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v.2

Övning 4 SK1111

Repetition

I. ELSTATIK forts. (kap. 23-24)

Energibetraktelse

Skillnad i potentiell energi!

$\frac{\text{Energiskillnad}}{\text{Laddningsenhet}}$.

Arbete

$$\boxed{W_p = U_a - U_b = \int_a^b \bar{F}\cdot d\bar{l}}$$

Potential

$$\boxed{V = V_a - V_b = -\int_b^a \bar{E}\cdot d\bar{l}}$$

Relation: $W_p = qV \xleftarrow{\text{Integration}} F = qE$

Differential form:

$$F = \frac{dW}{dl} \qquad E = -\frac{dV}{dl}$$

Typisk arbetsgång:

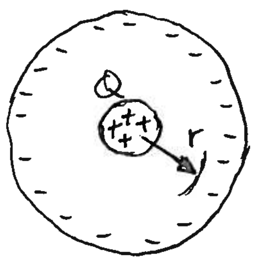
Laddning $Q \longrightarrow$ Fält $\bar{E} \longrightarrow$ Potential $V \longrightarrow$ Kapacitans C

Gauss lag:

$$\Phi = \oint\!\!\oint \bar{E}\cdot d\bar{A} = \frac{Q}{\varepsilon_0}$$

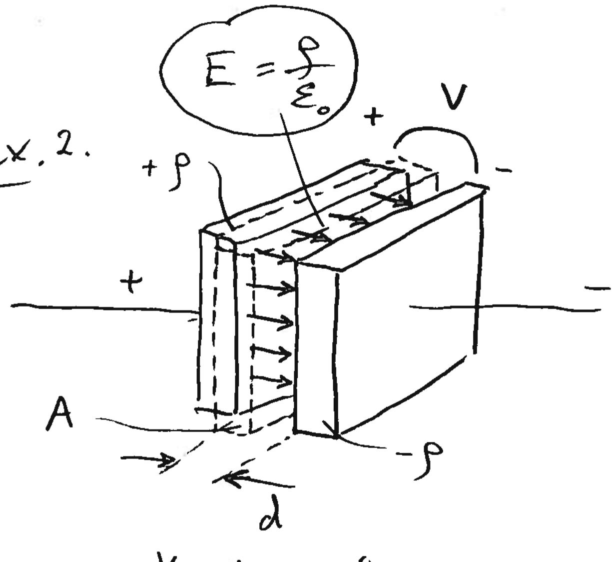
ex. 1.

$V(r)$?



Vanligtvis $V_b = 0$ i oändligheten

ex. 2.



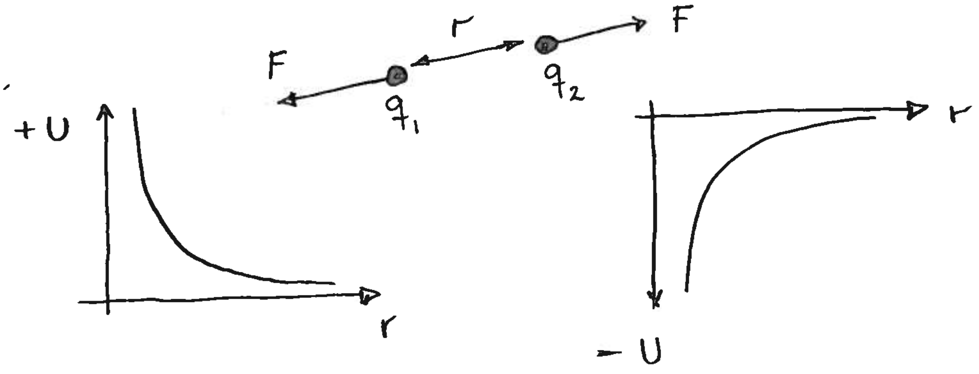
Kondensator

Potentiell energi

$$\boxed{U(r) = \frac{q_1 q_2}{4\pi\varepsilon_0 r}}$$

$U(\infty) = 0$!

ex.

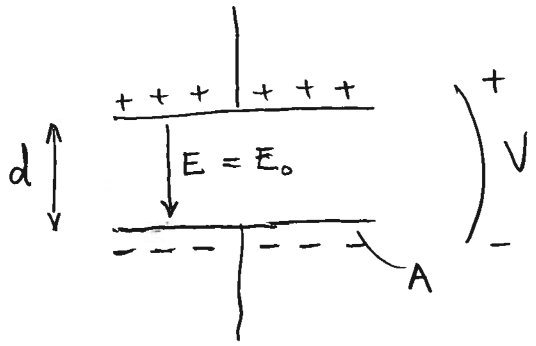


q_1, q_2 lika tecken $\qquad$ q_1, q_2 olika tecken

Kapacitans

$$\boxed{\begin{aligned} C &= \frac{Q}{V} \\ C &= \frac{\varepsilon_0 A}{d} \end{aligned}}$$

vakuum !

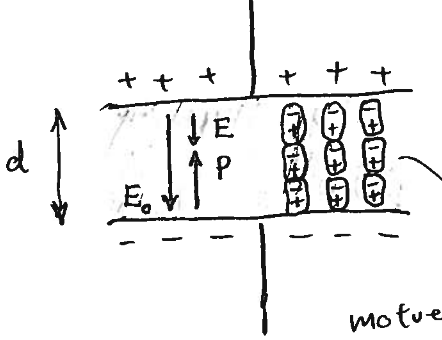


(Generellt i dielektrium ersätt $\varepsilon_0 = \varepsilon_0 \varepsilon_r$; $E = E_0 / \varepsilon_r = D / \varepsilon_0 \varepsilon_r$

ε_r : relativ permeabilitet ≥ 1

$E = E_0 - P/\varepsilon_0$

Dielektrikum ! $\boxed{C = \frac{\varepsilon_0 \varepsilon_r A}{d}}$

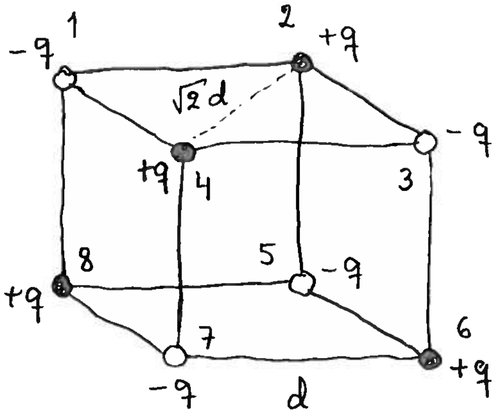


dielektrisk (polariserande) material motverkar E-fält.

Lagrad energi : $\boxed{U = \frac{1}{2} C V^2}$

Stor ε_r kan lagra mer energi ! med samma spänning.

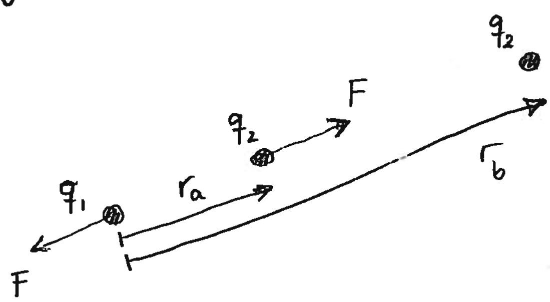
23:57) Potentiell energi i saltkristall. ③



Sökt: Energi $U(d)$? Känt: $U(\infty) = 0$

Idé: Vilket arbete utförs om vi för samman alla laddningar var för sig ifrån oändligheten?

- Coulombs lag: $\bar{F} = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r^2}\bar{e}_r$



- Arbete: $W_{ab} = \displaystyle\int_{r_a}^{r_b} \bar{F}\cdot d\bar{r}' =$

$$= \frac{q_1 q_2}{4\pi\varepsilon_0}\int_{r_a}^{r_b} \frac{1}{r'^2}\bar{e}_r \cdot dr'\,\bar{e}_r = \frac{q_1 q_2}{4\pi\varepsilon_0}\left[-\frac{1}{r'}\right]_{r_a}^{r_b} = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

- Energiförändring: $U_a - U_b = W_{ab} = U(r_a) - U(r_b)$; $U(r_b) = 0$ då $r_b \to \infty$

$$\Rightarrow \boxed{\text{Potentiell energi} \quad U(r) = \frac{q_1 q_2}{4\pi\varepsilon_0 r}}$$

a) För partikel n: $U_n = U_{n,n-1} + U_{n,n-2} + U_{n,n-3} + \dots + U_{n,0}$

Vi har $U_1 = 0$ då initialladdning $n=0$ saknas.

$$U_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{-qq}{d}\right)$$

$$U_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{-qq}{d} + \frac{(-q)(-q)}{\sqrt{2}d}\right)$$

④

forts,

$$U_4 = \frac{1}{4\pi\varepsilon_0}\left[\frac{-qq}{d} + \frac{qq}{\sqrt{2}d} + \frac{-qq}{d}\right]$$

$$U_5 = \frac{1}{4\pi\varepsilon_0}\left[\frac{-qq}{\sqrt{3}d} + \frac{(-q)(-q)}{\sqrt{2}d} + \frac{-qq}{d} + \frac{(-q)(-q)}{\sqrt{2}d}\right]$$

$$U_6 = \frac{1}{4\pi\varepsilon_0}\left[\frac{-qq}{d} + \frac{qq}{\sqrt{2}d} + \frac{q(-q)}{d} + \frac{qq}{\sqrt{2}d} + \frac{-qq}{\sqrt{3}d}\right]$$

$$U_7 = \frac{1}{4\pi\varepsilon_0}\left[\frac{-qq}{d} + \frac{(-q)(-q)}{\sqrt{2}d} + \frac{-qq}{d} + \frac{(-q)(-q)}{\sqrt{2}d} + \frac{-qq}{\sqrt{3}d} + \frac{(-q)(-q)}{\sqrt{2}d}\right]$$

$$U_8 = \frac{1}{4\pi\varepsilon_0}\left[\frac{-qq}{d} + \frac{qq}{\sqrt{2}d} + \frac{-qq}{d} + \frac{qq}{\sqrt{2}d} + \frac{-qq}{\sqrt{3}d} + \frac{qq}{\sqrt{2}d} + \frac{-qq}{d}\right]$$

$$U_{tot} = \sum U_n = \frac{q^2}{4\pi\varepsilon_0}\left[-\frac{12}{d} + \frac{12}{\sqrt{2}d} - \frac{4}{\sqrt{3}d}\right] = -5.82 \cdot \frac{q^2}{4\pi\varepsilon_0 d}$$

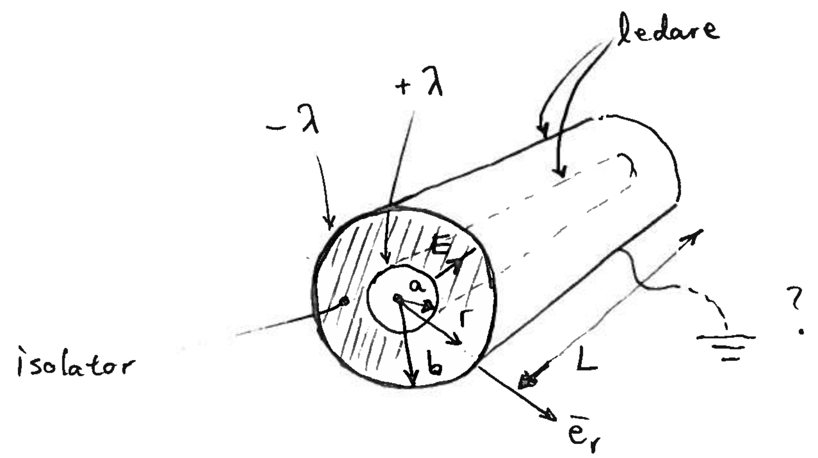
Negativ!

b) Saltkristaller kan byggas upp spontant i naturen eftersom den s.k. gitterenergin är negativ.

Avger energi vid kristallbildning!

Omvänt, när salt löses i en vätska utnyttjas hydratiseringsenergin samtidigt som temperaturen sjunker.

23.61) Elektrisk potential i cylinder



<u>Sökt</u>: Potential $V(r)$ och $E(r)$ $\forall\ 0 < r < \infty$?

$V_a - V_b$?

<u>Känt</u>: Laddning λ. Höljets potential noll.

Idé: $Q \xrightarrow[\text{Gauss lag}]{} E \xrightarrow[\text{Potential}]{} V$

Metallcylinderns laddningar ligger på ytan så att E-fältet är noll inuti cylidern.

$$\boxed{\text{Gauss lag} \quad \oint\!\!\oint \bar{E}\cdot d\bar{A} = \frac{Q}{\varepsilon_0}}$$

[E-fältet radiellt av symmetriskäl. $\bar{E} = E_r\,\bar{e}_r$, $d\bar{A} = dA\cdot\bar{e}_r$]

<u>Område</u>:

<u>$r<a$</u>

$$\oint\!\!\oint_A \bar{E}\cdot d\bar{A} = 0 \quad \text{då} \quad Q_{\text{innanför}} = 0$$

$$\Rightarrow \boxed{E_{r<a} = 0}$$

<u>$a \le r \le b$</u>

$$\oint\!\!\oint_A \bar{E}\cdot d\bar{A} = \oint\!\!\oint_A E_r\,\bar{e}_r\, dA\,\bar{e}_r = E_r \iint_A dA =$$

$$= E_r \cdot 2\pi r L = \frac{\lambda L}{\varepsilon_0}$$

$Q = \lambda L$

$$\Rightarrow \boxed{E_{a<r<b} = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

$r > b$ $\quad \oint\!\!\oint_A \bar{E}\cdot d\bar{A} = \frac{\lambda L - \lambda L}{\varepsilon_0} = 0$

$$\Rightarrow \boxed{E_{r>b} = 0}$$

a) Potential

$$\boxed{V_a - V_b = -\int_b^a \bar{E}\cdot d\bar{l}}$$

Låt V_r vara potentialen på radien r. $V_b = 0$!

$\bar{E} = E_r \cdot \bar{e}_r$, $d\bar{l} = dr \cdot e_r$

Område

ii) $a<r<b$: $V_r = V_r - V_b = -\int_b^r \bar{E}\cdot d\bar{l} = -\int_b^r E_r\, dr' =$

$$= -\int_b^r \frac{\lambda}{2\pi\varepsilon_0 r'}\,dr' = -\frac{\lambda}{2\pi\varepsilon_0}\left[\ln(r')\right]_b^r = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{b}{r}\right)$$

i) $r<a$: $V_r = -\left(\int_b^a \bar{E}\cdot d\bar{l} + \int_a^r \bar{E}\cdot d\bar{l}\right) = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$

$E_r = 0$!

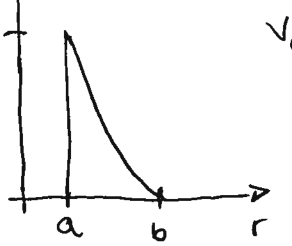
iii) $r>b$: $V_r = -\int_b^r \bar{E}\cdot d\bar{l} = 0$

$E_r = 0$!

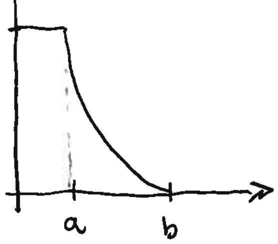
b) $$\boxed{V_{ab} = V_{r=a} = \frac{\lambda}{2\pi\varepsilon_0}\ln(b/a)}$$ V.S.V.

c) Givet V_{ab} har vi: $\lambda = \frac{V_{ab}\cdot 2\pi\varepsilon_0}{\ln(b/a)} \Rightarrow \boxed{E_{a<r<b} = \frac{V_{ab}}{\ln(b/a)\, r}}$

E-fält



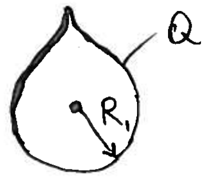
Potential



d) Om $-\lambda = 0$?!

$\Rightarrow$ V_{ab} oförändrad men potentialerna V_a, V_b olika då $E(r>b) \neq 0$!

23.80) Potential regndroppe



Sökt : Potential på ytan $V(R)$?

känt : Laddning $Q = -1{,}20$ pC , $r = 0{,}650$ mm

Ide' : $Q \xrightarrow[\text{Gauss lag}]{} E \xrightarrow[\text{potential}]{} V$

$$\boxed{\oint\!\!\oint \bar{E} \cdot d\bar{A} = \frac{Q}{\varepsilon_0}} \quad (1) \qquad \boxed{V_a - V_b = -\int_b^a \bar{E} \cdot d\bar{l}} \quad (2)$$

Potential i ∞ : $V_b = 0$; radie r : $V_r = V_a$

$$(1) \& (2) \Rightarrow V_r = -\int_\infty^r \frac{Q}{4\pi\varepsilon_0 r'^2}\, dr' = \left[\frac{Q}{4\pi\varepsilon_0 r'}\right]_\infty^r = \frac{Q}{4\pi\varepsilon_0 r}$$

a) $$\boxed{V_{R_1} = \frac{Q}{4\pi\varepsilon_0 R}} = -16.6 \text{ V}$$

OBS: V är :
- potential
- enhet
- volym

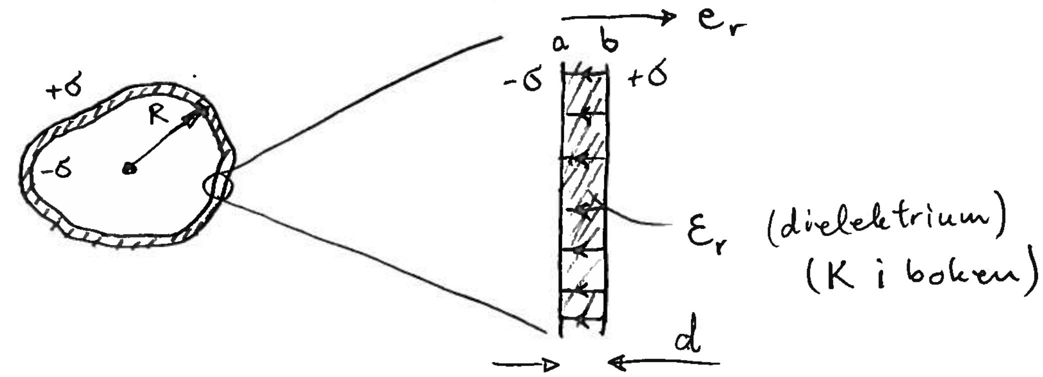
b) Volym en regndroppe : $V_0 = \frac{4\pi R^3}{3}$

Radie två regndroppar $R_2 = \sqrt[3]{\frac{3 \cdot 2 V_0}{4\pi}} = \sqrt[3]{2} \cdot R$

$$\Rightarrow \boxed{V_{R_2} = \frac{2Q}{4\pi\varepsilon_0 \sqrt[3]{2} R}} = -26.4 \text{ V}$$

24.73)
ed. 11.

Cellmembran som kondensator!



Sökt: a) E-fältet i cellväggen?
b) Potentialskillnad Kapacitans?
c) Total energikapacitet?

Känt: $\sigma = 0{,}5 \cdot 10^{-3}$ C/m^2 ; $\varepsilon_r = 5{,}4$; $V_{cell} = 10^{-16}$ m^3, $d = 5 \cdot 10^{-9}$ m

Idé: $Q \xrightarrow[\text{Gauss lag}]{} E \xrightarrow[\text{Potential}]{} V \xrightarrow[\text{energi}]{} W$

Gauss lag:

$$\oint \bar{E} \cdot d\bar{A} = \frac{Q}{\varepsilon_0 \varepsilon_r}$$

Generell version för dielektrium!

a) Radiell symmetri: $E_r \cdot 4\pi R^2 = -\dfrac{\sigma \cdot 4\pi R^2}{\varepsilon_0 \varepsilon_r}$

$$\Rightarrow \boxed{E_r = -\frac{\sigma}{\varepsilon_0 \varepsilon_r}} = -1{,}0 \cdot 10^7 \text{ V/m}$$

b) Vi har $V = V_b - V_a = -\displaystyle\int_a^b \bar{E} \cdot d\bar{l} = \left\{ \begin{array}{l} \bar{E} = E_r \bar{e}_r \\ d\bar{l} = dr\, \bar{e}_r \end{array} \right\}$

$$= -\int_0^d E_r \cdot dr = \int_0^d \frac{\sigma}{\varepsilon_0 \varepsilon_r} dr = \frac{\sigma}{\varepsilon_0 \varepsilon_r} \left[r \right]_0^d = \frac{\sigma d}{\varepsilon_0 \varepsilon_r} \quad (1)$$

$$= 0{,}05 \text{ V} > 0 \Rightarrow V_b > V_a$$

c) Lagrad energi: $U = \frac{1}{2} C V^2 = \left\{ C = \frac{Q}{V} \right\} = \frac{1}{2} Q V$

Vi har $Q = \sigma \cdot 4\pi R^2$

$\left(\text{där } R = \sqrt[3]{\frac{3 V_{cell}}{4\pi}} \right)$

och $V = \frac{\sigma d}{\varepsilon_0 \varepsilon_r}$ från (1)

$\Rightarrow \quad U = \frac{2\pi \sigma^2 R^2 d}{\varepsilon_0 \varepsilon_r} = \underline{\underline{1{,}36 \cdot 10^{-15} \text{ J}}}$

extra)

Definition: $C = \frac{Q}{V} = \frac{\cancel{\sigma} \cdot 4\pi R^2}{\cancel{\sigma} d} \varepsilon_0 \varepsilon_r$

$\Rightarrow \quad \boxed{\text{Kapacitans} \quad C = \frac{\varepsilon_0 \varepsilon_r A}{d}}$ där A: kondensatorns area.

$\left(C = 1 \cdot 10^{-12} \text{ F} = \underline{\underline{1 \text{ pF}}} \right)$