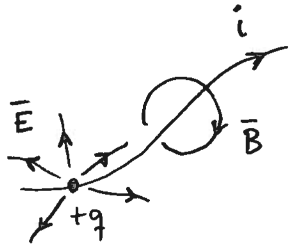
Övning 6 SK1111

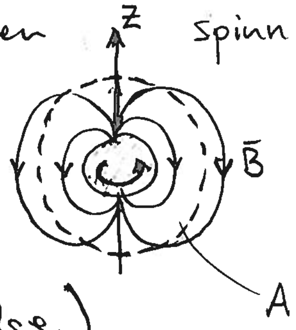
Repetition

III MAGNETOSTATIK (kap. 27 - 28)

Laddningar i rörelse → magnetfält $\bar{B}$ + $\bar{E}$-fält



ex. Dipolen, spinn hos elektron eller atomkärna



(IDAG: Endast konstant rörelse)

$$\frac{\partial B}{\partial t} = 0 \quad \nabla_6$$

Magnetostatisk lag I

Inga monopoler

$$\boxed{\oint_A \bar{B} \cdot d\bar{A} = 0}$$

Maxwell's ekv. 2

(jmf. Gauss lag. $\oint_A \bar{E} \cdot d\bar{A} = \frac{Q}{\varepsilon_0 \varepsilon_r}$)

Kraftverkan

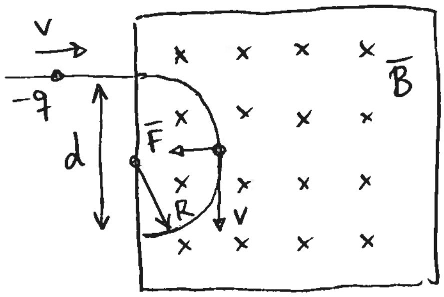
Lorentzkraften

$$\boxed{\bar{F} = q(\bar{E} + \bar{v} \times \bar{B})}$$

$\Rightarrow d\bar{F} = i\,d\bar{l} \times \bar{B}$

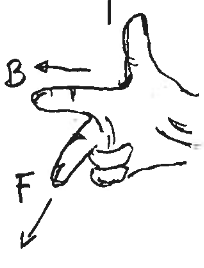
(då $dq = i\frac{dl}{v}$; $\bar{E} = 0$)

ex Masspektrometer $q, \bar{v}, \bar{B} \to \bar{F}, m_q \to d$



Högerhandsregeln (pos. laddn.)

Vänster — u — (neg. laddn.)



Newton's 2:a lag

$$F = ma = \frac{mv^2}{R}$$

OBS! Samband Elektriska och Magnetiska fält

Kuriosa. via speciella relativitetsteorin ?

$$\bar{F} = q(\bar{E} + \bar{v} \times \bar{B})$$

Om en kraft som verkar på en rörlig laddad partikel ser ut att härröra från elektriskt eller magnetiskt fält beror på val av referensram (inertialsystem).

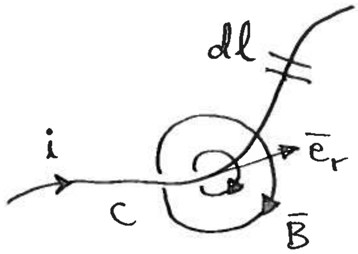
I partikelns referensram blir omgivande laddningsfördelningar påverkade av längdkontraktion.

Fält från ledare

Naturkonstant!
μ_0 : permeabilitet i vakuum $= 4\pi \cdot 10^{-7}$ Tm/A
(magnetiska fältets ledningsförmåga)

Biot-Savart's lag

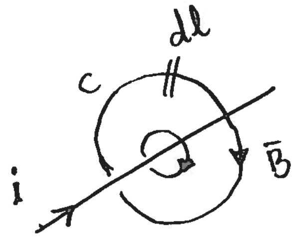
$$\bar{B} = \frac{\mu_0 i}{4\pi} \int_C \frac{d\bar{l} \times \bar{e}_r}{r^2}$$



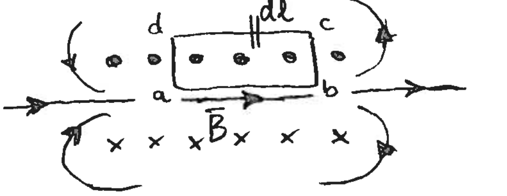
Magnetostatisk lag II

Ampere's lag

$$\oint_C \bar{B} \cdot d\bar{l} = \mu_0 i$$



ex Solenoid n varv/meter



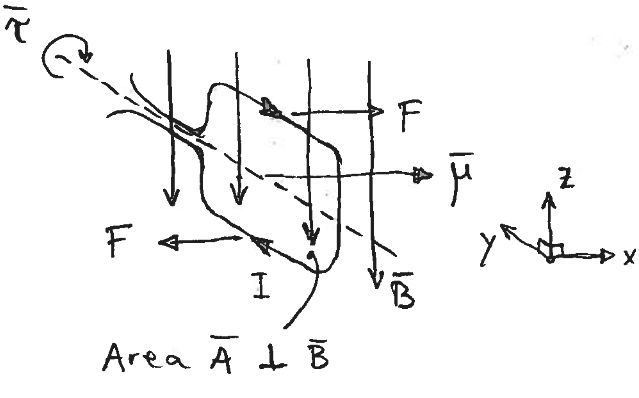
Homogent fält $\bar{B} \Rightarrow i \rightarrow \bar{B}$

$$i = \frac{|\bar{B}|}{\mu_0 n}$$

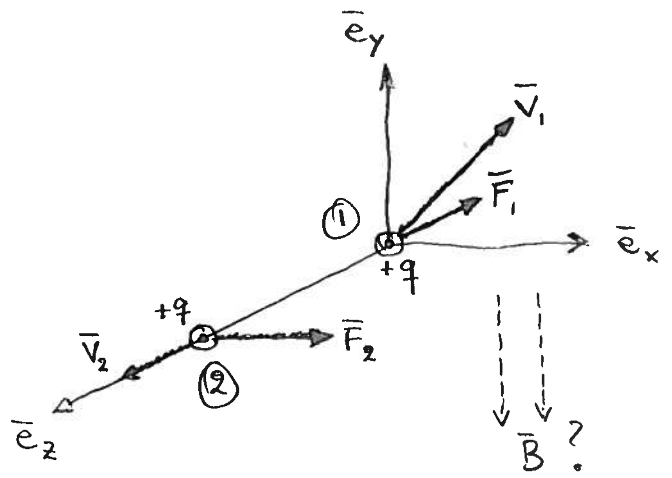
Magnetiskt vridmoment

$$\bar{\tau} = \bar{\mu} \times \bar{B} \qquad [\mathrm{Nm}]$$

där momentet $\mu = I\bar{A}$



27.53) Lorentz-krafter



Sökt : a) B? b) $\bar{F}_1$ uttryckt i $|F_2|$?

Känt : $\bar{v}_1$, $\bar{v}_2$, $|v_1| = |v_2|$, $\bar{F}_1$, $\bar{F}_2$, $+q$

Idé Laddad partikel rör sig i magnetfält B

$\Rightarrow$ Lorentz kraft : $\boxed{\bar{F} = q(\bar{E} + \bar{v} \times \bar{B})}$

Inget E-fält $\Rightarrow$ $\bar{F} = q\bar{v} \times \bar{B}$ $\quad \bar{B} = B_x\bar{e}_x + B_y\bar{e}_y + B_z\bar{e}_z$

Vektorkomponenter :

$$F_x\bar{e}_x + F_y\bar{e}_y + F_z\bar{e}_z = q(v_x\bar{e}_x + v_y\bar{e}_y + v_z\bar{e}_z) \times (B_x\bar{e}_x + B_y\bar{e}_y + B_z\bar{e}_z)$$

[Kryssprodukt-regler ; $\bar{e}_x = \bar{e}_y \times \bar{e}_z = -\bar{e}_z \times \bar{e}_y$
$\bar{e}_y = \bar{e}_z \times \bar{e}_x = -\bar{e}_x \times \bar{e}_z$
$\bar{e}_z = \bar{e}_x \times \bar{e}_y = -\bar{e}_y \times \bar{e}_x$]

$$\Rightarrow \boxed{F_x = qv_yB_z - qv_zB_y \; ; \; F_y = qv_zB_x - qv_xB_z \; ; \; F_z = qv_xB_y - qv_yB_x}$$

a) Partikel 1 : $\bar{F}_1 = -F_1\bar{e}_z$; $\bar{v}_1 = v_1(\cos\pi/4\,\bar{e}_x + \sin\pi/4\,\bar{e}_y)$

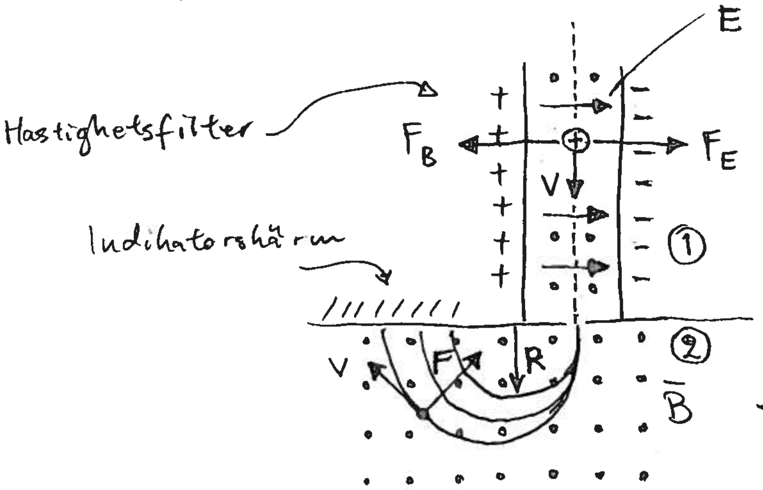
$$\begin{cases} F_z = -F_1 = qv_1(B_y - B_x)/\sqrt{2} & (1) \\ F_x = F_y = 0 = qv_1 B_z/\sqrt{2} & \Rightarrow \boxed{B_z = 0} \end{cases}$$

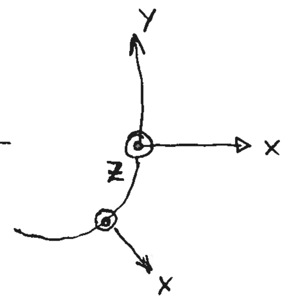
Partikel 2 : $\bar{F}_2 = F_2\bar{e}_x$ $\quad \bar{v}_2 = v_2\bar{e}_z$

$$\begin{cases} F_x = F_2 = 0 - qv_2B_y & \Rightarrow B_y = -\dfrac{F_2}{qv_2} \\ F_y = 0 = qv_2B_x - 0 & \Rightarrow B_x = 0 \end{cases} \quad (2)$$

b) (1) & (2) $\Rightarrow$ $-F_1 = \dfrac{qv_1}{\sqrt{2}}\left(-\dfrac{F_2}{qv_2}\right)$ $\quad \therefore \quad \underline{F_1 = F_2/\sqrt{2}}$

27.65) Masspektrometer





Sökt: Radien R för isotoperna ^{82}Kr, ^{84}Kr, ^{86}Kr

Känt: $\bar{E} = E\bar{e}_x$; $\bar{B} = B\bar{e}_z$ / $E = 1.88 \cdot 10^4$ V/m ; $B = 0{,}701$ T

Idé: Finn v → Lorentz kraftekv. → R !

Lorentz kraftekv.

$$\boxed{\bar{F} = q(\bar{E} + \bar{v} \times \bar{B})}$$

Krypton mass:
$m_{82} = 82u$
$m_{84} = 84u$
$m_{86} = 86u$
$1u = 1.66 \cdot 10^{-27}$ kg

Område 1: $\bar{F} = 0$; $\bar{v} = -v\bar{e}_y$

Vi har $0 = q(E\bar{e}_x - v\bar{e}_y \times B\bar{e}_z)$

$$\Rightarrow \boxed{v = E/B}$$

Område 2:

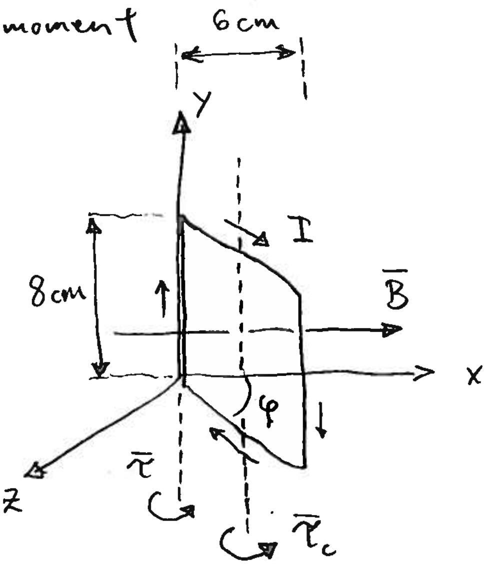
Newtons 2:a lag $F = ma$ $\Rightarrow$ $\boxed{\bar{F} = -\frac{mv^2}{R}\bar{e}_x}$

Vi har $-\frac{mv^2}{R}\bar{e}_x = q(0 - v\bar{e}_y \times B\bar{e}_z)$

$$\Rightarrow \frac{mv^2}{R} = qvB \Rightarrow \boxed{R = \frac{mv}{qB} = \frac{mE}{qB^2}}$$: svar

$R_{82} = 32.5$ mm ; $R_{84} = 33.3$ mm ; $R_{86} = 34.1$ mm

27.76) Vridmoment



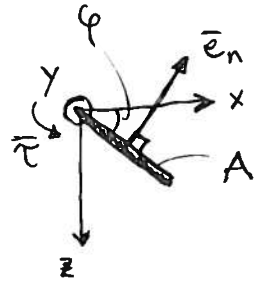
Sökt: a) Moment τ då $\varphi = 30°$ och $\bar{B} = B\bar{e}_x$

b) — " — $\bar{B} = -B\bar{e}_z$

c) — " — τ_c på centrumaxel.

känt: $I = 15$ A ; $B = 0{,}48$ T

$$\boxed{\text{Magnetiskt vridmoment} \quad \bar{\tau} = \bar{\mu} \times \bar{B}}$$ där momentet $\mu = I\bar{A} = IA\bar{e}_n$



Vi har $\bar{e}_n = -\cos\varphi\,\bar{e}_z + \sin\varphi\,\bar{e}_x$

Skapat vridmoment:

a) $\bar{\tau} = IA(-\cos\varphi\,\bar{e}_z + \sin\varphi\,\bar{e}_x) \times B\bar{e}_x = -IAB\cos\varphi\,\bar{e}_y$

$\therefore\ \tau = IAB\cos 30° = \underline{0{,}03 \text{ Nm}}$ neg. runt y-axeln.

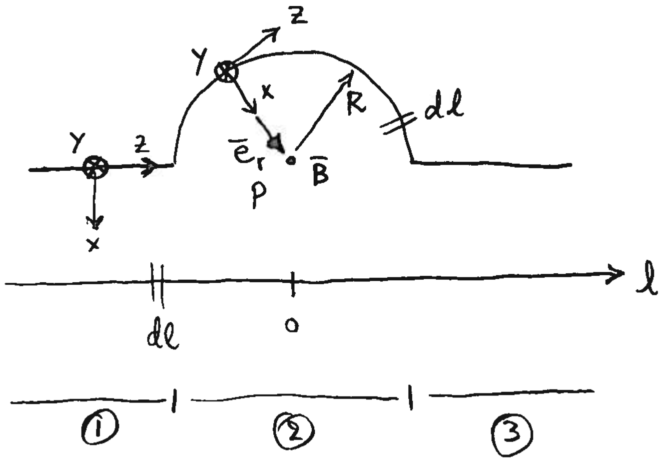
b) $\bar{\tau} = IA(-\cos\varphi\,\bar{e}_z + \sin\varphi\,\bar{e}_x) \times (-B\bar{e}_z) = IAB\sin\varphi\,\bar{e}_y$

$\therefore\ \tau = IAB\sin 30° = \underline{0{,}017 \text{ Nm}}$ pos. runt y-axeln.

c) $\bar{\tau}_c = \underbrace{2 \cdot I\frac{A}{2}}_{= IA}(-\cos\varphi\,\bar{e}_z + \sin\varphi\,\bar{e}_x) \times \bar{B}$

$\therefore$ Samma som ovan!

28.30) B-fält från krökt ström



Sökt: Magnetiskt fält $\bar{B}$ i pkt P.

Biot-Savarts lag

$$\boxed{d\bar{B} = \frac{\mu_0 i}{4\pi}\frac{d\bar{l} \times \bar{e}_r}{r^2}}$$

Tre bidragande områden:

① $\begin{cases} d\bar{l} = dl\,\bar{e}_z & , \bar{e}_r = \bar{e}_z \\ \bar{r} = -l\,\bar{e}_z \end{cases}$

② $\left\{\begin{array}{ll} d\bar{l} = dl\,\bar{e}_z & , \bar{e}_r = \bar{e}_x \\ \bar{r} = R\,\bar{e}_x \end{array}\right\}$ Roterande koordinatsystem!

③ $\begin{cases} d\bar{l} = dl\,\bar{e}_z & , \bar{e}_r = -\bar{e}_z \\ \bar{r} = l\,\bar{e}_z \end{cases}$

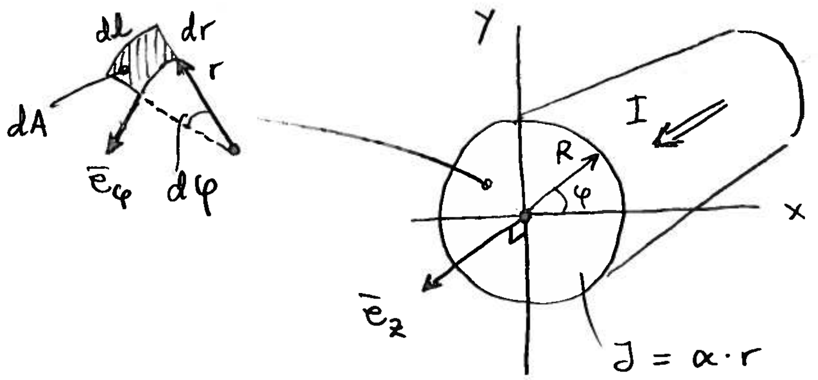
$\Rightarrow$

$$\bar{B} = \int dB = \frac{\mu_0 i}{4\pi}\left[\underbrace{\int_{-\infty}^{-R}\frac{dl\,\bar{e}_z \times \bar{e}_z}{l^2}}_{=0} + \int_0^{\pi R}\frac{dl\,\bar{e}_z \times \bar{e}_x}{R^2} + \underbrace{\int_R^{\infty}\frac{dl\,\bar{e}_z \times (-e_z)}{l^2}}_{=0}\right]$$

$$= \frac{\mu_0 i}{4\pi}\int_0^{\pi R}\frac{1}{R^2}\,dl\,\bar{e}_y$$

$$= \frac{\mu_0 i}{4\pi R^2}\pi R\,\bar{e}_y = \underline{\underline{\frac{\mu_0 i}{4R}\bar{e}_y}} \quad : \text{Svar}$$

28.71) Strömtäthet → Magnetfält



Sökt: a) α? b) $B(r)$ Känt: $R, I, \bar{J} = \alpha r \bar{e}_z$

Def. Strömtäthet

$$\boxed{I = \iint_A \bar{J} \cdot d\bar{A}}$$

där $d\bar{A} = dA\,\bar{e}_z = dr\,dl\,\bar{e}_z = dr\,r\,d\varphi\,\bar{e}_z$

⇒

a) $$I = \int_{r=0}^{R} \int_{\varphi=0}^{2\pi} \alpha r \bar{e}_z \cdot dr\,r\,d\varphi\,\bar{e}_z = \alpha \int_0^R r^2 dr \int_0^{2\pi} d\varphi =$$

$$= \alpha \left[\frac{r^3}{3}\right]_0^R \left[\varphi\right]_0^{2\pi} = \alpha \cdot 2\pi \cdot \frac{R^3}{3}$$

$$\Rightarrow \quad \alpha = \frac{3I}{2\pi R^3} \quad \therefore \quad \underline{\bar{J} = \frac{3I}{2\pi R^3} r \bar{e}_z}$$

b) Amperes lag

$$\boxed{\oint \bar{B} \cdot d\bar{l} = \mu_0 i}$$

där $\bar{B} = B(r)\,\bar{e}_\varphi$

$d\bar{l} = r\,d\varphi\,\bar{e}_\varphi$

$\underline{r \leq R}$

VL: $$\oint B(r)\,\bar{e}_\varphi \cdot r\,d\varphi\,\bar{e}_\varphi = \int_0^{2\pi} B(r)\,r\,d\varphi = B(r) \cdot 2\pi r$$

HL: $$\mu_0 i = \mu_0 \iint_A \bar{J}(r) \cdot d\bar{A} = \mu_0 \int_0^r \int_0^{2\pi} \frac{3I}{2\pi R^3} r' \bar{e}_z \,dr'\,r'\,d\varphi\,\bar{e}_z$$

$$= \mu_0 \frac{3I}{2\pi R^3} \int_0^r r'^2 dr' \int_0^{2\pi} d\varphi = \mu_0 \frac{3I}{2\pi R^3} \left[\frac{r'^3}{3}\right]_0^r \left[\varphi\right]_0^{2\pi} = \mu_0 \frac{I r^3}{R^3}$$

VL = HL $\Rightarrow$ $\boxed{B(r) = \mu_0 \frac{I r^2}{2\pi R^3}}$: svar

<u>$r \geq R$</u>

$$\oint \bar{B} \cdot d\bar{l} = \mu_0 I$$

$$B(r) \cdot 2\pi r = \mu_0 I$$

$$\boxed{B(r) = \frac{\mu_0 I}{2\pi r}}$$: svar