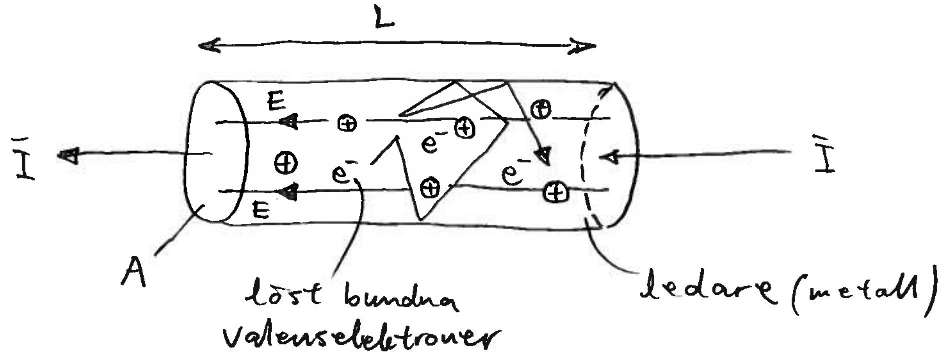
Övning 5 SK1111

Repetition

II. ELSTATIK (kap. 25-26)

Laddningar i rörelse $\Longrightarrow$ Strömmar ! (+ statiska magnetfält (senare))

(fortfarande tidsoberoende fält)



Ström

$$\boxed{I = \frac{dq}{dt} = -\frac{de}{dt}}$$

I motriktad elektronströmmen !

Drifthastighet: $\bar{v}_d = -\mu_e E \approx 10^{-4}$ m/s lågt ! (vid pålagt E-fält)

μ_e : mobilitet för elektroner

Strömtäthet

$$\boxed{I = \iint_A \bar{J} \cdot d\bar{A}}$$

där $\bar{J} = \rho_e \bar{v}_d$

ρ_e : laddningstäthet $[C/m^3]$

Ohms lag: Vi har $\bar{J} = -\rho_e \mu_e E$

Def. $\sigma = -\rho_e \mu_e$: konduktivitet ($\rho = \frac{1}{\sigma}$: resistivitet)

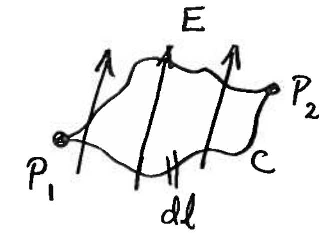
$$\boxed{\bar{J} = \sigma \bar{E}} \xrightarrow{\text{integration}} \boxed{I = \frac{U}{R}} \qquad R = \rho \frac{L}{A}$$

Mikroskopisk — Makroskopisk

Kirchoffs lagar:

Elstatisk lag II

Potential igen... $V_2 - V_1 = -\int_C \bar{E} \cdot d\bar{l}$

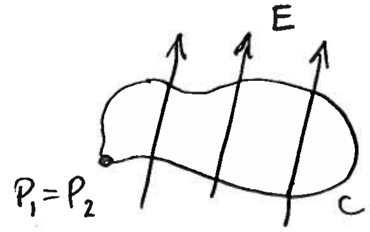


Låt $P_1 = P_2$; sluten kurva C.

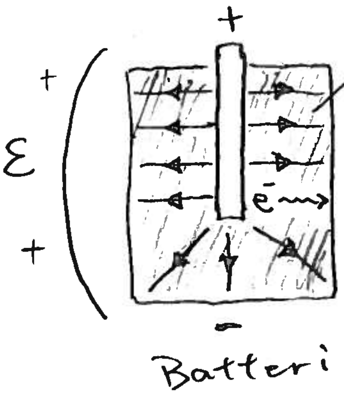
$\Rightarrow$

Konservativt fält E

$$\boxed{V_2 - V_1 = -\oint_C \bar{E} \cdot d\bar{l} = 0}$$



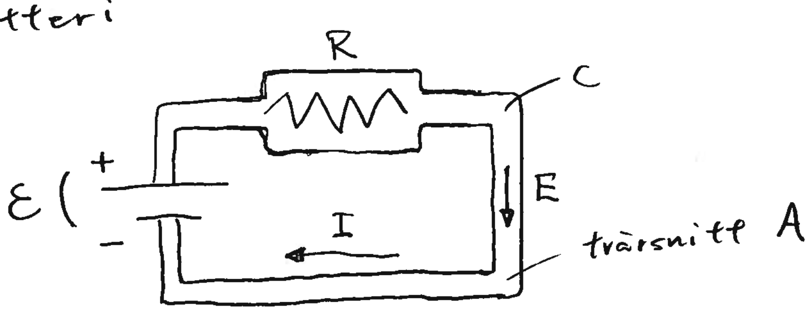
EMK (elektromotorisk kraft)



$\mathcal{E} = \frac{dW}{dq}$; elektromekaniskt arbete (jmf. $W_p = qV$)

Notera!: Elektroner går med fältet! ett arbete utförs.

$$\boxed{\mathcal{E} = -\oint \bar{E} \cdot d\bar{l}}$$

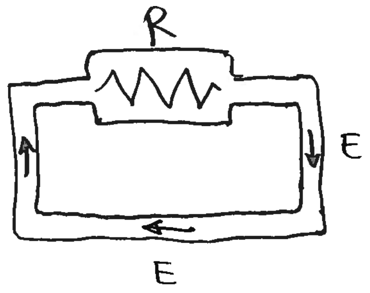


$$\oint_C \bar{E}\, d\bar{l} = \oint_C \frac{J}{\sigma}\, dl = \oint_C \frac{JA}{\sigma A}\, dl = I \oint_C \frac{1}{\sigma A}\, dl = I \cdot R = \mathcal{E}$$

Kirchoffs spänningslag

$$\boxed{\sum_j \mathcal{E}_j = \sum_k R_k I_k \iff \oint_C \bar{E} \cdot d\bar{l} = \mathcal{E}}$$

ex.
utan emk.



$$\oint \bar{E}\cdot d\bar{l} = \oint \frac{J}{\sigma} dl = 0$$

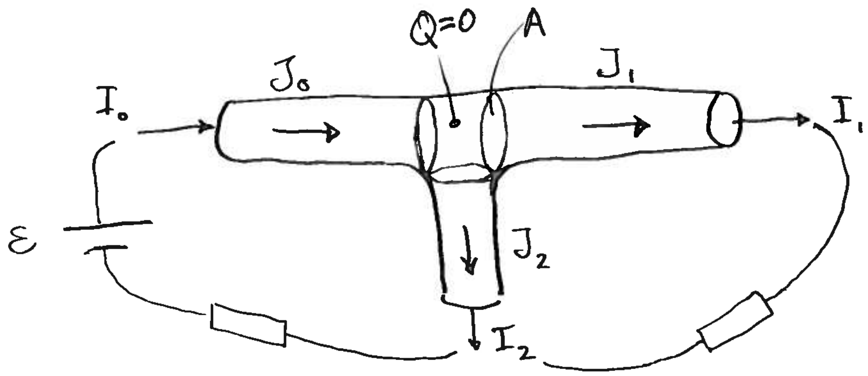
Om $\sigma \neq \infty$ $\Rightarrow$ $J = 0$! Ingen ström.

Om $\sigma \to \infty$?? $J \neq 0$; Supraledning !

Källfritt Gauss lag

$$\oiint_A \bar{E}\cdot d\bar{A} = \frac{Q}{\varepsilon_0} = 0$$

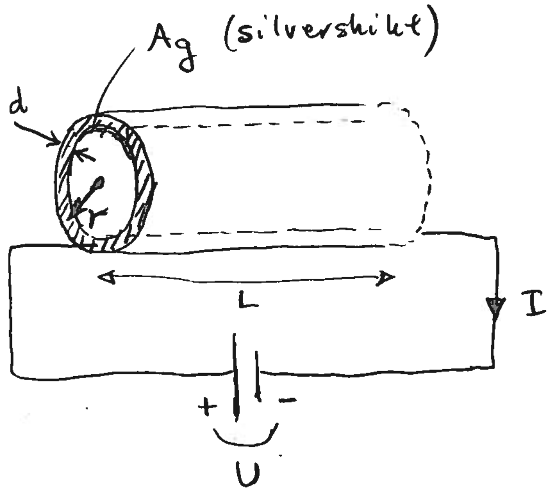
Vi har $J = \sigma E$ $\Rightarrow$ $\oiint_A J\cdot dA = 0$



Kirchoffs strömlag

$$\sum_j I_j = 0 \quad \Longleftrightarrow \quad \oiint \bar{J}\cdot d\bar{A} = 0$$

25.58) Resistivitet – Ohms lag



Sökt: I ?   Känt: $L = 25\ m$, $r = 2\ cm$
$d = 0{,}1\ mm$, $U = 12\ V$

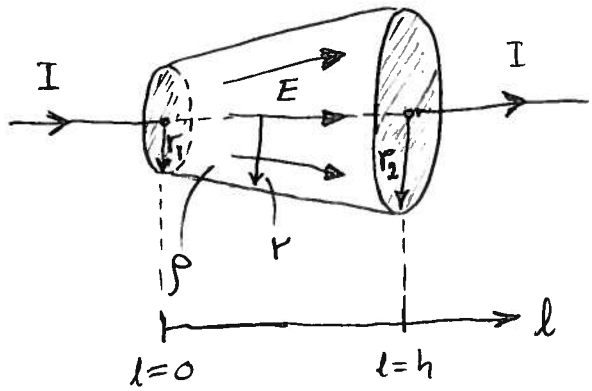
Makroskopiska Ohm's lag: $\boxed{I = \frac{U}{R}}$

$\Rightarrow$ R ? ! Vi känner resistiviteten hos Ag: $\rho = 1{,}47 \cdot 10^{-8}\ \Omega m$

Resistans: $\boxed{R = \rho \cdot \frac{L}{A}}$

$\therefore \quad I = \frac{UA}{\rho L} \approx \frac{U \cdot 2\pi r \cdot d}{\rho L} = 410\ A$

25.63) Konformad resistans



Notera!

$$r(l) = r_1 + \frac{(r_2 - r_1)}{h} l$$

Sökt: a) R? b) $R \to \rho \frac{h}{A}$ då $r_1 = r_2$

Känt: Resistorns form!

☆ Makroskopisk ohm's lag: $\boxed{I = \frac{U}{R}}$

$$\Rightarrow R = \frac{U}{I} = \frac{1}{I} \int_0^h E \, dl \quad (1)$$

! $\sigma = \frac{1}{\rho}$

σ: konduktivitet

ρ: resistivitet

☆ Mikroskopisk Ohm's lag: $\boxed{J = \sigma E} \Rightarrow E = \frac{J}{\sigma} \quad (2)$

a) (1) & $(2) \to \quad R = \underbrace{\frac{1}{I}}_{\text{konst. över } l} \int_0^h \frac{J}{\sigma} dl = \int_0^h \frac{J}{\sigma I} dl$

$$= \left\{ I = \int_A J(l) \, dA \right\} = \frac{1}{\sigma} \int_0^h \frac{J(l)}{\int_{A(l)} J(l) dA} dl = \frac{1}{\sigma} \int_0^h \frac{1}{\int_{A(l)} dA} dl =$$

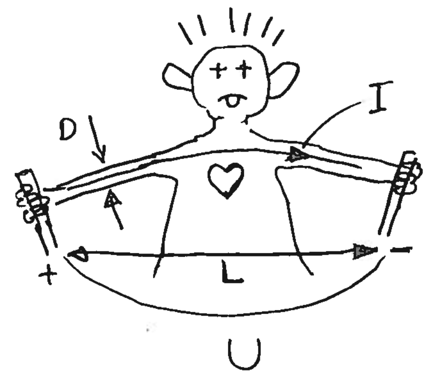
konst. m.a.p area vid visst dl!

$$= \frac{1}{\sigma} \int_0^h \frac{1}{\pi (r(l))^2} dl = \frac{1}{\sigma} \int_0^h \frac{1}{\pi \left(r_1 + \left(\frac{r_2 - r_1}{h}\right) l\right)^2} dl =$$

$$= -\frac{1}{\pi \sigma} \left[\frac{h}{r_2 - r_1} \cdot \frac{1}{r_1 + \left(\frac{r_2 - r_1}{h}\right) l} \right]_0^h = \frac{h}{\sigma \pi r_2 r_1} = \frac{\rho h}{\pi r_2 r_1}$$

b) Om $r_1 \to r_2 \Rightarrow R \to \rho \frac{h}{\pi r^2} = \rho \frac{h}{A}$ V.S.V.

25.71) Dödlig ström - resistans.



Sökt: a) R mellan två händer?

b) Vilket U ger $I_{död} = 100\ mA$?

c) Effekt P?

Känt: Modell: cylinder med $L = 1{,}6\ m$ och $D = 0{,}1\ m$

(Försumma ytresistansen hos huden)

Resistivitet $\rho = 5\ \Omega m$

a) $R = \rho \frac{L}{A} = \rho \frac{L}{\pi (D/2)^2} = \underline{\underline{1{,}0\ k\Omega}}$

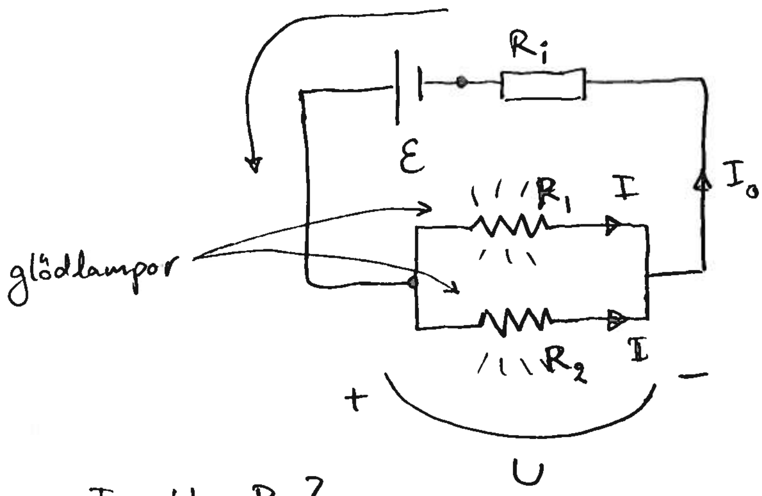
b) Ohms lag: $U = RI$

$\Rightarrow U_{död} = R I_{död} = 102\ V$

Växelspänning 240 V ?!

c) $P = U \cdot I = RI^2 = 10\ W$

26.57) Parallellkoppling av resistans.



<u>Sökt</u>: I, U, P ?

<u>Känt</u>: $R_i = 0{,}8\ \Omega$, $R_1 = R_2 = 2.0\ \Omega$, $\mathcal{E} = 8{,}0$ V

✱ Kirchoff's spänningslag ger:

$$\mathcal{E} - R I_0 - R_i I_0 = 0$$

$$\left(\text{Parallellt} : \Rightarrow \quad R = \left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1} \right)$$

Ekvation ger $$I_0 = \frac{\mathcal{E}}{\left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1} + R_i} = 4{,}44 \text{ A}$$

a) Vi har $$U = R I_0 = \left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1} I_0 = \underline{\underline{4{,}44 \text{ V}}}$$

$$\Rightarrow \quad I = U/R_1 = U/R_2 = \underline{\underline{2.22 \text{ A}}}$$

$$\& \quad P = R_1 I^2 = R_2 I^2 = \underline{\underline{9.9 \text{ W}}}$$

b) Låt $R_2 = \infty$ (ena lampan går sönder)

$$\Rightarrow \quad I_0 = \frac{\mathcal{E}}{R_1 + R_i} = I = 2.86 \text{ A}$$

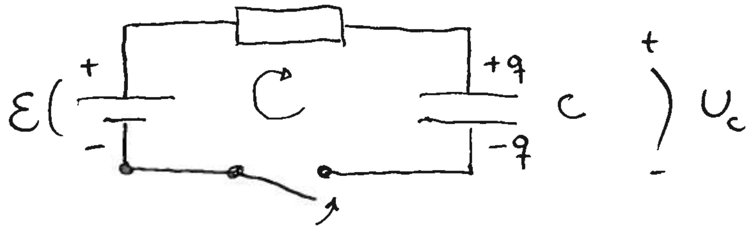
$$\& \quad P = R_1 I^2 = \underline{\underline{16.3 \text{ W}}}$$

ökning - lampa lyser starkare!

Vad händ om $R_i = 0$!?

28.46) RC - krets

Halliday
Resnick
Walker
ed. 6



Sökt : τ, tidskonstanten ; q_{max} ; $t = t_1$ då $q_1 = 16\ \mu C$

Känt : $\mathcal{E} = 12\ V$, $R = 1{,}4\ M\Omega$, $C = 1.8\ \mu F$

* Kirchoffs spänningslag ger :

$$\mathcal{E} - Ri - U_c = 0$$

Sambanden $i = \frac{dq}{dt}$ & $U_c = \frac{q}{C}$

$\Rightarrow$ Inhomogen diffekv. $\boxed{\frac{dq}{dt} + \frac{1}{RC} q = \frac{\mathcal{E}}{R}}$

Lösning : $q(t) = q_p + q_h$

Homogen ; Karaktäristisk ekv. $r^2 + \frac{1}{RC} r = 0$

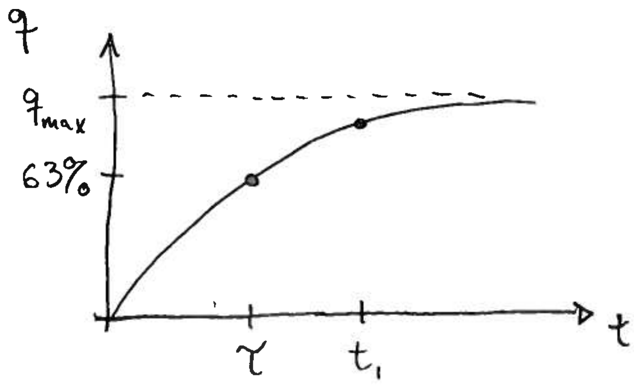
$\Rightarrow \left(r + \frac{1}{RC}\right) r = 0$ har rötterna $\begin{cases} r_1 = 0 \\ r_2 = -\frac{1}{RC} \end{cases}$

Partikulär : $q_p = \mathcal{E} C$

$$\Rightarrow \quad q(t) = q_h + q_p = A e^{r_1 t} + B e^{r_2 t} + \mathcal{E} C$$

[Villkor: $q(t \to \infty) \to \mathcal{E}C \Rightarrow A = 0$; $q(t=0) = 0 \Rightarrow B = -\mathcal{E}C$]

$$\therefore \quad \boxed{q(t) = \mathcal{E}C\left(1 - e^{-\frac{1}{RC}t}\right)}$$



a) def. tidskonstant $\tau = RC$

den tid då q nått 63% av slutvärde.

$$\left(1 - e^{-\frac{1}{RC}\tau}\right) = 1 - e^{-1} = 0{,}63 = 63\%$$

$\rightarrow \quad \tau = 2{,}52 \text{ s}$

b) $q_{max} = q(t \rightarrow \infty) = \mathcal{E}C = 21{,}6\ \mu C$

c) $q(t = t_1) = q_1$

$$\Rightarrow \quad q_1 = q_{max}\left(1 - e^{-\frac{t_1}{\tau}}\right)$$

$$\therefore \quad t_1 = -\tau \ln\left(1 - \frac{q_1}{q_{max}}\right) = 3{,}40 \text{ s}$$