

LOCAL STRUCTURE OF ALGEBRAIC STACKS: EXERCISES
LECTURE 3: DEFORMATION THEORY AND COHERENT
COMPLETENESS

Exercise 3.1 (Cotangent complexes). Let G be a *smooth* group scheme over a field k . Let $U = \text{Spec } A$ be a smooth affine scheme with a G -action.

- (a) Show that $L_{BG/k} = \mathfrak{g}^\vee[-1]$ with the adjoint G -action.
- (b) Show that $L_{[U/G]/k} = (\Omega_{U/k} \rightarrow \mathfrak{g}^\vee)$ (a complex concentrated in degrees 0, 1).

Exercise 3.2. Let \mathcal{X} be a cohomologically affine stacks with good moduli space $\pi: \mathcal{X} \rightarrow X = \text{Spec } \Gamma(\mathcal{X}, \mathcal{O}_{\mathcal{X}})$. Let $\mathcal{X}_0 \hookrightarrow \mathcal{X}$ be a closed substack such that $(\mathcal{X}, \mathcal{X}_0)$ is complete.

- (a) Show that $(\mathcal{X}, \pi^{-1}(\pi(\mathcal{X}_0)))$ is complete.
- (b) Show that $(X, \pi(\mathcal{X}_0))$ is complete. *Hint: Use (a) and fully faithfulness of the completion functor.*

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