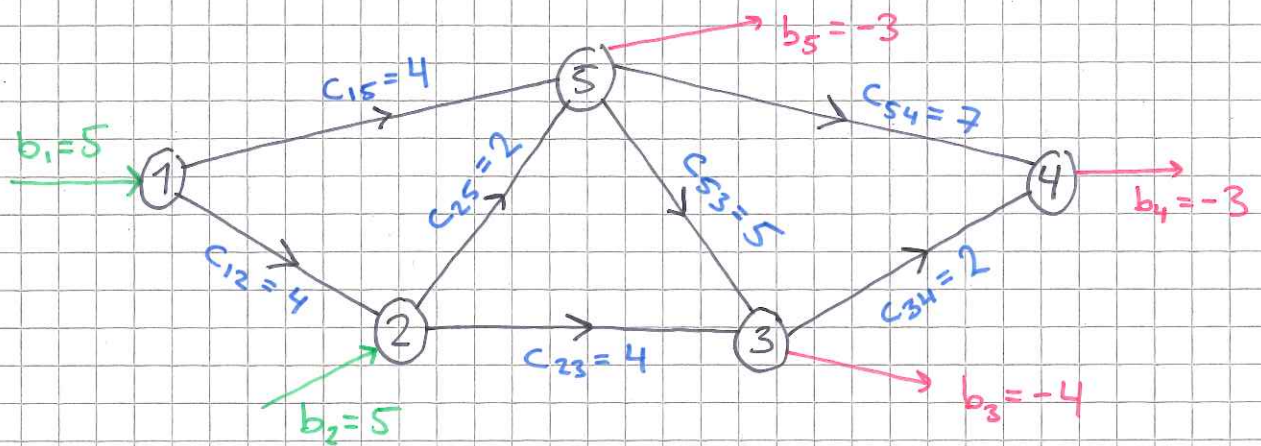


## Exercise session 2

13/11

① Find the minimum cost flow for

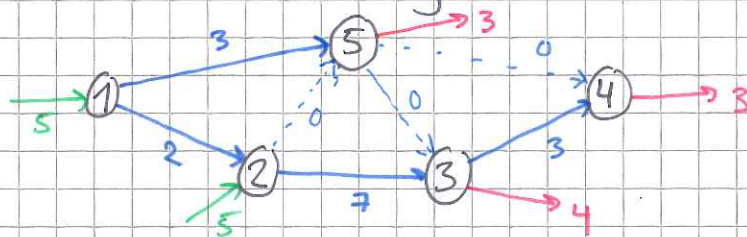


A network with nodes  $N = \{1, 2, 3, 4, 5\}$  and the set of directed edges

$$E = \{(1, 2), (1, 5), (2, 3), (2, 5), (3, 4), (5, 3), (5, 4)\}$$

a) Given the order of the edges in  $E$  write down the incidence matrix  $\hat{A} \in \mathbb{R}^{5 \times 7}$

b) Show that the following solution is optimal



c) Let  $c_{53} = 3$ . Show that the optimal solution in b) is no longer optimal. Find an optimal solution starting from the solution in b) using the Simplex-method.

② Given

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 3 & 1 \\ 3 & 3 & 4 & 2 \end{pmatrix}$$

a) Find bases for  $\ker(A)$  and  $\text{ran}(A)$

b) Find bases for  $\ker(A^T)$  and  $\text{ran}(A^T)$