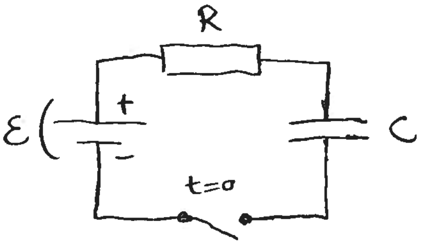
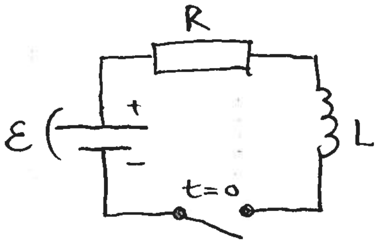
Daniel Ljunggren Okt. 2010

Övning 8 SK1111

Repetition

V SVÄNGNINGSKRETSAR (RLC) (kap. 26, 30)

Kretsanalys

RC-krets	RL-krets
	

$i = \frac{dq}{dt}$

$$\mathcal{E} = \frac{q}{C} + iR \qquad \qquad \mathcal{E} = L\frac{di}{dt} + iR$$

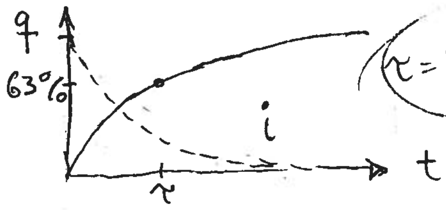
ger

$$\boxed{\frac{dq}{dt} + \frac{1}{RC}q = \frac{\mathcal{E}}{R}} \qquad \qquad \boxed{\frac{di}{dt} + \frac{R}{L}i = \frac{\mathcal{E}}{L}}$$

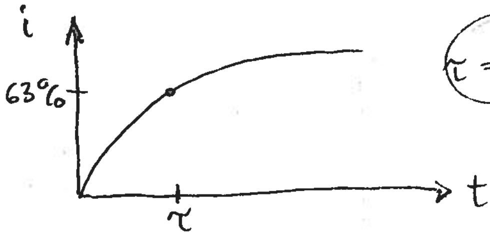
∴ Inhomogen differentialekvation!

☆ Lösning:

$$q(t) = \mathcal{E}C(1 - e^{-t/RC}) \qquad \qquad i(t) = \frac{\mathcal{E}}{R}(1 - e^{-\frac{R}{L}t})$$



$\tau = RC$



$\tau = L/R$

☆ Effekt: $P = U \cdot I \implies$

$$P_E = \frac{dW_E}{dt} = \frac{q}{C} \cdot \frac{dq}{dt} \qquad \qquad P_B = \frac{dW_B}{dt} = i \cdot L\frac{di}{dt}$$

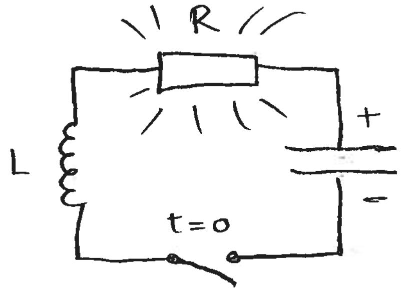
☆ Energi: $W = \int P \implies$

$$\boxed{W_E = \frac{q^2}{2C}} \qquad \qquad \boxed{W_B = \frac{Li^2}{2}}$$

RLC-kretsen

dämpning

rörelse energi



potentiell energi

Energibetraktelse

$$W = W_B + W_E = \frac{Li^2}{2} + \frac{q^2}{2C}$$

Energiförlust (dämpning) i R !

$$\Rightarrow \quad \frac{dW}{dt} = -Ri^2 \quad ; \quad i = \frac{dq}{dt}$$

ger

$$\boxed{L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0}$$

∴ Homogen diffekvation !

☆ Lösning: $q(t) = q_{max}\, e^{-\frac{R}{2L}t} \cos(\omega' t)$

OSCILLATOR ! (svängningskrets)

där $\omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$

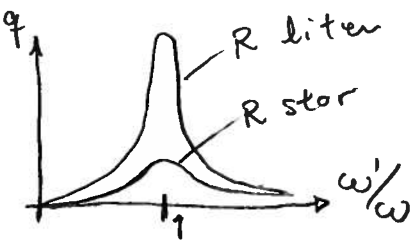
Frekvens filter !

C : släpper igenom höga frekvenser

L : släpper igenom låga frekvenser

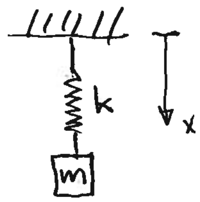
R=0 ger svängning vid resonansfrekvens

$$\omega = \omega' \; \frac{1}{\sqrt{LC}}$$



[Jmf. mekanisk oscillator

$$m\frac{d^2x}{dt^2} + kx = 0 \qquad \frac{1}{\sqrt{LC}} = \sqrt{\frac{k}{m}}$$



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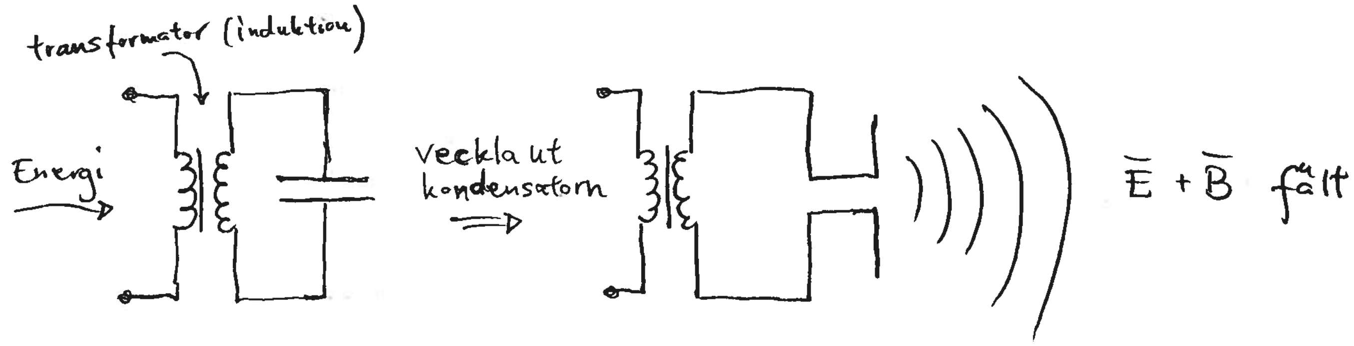
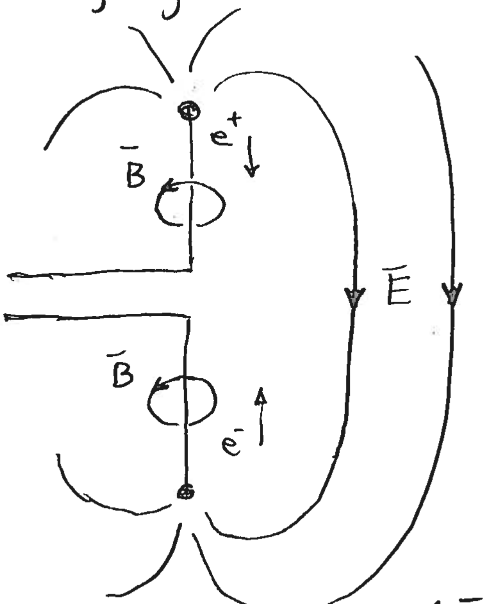
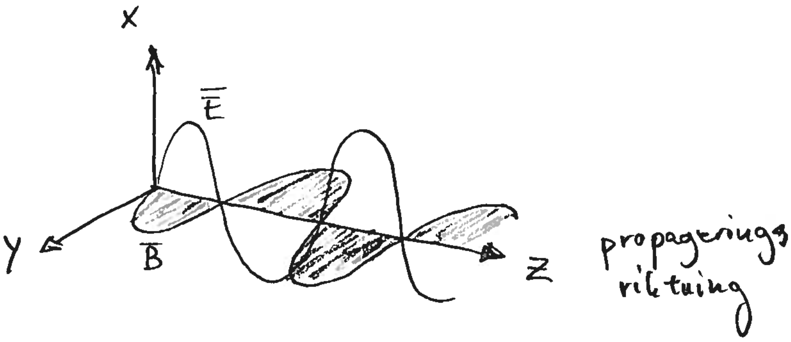
VI ELEKTROMAGNETISK VÅGUTBREDNING (kap. 32)

(tidsvarierande fält, svängningar)



Svängningskrets → Strålande svängningskrets





$\bar{E}$ och $\bar{B}$ alltid vinkelräta!

($\bar{B}$ – fält rörelseenergi q
$\bar{E}$ – fält potentiell energi q)

Analys av Maxwell's ekvationer ger

Vågekvationen

$$\frac{\partial^2 \bar{E}}{\partial t^2} = \frac{1}{c^2}\frac{\partial^2 \bar{E}}{\partial x^2}$$

1 dim

p.s.s. för $\bar{B}$

Lösning:

$$\bar{E} = E_0 \cos(kz - \omega t)\,\bar{e}_x$$
$$\bar{B} = B_0 \cos(kz - \omega t)\,\bar{e}_y$$

3-dim: $\bar{E} = Re\left\{E_0 e^{-i(\bar{k}\cdot\bar{r} - \omega t)}\bar{e}_x\right\}$

3-dim: $\bar{B} = Re\left\{B_0 e^{-i(\bar{k}\cdot\bar{r} - \omega t)}\bar{e}_y\right\}$

Poyntings vektor – energi transport

$$\bar{S} = \frac{1}{\mu_0}\bar{E} \times \bar{B}$$

Intensitet

$$I = |\bar{S}|_{avg} = \frac{1}{2c\mu_0}|\bar{E}|^2$$

Komplex vågfunktion!

k: vågtalet $k = 2\pi/\lambda$

ω: vinkelfrekv. $\omega = 2\pi f$

Våghastighet: $v = \frac{k}{\omega}$, $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{|E_0|}{|B_0|}$

Komplexa metoden (visardiagram)

Eulers formel

$$\boxed{e^{jx} = \cos x + j\sin x}$$

$j = \sqrt{-1}$ (i ström)

Låt $x = \omega t + \varphi \Rightarrow \cos(\omega t + \varphi) = \mathrm{Re}\{e^{jx}\}$

✱ Applicerat på spänning (och ström) kan vi då skriva

- $u(t) = U_0 \cos(\omega t + \varphi) = \mathrm{Re}\{U_0 e^{j\varphi} e^{j\omega t}\}$

  Låt $\bar{U}(t) = U_0 e^{j\varphi} e^{j\omega t}$ → VISARE (vektor, phasor):

- Skippa tidsberoendet!

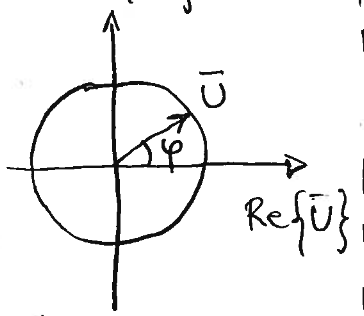
Komplex amplitud

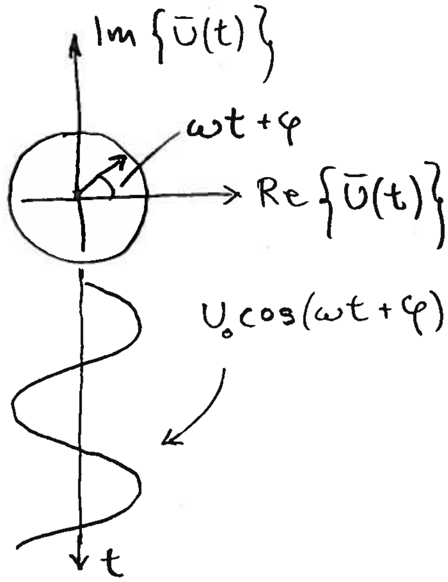
$$\boxed{\bar{U} = U_0 e^{j\varphi}}$$

där

$|\bar{U}| = U_0$: spänningens amplitud

$\arg(\bar{U}) = \varphi$: – " – fasvinkel





- $j\omega$-metoden

- Vaddå $j\omega$? → Kommer av derivata!

✱ Låt $i(t) = I_0 \cos(\omega t + \varphi) \Rightarrow \dfrac{di}{dt} = I_0 \omega \cos(\omega t + \varphi + \pi/2)$

Komplex amplitud: $\bar{I} = I_0 e^{j\varphi}$ ger $\dot{\bar{I}} = I_0 \omega e^{j(\varphi + \pi/2)}$

$= j\omega I_0 e^{j\varphi} = \underline{\underline{j\omega \bar{I}}}$

! Derivation i tiden svarar i komplexräkning mot multiplikation $j\omega$!

ex. $Z = R + jX$ (impedans = resistans + reaktans)

Induktans: $u = L\dfrac{di}{dt}$

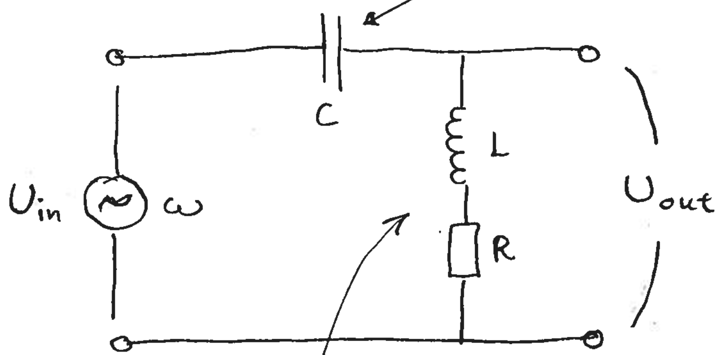
$Z_L = \dfrac{U}{I} = L\dfrac{\dot{\bar{I}}}{\bar{I}} = \underline{\underline{j\omega L}} = jX_L$

Kapacitans: $u = q/C$

$Z_C = \dfrac{U}{I} = \dfrac{q/C}{dq/dt} = \dfrac{\bar{Q}/C}{\dot{\bar{Q}}} = \underline{\underline{\dfrac{1}{j\omega C}}} = jX_C$

31.49) Hög-pass-filter

Blockerar låga frekvenser!



Kortsluter låga frekvenser!

Sökt: $\left|\frac{U_{out}}{U_{in}}\right|$ som funktion av ω?

Kirchoffs spänningslag $\sum \mathcal{E}_j = \sum Z_k I_k$

ger $U_{in} - U_c - \underbrace{(U_L + U_R)}_{U_{out}} = 0$

$$\Rightarrow \frac{|U_{out}|}{|U_{in}|} = \frac{|U_L + U_R|}{|U_c + U_L + U_R|} = \underset{\text{samma } I \text{ !}}{\{U = ZI\}} = \frac{|Z_L + Z_R|}{|Z_c + Z_L + Z_R|}$$

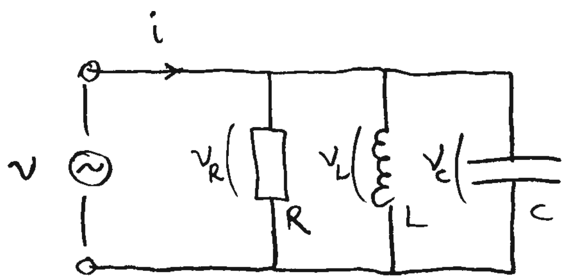
Vi har:
$$\begin{cases} Z_c = \dfrac{1}{j\omega C} \\ Z_L = j\omega L \\ Z_R = R \end{cases}$$

$$\Rightarrow \frac{|U_{out}|}{|U_{in}|} = \frac{R + j\omega L}{R + \frac{1}{j\omega C} + j\omega L} = \frac{\sqrt{R^2 + \omega^2 L^2}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

ω litet: $|U_{out}|/|U_{in}| \approx \sqrt{\dfrac{R^2}{R^2 + \frac{1}{\omega^2 C^2}}} \approx \sqrt{\dfrac{R^2\omega^2C^2}{R^2\omega^2C^2 + 1}} \approx \underline{\underline{R\omega C}}$

ω stort: $|U_{out}|/|U_{in}| \approx \sqrt{\dfrac{R^2 + \omega^2 L^2}{R^2 + \omega^2 L^2}} \approx \underline{\underline{1}}$ V.S.V.

31.54) RLC - parallellkrets



Sökt: a) Visa att $v_L = v_R = v_c = v$

och $i_R + i_L + i_c = i$

b) Faser i_R, i_L, i_c m.a.p. v ?

c) Visa att amplituden $I = \sqrt{I_R^2 + (I_c - I_L)^2}$

d) — " — $Z = \left(\frac{1}{R^2} + (\omega C - \frac{1}{\omega L})^2\right)^{-1/2}$

a) Enligt Kirchoffs spänningslag: $\sum \varepsilon_i = \sum Z_k i_k$

är alla momentana och effektiva spänningar lika i en parallellkrets.

Kirchoffs strömlag $\sum i_k = 0$

ger $i = i_R + i_L + i_c$

b) För total impedans har vi: $i = \frac{v}{Z} = \frac{v}{|Z|} e^{-j\arg(Z)}$

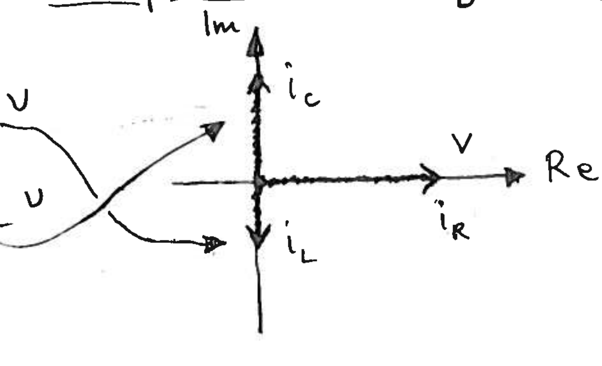
För resistans: $i_R = \frac{v}{R}$; och reaktans: $i_L = \frac{v}{jX_L}$; $i_c = \frac{v}{jX_c}$

$Z = R + jX$; $X_L = \omega L$, $X_c = -\frac{1}{\omega C}$

$$\Rightarrow \begin{cases} i_R = \frac{v}{R} e^{j\cdot 0} & \text{i fas } v \\ i_L = \frac{v}{\omega L} e^{-j\pi/2} & \frac{\pi}{2} \text{ efter } v \\ i_c = \omega C v e^{j\pi/2} & \frac{\pi}{2} \text{ före } v \end{cases}$$

komplexa amplituder

Komplext visardiagram!



forts.

c) Allmänt : $i(t) = \underbrace{I_0 e^{j\varphi}}_{\text{Komplex amplitud.}} e^{j\omega t} = \bar{I}\, \underbrace{e^{j\omega t}}$

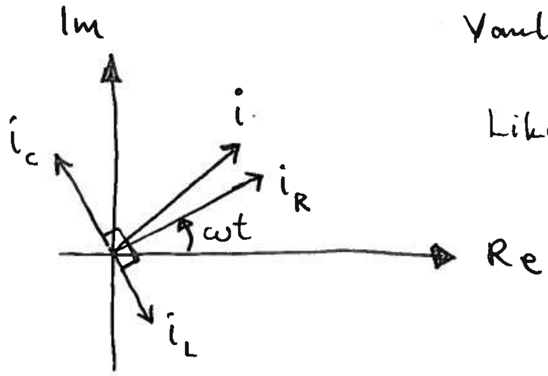
Vanligtvis skippas denna term! Lika för harmonisk svängning.

Def. reel amplitud

$I_0 = |\bar{I}| = |i|$

Def. fas :

$\varphi = \arg(I) = \varphi$



Geometri, vektoraddition ger

$$I_0 = |i| = |i_R + i_C + i_L| =$$

$$= \left| \frac{V}{R} + j\omega C V - j\frac{V}{\omega L} \right| =$$

$$= \sqrt{\left(\frac{V}{R}\right)^2 + \left(\omega C V - \frac{V}{\omega L}\right)^2} =$$

$$= \sqrt{I_R^2 + (I_C - I_L)^2} \quad \star$$

d) $I_0 = \dfrac{V}{|Z|}$ där

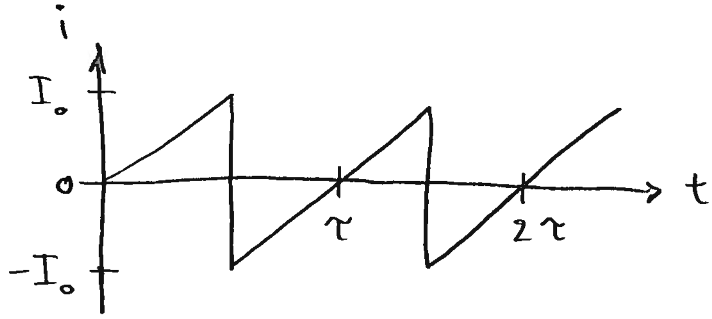
$$|Z| = \left| \left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C \right)^{-1} \right|$$

$$\Rightarrow \quad I_0 = V \cdot \frac{1}{|Z|} = V \cdot \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

$$= \sqrt{\frac{V^2}{R^2} + \left(\omega C V - \frac{V}{\omega L}\right)^2} = \star$$

V.S.V.

31.63) Medelvärde och effektivvärde.



Sökt: i_{avg} och i_{RMS} ?

a) Absolutbeloppet av momentana medelvärdet = 0 p.g.a Symmetri!

$$|i_{avg}| = \frac{I_0 + (-I_0)}{2} = 0$$

b) Är strömmen periodisk med periodtiden T kan vi tala om ett medelvärde av den utvecklade effekten genom en resistans:

$$\boxed{P = \frac{1}{T}\int_0^T p(t)\,dt}$$

(Notera $i(t) = I_0$ $\Rightarrow p(t) = RI_0^2 = P_0$)

där $p(t) = R \cdot (i(t))^2$

$$\Rightarrow P = \frac{1}{T}\int_0^T R\,(i(t))^2\,dt = R\,I_{eff}^2$$

Strömmens effektivvärde!

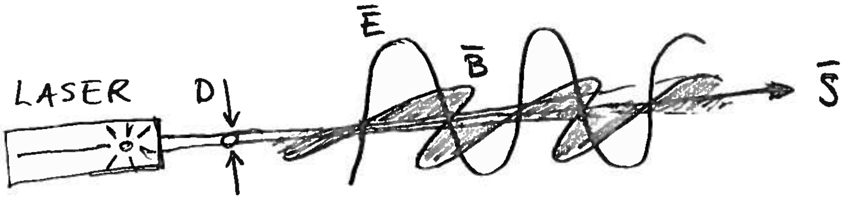
$$\therefore \quad I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = \left\{ \text{halva perioden: } i(t) = \frac{2I_0 t}{T} \right\} =$$

$$= \sqrt{\frac{1}{T/2}\int_0^{T/2}\left(\frac{2I_0 t}{T}\right)^2 dt} = \sqrt{\frac{2}{T}\,\frac{4I_0^2}{T^2}\left[\frac{t^3}{3}\right]_0^{T/2}}$$

$$= \frac{I_0}{\sqrt{3}} \quad : \text{Svar}$$

! Notera För $i(t) = I_0 \cos(\omega t)$ gäller $\boxed{I_{eff} = \frac{I_0}{\sqrt{2}}}$

32.41) Intensitet, fältstyrka i laserljus.



Känt: Effekt: $P = 3.20$ mW

Stråldiameter $D = 2.50$ mm

a) E_0 och B_0 ?

Vi har energiflöded från Poyntings vektor:

$$\boxed{\bar{S} = \frac{1}{\mu_0} \bar{E} \times \bar{B}}$$

$$\begin{cases} \bar{E} = E_0 \sin(kz - \omega t)\, \bar{e}_x \\ \bar{B} = B_0 \sin(kz - \omega t)\, \bar{e}_y \end{cases}$$

$$\Rightarrow \bar{S} = \frac{1}{\mu_0} E_0 \sin(kz-\omega t) B_0 \sin(kz-\omega t)\, \bar{e}_x \times \bar{e}_y$$

$$= \frac{E_0 B_0}{\mu_0} \sin^2(kz-\omega t)\, \bar{e}_z$$

Def. Intensitet: $\boxed{I = |S|_{avg}}$

vakuum:

$$\text{ger } I = \left\langle \left| \frac{E_0 B_0}{\mu_0} \underbrace{\sin^2(kz-\omega t)}_{\text{medel} = \frac{1}{2}} \right| \right\rangle = \frac{E_0 B_0}{2\mu_0} = \left\{ B_0 = \sqrt{\varepsilon_0 \mu_0}\, E_0 \right\}$$

$$= \frac{1}{2} \sqrt{\frac{\varepsilon_0}{\mu_0}}\, E_0^2$$

$$I = P/A = \frac{\text{effekt}}{\text{area}} = \frac{P}{\pi (D/2)^2}$$

$$\begin{cases} E_0 = \sqrt{2I \sqrt{\frac{\mu_0}{\varepsilon_0}}} = 701 \text{ V/m} \\ B_0 = \sqrt{\varepsilon_0 \mu_0}\, E_0 = 2.34 \cdot 10^{-6} \text{ T} \end{cases}$$

Notera!

ljushastigheten:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

∴ forts.

b) Potentiell energi lagrat i $\bar{E}$-fält

Kinetisk — " — $\bar{B}$-fält

Momentant: $\boxed{W_E = \frac{1}{2}\varepsilon_0 |E|^2}$; $\boxed{W_B = \frac{1}{2\mu_0}|B|^2}$

Medel: $|W_E|_{avg} = \frac{1}{2}\varepsilon_0 \frac{1}{2} E_0^2 = 1{,}09\ \mu J/m^3$

$|W_B|_{avg} = \frac{1}{2\mu_0}\frac{1}{2} B_0^2 =$ — " —

eftersom $|E|^2_{avg} = \langle \sin^2(kz-\omega t)\rangle = \frac{1}{2}$

c) Total energi i 1 meter stråle ?

$W_E = W_B$: energidensitet / volymenhet.

$$W_{volym} = (|W_E|_{avg} + |W_B|_{avg}) \cdot V =$$

$$= \left\{ V = \pi (D/2)^2 \cdot L \right\} =$$

$$= \frac{1}{4}\left(\varepsilon_0 E_0^2 + \frac{1}{\mu_0} B_0^2\right)\frac{\pi D^2}{4} L =$$

$$= \left\{ B_0 = \sqrt{\varepsilon_0 \mu_0}\, E_0 \right\} =$$

$$= \frac{1}{8}\varepsilon_0 E_0^2 \pi D^2 L = 1{,}07 \cdot 10^{-11}\ J$$