**Homework exercises for lecture #2**

*TO BE HANDED IN ON FEB 10.*

1. Let $A$ be a discrete valuation ring. Let $p(t) \in A[t]$ be a polynomial. Consider the $A$-algebra $B = A[t]/(p)$ and the induced morphism of schemes $f\colon \operatorname{Spec}(B) \to \operatorname{Spec}(A)$. Show that

   (a) $f$ is finite if and only if the leading coefficient of $p$ is invertible.

   (b) $f$ is quasi-finite if and only if some coefficient of $p$ is invertible.

   Can you give similar characterizations if $A$ is an arbitrary ring?

2. Let $f\colon X \to Y$ be a morphism of schemes. Show that the following are equivalent

   (a) $f$ is universally injective,

   (b) $\Delta_f$ is surjective,

   (c) for every point $y \in |Y|$, either $f^{-1}(y) = \emptyset$ or $f^{-1}(y) = \{x\}$ where $\kappa(x)/\kappa(y)$ is purely inseparable.

   (d) for every field $K$, the map of sets $\operatorname{Hom}(\operatorname{Spec} K, X) \to \operatorname{Hom}(\operatorname{Spec} K, Y)$ is injective.