WIDEBAND PARAMETRIC IDENTIFICATION OF A POWER TRANSFORMER

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Abstract

This paper applies a recently developed technique for wideband frequency domain system identification to parametric modelling of a power transformer using frequency response data. It is well known that frequency response data from a typical power transformer contains many resonant modes and spans several decades in frequency. This in effect leads to a numerically ill-conditioned problem when attempting to fit a parametric model to the data. The method employed here to fit the parametric model utilises what has been termed frequency localising basis functions. These functions have been shown to improve the numerical conditioning of a least squares type estimator which correspondingly results in an increased accuracy of the estimated parameters. Two examples are used in this paper to highlight the veracity of the frequency localising basis functions technique. The first example uses simulation data, whilst the second utilises frequency response data from a large power transformer.

1. Introduction

Frequency Response Analysis (FRA) is a method commonly used for monitoring the condition of a power transformer. It is well known [1] that the frequency response of a transformer provides a 'signature' unique to its mechanical geometry. Hence, any change in the geometry will result in a change to the observed frequency response. This makes FRA an essential component of any transformer testing regime.

When monitoring transformers using FRA, the typical industry practice is to visually compare the frequency responses of 1). different phases in the same apparatus, or 2). the same phase on sister units, or 3). using FRA data collected from the same phase on the same transformer at an earlier period in time [2]. Variation of the comparative responses indicates a geometric change which can be indicative of a variety of faults and/or structural damage [3, 1].

Rather than rely solely on the judgement of trained personnel, there is an ongoing research effort to automate the FRA condition monitoring process. To accomplish this, a mathematical model is required to accurately emulate the frequency response data from the power transformer and hence provide a facility to determine the degree of difference between various data sets [4]. To accomplish this, an accurate parametric model based on the frequency response becomes a prerequisite for the development of a completely automated analysis technique.

Two commonly used tests employed in FRA are known as the swept frequency and impulse tests. The swept frequency test, as the name describes, injects a series of sine waves within a desired frequency range into the transformer. Both the input and output signals are measured as shown, for a typical FRA testing setup, in Figure 1. The ratio of the output to input response, in the frequency domain, for each injected frequency provides the frequency response of the system. Alternatively, the impulse test will inject a low voltage impulse signal into the transformer, where again, the frequency response of the system is obtained in a similar manner.

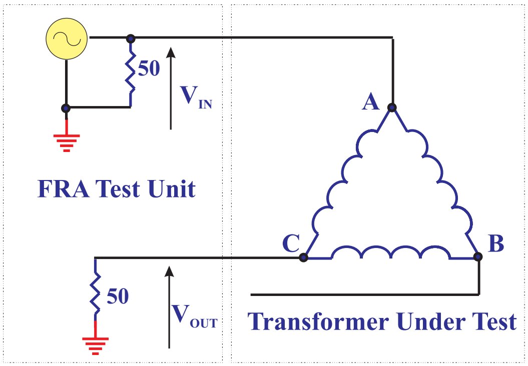

Fig. 1. Frequency Response Analysis Test Equipment Arrangement

In the conditioning monitoring of a power transformer, as stated above, FRA is utilised to detect variations in the structural geometry. To be able to detect small variations automatically an accurate parametric model of the transformer is required. Frequency responses of transformers are generally produced over a wideband, i.e. ranging from 10's of Hz to 10's of MHz. The size of this frequency range alone presents a problem for most parametric system identification methods. Further compounding this problem is the fact that the transformer response also contains a large number of resonant modes, usually greater than 20.

It is well known [5, 6, 7] that accuracy is inherently related to the degree of ill-conditioning in an estimator. In the problem we are considering ill-conditioning arises due to the large dynamic range of the entries in the regressor matrix. If, for example, the frequency range spans 3 decades and the system is of $5th$ order then the dynamic range will be, at a minimum, $10^{15}\ :\ 1$. Furthermore, the normal matrix in a least squares type estimator is highly correlated since every frequency will influence every coefficient being estimated.

The problem of obtaining a good parametric linear time invariant model, described by a transfer function involving the ratio of two polynomials, for a single-input-single-output (SISO) continuous time system, such as a power transformer, is very old. Levy [8] proposed a least squares based estimation technique, for use with experimentally obtained frequency domain data. It is well known that the normal matrix used in this type of estimator is sensitive to the dynamics and bandwidth of the system and can lead to ill-conditioning [5]. This typically manifests itself as poor or erroneous estimates of the system parameters.

There is substantial literature on the problem of how to improve the conditioning of the normal matrix. For example, [9] used frequency scaling and [10, 9] utilised orthogonal polynomials. Orthonormal basis functions have been shown to provide perfect conditioning of the normal matrix for specific input signals [11] and do exhibit some degree of robustness with respect to spectral colouring of the input. However, as shown in [12], there is still significant ill-conditioning associated with all the above mentioned methods for systems with a large dynamic range and more general inputs.

A technique proposed in [12], uses particular filters called 'frequency localising basis functions' (FLBFs). These functions span a desired frequency region, thus restricting the dynamic range over which each coefficient is estimated. This allows the normal matrix to take on a near block diagonal form, hence improving its conditioning when estimating over very large dynamic ranges. It has been shown in [13] that the frequency localising basis functions give rise to a bounded condition number, for output error models, irrespective of the dynamic range of the system.

Unlike the orthonormal basis functions, FLBFs are only 'nearly orthogonal', however this is over a wide range of inputs. Thus an exact property is traded for an approximate property with the aim of achieving numerical robustness.

Other techniques [14] have been proposed for the rational approximation of frequency response data. These techniques are quite complex in implementation when compared to the frequency localising basis functions. In fact, as will be shown section 2, FLBFs are essentially bandpass filters and hence relatively simple to implement.

In this paper, we describe a technique using FLBFs in a least squares type estimator for wideband (or large dynamic range) system identification. We then show how they can be successfully used to obtain an accurate parametric model of a power transformer over several decades of frequency.

The structure of the paper is as follows. In Section 2 we begin by formulating the problem and demonstrating how the frequency localising basis functions are utilised in a least squares type estimator. Next in Section 3 we consider higher order frequency localising basis functions, which can be used to represent systems with sharper resonances and also improve numerical conditioning when the input frequencies are close together. Section 4, presents a simulated example where the FLBFs are compared to some traditional methods over a large dynamic range. Then, in Section 5 we illustrate with a real world example, based FRA data from a power transformer, the accuracy obtained when utilising FLBFs in the estimation of a large dynamic range system. We present conclusions in Section 6.

2. Problem statement

In this section we outline the general estimation problem and provide a description of the frequency localising basis functions and how they are applied to a least squares estimator.

Consider a single-input-single-output linear continuous time system, with input $\{u(t)\}_{t\in\mathbb{R}}$ and output $\{y(t)\}_{t\in\mathbb{R}}$, defined by the strictly proper transfer function

$$G(s) := \frac{B(s)}{A(s)},$$

where

$$\begin{aligned} A(s) &:= s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0 \\ B(s) &:= b_m s^m + b_{m-1}s^{m-1} + \cdots + b_0; \\ &\qquad\qquad n, m \in \mathbb{N}\,(n > m). \end{aligned}$$

Let the input be a sum of sine waves of unit amplitude and equal phase at frequencies $\omega_1, \ldots, \omega_N$ ($N \in \mathbb{N}$) (i.e., $U(j\omega_k) = 1$ for $k = 1, \ldots, N$), and a model be given by

$$\hat{G}(s) := \frac{\hat{B}(s)}{\hat{A}(s)}.$$

Here, $\hat{A}$ and $\hat{B}$ are polynomials which minimise the cost

$$J := \sum_{k=1}^{N} \left| \frac{\hat{A}(j\omega_k)}{E(j\omega_k)} Y(j\omega_k) - \frac{\hat{B}(j\omega_k)}{E(j\omega_k)} \right|^2 \qquad (1)$$

where

$$\begin{aligned} \frac{\hat{A}(s)}{E(s)} &= 1 + \sum_{k=1}^{\tilde{n}} \alpha_k F_{2k-1}(s), \\ \frac{\hat{B}(s)}{E(s)} &= \sum_{k=1}^{\tilde{n}} \beta_k F_{2k}(s) \end{aligned}$$

and $\alpha_1, \ldots, \alpha_{\tilde{n}}, \beta_1, \ldots, \beta_{\tilde{n}} \in \mathbb{C}$ ($\tilde{n} \in \mathbb{N}$). The functions F_k, are those we term FLBFs [12], and are of the form

$$F_k(s) := s^{k-1} p_k \prod_{i=1}^{k} \frac{1}{s + p_i}; \qquad k = 1, \ldots, 2\tilde{n},$$

where $0 < p_1 < \cdots < p_{2\tilde{n}} < \infty$.

To obtain the values of $\alpha_1, \ldots, \alpha_{\tilde{n}}, \beta_1, \ldots, \beta_{\tilde{n}}$ which minimise J, we can use the Least Squares (LS) estimator:

$$\hat{\theta} = (\mathbf{X}^H \mathbf{X})^{-1} \mathbf{X}^H \mathbf{Y}$$

where ^{H} is the complex conjugate transpose, and

$$\begin{aligned} \hat{\theta} &:= [\, \alpha_1 \;\; \beta_1 \;\; \cdots \;\; \alpha_{\tilde{n}} \;\; \beta_{\tilde{n}} \,]^T \in \mathbb{C}^{2\tilde{n}\times 1} \\ \mathbf{X} &:= \begin{bmatrix} Y_1^1 & U_1^1 & \cdots & Y_1^{\tilde{n}} & U_1^{\tilde{n}} \\ \vdots & \vdots & & \vdots & \vdots \\ Y_N^1 & U_N^1 & \cdots & Y_N^{\tilde{n}} & U_N^{\tilde{n}} \end{bmatrix} \in \mathbb{C}^{N\times 2\tilde{n}} \\ \mathbf{Y} &:= \left[\, Y(j\omega_1) \;\; \cdots \;\; Y(j\omega_N) \,\right]^T \in \mathbb{C}^{N\times 1} \\ Y_k^i &:= -F_{2i-1}(j\omega_k) Y(j\omega_k) \\ U_k^i &:= F_{2i}(j\omega_k); \qquad k = 1, \ldots, N; \quad i = 1, \ldots, \tilde{n}. \end{aligned}$$

Note that we are not in anyway advocating explicitly to form and then invert the normal matrix. In practice, $\hat{\theta}$ should be computed using techniques such as Cholesky Factorisation, Householder Transformation or QR Factorisation [5], which do not form and invert $(X^H X)$.

To re-parameterise the model in terms of the parameters $\hat{a}_1, \ldots, \hat{a}_n, \hat{b}_1, \ldots, \hat{b}_m \in \mathbb{R}$ ($n, m \in \mathbb{N}$) involves only a simple transformation [12], i.e. for the parameters of $\hat{A}(s)$ let

$$\begin{aligned} M^k(s) &:= F_k(s) E(s) \\ &= m_{n-1}^k s^{n-1} + \cdots + m_{k-1}^k s^{k-1}; \\ &\qquad\qquad k = 1, \ldots, n, \end{aligned}$$

where k represents the kth basis function, then

$$\hat{\mathbf{a}} = M\underline{\alpha} + \mathbf{e},$$

where $\hat{\mathbf{a}}$ and $\mathbf{e}$ are the parameter vectors of $\hat{A}(s)$ and $E(s)$ respectively, $\underline{\alpha}$ is the vector of parameters α_k and

$$M := \begin{bmatrix} m_0^1 & 0 & \cdots & 0 \\ m_1^1 & m_1^3 & \ddots & \vdots \\ \vdots & \vdots & & 0 \\ m_{n-1}^1 & m_{n-1}^3 & \cdots & m_{n-1}^{2\tilde{n}-1} \end{bmatrix}.$$

The re-parameterisation for $\hat{B}(s)$ follows a similar procedure.

3. An extension to higher order filters

In order to improve the numerical conditioning of the least squares type estimator even further, frequency localising basis functions that have a sharper frequency response (or larger roll-off rate) can be used. This also has the added benefit of enhancing the parametric modelling of sharp resonant modes in a frequency response. These sharp resonances are often observed in a power transformer frequency response analysis. Hence indicating that it would perhaps be better to consider, for the transformer frequency response, the use of these sharper frequency localising basis functions.

We then consider the following frequency localising basis functions which can be set to have an arbitrary roll-off rate,

$$F_k(s) := s^{q(k-1)} p_k^q \prod_{i=1}^{k} \frac{1}{(s + p_i)^q}; \qquad (2)$$
$$k = 1, \ldots, 2\tilde{n},$$

where $q \in \mathbb{N}$ and $0 < p_1 < \cdots < p_{2\tilde{n}} < \infty$.

Here the variable q is used to set the desired sharpness (roll-off rate) of the frequency localising basis functions. To compare the responses for different values of q we provide Bode magnitude plots. Figure 2 shows the magnitude of the frequency response of the FLBFs, given by (2), for $p_1 = 1$, $p_2 = 10$ and $p_3 = 100$ and, with $q = 1$ and $q = 2$.

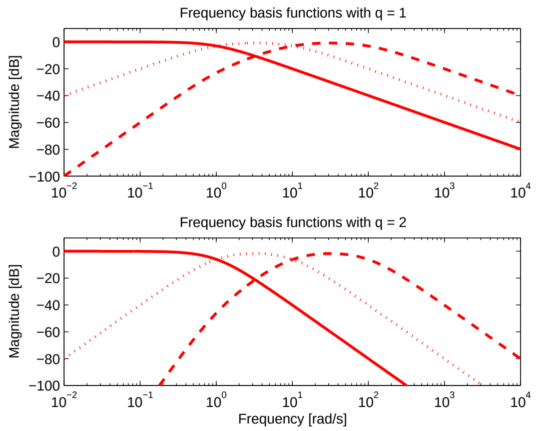

Fig. 2. Bode magnitude plots of frequency localising basis functions for $q = 1$ (top) and $q = 2$ (bottom). Solid line: $F_1(s)$, dotted line: $F_2(s)$, dashed line: $F_3(s)$.

4. Simulation example

The frequency localising basis function technique was compared to several other commonly used methods in [12]. To show the performance of the FLBFs versus the other methods in this paper we need to reproduce the example from [12]. This is done specifically as none of the other methods could produce any reliable results from the actual transformer data used in section 5. In fact most of the methods were so ill-conditioned that the algorithms failed completely.

To illustrate the performance of the FLBFs a highly resonant system of large order that spans several decades of frequency was chosen. The transfer function of this system is given by

$$G_k(s) = \sum_{k=1}^{9} \frac{b_k \omega_k^2}{(s^2 + 2\zeta_k \omega_k s + \omega_k^2)} \quad (3)$$

where $\{\omega_k\}$ spans 9 decades.

A bode magnitude plot of the true system is shown in Figures 3 and 4 as a dashed line. The frequency response of the system was evaluated at 36 logarithmically spaced points that span the complete frequency range dictated by the resonant modes in the transfer function. Note that no noise was added to the frequency response data of the system for this comparison. Models were then estimated using the following methods:

(1) Nonlinear Least Squares (NLS)
(2) Nonlinear Least Squares using Frequency Scaling (NLSFS)
(3) Tchebyshev Polynomials (TP)
(4) Laguerre Basis Functions with the poles chosen by a discretised search between the minimum and maximum frequency, so as to give the best fit (LBF)
(5) FLBF's with break points chosen logarithmically spaced between minimum and maximum frequency (FLBF)

It was also noted that Kautz basis functions where evaluated, but given the low number of excitation signals the authors were unable to obtain a satisfactory fit.

Estimation was carried out based on measured frequency response data using the cost function (1). Figures 3 and 4 compare the magnitudes of the estimated frequency response for each method listed above with that of the true system. The true system is plotted as a dashed line, the estimates as a solid line in all cases.

It can be seen from Figure 4 that the FLBF's are the only procedure to give an acceptable fit to the data over the full frequency range. The authors [12] acknowledged that it may well be possible to choose the various "free parameters" in the other methods to obtain a good fit. Some steps were taken to achieve this but clearly a claim cannot be made to have exhausted all possibilities. However, it does seem that the FLBF's are particularly easy to "tune" and give excellent results.

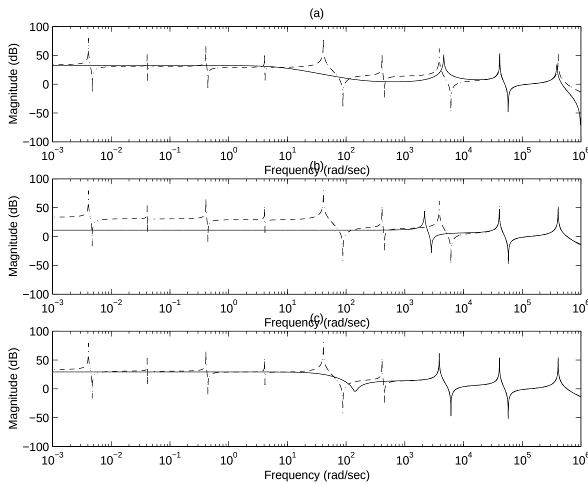

Fig. 3. Magnitude plots of estimates using (a) Nonlinear least squares, (b) Nonlinear least squares with frequency scaling, (c) Tchebyshev polynomials. The true system appears in all plots as a dashed line.

5. Power transformer example

To demonstrate the potential of the frequency localising basis functions, in a least squares type estimator, to produce accurate parametric models for wideband

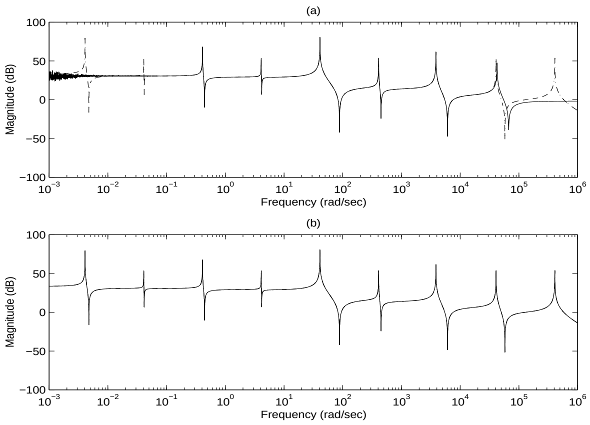

Fig. 4. Magnitude plots of estimates using (a) Laguerre basis functions, (b) Frequency Localising Basis Functions. The true system appears in all plots as a dashed line.

(large dynamic range) systems, we provide an example based on real experimental data [1] . This data is the result of a frequency response test on a large power transformer. It is well known that fitting a parametric model to this type of data using standard techniques is extremely difficult [15].

The data used in this example was collected from a Frequency Response Analysis test performed on the A-phase high voltage winding of an ABB power transformer with a rating of 132kV, 60MVA. At the time of test the transformer was filled with oil and had all bushings in place. The data was collected from a swept frequency test with a minimum test frequency of 20Hz (125 rad/sec) and a maximum test frequency of 974KHz (6.2×10^6 rad/sec). Note that some low frequency data points were deliberately removed from the response due to possible contamination with respect to 50Hz interference from the mains power supply.

In the first instance we consider frequency localising basis functions with $q = 1$. The response of these basis functions are as shown in top plot of Figure 2. Using the cost function given by (1) a parametric model of the transformer was estimated. Figures 5 and 6 provide the Bode magnitude and phase plots respectively for both the frequency response data and the estimated model. It can be clearly seen that the FLBFs provide an extremely good model over the 5 decades of frequency and 20 resonant modes. The dynamic range for this case is extremely large by any standard of measure.

It can be seen in Figures 5 and 6 that at the high frequency end, where there a several sharp resonance peaks close together, that the fit could perhaps be improved. Here we are now able to demonstrate the use of FLBF's with $q = 2$, i.e. the basis functions have a sharper response as shown in the bottom plot

[1] The authors would like to thank Connell Wagner of Australia for making available the data.

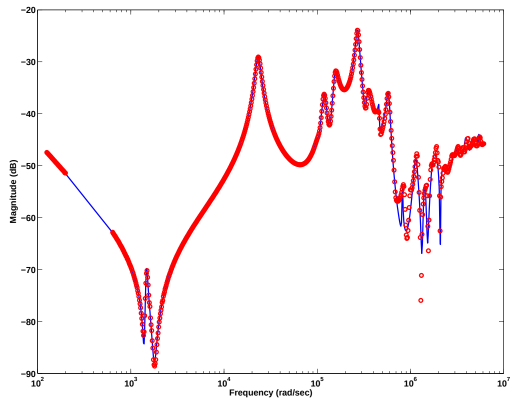

Fig. 5. Magnitude Response. Frequency response data (o), Parametric model (-).

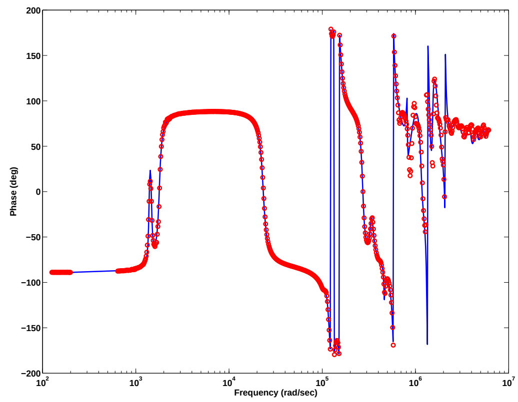

Fig. 6. Phase Response. Frequency response data (o), Parametric model (-).

Table 1. Quantitative Comparison of Frequency Localising Basis Functions

FLBF	L_2 Cost	L_∞ Cost
q=1	2.2133×10^{-6}	0.0046
q=2	0.7897×10^{-6}	0.0017

of Figure 2. The same cost function is used to then estimate the parametric model with the exception that the basis functions now have $q = 2$. The results are shown in Figures 7 and 8 for the Bode magnitude and phase plots. It is clearly evident that an improvement has been achieved in the model fit at the higher frequencies where the there are more 'features' in the observed response.

We also compare the results quantitatively for the different frequency localising basis functions (i.e. $q = 1$ and $q = 2$) using two measures; the mean square error and maximum error between the estimated model and the frequency response data at the frequencies of excitation. The results are given in Table 1. Although the fit obtained when using $q = 2$ looks much better on the Bode diagrams, there is not a large difference in the actual quantitative measures.

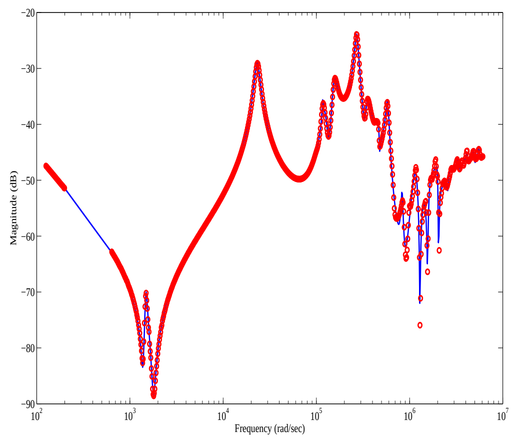

Fig. 7. Magnitude Response. Frequency response data (o), Parametric model (-).

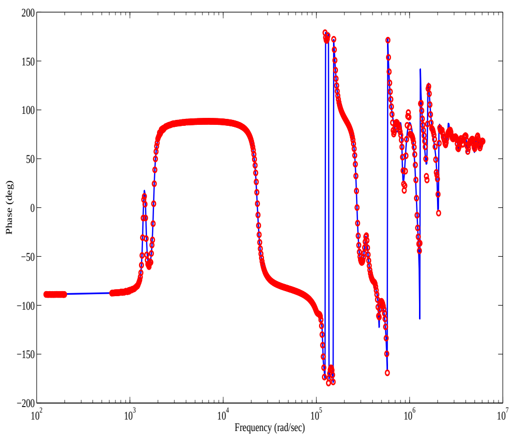

Fig. 8. Phase Response. Frequency response data (o), Parametric model (-).

6. Conclusion

In this paper we have described a recently developed technique for wideband system identification namely, Frequency Localising Basis Functions. We then applied this technique to frequency response data from a power transformer where it was shown that an accurate parametric model can be obtained over the entire frequency range. It is proposed that such a model could be beneficial in an automated method of comparing the frequency response of transformers to improve diagnostics by observing the changes in the parameters from the estimated model.

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