Semicompliant Force Generator Mechanism Design for a Required Impact and Contact Forces

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A novel design of compliant slider-crank mechanism is introduced and utilized as an impact and contact-force generator. This class of compliant slider mechanisms incorporates an elastic coupler, which is an initially straight flexible beam and buckles when it hits the stopper. The elastic pin-pin coupler, a buckling beam, behaves as a rigid body prior to the impact pushing the rigid slider. At a certain crank angle, the slider hits a stopper generating an impact force. This force can be changed by regulating the angular velocity of the crank and by achieving a desired velocity of the slider. Moreover, after the impact when the slider establishes a permanent contact with the stopper, the maximum contact force can also be adjusted by calculating the coupler dimensions (the length, the width, the thickness, and the amount of compression). The contact duration, the crank angular rotation range, can also be changed and attuned in this mechanism by moving the location of the impacted object. Several mechanism designs with the same working principle are introduced. A prototype compliant slider-crank mechanism is constructed and proved the conceptual contributions of the mechanism. [DOI: 10.1115/1.4002076]

1 Introduction

Compliant mechanisms usually are made from elastic elements which deflect to accomplish a required motion and might incorporate rigid links and flexible links together. Compliant mechanisms transfer an input force or displacement to another point by exploiting their elastic body deformation [1]. The motion of the mechanism is partially (or completely) produced by the elasticity of its links beside the mobility provided by its more conventional rigid-body counterparts.

The concept of compliant mechanism is not new [2]. It started in the 1960s and the compliant mechanisms gained popularities during the 1990s because of its suitability to be manufactured from lesser pieces or as a one piece using an injection molding technique. The compliant mechanisms started to draw more attention during the last decade because they are well suited for microelectromechanical systems (MEMSs) and attracting more attention recently because of its superior properties and suitability for micro sensor designs.

The compliant mechanisms have many advantages over the conventional rigid-body mechanisms. These advantages can be classified in two categories: the cost reduction and the increased performance. In the first category, decreased assembly time, simplified manufacturing processes, and requiring fewer parts can be counted. In the second category, increased precision due to reduced number of joints, accuracy, less wear, reduced weight, and friction, decreased built-in restoring force can be counted. Compliant mechanisms also have fewer movable joints, which result in less wear and have no need for lubrication [2–8].

In compliant mechanisms, energy is stored in the form of strain energy due to the deflection of flexible members. In some cases, this form of potential energy is stored in the whole body where the compliance is distributed. However, the stored energy in flexible members can be unfavorable in some applications where uncontrolled transform of the strain energy to the kinetic energy occurs. On the other hand, this sudden energy release and storage can be utilized for beneficial purposes. This aspect helps to broaden the range of the compliant mechanism design. The force deflection feature and stored strain-energy deflection feature of the flexible members may be taken into account for beneficial purposes [3].

The compliant mechanisms allow the designer a great freedom in the number of possible solutions. Nevertheless, the compliant mechanism design freedom is usually compensated by the difficulties in the analysis of the compliant mechanism members.

Generally, the compliant mechanism analysis is geometrically nonlinear [7] due to the large bending/deflections of its flexible elements. When the elastic deflections are large enough in the nonlinear range, the dynamic analysis is going to be quite complicated. The dynamic response of the compliant mechanism comparing to the quasistatic response of the mechanism can be quite different and the dynamic response of the mechanism may not be ignored. In order to synthesize the compliant mechanisms dynamically, the simulation of the mechanism should be usually done for several different variables due to the difficulties to synthesize the mechanism considering the complexity of the dynamic analysis.

In the mechanism's world, the slider-crank mechanism is one of the most common ones. This is the easiest mechanism to analyze because of the triangular form of its links. The slider-crank mechanism is known by converting a rotational motion into a linear motion. The steady state solutions of the slider-crank mechanism have been studied by Jasinski et al. [9], Zhu and Chen [10], and Badlani et al. [11]. The typical velocity control application of the slider-crank mechanism is found in petrol and diesel engines. This aspect is also covered numerically by changing the angular velocity of the novel compliant slider-crank mechanism introduced in this research.

In what follows, we summarize the state of the art research on the impact and on the control aspects related to flexible/compliant mechanisms and to flexible robot manipulators.

A variable structure control and stabilizing feedback control design by using pole placement technique [12] are studied and applied to the tracking control of the flexible slider-crank mechanism subject to an impact force. Simulation results are provided to demonstrate the performance of the motor-controller flexible slider-crank mechanism not only accomplishing good tracking trajectory of the crank angle but also eliminating vibrations of the flexible connecting rod.

Constant-force compliant mechanisms that involve the addition of a translational spring element to a slider-crank constant-force mechanism are studied [13]. The addition of a translational spring

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that also acts as a slider link provides new design configurations. The translational spring element would have the potential to completely remove the friction from the mechanism and provide a constant-force solution that could replace the current solutions and find applications in hydraulic or pneumatic devices. The same concept of removing the slider and attaching a translational spring may also be used in our research to reduce the number of links.

The nonlinear dynamics [14] of a slider-crank mechanism with a flexible rod is considered where the flexible rod mechanism is modeled with lumped masses and periodically impacted by an external flexible sphere. The impact is modeled using a kinematic coefficient of restitution. The chaotic behavior of the system and the stability of the motion are studied.

The dynamic behavior [15] of a planar flexible slider-crank mechanism with clearance is studied where the motion is characterized by the occurrence of three phases: a free motion, a continuous contact motion, and an impact motion. Both, the rigid and the elastic behavior of the mechanism are studied. It is shown that the coupler flexibility plays the role of a suspension for the mechanism in the presence of a clearance.

Compliant applications of the dual stiffness concepts are first introduced in dual stiffness floor [16], compliant dwell mechanisms incorporating the buckling beams [4,17], and snap through buckling arcs [4]. Currently, the dual stiffness concepts are applied to the robot joints. A novel safe joint mechanism composed of linear springs and a modified slider-crank mechanism is proposed in Ref. [18].

Safe joint mechanisms have high stiffness to an external force that is less than the critical impact force but they have an abrupt drop in the stiffness when the external force exceeds this critical value. This dual stiffness behavior guarantees positioning accuracy and collision safety.

The mechanisms that are called constant-force mechanisms [19] generate reaction force at the out port of the mechanism that does not change for the most of the input range. Constant-force generation might be useful in many applications and include such devices as electrical connectors in which a constant resistance force is desired during the connection procedure and throughout the life of the connector. The proposed mechanism, in this investigation, is not a constant-force mechanism but the concept behind this project is somewhat similar to that of the constant-force mechanism. Producing a required force concept is the same as that of the constant-force mechanism. A constant-force profile is generated in the constant compliant force mechanism, whereas a required impact and contract force can be obtained in the impact and contact-force generator mechanism. However, the general idea of generating a required impact force and contact force is unique.

The object of this investigation is to model the dynamic behavior of the compliant slider-crank mechanism, which is utilized as a required impact generator and contact-force generator. In our design, the compliant slider-crank system incorporates an elastic buckling coupler beam connected to a rigid crank. Figure 1 shows the three different configurations of the compliant slider-crank mechanism incorporating a flexible buckling beam as a coupler. A flexible curved coupler beam is not suitable considering large deflections (large crank angle duration) to this class of mechanism due to two main reasons:

- (1) It vibrates along its longitudinal axis (the axis connecting the crank end joint to the slider) before hitting the stopper.
- (2) It might produce unexpected/chaotic response even at moderate crank speeds.

The proposed design and experimental setup used in this research might be more suitably called as a flexible mechanism rather than a compliant mechanism. However, the flexible mechanisms also refer to the rigid mechanisms with unwanted flexible deflections in literature. The proposed mechanism in this research is a semicompliant mechanism made of four links. The four-link



Fig. 1 Three configurations of the compliant slider-crank mechanism setup

design and the prototype are used to prove the conceptual contribution of the mechanism. The mechanism could be made from lesser pieces by manufacturing coupler and slider together or the use of a slider might be completely eliminated by employing a parallel arm mechanism, as shown in Fig. 2.

The mechanism shown in Fig. 1 is chosen for the current investigation due to its simplicity to formulate, to analyze, to fabricate, and to demonstrate the conceptual contribution of the impact contact force generator (ICFG) mechanism class.

The remainder of this paper is organized as follows. In Sec. 2, the compliant impact-contact-force generator is introduced and its theoretical dynamic analysis is presented. In Sec. 3, the simulation analysis of the theoretical model is performed considering two cases, a constant (a step input) angular velocity and a linearly changing (a ramp input) angular velocity. In Sec. 4, the prototype mechanism and the experimental setup, first, are presented and the simulation results, second, are compared with the experimental results under the similar conditions. Section 4 is followed by the discussion in Sec. 5, and the concluding remarks in Sec. 6.

2 The Compliant Impact-Contact-Force Generator Mechanism

The flexible pin-pin coupler behaves as rigid body prior to the impact pushing the rigid slider. The reciprocal motion of the slider can be obtained in terms of the crank motion. The impact force can be determined and adjusted by changing the angular velocity of the crank. The angular velocity of the crank might be altered at different cycles to regulate the slider's velocity prior to the impact, therefore achieving a desired impact-force magnitude. Moreover, after the impact and the bouncing motion of the slider, the maximum contact force may also be preset by designing the flexible beam dimensions and the deflections of the buckled beam. Contact duration may be changed and controlled by two possible ways:



Fig. 2 Another compliant ICFG mechanism design made from two pieces

- First, the crank angle span during the contact can also be set to a certain measure by changing the fixed location of the impacted object.
- Second, the contact duration can also be adjusted (lengthened or shortened) by regulating the crank angular velocity.

This project presents a new concept of exploiting compliant mechanisms as an impact force and contact generator. A compliant slider-crank mechanism with an initially straight pin-pin buckling flexible beam coupler is employed for this purpose. The compliant slider-crank mechanism in literature [3] has flexible curved couplers beams, which behaves always elastic. On the other hand, the coupler of the mechanism presented in this research behaves as a rigid body prior to the impact eliminating longitudinal vibration along its axis and generating higher impact forces at the initial contact.

The behavior of the semi compliant mechanism will be investigated in three parts:

- *Rigid-body mechanism synthesis prior to the impact.* The flexible initially straight pin ended coupler behaves as a rigid body prior to the impact and it also behaves rigid under the dynamic inertia loads less than the critical Euler buckling load. Note that both the linear and the nonlinear theories give the same critical buckling for the straight beam $p_{\rm cr}=PL^2/EI=\pi^2$ ($p_{\rm cr}$ refers to the normalized load). The linear buckling theory cannot provide large deflection information at the postbuckling range due to its curvature (curvature in linear theory $\kappa=d^2w/dx^2$) assumption of negligible rotations ($\theta=dw/dt$). The results obtained from the nonlinear elastica theory are used in this research to calculate the postbuckling deflection of the flexible coupler and the contact forces.
- *Formulation of impact forces.* Two different approaches are implemented to calculate the impact-force magnitude. In the first approach, linear momentum and linear impulse relation of the impacted bodies are considered. In the second approach, an impact with a linear damper and a linear spring system connected in parallel without a mass are considered.
- *Calculation of the contact force.* After a continuous contact between the slider and the stopper is established, the non-linear elastica theory is used to compute the contact-force history.

A dynamic model considering the aforementioned cases is developed and then simulated. The simulation results are then compared with the prototype's experimental test results. Theoretical analyses of the above cases are presented in Sec. 2.

2.1 Theoretical Analysis. There are several ways to perform dynamic analysis including graphical methods, analytical methods, or simply using commercially available software packages. Graphical methods are helpful to obtain a fast and efficient solution for a particular position. Analytical methods are useful to obtain the solution of the system for the whole range of the motion. Software packages are powerful tools for analysis and design purposes.

Both vector loop closures and closed form solutions are used for the kinematic position analysis of slider-crank mechanism. The velocity and the acceleration analysis of the mechanism then may be obtained by differentiating position solution with the respect to the time [20].

The compliant impact- and contact-force generator introduced in this project runs in the three different working modes in each crank cycle as explained below:

- In the first working mode, before the impact, the flexible buckling link behaves as a rigid body and it pushes the slider resulting in a common slider-crank mechanism. A dynamic analysis is performed for this part.
- In the second working mode, the slider hits the stopper and



Fig. 3 Slider-crank mechanism

then the flexible beam buckles. Two different impact models are used for this part. In the first model, linear momentum and linear impulse equations are employed; in the second model, the stopper is assumed to have a linear spring and linear damper characteristics and modeled as a lumped parameter parallel spring and damper system. When linear momentum and linear impulse equations are used to determine the impact force, the shape of the force profile needs to be specified in order to calculate the maximum impact force. The impact duration is also another important parameter that needs to be determined. It can be measured experimentally from the force and the displacement sensors' readings and then substituted into the mathematical model. The magnitude of the impact force can be adjusted by changing the angular velocity of the crank and obtaining a desired slider velocity. A dynamic analysis is performed until the slider establishes a permanent contact with the stopper.

The third working mode starts right after the permanent contact of the impacting bodies occurs. During the continuous contact, flexible link applies a variable contact force. This force increases until the crank reaches a certain angle, and then decreases as the flexible beam relaxes. The contact force first increases during the compression of the flexible beam, and then decreases during the relaxation part of the flexible beam; after that, the contact ends buckling beam snaps back/returns to its original rigid-body configuration. The first working mode starts all over again. The contact force can be computed by performing the following steps: (1) calculating the distance between the two pin joints of the flexible beam, (2) using the nonlinear load deflection relation of the flexible coupler beam (a two-force member) calculating the force between the pin joints, and (3) calculating the horizontal component of the coupler force (the relevant contact force).

The equations concerning to these three different working modes are stitched together and are switched, when the next working mode starts, in order to obtain a full simulation of the system during the complete crank rotation.

The kinematic diagram of the compliant impact- and contactforce generator is shown in Fig. 3 before the slider hits the stopper.

2.1.1 Rigid-Body Dynamics. In order to perform kinematic analysis of the slider-crank mechanism for the first working mode, the well known loop closure equations may be written as

$$\boldsymbol{R}_2 + \boldsymbol{R}_3 = \boldsymbol{R}_1 \tag{1}$$

The horizontal and the vertical components of the loop closure equation are given below:

$$r_2 \cos \theta_2 + r_3 \cos \theta_3 = r_1 \tag{2}$$

$$r_2 \sin \theta_2 + r_3 \sin \theta_3 = 0 \tag{3}$$

Taking the first and the second derivatives of position analysis with respect to time, the following relations for the velocity and the acceleration could be obtained:

$$-r_2\omega_2\sin\theta_2 - r_3\omega_3\sin\theta_3 = \dot{r}_1 \tag{4}$$

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Fig. 4 The FBD of the mechanism

$$r_2\omega_2\cos\theta_2 + r_3\omega_3\cos\theta_3 = 0 \tag{5}$$

$$\ddot{r}_1 + r_3\alpha_3 \sin \theta_3 + r_2\alpha_2 \sin \theta_2 = -r_2\omega_2^2 \cos \theta_2 - r_3\omega_3^2 \cos \theta_3$$
(6)

$$-r_3\alpha_3\cos\theta_3 + r_2\alpha_2\cos\theta_2 = -r_2\omega_2^2\sin\theta_2 - r_3\omega_3^2\sin\theta_3 \quad (7)$$

The free body diagram (FBD) of the compliant slider mechanism is shown below in Fig. 4.

Using Newton's second law and neglecting the mass and the inertia of the flexible coupler (a two-force member), the following force and moment equations might be obtained for the link 2 (the crank), the link 3 (the flexible bucking beam), and the link 4 (the slider).

For link-2,

$$F_{12x} + F_{32x} = M_2 A_{c2x} \tag{8}$$

$$F_{12y} + F_{32y} = M_2 A_{c2y} \tag{9}$$

$$-F_{32x}r_2\sin\theta_2 + F_{32y}r_2\cos\theta_2 + \tau_m = I_2\alpha_2$$
(10)

For link-3, before the buckling, link-3 acts as a rigid body and transmits force from link-2 to link-4; after the buckling it behaves as a nonlinear spring $F_{32}=f_{NLS}(R_{30}-R_3)$.

For link-4, neglecting the friction force (a dry friction or a viscous friction might be added to this equation),

$$-F_{32x} - F_{\text{ext}} = M_4 \ddot{r}_1 \tag{11}$$

$$-F_{32y} + F_{14y} = 0 \tag{12}$$

Acceleration components of the crank mass center are given as

$$A_{c2,x} = -r_{c2}\alpha_2 \sin \theta_2 - r_{c2}\omega_2^2 \cos \theta_2$$
(13)

$$A_{c2,y} = r_{c2}\alpha_2 \cos \theta_2 - r_{c2}\omega_2^2 \sin \theta_2 \tag{14}$$

When the crank angle reaches to a certain angle, the impact phenomena will occur, which causes the flexible beam to buckle. The impact force can be adjusted by altering the velocity of the slider.

2.1.2 Impact Models. The impact of the slider might be modeled by either using linear impulse and linear moment relationship or choosing an elastic model (a linear spring and a linear damper connected in series) of the impacted body similar to Kelvin–Voigt model [21]. Both of these approaches are incorporated to our simulation program

Impact model I. Using linear momentum and linear impulse relationships, the coefficient of restitution I_e may be written as



Fig. 5 Elastic impact model

$$I_{e} = \frac{\int_{t_{o}}^{t} F_{r} dt}{\int_{t_{o}}^{t} F_{d} dt} = \frac{m_{sld}(v_{o} - v_{1}')}{m_{sld}(v_{1} - v_{o})} = \frac{v_{o} - v'}{v_{1} - v_{o}}$$
(15)

where v_1 and v'_1 represent the velocities of the slider prior to the impact and right after the impact, respectively and v_0 represents the common velocity of both bodies which is zero in our case.

Impact phenomenon is always accompanied by energy loss, which may be calculated by subtracting the kinetic energy of the system just after the impact from that of the just before the impact. According to classical impact theory, the value of coefficient of restitution lies between $0 < I_e < 1$. In this research, the collision occurs between the extension of the slider and the force sensor (wood and steel surfaces). The linear impulse and the linear momentum relationship for the slider may be written as

$$mv_1 + \int Fdt = mv_1' \tag{16}$$

where $\int F dt$ represents the total impulse applied to the slider during the impact including, the impulse due to impact with the stopper, the impulse due to pin-pin spring force, and the impulse due friction which may be neglected because of its smaller magnitude.

When computing the impact force using the above equations (15) and (16), the shape of the impact force has to be specified (a pulse, a half sine wave, etc.) and the impact time. More realistic impact-force shapes might be selected considering the following issues:

- 1. by assuming the velocity and the acceleration profile of the slider during the impact
- 2. by observing the experimental velocity and acceleration profiles of the slider while impacting with the stopper
- 3. by measuring impact time, which is unknown before the collision experiment are performed for the impacting bodies

Impact model II. The stopper (the force sensor) can be represented by a linear damper and linear spring system connected in parallel, as shown in Fig. 5. A similar application of the model, called the snubber model, and its MATLAB and SIMULINK files can be found in Dynamic Modeling and Control of Engineering Systems [22].

The slider hits the stopper when the slider position x_{sld} reaches the stopper position x_{stp} . It leaves the contact (it bounces back) when the reaction force applied by the stopper becomes zero, or it continues to remain in contact if the stopper force does not go



Fig. 6 The kinematic diagram of the compliant slider-crank mechanism at two positions

below the vertical component (vertical according to Fig. 5) of the critical buckling beam load. The slider's velocity and the dynamic contact-force relation applied by the stopper to the slider might be represented by the following relations:

$$\frac{dv_{sld}}{dt} = \frac{1}{m_{sld}} (F_{NLX} - F_{stp}) \tag{17}$$

Condition 1. For $x_{sld} < x_{stp}$,

$$F_{stp} = 0 \tag{1}$$

(8)

Condition 2. For $x_{sld} \ge x_{stn}$,

$$F_{stp} = \max[0, k_{stp}(x_{sld} - x_{stp}) + b_{stp}v_{sld}]$$
(19)

Equations (18) and (19) are also utilized later to obtain the equivalent parameters of the stopper and to compare the simulation results with those of the experimental cases. Although the above equations (17)–(19) are linear in their nature, switching between these equations causes nonlinearity in the system.

2.1.3 Contact Model. When the pin-pin flexible beam deflects, a nonlinear force (function of the deflection) is applied to the slider. The buckling force history may be calculated considering the pinned-pinned end conditions of the flexible beam using the elastica theory. A polynomial-fit function [23,24] to the exact non-linear large deflection analysis of the flexible pinned-pinned buck-ling beams is used in this research. In order to simplify the solution of the compliant system, the polynomial fit to the nonlinear elastica theory and the nonlinear algebraic/transcendental equations are used to perform the kinematic simulation just after the continuous contact between impacting bodies are established

In Fig. 6, the kinematic diagram of the mechanism considering the initial contact angle at the beginning of the impact is represented with a solid line, and the kinematic diagram considering the buckling beam motion is represented with a dashed line.

In Fig. 6, $\varphi = 360^{\circ} - \theta_3 = -\theta_3$ is R_3 's complementary angle with the horizontal, θ_{2cr} is the certain angle where initial contact is established, and *e* is the distance between the crank's end joint and the stopper. The undeflected length (the original length) of the flexible beam may be written as

$$L_{\text{orig}} = L_{\text{beam}}(\theta_2) + U \tag{20}$$

where $L_{\text{beam}} = R_3$ represents the deflected beam's length and U represents the flexible beam's deflection.

$$L_{\text{beam}}(\theta_2) = \sqrt{(R_2 \sin \theta_2)^2 + (e - R_2 \cos \theta_2)^2}$$
(21)

From the geometrical analysis of the mechanism (see Fig. 6), the deflection of the flexible beam U can be expressed as

$$U = L_{\text{orig}} - \sqrt{(R_2 \sin \theta_2)^2 + (e - R_2 \cos \theta_2)^2}$$
(22)

The complementary contact angle φ can be found as

$$\varphi = \tan^{-1} \left(\frac{R_2 \sin \theta_2}{e - R_2 \cos \theta_2} \right)$$
(23)

When the flexible beam buckles, it applies a nonlinear force function at its postbucking range to the slider. The flexible pinnedpinned coupler is a two-force member and its normalized force



Fig. 7 Normalized load versus normalized displacement

deflection is calculated using the nonlinear elastica theory. A polynomial curve fit plot to the exact nonlinear solution plot is shown in Fig. 7 and the corresponding normalized equation is given below.

$$p(u) = 7.1908u^4 - 6.2016u^3 + 6.2532u^2 + 4.3898u + 9.8879$$
(24)

where *u* and *p* represent the normalized deflection u=U/L and the normalized load $p=PL^2/EI$. The reader should refer to Refs. [23,24] for a more detailed analysis.

The schematic of the mechanism's stopper is shown below (see Fig. 8) during the contact, and the following equations might be used to calculate the contact force.

$$F_{\text{pin-pin}} = \frac{p(u)EI}{L_{\text{beam}}^2}$$
(25)

$$F_{\text{contact}} = F_{\text{pin-pin}} \cos \varphi \tag{26}$$

3 Simulation Results

The equation set, including all equations between Eq. (2) and Eq. (26), is used to obtain the numerical simulation results of the compliant slider-crank mechanism hitting a stopper. The simulation-block-diagram method is used in MATLAB and SIM-ULINK to solve Eqs. (2)-(26). The simulation-block diagram of the mechanical system consists of three subsystems. In the first subsystem, the dynamic analysis of the slider-crank mechanism is included. In the second subsystem, the impact force might be calculated by choosing two approaches. In the third subsystem, the contact force is calculated by using polynomial fits to exact nonlinear elastica solution. These three sets of subsystems are stitched together by using "if-else statement." Two cases are separately simulated: The first case involves in a constant angular velocity of the crank, and the second case involves in a constant angular acceleration of the crank (a linear/ramp input of the crank's angular velocity).

The simulation result of the mechanism is presented in Fig. 9 for a constant angular velocity (2 rad/s). The impact force is approximately 57 N as seen from the Fig. 9 and the period of the movement of the slider is 3.14 s for 1 revolution.



Fig. 8 The slider generates a contact-force

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Fig. 9 The impact-/contact-force magnitudes for a constant angular velocity ω_2 =2.0 rad/s (case 1)

The power requirement of the rotational actuator for case 1 is calculated and plot below in Fig. 10, showing that the maximum power requirement for a constant angular velocity $\omega_2=2.0$ rad/s is 5.5 W. Since the experimental setup is intended to be used in similar angular velocities, a DC motor with 17 W power is chosen.

The magnitude of the impact force can be adjusted by regulating the slider's velocity. The impact force and the contact force are shown in Fig. 11 for a ramp angular velocity input. The input angular velocity linearly changes between 0.5 rad/s and 5.5 rad/s for the current case. The magnitudes of the impulse forces increase when the angular velocity of the crank increases. Moreover, the periods of the impulses decreases as a result the increasing angular velocity of the crank. However, it is easily seen that the duration time (not the crank duration angle which is always constant for a specified dimensions) of the contact force decreases when the angular velocity of the crank rises.

3.1 A Methodology to Determine Impact Forces. Calculating a desired impact force is more complicated than achieving a desired impact force experimentally. Predetermining the impact force, before the experimental setup is build, is difficult if not



Fig. 10 The required power of the mechanism for case 1



Fig. 11 The impact-/contact-force magnitudes for a ramp angular velocity input (0.5–5.5 rad/s) (case 2)

impossible because it depends on the unknown values of the prototype mechanism.

Two methods are used to calculate the impact forces. There are several parameters related to these methods that need to be determined before running the simulations. Considering method 1, the impact duration is not known, the shape of the force profile needs to be decided, and the coefficient of the restitution I_{ρ} is ascertain, which depends on not only the materials of the impacting bodies but also their shape. Considering method 2, the values of lumped parameters $(b \text{ and } k) \mod k$ model may not be predetermined before the experiments are carried out. First, the construction of the mechanism and collection of data from the experimental setup are required. However, if they (b and k) were determined from the experimental setup, the magnitude of the impact force still needs to be determined by a simulation based on calculation procedure, since a simple calculation is not enough. The flexible beam horizontal force component and its stiffness also play an important role on the magnitude of the impact force and on the impact time since the flexible beam is pushing the slider. Since there are five important parameters that define the maximum impact force (v, b, d)k, $F_{(pin-pin)X}$, and $k_{elastica}$), it is unpractical to calculate and to present synthesize charts.

3.2 Stress Calculation. The flexible buckling beam maximum stress analysis is presented in this section. The three working modes of the mechanisms are related to the flexible buckling beam's stresses. In the first operating mode, the flexible buckling beam is under the inertial loads. In the second operating mode, the beam is under the axial compression (impact) load. In the third operating mode, the beam deflects; therefore, it is under bending stresses. Before the flexible beam deflects, the stress component is the axial stress and may be calculated as

$$= P/A \tag{27}$$

where *P* is the applied axial load and *A* is the cross sectional area $(A=bh \text{ and } I=bh^3/12)$. The axial compression stress during the impact is about $57/(0.03 \times 0.0003)=6.33$ MPa, which could be neglected compared to bending stresses.

After the flexible beam deflects, the primary stresses are due to the bending stresses. The buckled flexible beam's curvature changes as the flexible beam deflects. The bending moment along an initially straight beam [23] may be written as

$$M = 2k\sqrt{EIP}\cos\phi \tag{28}$$

The pin ends are inflection points, hence do not carry end moments. Therefore, the pin ends' angles ϕ changes between 0.5π and 1.5π . Considering only bending stresses assuming the beam middle axis is inextensible (Bernoulli beam) and ignoring stresses due to axial load for a pinned-pinned flexible initially straight flexible beam; maximum bending moment occurs in the middle of the beam where $\phi = \pi$. The primary bending stress is expressed by the following formula:

$$\sigma_b = \frac{M}{I} y \tag{29}$$

Using the aforementioned equations, the following maximum bending stress of the rectangular cross section area may be calculated as

$$\sigma_{\rm max} = 2k\sqrt{3EP/bh} \tag{30}$$

where *b* and *h* are the cross sectional dimensions width and thickness, and *k* is the shape factor, respectively. Considering Eq. (30), *k* is the shape factor and *P* is the resultant force acting along the pin joints ($F_{pin-pin}$). The shape factor of the pin-pin elastica is given by the following rational fit. The reader should refer to Ref. [24] for a more detailed analysis.

$$k = \frac{0.087u^3 + 1.332u^2 + 0.1762u}{u^2 + 0.7351u + 0.01656}$$
(31)

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Fig. 12 Maximum stress history of the buckling beam

where *u* is the normalized beam deflection u = U/L.

The maximum stress history of the buckling beam is calculated for full crank rotation. The maximum stress must be kept below the yield stress within the elastic range of the material. The maximum stress is calculated as σ_m =398.75 MPa, which is the below yielding strength of the 1095 spring steel (Howell 2001, Appendix C.5).

3.2.1 Fluctuating Stresses and Fatigue Failure in Compliant Members. Fluctuating stresses are commonly seen in compliant mechanism elastic members. These stresses may be expressed in terms of a mean stress component σ_m and an alternating stress component σ_a . If the stress condition is below the two lines described in modified Goodman diagram for fatigue failure [3], the compliant member is expected to have an infinite life. The safety factor *SF* for the modified Goodman and the yielding lines may be expresses as

$$\frac{1}{SF} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}$$
(32)

The endurance limit S_e can be calculated by the following formula:

$$S_e = C_{\text{surf}} C_{\text{size}} C_{\text{load}} C_{\text{reliab}} C_{\text{misc}} S_{e'}$$
(33)

where the uncorrected endurance limit may be approximated with $S_{e'} \approx 0.5S_{ut} = 507$ MPa using the ultimate tensile strengths $S_{ut} = 1014$ MPa and $S_y = 500$ MPa of the 1095 spring steel (Howell 2001, Appendix C.5). The corrected endurance limit is $S_e = 0.5248 \times 507 = 266$ MPa. The parameters given in Eq. (33) are calculated for the buckled beam as follows:

- surface factor $C_{surf}=0.8712:1.58\times(S_{ut})^{-0.086}$ MPa for polished members, and spring steels are available as polished sheets
- size factor $C_{\text{size}}=1.0$ for the small pieces if the equivalent diameter is less than 2.79 mm, $d_{\text{eq}}=0.808(bh)^{0.5}$ =2.42 mm for a rectangular cross section
- load factor C_{load} =1.0 because the stresses in buckled beam are due to bending
- reliability factor $C_{\text{reliab}}=0.753$ for 99.9% reliability
- stress concentration *effect* $C_{\text{misc}} = 0.800$; $C_{\text{misc}} = 1/K_{\text{geo}}$ = 1/1.25 where K_{geo} is the geometric stress concentration factor

The maximum stress history of the buckling beam is plotted (see Fig. 12) for 3 cycles of crank rotation. The modified Goodman theory uses a sinusoidal fluctuating stress history. Therefore, a sinusoidal stress having the same amplitude with the buckling beam stress is used for the calculations, as shown in Fig. 12. Since the assumed sinusoidal stress history is greater than the actual stress history, this calculation will provide more conservative results. The maximum stress σ_m =398.75 MPa and the alternating stress component σ_a =198.38 MPa. The abovementioned factors

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Fig. 13 The compliant slider-crank prototype mechanism

results in the safety factor SF=1.14. If SF result is greater than 1, it is the indication of infinite life for fatigue failure.

4 Experimental Setup

The experimental setup and the prototype semicompliant mechanism are described in this section. Figures 13 and 14 show a picture of the compliant mechanism prototype and its schematic, respectively. This prototype is used to validate the conceptual contributions of the current research. The mechanism is mainly composed of a crank, a flexible initially straight buckling beam and a slider.

The prototype mechanisms' dimensions and related parameters are listed in Table 1. E is the modulus of elasticity. The variables b, h, and I are the flexible beam width, thickness, and moment of area, respectively.

The theoretical values of the flexible beam maximum contact forces are compared for different plane thicknesses of the flexible beam before running the dynamic simulations using the values in Table 1. The magnitude of contact force generated by the pin-pin buckling beam depends on its dimensions: the length, in plane thickness, and out of plane width. The theoretical values of the maximum contact force at the location of the maximum deflection are computed for 0.1 mm, 0.3 mm, and 0.5 mm in plane thicknesses and they are 0.0655 N, 1.7684 N, and 8.1871 N, respectively. The maximum contact force rises with the cubic power of the thickness of the flexible beam due to the second moment of area depends on the cubic power of in plane thickness for the rectangular cross sections. The same force increases linearly with the out of plane width of the flexible beam. As a result, the desired maximum contact force can be obtained by changing these dimensions. In addition to these parameters, the amount of maximum deflection and the pin-pin coupler angle are the other factors that determine this force. The location of the stopper is another geometric parameter that determines the contact force.



Fig. 14 Schematics of the slider-crank mechanism

Table 1 The slider-crank mechanism's dimensions

Parameter	Value	
<i>r</i> ₂	140 mm	
r ₃	310 mm	
Ē	210×10^{9} MPa	
b	30 mm	
h	0.3 mm	
Ι	0.0675 mm^4	



Fig. 15 The rigid mode of the slider-crank mechanism

The contact duration respect to crank angle may also be adjusted by changing the location of the stopper relative to the slider. The limit locations lie between the two limit positions of the slider corresponding 360 deg and 0 deg crank angle contact durations.

The experimental setup was constructed to allow testing for different motor angular velocities. A DC motor is with built-inhouse gearbox is used to increase the torque of the motor. Three sensors are incorporated into the mechanical setup including the following:

- A separate encoder is attached to the crankshaft to measure the crank angle precisely and to calculate the angular crank velocity. Even though a DC motor within house encoder is used for the experiments, the response of the DC motor's encoder was not good enough to our expectations. Therefore, another more precise encoder is attached to the crankshaft.
- An ultrasonic position sensor is used to measure the position of the slider and to calculate the slider's velocity. The position sensor is mounted behind the stopper.
- A force sensor is used as a stopper to measure forces generated by the impact and the contact, which measures the forces up to 44.5 N (100 lbf).

Both the position sensor and the force sensor responses are also used to measure the impact time. The data from the measurements of the impact forces, contact forces and, positions are acquired with IOtech personal DAC 3000 series data acquisition card. Data processing is performed by MATLAB and SIMULINK software packages.

The working principle of the mechanism is different from that of the conventional slider-crank mechanism. This mechanism behaves in two different modes: the rigid mode and the flexible mode. The working modes of the mechanism sketches were previously shown in Fig. 1 and relevant pictures of the prototype are presented in Figs. 13 and 15.

At the crank angle range of approximately between +60 deg and -60 deg, the mechanism behaves like a flexible slider-crank. When the crank angle reaches +60 deg, the slider hits the stopper and the system generates an impact force applied to the stopper. After the bouncing motion of the stopper halts, a continuous contact force is formed by the slider applied to the stopper.

For the other crank angle locations, the compliant mechanism behaves like a rigid slider-crank mechanism. The magnitude of the impact force and the contact time can be adjusted by changing the angular velocity of the actuator (DC motor). The crank angular velocity is regulated by changing the input voltage of the DC motor. This system starts to work near 2.0 V input voltage and continues to measure the impact and contact force up to the some limit voltage, above this voltage the force sensor output is of its required measurement range.

4.1 Experimental Results. Experimental results are obtained for two different voltages of the DC motor (having different distinct angular velocity histories). Therefore, the effects of the voltage/angular velocity on the generated impact forces can be observed. 2.5 V and 2.9 V constant inputs are supplied to the DC motor. The data collected from the encoder and the ultrasonic distance sensors are processed to obtain the angular velocity of the



Fig. 16 Crank angular velocity (2.5 V input)

crank and the velocity of the slider.

The crank's angular velocity and the slider's velocity are obtained numerically from the angular and linear displacement data of the relevant sensors by filtering the raw data with the following differential operator.

$$D = \frac{d}{dt}[V(t)] = \frac{\tau s}{\tau s + 1}[V(s)] = \frac{12.5s}{12.5s + 1}[V(s)]$$
(34)

The aforementioned differential operator D guarantees the causality and selecting τ =12.5 s filters some of the discontinuities of the differential operator and filters some noise. In order to obtain a noise free angular velocity results and the slider translational velocity results, the data are further filtered by the following operator

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{100}{s^2 + 2\times 0.707 \times 10s + 100}$$
(35)

The low-pass filter G(s) has the cutoff frequency at $\omega = 10$ rad/s and $\xi = 0.707$, which assures continuous noise free response of the crank's angular velocity and the slider's translational velocity, respectively. After the second filtering operation, a zero phase filter is further applied to correct the phase difference obtained by the previous second order filtering. Figure 16 compares the calculated angular velocity response of the crank raw data and the processed data at the steady state running conditions.

Figures 17 and 18 demonstrate the experimental results for the slider's position and velocity, respectively.

Since the presented research claims that its novelty is in designing for a given impact and contact force, the simulation results should be compared with the experimental test results under the similar conditions if it were not very difficult or impossible to be compared under the same conditions. A constant angular velocity approximated from the experimental results is used for the simulations because the DC motor angular velocity control is out of the scope of the current research. The desired impact forces, which can be produced by controlling the slider velocity, in each cycle for the same (or similar) compliant mechanism configuration (e.g., a four bar with flexible coupler) is left to a possible future work.



Fig. 17 Slider position (2.5 V Input)



Fig. 18 Slider's velocity filtered results (2.5 V input)

4.2 Calculating Equivalent Damping and Stiffness of the Stopper. Two voltages other than the experimental case voltages (2.5 V and 2.9 V) are used to determine the equivalent damping and the stiffness coefficient of the stopper. For each voltage value, ten readings are collected and the average value of corresponding case readings is calculated (see Table 2).

The force exerted by the stopper model on the slider is given by

$$F_{stp} = b_{stp} \dot{x} + k_{stp} x \tag{36}$$

Using discrete representation of the equation given above, the following relation might be obtained.

$$F_{stp} = b_{stp} \dot{x} + k_{stp} \dot{x} \Delta t \tag{37}$$

where Δt is the sampling time and equals to 0.002 s in our case. Using the values given in Table 2, the following equations are obtained:

$$f_1(b_{stp}, k_{stp}) = 0.1309 \times b_{stp} + 0.1309 \times 0.002 \times k_{stp} - 22.8085 = 0$$
(38)

$$f_2(b_{stp}, k_{stp}) = 0.2165 \times b_{stp} + 0.2165 \times 0.002 \times k_{stp} - 37.3589 = 0$$
(39)

Table 2 Average experimental readings

DC motor (V)	Impact force (N)	$\frac{dx/dt}{(m/s)}$
2.2	22.8085	0.1309
2.7	37.3589	0.2165



Fig. 20 Force history results comparing the experiment and the simulation (2.5V input-1.15 rad/s)

The maximum value of the impact force reading and the sampling time is used to represent the above equations in discrete form. Since the sampling rate is the same/constant, the determinant of above equations equals to zero.

$$\det \begin{bmatrix} 0.1309 & 0.1309 \times 0.002 \\ 0.2165 & 0.2165 \times 0.002 \end{bmatrix} = 0$$

Using Eq. (38), the solution range of the b_{stp} and k_{stp} are found by plotting the results for a range of solution at the positive solution domain, and taking absolute value of the function $f_1(b_{stp}, k_{stp})$, so the solution line that cuts the z=0 plane can be observed.

The solution range of $f_1(b_{stp}, k_{stp})$ is between 0 N/m s and 180 N/m s for b_{stp} and it is between 0 N/m and 87000 N/m for k_{stp} , as seen from the Fig. 19. Two planes represented by Eqs. (38) and (39) cut each other in a single line (see Fig. 19) and the resulting line has a near zero value at the point where $b_{stp}=110$ N/m s and $k_{stp}=30000$ N/m. These values are then plugged into equations that represent the stopper impact model II.

4.3 Validation of the Theory and the Simulation Results. Simulation results are obtained considering for two cases at the constant angular velocities approximating the experimental angular velocities of 2.5 V and 2.9 V dc inputs. The results are then compared for both cases, as shown in Figs. 20 and 21, respectively.

The force profiles match nicely with the constant angular velocity simulation results and the constant voltage input (to the DC motor) experimental results for both cases. In case 1 (see Fig. 20), constant speed simulation results concerning the impact-force



Fig. 19 (a) Absolute value of $f_1(b_{stp}, k_{stp})$ function representing Eq. (38) (b) Solution domain of $f_1(b_{stp}, k_{stp})$ and $f_2(b_{stp}, k_{stp})$ planes representing Eqs. (38) and (39).



Fig. 21 Force history results comparing the experiment and the simulation (2.9V input-1.45 rad/s)

magnitudes are the same as the experimental results and slight changes at the experimental maximum impact forces are also observed. In case 2 (see Fig. 21), experimental maximum impact forces are slightly higher than those of the simulation results.

An enlarged view of the first hump in case 1 is presented in Fig. 22. Experimental results show multiple near zero contact-force regions just after the impact. On the other hand, the simulation results switch to the third phase of calculation procedure, as explained in Sec. 2.1.3, impact model II. At the continuous contact region, the experimental-contact-force profile results are slightly higher than the simulation results until the maximum contact force is reached and slightly lower than the simulation results after that. The sudden change of the crank angular velocity just after the impact is thought to be the cause of this difference. At the end of the contact region, the experimental force results go to zero before the simulation results, which is due to slight misalignment of prototype mechanism's slider.

5 Discussions

The models and the prototype investigated in this research are considered to be a mechanical system because it does not include a DC motor model. However, the control aspect of the DC motor's angular velocity (a mechatronic system), therefore, the controlled slider impact velocity causing different required impact forces in each cycle, is left for a possible future study. Moreover, the contact duration may be adjusted in time domain by controlling the angular velocity.

The inertia of the crank and the inertia of the slider will affect the contact-force profile at moderate crank speeds. Even these inertia forces might reach the critical bucking beam load before the slider hits the stopper. This phenomenon could occur if either the bucking beam is too thin or the mass of the crank and of the slider is too high relative to the bucking beam flexibility. The simulation program of the current research may easily be modified considering the moderate crank speeds, but a sturdier prototype has to be manufactured and sensors with wider force measuring range and higher precisions and faster response times are needed.



Fig. 22 Zoom in to the first hump in Fig. 1, case 1 (2.5 V)



Fig. 23 A new design of the compliant ICFG suitable for both macro and MEMS applications

The dynamics response of the current complaint mechanism could be quite different considering the higher angular velocities. Several aspects of the dynamic behavior of the compliant impactcontact-force generator are discussed below.

First, the load deflection characteristics obtained by the nonlinear elastica solution is valid if the elastic beam buckles in its first mode having at most two inflection points along its length. The load deflection characteristic used in this research considers the first bucking mode only. The first mode solution assumption is valid up to a certain frequency, (until moderate crank speeds) and is not valid at the crank speed higher than the first vibration mode of the buckling beam concerning large amplitude vibrations [25]. To the best of authors' knowledge, an analytical load deflection solution of the elastica at high frequencies does not exist in literature; therefore, nonlinear finite element methods have to be used for high-speed angular velocity simulations. The pin-pin elastic buckling beam (considering the large deflections) under the sinusoidal load will behave chaotically at high crank speeds, which makes the current mechanism application useless at high-speed applications due to its unpredictable behavior. Still, there is a possibility of the periodic responses at certain high frequency crank angular velocity ranges. The complete affect of the forcing frequency considering the moderate-higher crank angular velocity range over the nonlinear vibration system requires parametric stability analysis [26], which includes the solution of Mathieu type differential equations and plotting Strut diagrams, but it is not in the scope of this investigation. Chaotic behaviors are also observed in vibro-impact models with linear springs subjected to sinusoidal loads with high vibration frequencies [21,27].

The ICFG concept may find applications in microdimensions also. The following design shown in Fig. 23 is proposed and it is suitable to be manufactured both macro- and microdimensions. The shuttle part of the mechanism with the compliant arcs are already considered and fabricated as a part of a bistable MEMS [3,28] that also is used as a part of the complaint double dwell mechanism introduced in Ref. [4] and as an example of several MEMS crash sensor designs presented in Ref. [29]. In microdimensions, the proposed mechanism can be actuated either by the gears, as shown in Fig. 23, or by comb drives [3]. The ICFG mechanism may find applications where two actions are required together; the first one is a high force requirement task in a short duration, such as punching, cutting, or breaking, and the second one is low force requirement task in longer durations, such as holding, gluing, and applying pressure.

6 Conclusions

This investigation presented a novel use of a slider-crank mechanism incorporating initially straight flexible buckling beam

used as a force generator for required impact and contact forces. A dynamic model has been developed and implemented. The prototype proved the conceptual approach behind the design and the synthesis of the compliant ICFG mechanism. Different impact forces are obtained by changing the crank angular velocity experimentally and numerically.

In this mechanism, the slider attached to the crank with a flexible beam. The flexible beam acts as a rigid body and does not buckle until the certain angle is reached. When the slider hits the stopper, the slider generates a desired impact force, after that flexible beam buckles and applies an increasing contact force to the impacted object. This force cycle characteristics might find application areas, where objects with different thicknesses, hardnesses and delicacies need to be cut/punch and attached/glued/molded to another object or a surface.

The simulation results have been compared with those of the experimental results under similar conditions and matches nicely validating the theoretical analysis and the simulation programs. Different mechanism designs are presented in this paper and some of these are suitable for MEMS fabrications and applications.

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Nomenclature

- R_1 = ground link length
- R_2 = crank length
- R_3 = flexible link length
- θ_1 = ground link orientation
- $\theta_2 = \text{crank angle}$
- θ_3 = flexible link angle
- ω_2 = angular velocity of crank
- ω_3 = angular velocity of flexible link
- A_2 = angular acceleration of crank
- α_3 = angular acceleration of flexible link
- F_{ij} = force form components *i* to *j*
- F_{ext} = external force
- $F_{\rm imp}$ = impact force τ_m = motor torque
- M_i = mass of the *i*th link
- A_{c2} = acceleration of the crank mass center
- I_{e} = coefficient of restitution
- F_r = impact force during the restoration
- F_d = impact force during the deformation
- \ddot{V} = velocity
- m_{sld} = mass of the slider
- x_{sld} = position of the slider
- v_{sld} = velocity of the slider
- x_{stp} = position of the stopper
- b_{stp} = equivalent damping of the stopper
- k_{stp} = equivalent stiffness of the stopper
- L_{orig} = length of flexible beam before buckling: R_{30}
- $L_{\text{beam}}(\theta_2)$ = length of flexible beam after buckling: R_3
 - P = flexible buckling beam load represented with a polynomial
 - p = normalized load
 - U = flexible buckling beam deflection
 - u = normalized deflection
 - e = displacement between the crank joint and the stopper
 - applied complementary force angle with the $\omega =$ horizontal
 - θ_{2cr} = crank angle when the slider hits the stopper
 - EI = flexible beam's elastic rigidity

$$F_{\text{pin-pin}} = \text{buckling beam force}$$

 $F_{NLX} = \text{the component of the } F_{\text{pin-pin}}$ force along the
slider direction
 $F_{\text{contact}} = \text{contact force}$

References

- [1] Midha, A., Howell, L. L., and Norton, T. W., 2000, "Limit Positions of Compliant Mechanisms Using the Pseudo-Rigid-Body Model Concept," Mech. Mach. Theory, 35, pp. 99-115.
- [2] Her, I., and Midha, A., 1987, "A Compliance Number Concept for Compliant Mechanisms and Type Synthesis," ASME J. Mech., Transm., Autom. Des., 109(3), pp. 348-355.
- [3] Howell, L. L., 2001, Compliant Mechanisms, Wiley, New York
- [4] Sönmez, Ü., 2007, "Introduction to Compliant Long Dwell Mechanism Designs Using Buckling Beams and Arcs," ASME J. Mech. Des., 129, pp. 831-843.
- [5] Sevak, N. M., and McLarnan, C. W., 1974, "Optimal Synthesis of Flexible Link Mechanisms With Large Static Deflection," ASME Paper No. 74-Der-83.
- [6] Howell, L. L., and Midha, A., 1994, "The Development of Force-Deflection Relationships for Compliant Mechanisms," ASME Machine Elements Machine Dynamics DE, 71, pp. 501-508.
- [7] Parkinson, B. M., Jenson, B. D., and Roach, G. M., 2000, "Optimization-Based Design of a Fully-Compliant Bistable Micro Mechanism," ASME Paper No. DETC2000/MECH-14119.
- [8] Masters, N. D., and Howell, L. L., 2003, "A Self-Retracting Fully-Compliant Bistable Micro Mechanism," J. Microelectromech. Syst., **12**, pp. 273–280. [9] Jasinski, P. W., Lee, H. C., and Sandor, G. N., 1971, "Vibrations of Elastic
- Connecting Rod of a High-Speed Slider Crank Mechanism," ASME J. Eng. Ind., 93, pp. 636-644.
- [10] Zhu, Z.G., and Chen, Y., 1983, "The Stability of the Motion of a Connecting Rod," ASME J. Mech., Transm., Autom. Des., 105, pp. 637-640.
- [11] Badlani, M., and Kleinhenz, W., 1970, "Dynamic Stability of Elastic Mechanisms," ASME J. Mech., Transm., Autom. Des., 101, pp. 149-153.
- [12] Fung, R.-F., Sun, J.-H., and Wu, J.-W., 2002, "Tracking Control of the Flexible Slider-Crank Mechanism System Under Impact," J. Sound Vib., 255(2), pp. 337-355
- [13] Frischknecht, B. D., Howell, L. L., and Magleby, S. P., 2004, "Crank-Slider With Spring Constant Force Mechanism," Proceedings of the ASME Design Engineering Technical Conference, Vol. 2A, pp. 809-817.
- [14] Dupac, M., and Marghitu, D. B., 2006, "Nonlinear Dynamics of a Flexible Mechanism With Impact," J. Sound Vib., 289(4-5), pp. 952-966.
- [15] Khemili, I., and Romdhane, L., 2008, "Dynamic Analysis of a Flexible Slider-Crank Mechanism With Clearance," Eur. J. Mech. A/Solids, 27(5), pp. 882-
- [16] Casalena, J. A., Badre-Alam, A., Ovaert, T. C., Cavanagh, P. R., and Streit, D. A., 1998, "The Penn State Safety Floor: Part II-Reduction of Fall-Related Peak Impact Forces on the Femur," ASME J. Biomech. Eng., 120(4), pp. 527-532
- [17] Badre-Alam, A., and Streit, D. A., 1994, "Long-Dwell, Finite-Dwell Linkages," ASME Mechanism Synthesis and Analysis Conference, pp. 479-485.
- [18] Park, J.-J., Lee, Y.-J., Song, J.-B., and Kim, H.-S., 2008, "Safe Joint Mechanism Based on Nonlinear Stiffness for Safe Human-Robot Collision," Proceedings of the IEEE International Conference on Robotics and Automation, Art. No. 4543529, pp. 2177-2182.
- [19] Boyle, C., Howell, L. L., Magleby, S. P., and Evans, M. S., 2003, "Dynamic Modeling of Compliant Constant-Force Compression Mechanisms," Mech. Mach. Theory, 38, pp. 1469-1487.
- [20] Erdman, A. G., and Sandor, G. N., 1997, Mechanism Design: Analysis and Synthesis, Vol. 1, 3rd ed. Prentice-Hall, Upper Saddle River, NJ.
- [21] Kocanda, L., 2002, "Continuous Contact Force Models for Impact Analysis in Multibody Systems," Journal of Nonlinear Dynamics, 5(2), pp. 193–207.
- [22] Kulakowski, B. T., Gardner, J. F., and Shearer, J. L., 2007, Dynamic Modeling and Control of Engineering Systems, Cambridge University Press, Cambridge. UK
- [23] Sönmez, Ü., 2000, "Compliant Mechanism Design and Synthesis Using Buckling and Snap-Through Buckling of Flexible Members." Ph.D. thesis, Pennsylvania State University, State College, PA.
- [24] Sönmez, Ü., and Tutum, C., 2008, "A Compliant Bistable Mechanism Design Incorporating Elastica Buckling Beam Theory and Pseudo-Rigid-Body Model," ASME J. Mech. Des., 130, pp. 042304.
- [25] Sönmez, Ü., 2007, "Buckling Dynamics of Imperfect Elastica Subject to Various Loading Conditions," Int. J. Mech. Mater. Des., 3(4), pp. 347-360.
- [26] Bazant, Z. P., and Cedolin, L., 1991, Stability of Structures, Oxford University Press, Oxford, Sec. 33.
- [27] Rega, G., Vestroni, F., and Peterkat, F., 2003, "Dynamics of Mechanical Systems With Soft Impacts," Proceedings of the IUTAM Symposium on Chaotic Dynamics and Control of Systems and Processes in Mechanics, Rome, Italy, Vol. 122, pp. 313-322.
- [28] Gomm, T., Howell, L. L., and Selfridge, R. H., 2002, "In-Plane Linear Displacement Bistable Micro Relay," J. Micromech. Microeng., 12, pp. 257-264.
- [29] Sönmez, Ü., 2007, "Compliant MEMS Crash Sensor Designs: The Preliminary Simulation Results," Proceedings of the 2007 IEEE Intelligent Vehicles Symposium, Istanbul, Turkey, Jun. 13-15, pp. 303-308.