that the cluster Ξ_l with $l \in \{1, \ldots, L\}$ satisfies the assumption of Case A, while there exists another cluster Ξ_m with $m \neq l$ such that Ξ_m does not satisfy this assumption. In other words, for all $v_i, v_j \in \Xi_l$ it holds that $\phi_i = \phi_j$, while $\exists v_i, v_j \in \Xi_m$ with $m \neq l$ such that $\phi_i \neq \phi_j$. Consequently, Theorem 2 guarantees local satisfaction of ϕ_i for all $v_i \in \Xi_l$. Assumption 4 in Theorem 2 restricts the formula $\phi_i := \phi$ to be locally feasible. This assumption can be relaxed at the expense of not locally satisfying ϕ_i by relaxing r_i and $\rho_i^{\max}$.

Corollary 1: Assume that all assumptions of Theorem 2 hold for each agent $v_i \in \mathcal{V}_\phi$ except for Assumption 4 and the choice of $\rho_i^{\max}$ and r_i. If $\rho_i^{\max} \in (\rho^{\psi_i}(\boldsymbol{x}_{\phi_i}(0)), \rho_i^{\mathrm{opt}})$ and $r_i \in (-\infty, \rho_i^{\max})$, then $r := r_i \leq \rho^\phi(\boldsymbol{x}_\phi, 0) \leq \rho_i^{\max} =: \rho_{\max}$.

Proof: The proof can be found in [11]. ■

B. An Online Detection & Repair Scheme

Assume now that the cluster Ξ_l with $l \in \{1, \ldots, L\}$ may not satisfy the assumption of Case A in Problem 1. We propose that each agent $v_i \in \Xi_l$ initially applies the control law (11) with parameters as in (5)-(10). The control law (11) consists of two components; $\epsilon_i(\boldsymbol{x}_{\phi_i}, t)$ determines the control strength. The closer $\xi_i(\boldsymbol{x}_{\phi_i}, t)$ gets to $\Omega_\xi := \{-1, 0\}$, i.e., the funnel boundary, the bigger gets $\epsilon_i(\boldsymbol{x}_{\phi_i}, t)$ and consequently also $\|\boldsymbol{u}(\boldsymbol{x}_{\phi_i}, t)\|$. Note that $\|\boldsymbol{u}(\boldsymbol{x}_{\phi_i}, t)\| \to \infty$ as $\xi_i(\boldsymbol{x}_{\phi_i}, t) \to \Omega_\xi$. The control direction is determined by $\frac{\partial \rho^{\psi_i}(\boldsymbol{x}_{\phi_i})}{\partial \boldsymbol{x}_i}$, i.e., in which direction control action should mainly happen. In summary, the control law always steers in the direction away from the funnel boundary, and the control effort increases close to the funnel boundary. We reason that applying the control law (11) is hence a good initial choice such that ϕ_i will be locally satisfied if the participating agents $\mathcal{V}_{\phi_i} \setminus \{v_i\}$ behave reasonably. The resulting trajectory $\boldsymbol{x}_{\phi_i}$ may, however, hit the funnel boundary, i.e., $\xi_i(\boldsymbol{x}_{\phi_i}, t) = \{-1, 0\}$, and lead to critical events.

Example 2: Consider three agents v_1, v_2, and v_3. Agent v_2 is subject to the formula $\phi_2 := F_{[5,15]}(\|\boldsymbol{x}_2 - \begin{bmatrix} 90 & 90 \end{bmatrix}\| \leq 5)$, while agent v_3 is subject to $\phi_3 := F_{[5,15]}(\|\boldsymbol{x}_3 - \begin{bmatrix} 90 & 10 \end{bmatrix}\| \leq 5)$, i.e., both agents are subject to non-collaborative formulas. Agent v_1 is subject to the collaborative formula $\phi_1 := G_{[0,15]}(\|\boldsymbol{x}_1 - \boldsymbol{x}_2\| \leq 10 \wedge \|\boldsymbol{x}_1 - \boldsymbol{x}_3\| \leq 10)$. Note that the set of formulas $\{\phi_1, \phi_2, \phi_3\}$ is not globally feasible, although each formula itself is locally feasible. Under (11), agents v_2 and v_3 move to $\begin{bmatrix} 90 & 90 \end{bmatrix}$ and $\begin{bmatrix} 90 & 10 \end{bmatrix}$, respectively. Agent v_1 can consequently not satisfy ϕ_1 and only decrease the robustness such that $r_i < 0$ to achieve $r_i \leq \rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) \leq 0$ similar to Corollary 1.

Even if the set $\{\phi_1, \ldots, \phi_M\}$ is globally feasible, there are reasons why critical events may occur as illustrated next.

Example 3: Consider two agents v_4 and v_5 with $\phi_4 := F_{[5,10]}(\|\boldsymbol{x}_4 - \boldsymbol{x}_5\| \leq 10 \wedge \|\boldsymbol{x}_4 - \begin{bmatrix} 50 & 70 \end{bmatrix}\| \leq 10)$ and $\phi_5 := F_{[5,15]}(\|\boldsymbol{x}_5 - \begin{bmatrix} 10 & 10 \end{bmatrix}\| \leq 5)$. Under (11), agent v_5 moves to $\begin{bmatrix} 10 & 10 \end{bmatrix}$ by at latest 15 time units. However, agent v_4 is forced to move to $\begin{bmatrix} 50 & 70 \end{bmatrix}$ and be close to agent v_5 by at latest 10 time units. This may lead to critical events where (4) is violated for agent v_4. If agent v_5 cooperates, it can first help to locally satisfy ϕ_4, e.g., by using *collaborative control* as in Theorem 2, and locally satisfy ϕ_5 afterwards.

We propose an online detection & repair scheme by using a local hybrid system $\mathcal{H}_i := (C_i, F_i, D_i, G_i)$ for each agent $v_i \in \Xi_l$. We *detect* critical events that may lead to trajectories that do not locally satisfy ϕ_i by the jump set D_i. Then, agent v_i locally *repairs* the funnel, i.e., the design parameters t_i^*, $\rho_i^{\max}$, r_i, and γ_i, in a first stage. If this is not successful, *collaborative control* as in Theorem 2 will be considered in a second stage (Example 3). If *collaborative control* is not applicable, r_i is successively decreased by $\delta_i > 0$ in the third stage (Example 2), where δ_i is a design parameter.

Let $\boldsymbol{p}_i^\gamma := \begin{bmatrix} \gamma_i^0 & \gamma_i^\infty & l_i \end{bmatrix}$ and $\boldsymbol{p}_i^{\mathrm{f}} := \begin{bmatrix} t_i^* & \rho_i^{\max} & r_i & \boldsymbol{p}_i^\gamma \end{bmatrix}$ contain the parameters that define (4), and let $\boldsymbol{p}_i^{\mathrm{r}} := \begin{bmatrix} \mathfrak{n}_i & \mathfrak{c}_i \end{bmatrix}$; $\mathfrak{n}_i$ indicates the number of repair attempts in the first repair stage, while $\mathfrak{c}_i$ is used in the second repair stage ($\mathfrak{c}$ for collaborative). If $\mathfrak{c}_i \in \{1, \ldots, M\}$, *collaborative control* as in Theorem 2 is used to collaboratively satisfy $\phi_{\mathfrak{c}_i}$. If $\mathfrak{c}_i = 0$, then agent v_i tries to locally satisfy ϕ_i by itself and if $\mathfrak{c}_i = -1$, then agent v_i is free, i.e., not subject to a task.

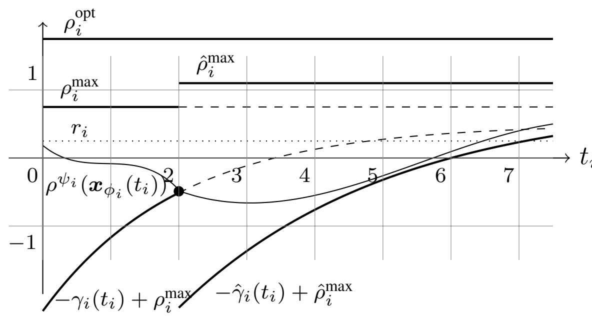


Fig. 1: Funnel repair in the first stage for $\phi_i := F_{[4,6]}\psi_i$.

We define the hybrid state as $\boldsymbol{z}_i := \begin{bmatrix}\boldsymbol{x}_i & t_i & \boldsymbol{p}_i^{\text{f}} & \boldsymbol{p}_i^{\text{r}}\end{bmatrix} \in \mathcal{Z}_i$, where t_i is a clock, $\mathcal{Z}_i := \mathbb{R}^n \times \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0}^6 \times \mathbb{Z}^2$ and $\boldsymbol{z}_i(0,0) := \begin{bmatrix}\boldsymbol{x}_i(0) & 0 & \boldsymbol{p}_i^{\text{f}}(0) & \boldsymbol{0}_2\end{bmatrix}$ with $\mathbb{Z}$ being the set of integers. The elements in $\boldsymbol{p}_i^{\text{f}}(0)$ are as chosen according to (5)-(10). Additionally, we choose $\boldsymbol{p}_i^{\text{f}}(0) = \boldsymbol{p}_j^{\text{f}}(0)$ if Case A holds for all agents $v_i, v_j \in \Xi_l$. Next, define

$$\boldsymbol{u}_i^{\text{int}} = \begin{cases} \boldsymbol{0}_{m_i} & \text{if } \mathfrak{c}_i = -1 \\ -\epsilon_i(\boldsymbol{x}_{\phi_i}, t_i) g_i(\boldsymbol{x}_i)^T \frac{\partial \rho^{\psi_i}(\boldsymbol{x}_{\phi_i})}{\partial \boldsymbol{x}_i} & \text{if } \mathfrak{c}_i = 0 \\ -\epsilon_{\mathfrak{c}_i}(\boldsymbol{x}_{\phi_{\mathfrak{c}_i}}, t_i) g_i(\boldsymbol{x}_i)^T \frac{\partial \rho^{\psi_{\mathfrak{c}_i}}(\boldsymbol{x}_{\phi_{\mathfrak{c}_i}})}{\partial \boldsymbol{x}_i} & \text{if } \mathfrak{c}_i > 0 \end{cases}$$

so that the flow map can be written as

$$F_i := \begin{bmatrix} f_i(\boldsymbol{x}_i) + f_i^{\text{c}}(\boldsymbol{x}) + g_i(\boldsymbol{x}_i)\boldsymbol{u}_i^{\text{int}} + \boldsymbol{w}_i & 1 & \boldsymbol{0}_6 & \boldsymbol{0}_2 \end{bmatrix}.$$

External inputs are $\boldsymbol{w}_i$ and $\boldsymbol{x}_i^{\text{ext}}$. By assuming $v_i \in \Xi_l$, we define $\mathfrak{c}_i^{\text{ext}} := \begin{bmatrix}\mathfrak{c}_{j_1} & \dots & \mathfrak{c}_{j_{|\Xi_l|-1}}\end{bmatrix}$ and $\boldsymbol{p}_i^{\text{f,ext}} := \begin{bmatrix}\boldsymbol{p}_{j_1}^{\text{f}} & \dots & \boldsymbol{p}_{j_{|\Xi_l|-1}}^{\text{f}}\end{bmatrix}$ with $v_{j_1}, \dots, v_{j_{|\Xi_l|-1}} \in \Xi_l \setminus \{v_i\}$. Define the external input as $\boldsymbol{u}_i^{\text{ext}} := \begin{bmatrix}\boldsymbol{w}_i & \boldsymbol{x}_i^{\text{ext}} & \mathfrak{c}_i^{\text{ext}} & \boldsymbol{p}_i^{\text{f,ext}}\end{bmatrix}$.

The set $\mathcal{D}_i'$ is used to *detect* a critical event when the funnel in (4) is violated, i.e., when $\xi_i(t_i) \notin \Omega_\xi := (-1, 0)$.

$$\mathcal{D}_i' := \{(\boldsymbol{z}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}}) \in \mathfrak{H}_i | \xi_i(t_i) \in \{-1, 0\},\ \mathfrak{c}_i = 0\}.$$

1) Repair Stage 1: The first repair stage is indicated by

$$\mathcal{D}_{i,1}' := \mathcal{D}_i' \cap \{(\boldsymbol{z}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}}) \in \mathfrak{H}_i | \mathfrak{n}_i < N_i\}$$

where $N_i \in \mathbb{N}$ is a design parameter representing the maximum number of repair attempts in the first stage. If $(\boldsymbol{z}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}}) \in \mathcal{D}_{i,1}'$, we first relax the parameters t_i^*, $\rho_i^{\max}$, r_i, and γ_i in a way that still guarantees local satisfaction of ϕ_i. Pictorially speaking, we make the funnel in (4) bigger.

Example 4: Consider the formula $\phi_i := F_{[4,6]}\psi_i$ with $r_i := 0.25$ as the desired initial robustness, which is supposed to be achieved at $t_i^* \approx 4.8$. The original funnel is shown in Fig. 1 and given by $\rho_i^{\max}$ and $-\gamma_i + \rho_i^{\max}$ as in (4). Without detection of a critical event, it would hence hold that $\rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) \geq r_i$ since $\rho^{\psi_i}\big(\boldsymbol{x}_{\phi_i}(t_i^*)\big) \geq r_i$ would be achieved. However, at $t_r := 2$, where t_r indicates the time where a critical event is detected, the trajectory $\rho^{\psi_i}\big(\boldsymbol{x}_{\phi_i}(t)\big)$ touches the lower funnel boundary and repair action is needed. This is done by setting $\hat{t}_i^* := 6$ (time relaxation), $\hat{r}_i := 0.0001$ (robustness relaxation), $\hat{\rho}_i^{\max} := 1.1$ (upper funnel relaxation), and also adjusting $\hat{\gamma}_i$ (lower funnel relaxation). The funnel is hence relaxed to $\hat{\rho}_i^{\max}$ and $-\hat{\gamma}_i + \hat{\rho}_i^{\max}$ as depicted in Fig. 1. Due to repair action, $\boldsymbol{x}_{\phi_i}$ locally satisfies ϕ_i as shown in Fig. 1.

We first introduce the notation $\{\hat{\boldsymbol{z}}_i \in \mathcal{Z}_i | \hat{\boldsymbol{z}}_i = \boldsymbol{z}_i\ ;\ exception\}$ denoting the set of $\hat{\boldsymbol{z}}_i \in \mathcal{Z}_i$ such that $\hat{\boldsymbol{z}}_i = \boldsymbol{z}_i$ after the jump except for the elements in $\hat{\boldsymbol{z}}_i$ explicitly mentioned after the semicolon, here denoted by the placeholder $exception$. With Example 4 in mind, set

$$\mathcal{G}_{i,1}' := \Big\{ \hat{\boldsymbol{z}}_i \in \mathcal{Z}_i | \hat{\boldsymbol{z}}_i = \boldsymbol{z}_i\ ;\ \hat{t}_i^* := \begin{cases} b_i & \text{if } \phi_i = F_{[a_i,b_i]}\psi_i \\ t_i^* & \text{if } \phi_i = G_{[a_i,b_i]}\psi_i, \end{cases}$$
$$\hat{\rho}_i^{\max} = \rho_i^{\max} + \zeta_i^{\text{u}},\ \hat{r}_i \in (0, r_i),\ \hat{\boldsymbol{p}}_i^{\gamma} = \boldsymbol{p}_i^{\gamma,\text{new}},\ \hat{\mathfrak{n}}_i = \mathfrak{n}_i + 1 \Big\}$$

where the variables ζ_i^{u} and $\boldsymbol{p}_i^{\gamma,\text{new}}$ are defined in the sequel. In words, we set $\hat{t}_i^* := b_i$ if $\phi_i = F_{[a_i,b_i]}\psi_i$ (time relaxation) and keep $\hat{t}_i^* := t_i^* = a_i$ otherwise. The parameter r_i is decreased to $\hat{r}_i \in (0, r_i)$ (robustness relaxation), while ζ_i^{u} relaxes the upper funnel and needs to be such that $\hat{\rho}_i^{\max} := \rho_i^{\max} + \zeta_i^{\text{u}} < \rho_i^{\text{opt}}$ (upper funnel relaxation) according to (6), i.e., let $\zeta_i^{\text{u}} \in (0, \rho_i^{\text{opt}} - \rho_i^{\max})$. At t_r, the detection time of a critical event, we set $\gamma_i^{\text{r}} := \hat{\gamma}_i(t_r) := \hat{\rho}_i^{\max} - \rho^{\psi_i}(\boldsymbol{x}_{\phi_i}) + \zeta_i^{\text{l}}$ with

$$\zeta_i^{\text{l}} \in \begin{cases} \mathbb{R}_{>0} & \text{if } \hat{t}_i^* > t_i \\ (0, \rho^{\psi_i}(\boldsymbol{x}_{\phi_i}) - \hat{r}_i] & \text{otherwise,} \end{cases}$$

which resembles (8) (lower funnel relaxation); ζ_i^{u} and ζ_i^{l} determine the margin by how much the funnel is relaxed. Let $\boldsymbol{p}_i^{\gamma,\text{new}} := \begin{bmatrix}\gamma_i^{0,\text{new}} & \gamma_i^{\infty,\text{new}} & l_i^{\text{new}}\end{bmatrix}$ and select, similar to (9) and (10), $\gamma_i^{\infty,\text{new}} \in (0, \min(\gamma_i^{\text{r}}, \hat{\rho}_i^{\max} - \hat{r}_i)]$ and

$$l_i^{\text{new}} := \begin{cases} 0 & \text{if } -\gamma_i^{\text{r}} + \hat{\rho}_i^{\max} \geq \hat{r}_i \\ \frac{-\ln\left(\frac{\hat{r}_i + \gamma_i^{\infty,\text{new}} - \hat{\rho}_i^{\max}}{-(\gamma_i^{\text{r}} - \gamma_i^{\infty,\text{new}})}\right)}{\hat{t}_i^* - t_i} & \text{if } -\gamma_i^{\text{r}} + \hat{\rho}_i^{\max} < \hat{r}_i. \end{cases}$$

Finally, set $\gamma_i^{0,\text{new}} := (\gamma_i^{\text{r}} - \gamma_i^{\infty,\text{new}})\exp(l_i^{\text{new}} t_i) + \gamma_i^{\infty,\text{new}}$ to account for the clock t_i that is not reset ($\hat{t}_i := t_i$).

2) Repair Stage 2: Repairs of the second and third stage are detected by

$$\mathcal{D}_{i,\{2,3\}}' := \mathcal{D}_i' \cap \{(\boldsymbol{z}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}}) \in \mathfrak{H}_i | \mathfrak{n}_i \geq N_i\}.$$

The second stage will only be initiated if some timing constraints hold. Then, *collaborative control* as in Theorem 2 is used to satisfy ϕ_i. The second stage is detected by

$$\mathcal{D}_{i,2}' := \mathcal{D}_{i,\{2,3\}}' \cap \Big\{ (\boldsymbol{z}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}}) \in \mathfrak{H}_i | \forall v_j \in \mathcal{V}_{\phi_i} \setminus \{v_i\},$$
$$(\mathfrak{c}_j = -1) \text{ or } \left(\mathfrak{c}_j = 0, b_i < \begin{cases} b_j \text{ if } \phi_j = F_{[a_j,b_j]}\psi_j \\ a_j \text{ if } \phi_j = G_{[a_j,b_j]}\phi_j \end{cases} \right) \Big\},$$

i.e., each agent $v_j \in \mathcal{V}_{\phi_i} \setminus \{v_i\}$ is either free or postpones satisfaction of ϕ_j to collaboratively deal with ϕ_i first, while ensuring that there is enough time to deal with ϕ_j afterwards. If $(\boldsymbol{z}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}}) \in \mathcal{D}_{i,2}'$, agent v_i acts according to

$$\mathcal{G}_{i,2}' := \Big\{ \hat{\boldsymbol{z}}_i \in \mathcal{Z}_i | \hat{\boldsymbol{z}}_i = \boldsymbol{z}_i\ ;\ \hat{\rho}_i^{\max} = \rho_i^{\max} + \zeta_i^{\text{u}},$$
$$\hat{r}_i \in (0, r_i),\ \hat{\boldsymbol{p}}_i^{\gamma} = \boldsymbol{p}_i^{\gamma,\text{new}},\ \hat{\mathfrak{c}}_i = i \Big\}$$

where $\hat{\mathfrak{c}}_i := i$ initiates *collaborative control*, while again relaxing the funnel parameters as in the first repair stage. The jump set $\mathcal{D}'_{i,2}$ applies if agent v_i detects a critical event. Now changing the perspective to the participating agents $v_j \in \mathcal{V}_{\phi_i} \setminus \{v_i\}$, all agents v_j need to participate in *collaborative control*. Assume that $v_j \in \Xi_l$, then for agent v_j

$$\mathcal{D}''_{j,2} := \{(\boldsymbol{z}_j, \boldsymbol{u}_j^{\text{int}}, \boldsymbol{u}_j^{\text{ext}}) \in \mathfrak{H}_j | \mathfrak{c}_j \in \{-1, 0\}, \exists v_i \in \Xi_l \setminus \{v_j\},\ v_j \in \mathcal{V}_{\phi_i},\ \mathfrak{c}_i = i\},$$

is the jump set, which is activated when agent v_i asks agent v_j for *collaborative control*. If $(\boldsymbol{z}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}}) \in \mathcal{D}''_{j,2}$, set

$$\mathcal{G}''_{j,2} := \Big\{ \hat{\boldsymbol{z}}_j \in \mathcal{Z}_j | \hat{\boldsymbol{z}}_j = \boldsymbol{z}_j \ ;\ \hat{\boldsymbol{p}}_j^{\text{f}} = \boldsymbol{p}_i^{\text{f}},\ \hat{\mathfrak{c}}_j = \mathfrak{c}_i \Big\}$$

where $\hat{\mathfrak{c}}_j = \mathfrak{c}_i$ and $\hat{\boldsymbol{p}}_j^{\text{f}} = \boldsymbol{p}_i^{\text{f}}$ enforce that all conditions in Theorem 2 hold such that ϕ_i will be locally satisfied.

3) Repair Stage 3: If the timing constraints in $\mathcal{D}'_{i,2}$ do not apply, repairs of the third stage are initiated by

$$\mathcal{D}'_{i,3} := \mathcal{D}'_{i,\{2,3\}} \setminus \mathcal{D}'_{i,2}.$$

Agent v_i reacts in this case by reducing the robustness r_i by $\delta_i > 0$ as illustrated in Example 2 and according to

$$\mathcal{G}'_{i,3} := \Big\{ \hat{\boldsymbol{z}}_i \in \mathcal{Z}_i | \hat{\boldsymbol{z}}_i = \boldsymbol{z}_i \ ;\ \hat{\rho}_i^{\max} = \rho_i^{\max} + \zeta_i^{\text{u}}, \hat{r}_i = r_i - \delta_i,\ \hat{\rho}_i^{\max} = \rho_i^{\text{opt}} + \sigma_i,\ \hat{\boldsymbol{p}}_i^{\gamma} = \boldsymbol{p}_i^{\gamma,\text{new}} \Big\}.$$

where now $\gamma_i^{\text{r}} := \hat{\rho}_i^{\max} - \rho^{\psi_i}(\boldsymbol{x}_{\phi_i}) + \delta_i$ is used to calculate $\boldsymbol{p}_i^{\gamma,\text{new}}$, while $\sigma_i > 0$ will avoid Zeno behavior.

4) The Overall System: It now needs to be determined what happens when a task ϕ_i is locally satisfied. Define $\nu_i := \begin{cases} \mathfrak{c}_i & \text{if } \mathfrak{c}_i > 0 \\ i & \text{if } \mathfrak{c}_i = 0 \end{cases}$ and detect such events by

$$\mathcal{D}_{i,\text{sat}} := \Big\{ (\boldsymbol{z}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}}) \in \mathfrak{H}_i | r_{\nu_i} \leq \rho^{\psi_{\nu_i}}\big(\boldsymbol{x}_{\phi_{\nu_i}}\big) \leq \rho_{\nu_i}^{\max},\ \mathfrak{c}_i \geq 0, t_i \in \begin{cases} [a_{\nu_i}, b_{\nu_i}] & \text{if } \phi_{\nu_i} = F_{[a_{\nu_i}, b_{\nu_i}]}\psi_{\nu_i} \\ b_{\nu_i} & \text{if } \phi_{\nu_i} = G_{[a_{\nu_i}, b_{\nu_i}]}\psi_{\nu_i} \end{cases} \Big\} \setminus (\mathcal{D}'_i \cup \mathcal{D}''_{i,2}),$$

where the set substraction of $\mathcal{D}'_i \cup \mathcal{D}''_{i,2}$ exludes the case where $\mathcal{D}'_i$ or $\mathcal{D}''_{i,2}$ apply simultaneously with $\mathcal{D}_{i,\text{sat}}$. This would result in undesirable non-determism endangering the logic behind the hybrid system. If $(\boldsymbol{z}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}}) \in \mathcal{D}_{i,\text{sat}}$, let

$$\mathcal{G}_{i,\text{sat}} := \Big\{ \hat{\boldsymbol{z}}_i \in \mathcal{Z}_i | \hat{\boldsymbol{z}}_i = \boldsymbol{z}_i \ ;\ \hat{t}_i^* = \begin{cases} b_i & \text{if } \phi_i = F_{[a_i,b_i]}\psi_i \\ a_i & \text{if } \phi_i = G_{[a_i,b_i]}\psi_i, \end{cases} \hat{\rho}_i^{\max} = \tilde{\rho}_i^{\max},\ \hat{r}_i = \tilde{r}_i, \hat{\boldsymbol{p}}_i^{\gamma} = \boldsymbol{p}_i^{\gamma,\text{new}},\ \hat{\mathfrak{c}}_i = \begin{cases} 0 & \text{if } \mathfrak{c}_i > 0 \text{ and } \mathfrak{c}_i \neq i \\ -1 & \text{if } \mathfrak{c}_i = 0 \text{ or } \mathfrak{c}_i = i \end{cases} \Big\}$$

where $\tilde{\rho}_i^{\max}$ and $\tilde{r}_i$ are chosen according to (6) and (7), but evaluated with $\boldsymbol{x}_{\phi_i}(t_i)$ instead of $\boldsymbol{x}_{\phi_i}(0)$.

Note that $\mathcal{D}'_i = \mathcal{D}'_{i,1} \cup \mathcal{D}'_{i,2} \cup \mathcal{D}'_{i,3}$ with $\mathcal{D}'_{i,1} \cap \mathcal{D}'_{i,2} \cap \mathcal{D}'_{i,3} = \emptyset$. The hybrid system $\mathcal{H}_i$ is given by $D_i := \mathcal{D}'_i \cup \mathcal{D}''_{i,2} \cup \mathcal{D}_{i,\text{sat}}$ and $C_i := \mathcal{Z}_i \setminus D_i$. The jump map $G_i(\boldsymbol{z}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}})$ is given by $\mathcal{G}'_{i,1}$, $\mathcal{G}'_{i,2}$, $\mathcal{G}''_{i,2}$, $\mathcal{G}'_{i,3}$, and $\mathcal{G}_{i,\text{sat}}$ together with the corresponding jump sets $\mathcal{D}'_{i,1}$, $\mathcal{D}'_{i,2}$, $\mathcal{D}''_{i,2}$, $\mathcal{D}'_{i,3}$, and $\mathcal{D}_{i,\text{sat}}$, respectively. Note now that the sets $\mathcal{D}'_i$ and $\mathcal{D}_{i,\text{sat}}$ as well as $\mathcal{D}''_{i,2}$ and $\mathcal{D}_{i,\text{sat}}$ are non-intersecting. However, $\mathcal{D}'_i$ and $\mathcal{D}''_{i,2}$ are intersecting. Therefore, if $(\boldsymbol{z}_i, \boldsymbol{u}_i^{\text{int}}, \boldsymbol{u}_i^{\text{ext}}) \in \mathcal{D}'_i \cap \mathcal{D}''_{i,2}$, which will rarely happen in practice, we only execute the jump induced by $\mathcal{D}''_{i,2}$ to not endager the logic behind the hybrid system. Thereby, we can say that the sets $\mathcal{D}'_i$, $\mathcal{D}''_{i,2}$, and $\mathcal{D}_{i,\text{sat}}$ are technically non-intersecting.

Theorem 3: Assume that each agent $v_i \in \mathcal{V}$ is subject to ϕ_i of the form (3b) and controlled by $\mathcal{H}_i := (C_i, F_i, D_i, G_i)$, while Assumptions 1-5 are satisfied. The induced dependency clusters $\bar{\Xi} = \{\Xi_1, \ldots, \Xi_L\}$ are such that for each $\Xi_l \in \bar{\Xi}$ it holds that v_i and v_j can communicate for all $v_i, v_j \in \Xi_l$. For $v_i \in \Xi_l$ it then holds that $\rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) \geq r_i$, where either $r_i := r_i(0,0)$ (initial robustness) if $\phi_i = \phi_j$ for all $v_i, v_j \in \Xi_l$ or r_i is lower bounded and maximized up to a precision of δ_i otherwise. Zeno behavior is excluded.

Proof: The proof can be found in [11]. ■

V. SIMULATIONS

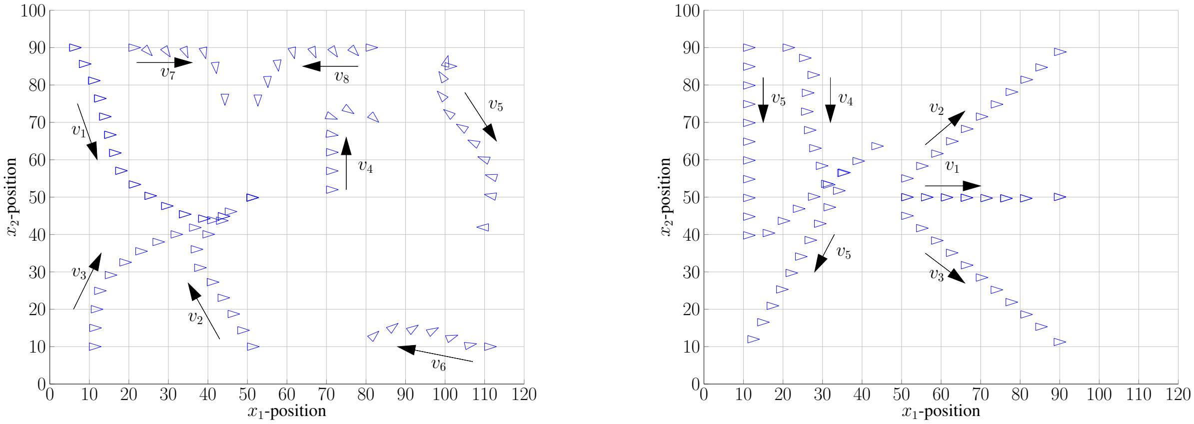
We consider omni-directional robots as in [12] with two states x_1 and x_2 indicating the robot position and one state x_3 indicating the robot orientation with respect to the x_1-axis. Let $x_{i,j}$ with $j \in \{1,2,3\}$ denote the j-th element of agent v_i's state and let $\boldsymbol{p}_i := \begin{bmatrix} x_{i,1} & x_{i,2} \end{bmatrix}$. We hence have $\boldsymbol{x}_i := \begin{bmatrix} \boldsymbol{p}_i & x_{i,3} \end{bmatrix} = \begin{bmatrix} x_{i,1} & x_{i,2} & x_{i,3} \end{bmatrix}$ with the dynamics

$$\dot{\boldsymbol{x}}_i = \begin{bmatrix} \cos(x_{i,3}) & -\sin(x_{i,3}) & 0 \\ \sin(x_{i,3}) & \cos(x_{i,3}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \Big(B_i^T\Big)^{-1} R_i \boldsymbol{u}_i,$$

where $R_i := 0.02$ is the wheel radius and $B_i := \begin{bmatrix} 0 & \cos(\pi/6) & -\cos(\pi/6) \\ -1 & \sin(\pi/6) & \sin(\pi/6) \\ L_i & L_i & L_i \end{bmatrix}$ describes geometrical constraints with $L_i := 0.2$ as the radius of the robot body.

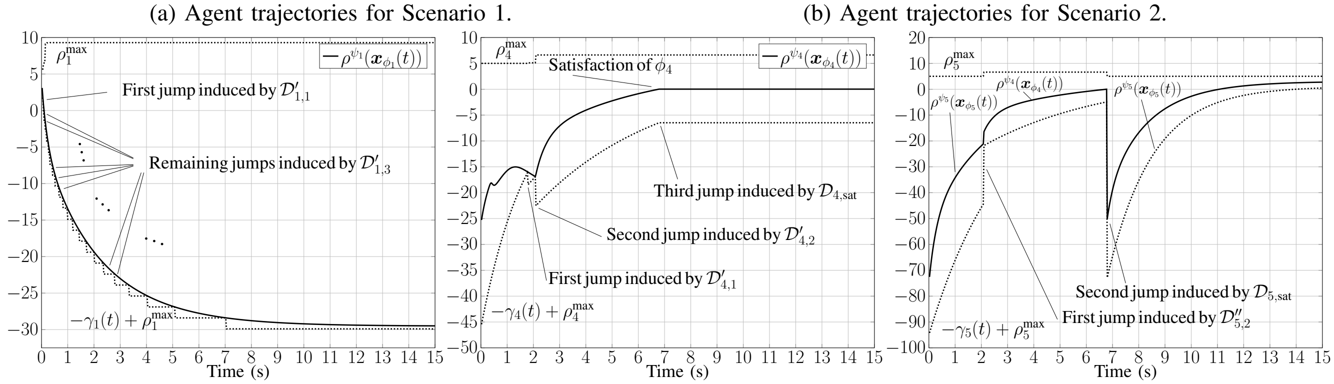
Scenario 1: In this scenario, the clusters $\Xi_1 := \{v_1, v_2, v_3\}$, $\Xi_2 := \{v_4, v_5, v_6\}$, and $\Xi_3 := \{v_7, v_8\}$ are subject to the same formula so that Theorem 1 applies. With $\boldsymbol{x}_A := \begin{bmatrix} 50 & 50 \end{bmatrix}$, $\boldsymbol{x}_B := \begin{bmatrix} 110 & 40 \end{bmatrix}$, $\boldsymbol{x}_C := \begin{bmatrix} 40 & 70 \end{bmatrix}$, and $\boldsymbol{x}_D := \begin{bmatrix} 55 & 70 \end{bmatrix}$, the formulas are $\phi_1 := \phi_2 := \phi_3 := F_{[10,15]}\psi_{l_1}$ with $\psi_{l_1} := (\|\boldsymbol{p}_1 - \boldsymbol{p}_2\| < 2) \wedge (\|\boldsymbol{p}_1 - \boldsymbol{p}_3\| < 2) \wedge (\|\boldsymbol{p}_2 - \boldsymbol{p}_3\| < 2) \wedge (\|\boldsymbol{p}_1 - \boldsymbol{p}_A\| < 2)$. For the second cluster, $\phi_4 := \phi_5 := \phi_6 := F_{[10,15]}\psi_{l_2}$ is used with $\psi_{l_2} := (\|\boldsymbol{p}_5 - \boldsymbol{p}_B\| < 5) \wedge (27 < x_{5,1} - x_{4,1} < 33) \wedge (27 < x_{5,1} - x_{6,1} < 33) \wedge (27 < x_{4,2} - x_{5,2} < 33) \wedge (27 < x_{5,2} - x_{6,2} < 33) \wedge (|\deg(x_{4,3}) + 45| < 5) \wedge (|\deg(x_{5,3}) - 180| < 5) \wedge (|\deg(x_{6,3}) - 45| < 5)$, where $\deg(\cdot)$ converts radian into degree. The third cluster employs $\phi_7 := \phi_8 := F_{[10,15]}\psi_{l_3}$ with $\psi_{l_3} := (\|\boldsymbol{p}_7 - \boldsymbol{p}_8\| < 10) \wedge (\|\boldsymbol{p}_7 - \boldsymbol{p}_C\| < 10) \wedge (\|\boldsymbol{p}_8 - \boldsymbol{p}_D\| < 10) \wedge (|\deg(x_{7,3}) + 90| < 5) \wedge (|\deg(x_{5,3}) + 90| < 5)$. The simulation result is shown in Fig. 2a. Note that all tasks are satisfied within the time interval $[10, 15]$.

Scenario 2: This scenario features two clusters $\Xi_1 := \{v_1, v_2, v_3\}$ and $\Xi_2 := \{v_4, v_5\}$ simulating Example 2 and 3, respectively. Recall therefore ϕ_1, ϕ_2, ϕ_3, ϕ_4, and ϕ_5 from Example 2 and 3. We set $\delta_i := 1.5$ and $N_i := 1$ for all agents $v_i \in \mathcal{V}$. Agent trajectories are shown in Fig. 2b, while Fig. 2c shows the funnel (4) for agent v_1. It is visible that agent v_1 first tries to repair its parameter in stage 1, and then initiates stage 3 to successively reduce the robustness r_1. Agent v_1



(a) Agent trajectories for Scenario 1.

(b) Agent trajectories for Scenario 2.



(c) Scenario 2: Funnel repairs for agent v_1 (d) Scenario 2: Funnel repairs for agent v_4 (e) Scenario 2: Funnel repairs for agent v_5

Fig. 2: Simulation results for Scenario 1 and 2.

hence finds a trade-off between staying close to agent v_2 and v_3 as shown in Fig. 2b. Agent v_4 first tries to repair its parameters in Stage 1, but then requests agent v_5 to use *collaborative control* to satisfy ϕ_4 as indicated in Fig. 2d and 2e. Agent v_5 satisfies ϕ_5 afterwards; ϕ_2, ϕ_3, ϕ_4, and ϕ_5 are locally satisfied with robustness $r_2 = r_3 = r_4 = r_5 = 0.5$, while for ϕ_1 it holds that $\rho^{\phi_i}(\boldsymbol{x}_{\phi_i}, 0) > r_1 = -30$.

VI. Conclusion

We presented a framework for the control of multi-agent systems, where each agent is subject to a local signal temporal logic task. By leveraging ideas from prescribed performance control, we developed a continuous feedback control law that achieves satisfaction of the local tasks under some conditions. If these conditions do not hold, we combined the developed control law with an online detection & repair scheme, expressed as a hybrid system. Advantages of our framework are low computation times and robustness.

Possible future extensions are the improvement of the repair stages in the online detection & repair scheme. A next step is also to perform physical experiments.

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