

Homework exercises for lecture #4

TO BE HANDED IN ON FEB 24.

1. Let A be a semi-local ring with Jacobson radical \mathfrak{r} . Show that $(\mathrm{Spec} A, \mathrm{Spec} A/\mathfrak{r})$ is a Henselian pair if and only if A is a product of local henselian rings.
2. Let A be a henselian local ring with residue field k and let $X \rightarrow \mathrm{Spec} A$ be a smooth morphism. Show that $X(A) \rightarrow X(k)$ is surjective. (*This is somewhat surprising: formal smoothness shows that $X(A/\mathfrak{m}^n) \rightarrow X(A/\mathfrak{m}^{n-1}) \rightarrow \cdots \rightarrow X(A/\mathfrak{m}) = X(k)$ is surjective for all n and this shows that $X(\hat{A}) \rightarrow X(k)$ is surjective. One could then deduce the result by Artin approximation but there's a much simpler proof of a different flavor.*)