Exercise session 4

Problem 1

Consider

$$\begin{aligned} \min \quad & \frac{1}{2}x^T Hx + c^T x \\ s.t. \quad & x \in \mathbb{R}^2 \end{aligned} \tag{QP}$$

with

$$H = \begin{bmatrix} 1 & 2 \\ 2 & a \end{bmatrix}.$$

Determine if (QP) has any global optimal solutions for
a) $a = 6$ and $c = (1 \quad 2)^T$.
b) $a = 4$ and $c = (1 \quad 2)^T$.
c) $a = 4$ and $c = (1 \quad 0)^T$.
d) $a = 2$ and $c = (1 \quad 2)^T$.

Problem 2

Consider

$$\begin{aligned} \min \quad & \frac{1}{2}x^T Hx + c^T x \\ s.t. \quad & x \in \mathbb{R}^3 \end{aligned}$$

with

$$H = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}.$$

a) Use LDL^T-factorization to determine if H is positive definite.
b) For $c = (2 \quad 2 \quad 4)^T$ determine if $-c \in \mathcal{R}(H)$. If yes, find an expression for all optimal solutions. If no, find a decent direction d.
c) Same as b) but for $c = (2 \quad 2 \quad 0)^T$.

Problem 3

Consider (P) in exercise 10.10a) on page 99 in ASKS.
a) Rewrite (P) as

$$\begin{aligned} \min \quad & \frac{1}{2}x^T Hx + c^T x + c_0 \\ s.t. \quad & Ax = b \end{aligned} \tag{QP=}$$

b) Solve (QP=) with the null-space method.