Decentralized Robust Control of Coupled Multi-Agent Systems
under Local Signal Temporal Logic Tasks

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Abstract—Motivated by the recent interest in formal methods-based control of multi-agent systems, we adopt a bottom-up approach. Each agent is subject to a local signal temporal logic task that may depend on other agents’ behavior. These dependencies pose control challenges since some of the tasks may be opposed to each other. We first develop a local continuous feedback control law and identify conditions under which this control law guarantees satisfaction of the local tasks. If these conditions do not hold, we propose to use the developed control law in combination with an online detection & repair scheme, expressed as a local hybrid system. After detection of a critical event, a three-stage procedure is initiated to resolve the problem. The theoretical results are illustrated in simulations.

I. INTRODUCTION

Multi-agent systems under objectives such as consensus, formation control, and connectivity maintenance have been well studied [1]. The need for more complex objectives in robotic applications has led to formal methods-based control strategies where temporal logics are used to formulate high-level temporal tasks. Top-down approaches have been considered, as for instance in [2], by decomposing a global task into local ones. Top-down approaches are usually subject to high computational complexity. On the other hand, the works in [3], [4] favor a bottom-up approach, where local tasks are independently distributed to each agent. Here, however, feasibility of each local task does not imply feasibility of the conjunction of all local tasks. The works in [2]-[4] rely on automata-based verification techniques that discretize the physical environment and agent dynamics. We instead consider continuous-time dynamics without the need for discretizing neither environment nor agent dynamics in space or time. This paper extends our work in [5] to multi-agent systems and is, to the best of our knowledge, the first to consider the notion of space robustness [7], a robustness metric stating that this control law guarantees satisfaction of the local tasks.

A local hybrid system [9] for each agent. Critical events will be detected and resolved in a three-stage procedure, gradually relaxing parameters such as robustness. One advantage of our decentralized approach is the low computational complexity due to the continuous feedback control laws. Furthermore, the team of agents is allowed to be heterogeneous with additional dynamic couplings among them. Robustness is considered with respect to disturbances and with respect to the signal temporal logic task. Multi-agent systems under signal temporal logic tasks have also been considered in [10] in a centralized approach, not investigating task dependencies, but with a special focus on communication.

Section II introduces preliminaries, while Section III states the problem definition. Section IV presents our solution to the stated problem, which is verified by simulations in Section V. Conclusions are given in Section VI.

II. PRELIMINARIES

Scalars and column vectors are denoted by non-bold letters \( x \) and bold letters \( \mathbf{x} \), respectively, while true and false are denoted by \( \top \) and \( \bot \). As usual, \( \mathbb{R} \) is the set of real numbers, while \( \mathbb{R}^n \) is the \( n \)-dimensional real vector space. The natural, non-negative, and positive real numbers are \( \mathbb{N}, \mathbb{R}_{\geq 0} \), and \( \mathbb{R}_{>0} \), respectively. For convenience, we abbreviate \( [x \ y] := [x^T \ y^T]^T \).

A. Signal Temporal Logic (STL)

Signal temporal logic (STL) is a predicate logic and consists of predicates \( \mu \) that are obtained after the evaluation of a function \( h : \mathbb{R}^n \rightarrow \mathbb{R} \) as \( \mu := \begin{cases} \top & \text{if } h(x) \geq 0 \\ \bot & \text{if } h(x) < 0. \end{cases} \)

With \( \mu \) being a predicate, the STL syntax is given by

\[
\phi := \top \mid \mu \mid \neg \phi \mid \phi \land \psi \mid \phi U_{[a,b]} \psi,
\]

where \( \phi \) and \( \psi \) are STL formulas. The satisfaction relation \( (x, t) \models \phi \) indicates if the signal \( x : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^n \) satisfies \( \phi \) at time \( t \). Then, the STL semantics are inductively defined as

\[
(x, t) \models \mu \text{ if and only if } h(x(t)) \geq 0, (x, t) \models \neg \mu \text{ if and only if } \neg h(x(t)) = 0 \text{, and (x, t) \models } \phi \land \psi \text{ if and only if (x, t) \models } \phi \text{ and (x, t) \models } \psi \text{ if and only if there exist } t_0 \in [t + a, t + b] \text{ s.t. } (x, t_0) \models \psi \land \forall t_2 \in [t, t_1), (x, t_2) \models \phi \text{.}
\]

Disjunction, eventually, and always-operator are derived as

\[
\phi \lor \psi := \neg(\neg \phi \land \neg \psi), F_{[a,b]} \phi := \top U_{[a,b]} \phi, \text{ and } G_{[a,b]} \phi := \neg F_{[a,b]} \neg \phi,
\]

respectively. Robust semantics, which are called

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space robustness $\rho^\phi(x,t)$ [7], are defined in Definition 1; $\rho^\phi(x,t)$ determines how robustly the signal $x : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ satisfies $\phi$ at time $t$. It holds that $(x(t),t) = \phi$ if $\rho^\phi(x,t) > 0$.

**Definition 1 (Space Robustness):** The semantics of space robustness are inductively defined as [7, Definition 3]:

- $\rho^\phi(x,t) := h(x(t))$
- $\rho^{\neg \phi}(x,t) := -\rho^\phi(x,t)$
- $\rho^{\psi_1 \land \psi_2}(x,t) := \min \{ \rho^\psi_1(x,t), \rho^\psi_2(x,t) \}$
- $\rho^{G[a,b] \psi}(x,t) := \max_{t \in [a,t]} \rho^\psi(x,t_1)$
- $\rho^{F[a,b] \psi}(x,t) := \min_{t \in [a,t]} \rho^\psi(x,t_1)$

We abuse the notation as $\rho^\phi(x(t)) := \rho^\phi(x,t)$ if $t$ is not explicitly contained in $\rho^\phi(x,t)$. For instance, $\rho^\phi(x(t)) := \rho^\phi(x(t),t) := h(x(t))$ since $t$ is not an explicit parameter.

**B. A Bottom-up Approach for Multi-Agent Systems**

Consider a multi-agent system with $M$ agents modeled by an undirected graph $G := (V,E)$ [11]. The vertex and edge sets are $V := \{v_1, \ldots, v_M\}$ and $E \subseteq V \times V$, respectively. Two agents $v_i, v_j \in V$ can communicate if and only if there is a path between $v_i$ and $v_j$. A path is a sequence $v_i, v_{k_1}, \ldots, v_{k_p}, v_j$ such that $(v_{k_1}, v_{k_2}), \ldots, (v_{k_p}, v_j) \in E$.

Let $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$, and $w_i \in \mathbb{W}_i$ be the state, input, and additive noise of agent $i$, with $\mathbb{W}_i \subseteq \mathbb{R}^{n_i}$ being a bounded set. Let $x := [x_1 \ x_2 \ \ldots \ x_M]$ be the stacked vector of all agents' states. Each agent $v_i$ obeys

$$\dot{x}_i = f_i(x_i) + f^i(x_i) + g_i(x_i)u_i + w_i,$$

where $f_i(x)$ describes preassumed dynamic couplings between agents. Also define $x^\text{int}_i := [x_{j_1} \ \ldots \ \ x_{j_{M-1}}]$ with $j_1, \ldots, j_{M-1} \in V \setminus \{v_i\}$, i.e., $x^\text{int}_i$ is a stacked vector containing the states of all agents except of $v_i$.

**Assumption 1:** The functions $f_i : \mathbb{R}^n \to \mathbb{R}^n$, $f^i : \mathbb{R}^M \to \mathbb{R}^n$, and $g_i : \mathbb{R}^n \to \mathbb{R}^{n_i \times m_i}$ are locally Lipschitz continuous, and $g_i(x_i)g_i(x_i)^T$ is a positive definite for all $x_i \in \mathbb{R}^{n_i}$.

We now tailor the STL semantics to a multi-agent bottom-up approach. Each agent $v_i \in V$ is subject to a local STL formula that is endowed with the subscript $i$, i.e., $\phi_i$. Based on [4, Definition 3], local satisfaction of $\phi_i$ by the signal $x_{\phi_i} : \mathbb{R}_{\geq 0} \to \mathbb{R}^{n_i}$ is defined in Definition 2. We will be more specific regarding $x_{\phi_i}$ and $p_i$ after Definition 4.

**Definition 2 (Local Satisfaction):** The signal $x_{\phi_i} : \mathbb{R}_{\geq 0} \to \mathbb{R}^{n_i}$ locally satisfies $\phi_i$ if and only if $(x_{\phi_i}(0),0) = \phi_i$.

**Definition 3 (Local Feasibility):** The formula $\phi_i$ is locally feasible if and only if $\exists x_{\phi_i} : \mathbb{R}_{\geq 0} \to \mathbb{R}^{n_i}$ such that $x_{\phi_i}$ locally satisfies $\phi_i$.

Each local formula $\phi_i$ depends on agent $v_i$, and may also depend on other agents $v_j \in V$. Consider $x_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ to be the solution to (1) associated with agent $v_i$.

**Definition 4 (Formula-Agent Dependency):** If $x_i(t)$ is not contained in $x_{\phi_i}(t)$ for all $t \in \mathbb{R}_{\geq 0}$ and local satisfaction of $\phi_i$, i.e., $(x_{\phi_i}(0),0) = \phi_i$, can be evaluated, then $\phi_i$ does not depend on $v_j$. Otherwise, i.e., knowledge of $x_j(t)$ is needed and hence $x_j(t)$ is contained in $x_{\phi_i}(t)$, then $\phi_i$ does depend on $v_j$ and we say that agent $v_j$ is participating in $\phi_i$.

### III. Problem Statement

In this paper, the following STL fragment is considered:

- $\psi := T \ | \ \mu \ | \ \neg \mu \ | \ \psi(1) \land \psi(2)$
- $\phi := G[a,b] \psi \ | \ F[a,b] \psi$
- $\theta_i := \land_{k=1}^K \phi(k)$ with $b(k) \leq a(k+1)$
where μ is a predicate and ψ(1), ψ(2),... are formulas of class ψ given in (3a), whereas φ(k) with k ∈ {1,...,K} are formulas of class φ given in (3b) with corresponding time intervals [a(k),b(k)]. Note the use of brackets, e.g. ψ(1), to distinguish from local formulas, e.g. ψ. For conjunctions of non-temporal formulas of class ψ, the robust semantics are approximated by a smooth function that preserves the property (x,0) |= ψ if ρψ(x) > 0 as in [5].

**Assumption 2:** The robust semantics for a conjunction of q non-temporal formulas of class ψ given in (3a), i.e., ρψ(1)∧...∧ψ(q)(x), are approximated by a smooth function as

\[ ρψ(1)∧...∧ψ(q)(x) ≈ -\ln \left( \sum_{i=1}^{q} \exp \left( -\rhoψ(i)(x) \right) \right). \]

The objective in this paper is to consider local formulas of class φ given in (3b) that are independently distributed to each agent vi ∈ V. The proposed solution can then be extended to local formulas of class θ given in (3e) as instructed in [5]. Each agent vi ∈ V is hence subject to a local formula φi of the form (3b). Let ψi correspond to φi.

**Assumption 3:** Each formula of class ψ given in (3a) that is contained in (3b) and associated with an agent vi is: 1) s.t. ρψi(x,0) is concave and 2) well-posed in the sense that (x,0) |= ψi implies ||x,0|| ≤ C < ∞ for some C ≥ 0.

Next, define the global optimum of ρψi(x,0) as ρψi := supx∈R,ρψ(x,0) ρψi(x,0), which is straightforward to compute due to Assumption 2 and 3. Assumption 4 now guarantees that φi is locally feasible since ρψi > 0 implies that ρψi(x,0) > 0 is possible.

**Assumption 4:** The optimum of ρψi(x,0) is s.t. ρψi > 0.

The goal is to derive a local control law \( u_i(x_i,t) \) for each agent vi such that \( r_i ≤ ρψi(x_i,0) ≤ ρψi \), where \( r_i ∈ R \) is a robustness measure, while \( ρψi ∈ R \) with \( r_i < ρψi \) is a robustness delimiter. We look at each dependency cluster separately and then distinguish between two cases.

**Problem 1:** Assume that each agent vi is subject to a local STL formula φi of the form (3b), hence inducing the maximal dependency clusters \( \Xi := \{ Ξ_{i1},...,Ξ_{IL} \} \) with \( L ≤ M \). For each cluster \( Ξ_{i} \) with \( i ∈ \{1,...,L\} \), derive a control strategy as follows. Case A) Under the assumption that each agent vi, vj ∈ Ξi is subject to the same formula, i.e., φi = φj, design a local feedback control law \( u_i(x_i,t) \) such that \( 0 < r_i ≤ ρψi(x_i,0) ≤ ρψi \) for all \( v_i ∈ Ξ_i \), which means local satisfaction of φi. Case B) Otherwise, i.e., \( \forall v_i,v_j ∈ Ξ_i \) such that \( φ_i ≠ φ_j \), each agent \( v_i ∈ Ξ_i \) nevertheless initially applies the derived control law \( u_i(x_i,t) \) for Case A. Design a local online detection & repair scheme for each agent \( v_i ∈ Ξ_i \) such that \( r_i ≤ ρψi(x_i,0) ≤ ρψi \), where \( r_i ∈ R \), possibly negative, is maximized up to a precision of \( δ_t > 0 \) with \( δ_t \) being a design parameter.

**IV. PROPOSED PROBLEM SOLUTION**

In the proposed solution to Case A in Problem 1 in Section IV-A, neither formula-agent dependencies nor dynamic couplings \( f_i(x) \) pose difficulties. For Case B, both types of dependencies may lead to trajectories that do not satisfy the formulas. The proposed solution, introducing an online detection & repair scheme, is given in Section IV-B. We now first present the main idea of our work on single-agent systems [5], which is based on prescribed performance control [8] and now extended to multi-agent systems. For a thorough illustration, the reader is referred to [5]. Define the performance function \( γ_i \) for agent \( v_i \) in Definition 9 and the transformation function \( S \) in Definition 10.

**Definition 9:** The performance function \( γ_i : R_{≥0} \rightarrow R_{≥0} \) is continuously differentiable, bounded, positive, non-increasing, and given by \( γ_i(t) := (γ_i^0 − γ_i^∞) \exp(-l_it) + γ_i^∞ \) where \( γ_i^0,γ_i^∞ ∈ R_{≥0} \) with \( γ_i^0 ≥ γ_i^∞ \) and \( l_i ∈ R_{≥0} \).

**Definition 10:** A transformation function \( S : (-1,0) \rightarrow R \) is a strictly increasing function, hence injective and admitting an inverse. In particular, let \( S(ξ) := ln(1+ξ) \).

Our objective, i.e., \( r_i ≤ ρψi(x_i,0) ≤ ρψi \), is achieved by prescribing a temporal behavior to \( ρψi(x_i(t)) \) through the design parameters \( γ_i \) and \( ρψi \) as

\[ -γ_i(t) + l_i + ρψi(x_i(t)) < ρψi \]

Note the use of \( ρψi(x_i(t)) \) and not \( ρψi(x_i,0) \). When \( x_i \) is seen as a state, define \( e_i(x_i) := ρψi(x_i,0) − ρψi \),

\[ ξ_i(x_i,t) := e_i(x_i) + γ_i(t), \]

and \( e_i(x_i,t) := S(ξ_i(x_i,t)) \). Now, (4) can be written as \( e_i(t) < 0 \). If \( e_i(t) \) is bounded for all \( t ≥ 0 \), then inequality (4) holds for all \( t ≥ 0 \). The connection between \( ρψi(x_i(t)) \) in (4) and \( ρψi(x_i,0) \) is made by \( γ_i \), which needs to be chosen as explained in detail in [5] to obtain \( 0 < r_i ≤ ρψi(x_i,0) ≤ ρψi \). Therefore, select the parameters

\[ t_i^* \in \begin{cases} a_i & \text{if } φ_i = G[a_i,b_i]ψ_i \\ [a_i,b_i] & \text{if } φ_i = F[a_i,b_i]ψ_i; \end{cases} \]

(5)

\[ ρψi \in \begin{cases} \max(0,ρψi(x_i(0))), & ρψi \end{cases} \]

(6)

\[ r_i \in (0,ρψi) \]

(7)

\[ γ_i^0 \in (max(ρψi,x_i(0)),∞) \]

(8)

\[ γ_i^∞ \in (0,\max(γ_i^0,ρψi−r_i)) \]

(9)

\[ l_i \in \begin{cases} R_{≥0} & \text{if } γ_i^0 + ρψi ≥ r_i \\ -ln(1+γ_i^∞) & \text{if } γ_i^0 + ρψi < r_i \end{cases} \]

(10)

where it has to hold that \( ρψi(x_i,0) > r_i \) if \( t_i^* = 0 \).

A. Global and Local Satisfaction Guarantees

Theorem 1 provides a global satisfaction guarantee if all clusters in \( Ξ \) satisfy the assumption of Case A in Problem 1.

**Theorem 1:** Let each agent \( v_i \in V \) be subject to \( φ_i \) as in (3b), hence inducing the maximal dependency clusters \( Ξ := \{ Ξ_{i1},...,Ξ_{IL} \} \). Assume that for each \( Ξ_i ∈ Ξ \) it holds that: for all \( v_i,v_j ∈ Ξ_i \) we have 1) \( v_i \) and \( v_j \) can communicate,
2) $\phi_i = \phi_j$, and 3) $t^*_i = t^*_j$, $\rho_{max}^i = \rho_{max}^j$, $r_i = r_j$, and $\gamma_i = \gamma_j$ are chosen as in (5)-(10). If for each agent $v_i \in V$ Assumptions 1-4 hold and each agent $v_i$ applies
\[
u_i(x_{\phi_i}, t) := -\epsilon_i(x_{\phi_i}, t)g_i(x_i)^T \frac{\partial \rho(v_i(x_{\phi_i}))}{\partial x_i},
\]
then it holds that $0 < r_i \leq \rho_{max}^i(x_{\phi_i}, 0) \leq \rho_{max}^i$ for all agents $v_i \in V$, i.e., each agent $v_i$ locally satisfies $\phi_i$, which in turn guarantees global satisfaction of $\{\phi_1, \ldots, \phi_M\}$. All closed-loop signals are well-posed, i.e., continuous and bounded.

**Proof:** The proof can be found in [11].

If $L = M$, i.e., each agent $v_i \in V$ is subject to a non-collaborative formula $\phi_i$, Theorem 1 trivially applies. For the next result, a stronger assumption on $f_j^i(x)$ is needed.

**Assumption 5:** The function $f_j^i : \mathbb{R}^{n_i} \rightarrow \mathbb{R}^n$ is bounded.

Now consider a formula $\phi$ of the form (3b) and assume that each $v_i \in V_{i\phi}$ is subject to $\phi_i := \phi$. Then Theorem 2 guarantees satisfaction of $\phi$ if all agents $v_i \in V_{i\phi}$ collaborate.

**Theorem 2:** Let each agent $v_i \in V$ satisfy Assumption 1 and 5. Consider a formula $\phi$ as in (3b) and let each agent $v_i \in V_{i\phi}$ be subject to $\phi_i := \phi$. Assume that for all $v_i, v_j \in V_{i\phi}$ it holds that: 1) $v_i$ and $v_j$ can communicate and 2) $t^*_i = t^*_j$, $\rho_{max}^i = \rho_{max}^j$, $r_i = r_j$, and $\gamma_i = \gamma_j$ are chosen as in (5)-(10). Assume further that all agents $v_k \in V \setminus V_{i\phi}$ apply a control law $u_k^i$ such that $x_k$ remains in a compact set $\Omega_k$. If for each agent $v_i \in V_{i\phi}$ Assumptions 2-4 hold and each $v_i \in V_{i\phi}$ applies (11), then $0 < r < 1 \leq r_i \leq \rho^i(\phi_i(x_{\phi_i}), 0) \leq \rho_{max} := \rho_{max}^i$, i.e., $(x_{\phi_i}, 0) \Rightarrow \phi$. All closed-loop signals are well-posed.

**Proof:** The proof can be found in [11].

The assumption of $u_k^i$ is not restrictive and excludes finite escape time. For instance, if $x_k := \hat{f}(x_k)$ is asymptotically stable, then $u_k^i(x_k) := -g_i(x_k)$ $x_k$ keeps the state $x_k$ in a compact set. If all agents $v_i \in V_{i\phi}$ apply the control law (11) under the conditions in Theorem 2 to satisfy $\phi$, we refer to this as collaborative control in the remainder. Theorem 2 has further implications with respect to Case A in Problem 1. Consider again the induced maximal dependency clusters $\Xi := \{\Xi_1, \ldots, \Xi_L\}$. Assume that the cluster $\Xi_l$ with $l \in \{1, \ldots, L\}$ satisfies the assumption of Case A, while there exists another cluster $\Xi_m$ with $m \neq l$ such that $\Xi_m$ does not satisfy this assumption. In other words, for all $v_i, v_j \in \Xi_l$ it holds that $\phi_i = \phi_j$, while $\exists v_i, v_j \in \Xi_m$ with $m \neq l$ such that $\phi_i \neq \phi_j$. Consequently, Theorem 2 guarantees local satisfaction of $\phi_i$ for all $v_i \in \Xi_l$. Assumption 4 in Theorem 2 restricts the formula $\phi_i := \phi$ to be locally feasible. This assumption can be relaxed at the expense of not locally satisfying $\phi_i$ by relaxing $r_i$ and $\rho_{max}^i$.

**Corollary 1:** Assume that all assumptions of Theorem 2 hold for each agent $v_i \in V_{i\phi}$ except for Assumption 4 and the choice of $\rho_{max}^i$ and $r_i$. If $\rho_{max}^i \in (\rho^i(\phi_i(x_{\phi_i}, 0), \rho_{\text{opt}}^i)$ and $r_i \in (-\infty, \rho_{max}^i)$, then $r_i \leq r_i \leq \rho^i(\phi_i, 0) \leq \rho_{max}^i := \rho_{max}$. $\epsilon_i$ is used in the second repair stage ($t^*_i$).

**Proof:** The proof can be found in [11].

**B. An Online Detection & Repair Scheme**

Assume now that the cluster $\Xi_l$ with $l \in \{1, \ldots, L\}$ may not satisfy the assumption of Case A in Problem 1. We propose that each agent $v_i \in \Xi_l$ initially applies the control law (11) with parameters as in (5)-(10). The control law (11) consists of two components: $\epsilon_i(x_{\phi_i}, t)$ determines the control strength. The closer $\xi_i(x_{\phi_i}, t)$ gets to $\Omega_c := \{-1, 0\}$, i.e., the funnel boundary, the bigger gets $\epsilon_i(x_{\phi_i}, t)$ and consequently also $\|u(x_{\phi_i}, t)\|$. Note that $\|u(x_{\phi_i}, t)\| \rightarrow \infty$ as $\xi_i(x_{\phi_i}, t) \rightarrow \Omega_c$. The control direction is determined by $\frac{\partial \rho^i(x_{\phi_i})}{\partial x_i}$, i.e., in which direction control action should mainly happen. In summary, the control law always steers in the direction away from the funnel boundary, and the control effort increases close to the funnel boundary. We reason that applying the control law (11) is hence a good initial choice such that $\phi_i$ will be locally satisfied if the participating agents $\forall v_i \setminus \{v_i\}$ behave reasonably. The resulting trajectory $x_{\phi_i}$ may, however, hit the funnel boundary, i.e., $\xi_i(x_{\phi_i}, t) = \{-1,0\}$, and lead to critical events.

**Example 2:** Consider three agents $v_1, v_2,$ and $v_3$. Agent $v_2$ is subject to the formula $\phi_2 := F_{[5,15]}([x_2 - [90 10]]) \leq 5$, while agent $v_3$ is subject to $\phi_3 := F_{[5,15]}([x_3 - [90 10]]) \leq 5$, i.e., both agents are subject to non-collaborative formulas. Agent $v_1$ is subject to the collaborative formula $\phi_1 := G_{[5,15]}([x_1 - x_2]) \leq 10 \land [x_1 - x_3] \leq 10$. Note that the set of formulas $\{\phi_1, \phi_2, \phi_3\}$ is not globally feasible, although each formula itself is locally feasible. Under (11), agents $v_2$ and $v_3$ move to $[90 10]$ and $[90 10]$, respectively. Agent $v_1$ can consequently not satisfy $\phi_1$ and only decrease the robustness such that $r_i < 0$ to achieve $r_i \leq \rho^i(\phi_i, 0) \leq 0$ similar to Corollary 1.

Even if the set $\{\phi_1, \ldots, \phi_M\}$ is globally feasible, there are reasons why critical events may occur as illustrated next.

**Example 3:** Consider two agents $v_4$ and $v_5$ with $\phi_4 := F_{[5,10]}([x_4 - 50 70]) \leq 10$ and $\phi_5 := F_{[5,15]}([x_5 - [50 70]]) \leq 10$. Under (11), agent $v_5$ moves to $[10 10]$ by at latest 15 time units. However, agent $v_4$ is forced to move to $[50 70]$ and be close to agent $v_5$ by at latest 10 time units. This may lead to critical events where (4) is violated for agent $v_4$. If agent $v_5$ cooperates, it can first help to locally satisfy $\phi_4$, e.g., by using collaborative control as in Theorem 2, and locally satisfy $\phi_5$ afterwards.

We propose an online detection & repair scheme by using a local hybrid system $H_i := \{C_i, F_i, D_i, G_i\}$ for each agent $v_i \in \Xi_l$. We detect critical events that may lead to trajectories that do not locally satisfy $\phi_i$ by the jump set $D_i$. Then, agent $v_i$ locally repairs the funnel, i.e., the design parameters $t^*_i$, $\rho_{max}^i$, $r_i$, and $\gamma_i$ in a first stage. If this is not successful, collaborative control as in Theorem 2 will be considered in a second stage (Example 3). If collaborative control is not applicable, $r_i$ is successively decreased by $\delta_i > 0$ in the third stage (Example 2), where $\delta_i$ is a design parameter.

Let $\tilde{p}_i^1 := [n_i \gamma_i \gamma_i] t_i$ and $\tilde{p}_i^2 := [t_i \rho_{max}^i r_i \frac{\partial \rho^i}{\partial x_i}]$ contain the parameters that define (4), and let $p_i^1 := [n_i \ n_i]$; $n_i$ indicates the number of repair attempts in the first repair stage, while $c_i$ is used in the second repair stage ($c$ for collaborative). If $c_i \in \{1, \ldots, M\}$, collaborative control as in Theorem 2 is used to collaboratively satisfy $\phi_i$. If $c_i = 0$, then agent $v_i$ tries to locally satisfy $\phi_i$ by itself and if $c_i = -1$, then agent $v_i$ is free, i.e., not subject to a task.
We define the hybrid state as $z_i := [x_i, t_i, p_i^f, p_i^g] \in Z_i$, where $t_i$ is a clock, $Z_i := \mathbb{R}^n \times \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \times \mathbb{Z}^2$ and $z_i(0,0) := [x_i(0), 0, p_i^f(0), 0]_2$ with $Z$ being the set of integers. The elements in $p_i^f(0)$ are chosen according to (5)-(10). Additionally, we choose $p_i^f(0) = p_i^{f,j}$ if Case A holds for all agents $v_i, v_j \in \Xi_i$. Next, define

$$u_i^{\text{int}} = \begin{cases} 0 & \text{if } c_i = -1 \\ -\epsilon_i(x_{\phi_i}, t_i)g_i(x_i)T\frac{\partial \rho^{\psi_i}(x_{\phi_i})}{\partial x_{\phi_i}} & \text{if } c_i = 0 \\ -\epsilon_i(x_{\phi_i}, t_i)g_i(x_i)T\frac{\partial \rho^{\psi_i}(x_{\phi_i})}{\partial x_{\phi_i}} & \text{if } c_i > 0 \end{cases}$$

so that the flow map can be written as

$$F_i := [f_i(x_i) + f^e_i(x) + g_i(x_i)u_i^{\text{int}} + w_i, 0, 0, 02].$$

External inputs are $w_i$ and $x^{\text{ext}}_i$. By assuming $v_i \in Z_i$, we define $c_i^{\text{ext}} := [c_{j_1}, \ldots, c_{j_{|\Xi_i|-1}}]$ and $p_i^{\text{ext}} := [p_i^{f,j_1}, \ldots, p_i^{f,j_{|\Xi_i|-1}}] \in \Xi_i \setminus \{v_i\}$. Define the external input as $u_i^{\text{ext}} := [u_i^{\text{ext}}, c_i^{\text{ext}}, p_i^{\text{ext}}].$

The set $D_i$ is used to detect a critical event when the funnel in (4) is violated, i.e., when $\xi_i(t_i) \notin \Xi_i := (-1,0)$. Then $D_i := \{z_i, u_i^{\text{int}}, u_i^{\text{ext}} \in \Sigma_i | c_i(t_i) = -1, 0, 0\}$. 

1) Repair Stage 1: The first repair stage is indicated by $D'_{i,1} := D_i \cap \{z_i, u_i^{\text{int}}, u_i^{\text{ext}} \in \Sigma_i | n_i < N_i \}$ where $N_i \in \mathbb{N}$ is a design parameter representing the maximum number of repair attempts in the first stage. If $(z_i, u_i^{\text{int}}, u_i^{\text{ext}}) \in D'_{i,1}$, we first relax the parameters $t_i, \rho_i^\max, r_i$, and $\gamma_i$ in a way that still guarantees local satisfaction of $\phi_i$. Pictorially speaking, we make the funnel in (4) bigger. 

Example 4: Consider the formula $\phi_i := F_{[4,6]} \psi_i$ with $r_i := 0.25$ as the desired initial robustness, which is supposed to be achieved at $t_i^* \approx 4.8$. The original funnel is shown in Fig. 1 and given by $\rho_i^\max$ and $-\gamma_i + \rho_i^\max$ as in (4). Without detection of a critical event, it would hence hold that $\rho_i^\psi(x_{\phi_i}, 0) \geq r_i$; since $\rho_i^\psi(x_{\phi_i}(t_i^*)) \geq r_i$ would be achieved. However, at $t_i := 2$, where $t_i$ indicates the time where a critical event is detected, the trajectory $\rho_i^\psi(x_{\phi_i}(t))$ touches the lower funnel boundary and repair action is needed. This is done by setting $t_i^* := 6$ (time relaxation), $\hat{r}_i := 0.0001$ (robustness relaxation), $\hat{\rho}_i^\max := 1.1$ (upper funnel relaxation), and also adjusting $\gamma_i$ (lower funnel relaxation). The funnel is hence relaxed to $\rho_i^\max$, and $-\hat{\gamma}_i + \hat{\rho}_i^\max$ as depicted in Fig. 1. Due to repair action, $x_{\phi_i}$ locally satisfies $\phi_i$ as shown in Fig. 1.

We first introduce the notation $\{\hat{z}_i \in Z_i | \hat{z}_i = z_i ; \text{exception}\}$ denoting the set of $\hat{z}_i \in Z_i$ such that $\hat{z}_i = z_i$ after the jump except for the elements in $\hat{z}_i$ explicitly mentioned after the semicolon, here denoted by the placeholder exception. With Example 4 in mind, set

$$G'_{i,1} := \{\hat{z}_i \in Z_i | \hat{z}_i = z_i ; t_i^* := \begin{cases} b_i & \text{if } \phi_i = F_{[a_i,b_i]} \psi_i \\
 & \text{if } \phi_i = G_{[a_i,b_i]} \psi_i, \\
r_i &= \text{initiates collaborative control} \right\}$$

where the variables $\zeta_i^u$, $\hat{r}_i$, $\hat{p}_i^{\gamma,\text{new}}$, $\hat{c}_i = i$.

Finally, set $\gamma_i^{0,\text{new}} := (\gamma_i^\max + \zeta_i^u, \hat{r}_i)$, $\hat{p}_i^{\gamma,\text{new}} = 0$ and select, similar to (9) and (10), $\gamma_i^{\infty,\text{new}} \in (0, \min(\gamma_i^\max - \hat{r}_i)]$ and

$$\mu_i^{\text{new}} := \begin{cases} 0 & \text{if } -\gamma_i^\max \geq \hat{r}_i \\
 & \text{if } -\gamma_i^\max > \hat{r}_i \end{cases}$$

Then, set $r_i^{\text{new}} := (r_i^\gamma - \gamma_i^{\infty,\text{new}}) \exp(t_i^{\gamma,\text{new}}) + \gamma_i^{\infty,\text{new}}$ to account for the clock $t_i$ that is not reset ($t_i := t_i^{\gamma,\text{new}}$).

1) Repair Stage 2: Repairs of the second and third stage are detected by

$$D'_{i,(2,3)} := \{z_i, u_i^{\text{int}}, u_i^{\text{ext}} \in \Sigma_i | n_i \geq N_i \}.$$
relaxing the funnel parameters as in the first repair stage. The jump set $D'_{i,2}$ applies if agent $v_j$ detects a critical event. Now changing the perspective to the participating agents $v_j \in \mathcal{V}_i \setminus \{v_i\}$, all agents $v_j$ need to participate in collaborative control. Assume that $v_j \in \Xi_i$, then for agent $v_j$

$$D'_{j,2} := \{(z_j, u^m_{j,2}, u^{ext}_{j,2}) \in \mathcal{S}_j | z_j \in \{-1, 0\},$$

$$\exists v_i \in \Xi_i \setminus \{v_j\}, v_j \in \mathcal{V}_i, c_i = c_j,$$

is the jump set, which is activated when agent $v_j$ asks agent $v_i$ for collaborative control. If $(z_i, u^m_{i,2}, u^{ext}_{i,2}) \in D'_{i,2}$, set

$$G'_{i,2} := \{ \hat{z}_j \in \mathcal{Z}_j | \hat{z}_j = z_j ; \hat{p}_j^i = \hat{p}_i^i, \hat{c}_j = c_i \},$$

where $\hat{c}_j = c_i$ and $\hat{p}_j^i = \hat{p}_i^i$ enforce that all conditions in Theorem 2 hold such that $\phi_i$ will be locally satisfied.

3) Repair Stage 3: If the timing constraints in $D'_{i,2}$ do not apply, repairs of the third stage are initiated by $D'_{i,3} := D'_{i,2} \setminus D'_{i,2}$.

Agent $v_i$ acts in this case by reducing the robustness $r_i$ by $\delta_i > 0$ as illustrated in Example 2 and according to

$$G'_{i,3} := \{ \hat{z}_i \in \mathcal{Z}_i | \hat{z}_i = z_i ; \hat{p}_i^i = \hat{p}_i^{\max} + \alpha_i,$$

$$\hat{r}_i = r_i - \delta_i, \hat{p}_i^{\max} = \hat{p}_i^{\max} + \sigma_i, \hat{p}_i^i = \hat{p}_i^{\new} \},$$

where $\gamma_i = \hat{p}_i^{\max} - \rho \psi_i(x_{\phi_i}) + \delta_i$ is used to calculate $\hat{p}_i^{\new}$, while $\sigma_i > 0$ will avoid Zeno behavior.

4) The Overall System: It now needs to be determined what happens when a task $\phi_i$ is locally satisfied. Define $v_i := \begin{cases} c_i & \text{if } c_i > 0 \\ i & \text{if } c_i = 0 \end{cases}$ and detect such events by

$$D_{i, sat} := \{(z_i, u^m_{i,2}, u^{ext}_{i,2}) \in \mathcal{S}_i | r_{v_i} \leq \rho \psi_i(x_{\phi_i}) \leq \rho \psi_{c_i}, c_i > 0,$$

$$t_i \in \{a_{v_i}, b_{v_i} \} \text{ if } \phi_{v_i} = F_{a_{v_i}, b_{v_i}} \psi_{v_i},$$

$$\text{if } \phi_{v_i} = G_{a_{v_i}, b_{v_i}} \psi_{v_i} \} \setminus (D'_{i} \cup D'_{i,2}),$$

where the set subtraction of $D'_{i} \cup D'_{i,2}$ excludes the case where $D'_{i}$ or $D'_{i,2}$ apply simultaneously with $D_{i, sat}$. This would result in undesirable non-determinism endangering the logic behind the hybrid system. If $(z_i, u^m_{i,2}, u^{ext}_{i,2}) \in D_{i, sat}$, let

$$G_{i, sat} := \{ \hat{z}_i \in \mathcal{Z}_i | \hat{z}_i = z_i ; \hat{p}_i^{i} = \hat{p}_i^{\max}, \hat{r}_i = \hat{r}_i,$$

$$\hat{p}_i^{i} = \hat{p}_i^{\new}, \hat{c}_i = \begin{cases} 0 & \text{if } c_i > 0 \text{ and } c_i \neq i \\ -1 & \text{if } c_i = 0 \text{ or } c_i = i \end{cases} \}$$

where $\hat{p}_i^{\max}$ and $\hat{r}_i$ are chosen according to (6) and (7), but evaluated with $x_{\phi_i}(t_i)$ instead of $x_{\phi_i}(0)$.

Note that $D'_{i} = D'_{i,1} \cup D'_{i,2}$ with $D'_{i,1} \cap D'_{i,2} \cap D'_{i,3} = \emptyset$. The hybrid system $H_i$ is given by $D_i := D'_{i,1} \cup D'_{i,2} \cup D_{i, sat}$ and $C_i := Z_i \setminus D_i$. The jump map $G_i(z_i, u^m_{i,2}, u^{ext}_{i,2})$ is given by $G_{i,1}, G_{i,2}, G_{i,3},$ and $G_{i, sat}$ together with the corresponding jump sets $D'_{i,1}, D'_{i,2}, D'_{i,3},$ and $D_{i, sat},$ respectively. Note now that the sets $D'_{i,1}$ and $D_{i, sat}$ as well as $D'_{i,2}$ and $D_{i, sat}$ are non-intersecting. However, $D'_{i,3}$ and $D_{i, sat}$ are technically non-intersecting. Therefore, if $(z_i, u^m_{i,2}, u^{ext}_{i,2}) \in D'_{i} \cap D'_{i,3}$, which will rarely happen in practice, we only execute the jump induced by $D'_{i,3}$ to not endanger the logic behind the hybrid system. Thereby, we can say that the sets $D'_{i,1}, D'_{i,2},$ and $D_{i, sat}$ are technically non-intersecting.

**Theorem 3:** Assume that each agent $v_i \in V$ is subject to $\phi_i$ of the form (3b) and controlled by $H_i := (C_i, F_i, D_i, G_i)$, while Assumptions 1-5 are satisfied. The induced dependency clusters $\Xi := \{\Xi_1, \ldots, \Xi_L\}$ are such that for each $\Xi_i \in \Xi$ it holds that $v_i$ and $v_j$ can communicate for all $v_i, v_j \in \Xi$. For $v_i \in \Xi_i$ it then holds that $\phi_i(x_{\phi_i}, 0) \geq r_i$, where either $r_i := r_i(0, 0)$ (initial robustness) if $\phi_i = \phi_j$ for all $v_i, v_j \in \Xi$ or $r_i$ is lower bounded and maximized up to a precision of $\delta_i$ otherwise. Zeno behavior is excluded.

Proof: The proof can be found in [11].

V. SIMULATIONS

We consider omni-directional robots as in [12] with two states $x_{1}$ and $x_{2}$ indicating the robot position and one state $x_{3}$ indicating the robot orientation with respect to the $x_{1}$-axis. Let $x_{i,j}$ with $j \in \{1, 2\}$ denote the $j$-th element of agent $v_i$'s state and let $p_i := [x_{i,1}, x_{i,2}, x_{i,3}, x_{i,4}, x_{i,5}].$ We hence have $x_i := [p_i] = [x_{i,1}, x_{i,2}, x_{i,3}, x_{i,4}, x_{i,5}]$, which describes geometrical constraints with $L_i := 0.20$ as the radius of the robot body.

**Scenario 1:** In this scenario, the clusters $\Xi_1 := \{v_1, v_2, v_3\}, \Xi_2 := \{v_4, v_5, v_6\},$ and $\Xi_3 := \{v_7, v_8\}$ are subject to the same formula so that Theorem 1 applies. With $x_A := [50, 50], x_B := [110, 40], x_C := [40, 70], x_D := [55, 70]$, the formulas are $\phi_1 := \phi_2 := \phi_3 := F_{[0, 10, 15]} \psi_{v_1}$ with $\psi_{v_1} := \{(p_{1} - p_{2}) < 2\}$ and $\{(p_{2} - p_{3}) < 2\}$,

$$\{(p_{1} = p_{2}) < 2\} \land \{(p_{2} = p_{3}) < 2\} \land \{(p_{1} = p_{3}) < 2\} \land \{(p_{2} = p_{1}) < 2\}.$$ We assume that $\phi_4 := \phi_5 := \phi_6 := F_{[0, 10, 15]} \psi_{v_2}$ is used with $\psi_{v_2} := \{(p_{3} - p_{5}) < 5\} \land \{(p_{5} - p_{7}) < 5\} \land \{(p_{7} - p_{9}) < 5\} \land \{(p_{9} - p_{11}) < 5\}$, where $\deg{\cdot}$ converts radians into degree. The third cluster employs $\phi_7 := \psi_{v_3} := \{(p_{7} - p_{9}) < 5\} \land \{(p_{9} - p_{11}) < 5\} \land \{(p_{11} - p_{13}) < 5\}$.

The simulation result is shown in Fig. 2a. Note that all tasks are satisfied within the time interval $[10, 15].$

**Scenario 2:** This scenario features two clusters $\Xi_1 := \{v_1, v_2, v_3\}$ and $\Xi_2 := \{v_4, v_5, v_6\}$ simulating Example 2 and 3, respectively. Recall therefore $\phi_1, \phi_2, \phi_3, \phi_4,$ and $\phi_5$ from Example 2 and 3. We set $\delta_1 := 1.5$ and $N_i := 1$ for all agents $v_i \in V.$ Agent trajectories are shown in Fig. 2b, while Fig. 2c shows the funnel (4) for agent $v_1$. It is visible that agent $v_1$ first tries to repair its parameter in stage 1, and then initiates stage 3 to successively reduce the robustness $r_1.$ Agent $v_1$
hence finds a trade-off between staying close to agent $v_2$ and $v_3$ as shown in Fig. 2b. Agent $v_4$ first tries to repair its parameters in Stage 1, but then requests agent $v_5$ to use collaborative control to satisfy $\phi_4$ as indicated in Fig. 2d and 2e. Agent $v_5$ satisfies $\phi_5$ afterwards; $\phi_2$, $\phi_3$, $\phi_4$, and $\phi_5$ are locally satisfied with robustness $r_2 = r_3 = r_4 = r_5 = 0.5$, while for $\phi_1$ it holds that $\rho^{\phi_1}(x_{\phi_1}, 0) > r_1 = -30$.

VI. CONCLUSION

We presented a framework for the control of multi-agent systems, where each agent is subject to a local signal temporal logic task. By leveraging ideas from prescribed performance control, we developed a continuous feedback control law that achieves satisfaction of the local tasks under some conditions. If these conditions do not hold, we combined the developed control law with an online detection & repair scheme, expressed as a hybrid system. Advantages of our framework are low computation times and robustness.

Possible future extensions are the improvement of the repair stages in the online detection & repair scheme. A next step is also to perform physical experiments.

REFERENCES