[20] S. Vishwanath, N. Jindal, and A. Goldsmith, "Duality, achievable rates, and sum-rate capacity of Gaussian MIMO broadcast channels," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2658–2668, Oct. 2003.
[21] W. Yu, "Sum-capacity computation for the Gaussian vector broadcast channel via dual decomposition," *IEEE Trans. Inf. Theory*, vol. 52, no. 2, pp. 754–759, Feb. 2006.
[22] C. Peel, B. Hochwald, and A. Swindlehurst, "A vector-perturbation technique for near-capacity multiantenna multiuser communication—Part I: Channel inversion and regularization," *IEEE Trans. Commun.*, vol. 53, no. 1, pp. 195–202, Jan. 2005.
[23] Code repository, Division of Communication Systems, Linköping Univ., Linköping, Sweden [Online]. Available: http://www.commsys.isy.liu.se/en/publications

Weighted Sum-Rate Maximization for MISO Downlink Cellular Networks via Branch and Bound

Satya Krishna Joshi, Pradeep Chathuranga Weeraddana, Marian Codreanu, and Matti Latva-aho

***Abstract*—The problem of weighted sum-rate maximization (WSRMax) in multicell downlink multiple-input single-output (MISO) systems is considered. The problem is known to be NP-hard. We propose a method, based on branch and bound technique, which solves globally the nonconvex WSRMax problem with an optimality certificate. Specifically, the algorithm computes a sequence of asymptotically tight upper and lower bounds and it terminates when the difference between them falls below a pre-specified tolerance. Novel bounding techniques via conic optimization are introduced and their efficiency is demonstrated by numerical simulations. The proposed method can be used to provide performance benchmarks by back-substituting it into many existing network design problems which relies on WSRMax problem. The method proposed here can be easily extended to maximize *any* system performance metric that can be expressed as a Lipschitz continuous and increasing function of signal-to-interference-plus-noise ratio.**

***Index Terms*—Branch and bound, global (nonconvex) optimization, multicell networks, second-order cone program (SOCP), weighted sum-rate maximization.**

I. Introduction

We consider the problem of weighted sum-rate maximization (WSRMax) for multicell downlink systems with linear precoding. The base stations (BSs) are assumed to have multiple antennas while all the receivers are equipped with single antenna. Although the WSRMax is central to many network optimization methods [1]–[8], it is known to be an NP-hard problem [9]. Therefore, we have to rely on global optimization approaches [10], [11] for computing an exact solution.

Manuscript received June 16, 2011; revised October 17, 2011; accepted December 20, 2011. Date of publication December 30, 2011; date of current version March 06, 2012. The associate editor coordinating the review of this manuscript and approving it for publication was Dr. Maja Bystrom. This research was supported by the Finnish Funding Agency for Technology and Innovation (Tekes), Academy of Finland, Nokia, Nokia Siemens Networks, Renesas Mobile Europe, Elektrobit, Graduate School in Electronics, Telecommunications, Automation (GETA) Foundations, EU projects Hydrobionets, and Hycon2.

S. K. Joshi, M. Codreanu, M. Latva-aho are with the Centre for Wireless Communications, University of Oulu, Oulu, Finland (e-mail: sjoshi@ee.oulu.fi; codreanu@ee.oulu.fi; matti.latvaaho@ee.oulu.fi).
P. C. Weeraddana is with the KTH Royal Institute of Technology, Electrical Engineering and ACCESS Linnaeus Center, Automatic and Networked Control Lab, Stockholm, Sweden (e-mail: chatw@kth.se).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.
Digital Object Identifier 10.1109/TSP.2011.2182346

In the case of single-input single-output systems, the problem of WSRMax by using global optimization approaches has been addressed in [12]–[16]. In multiple antenna systems, local methods have been proposed in, e.g., [17] and [18]. In this case, the decision variables space is large, i.e., joint optimization of transmit beamforming patterns, transmit powers, and link activation is required. Therefore, designing global optimal methods for WSRMax in multiantenna systems is a more challenging task.

The main contribution of the paper is to propose a solution method, based on the branch-and-bound (BB) technique, which solves globally the nonconvex WSRMax problem in multiple-input and single-output (MISO) systems within a predefined accuracy ϵ. Specifically, the proposed algorithm computes a sequence of asymptotically tight upper and lower bounds for the maximum weighted sum rate, and it terminates when the difference between the upper and lower bound is smaller than ϵ. Thus, our solution is certified to be at most ϵ-away from the global optimal value. It is worth noting that this paper extends our recent work [16] to MISO systems. In addition, we derive an improved bounding technique[1] that increases significantly the convergence speed as compared with the basic bounds of [16]. The proposed bounds and the transmit beamformers are computed via second-order cone programming (SOCP) [19].

The proposed method can be used to provide performance benchmarks by back-substituting it into many existing network design problems which relies on solving the WSRMax problem (e.g., it allows evaluation of the performance loss encountered by any heuristic method for WSRMax). The proposed framework is not restricted to WSRMax; it can be used to maximize *any* system performance that is Lipschitz-continuous and the increasing function of SINR values.

The remainder of this paper is organized as follows. The considered MISO system model and problem formulation are described in Section II. The branch-and-bound algorithm is introduced in Section III, and the computation of the proposed bounds is described in Section IV. The numerical results are presented in Section V, and Section VI concludes our paper.

Notations

All boldface lower case and upper case letters represent vectors and matrices, respectively, and calligraphy letters represent sets. The notation $\mathbb{R}_+^L$ denotes the set of real L-vectors with nonnegative entries, $\mathbb{C}^T$ denotes the set of complex T-vectors, $|x|$ denotes the absolute value of the scalar x, $\|\mathbf{x}\|_2$ and $[\mathbf{x}]_i$, respectively denote the Euclidean norm and the ith element of the vector $\mathbf{x}$, $\mathbf{I}$ denotes the identity matrix, and $\mathcal{CN}(\mathbf{m}, \mathbf{C})$ denotes the complex circular symmetric Gaussian vector distribution with the mean $\mathbf{m}$ and covariance matrix $\mathbf{C}$. The superscript $(\cdot)^{\mathrm{H}}$ and $(\cdot)^{\star}$ is used to denote a Hermitian transpose of a matrix and a solution of an optimization problem, respectively.

II. System Model and Problem Formulation

A multicell MISO downlink system, with N BSs each equipped with T transmit antennas, is considered. The set of all BSs is denoted by $\mathcal{N}$, and we label them with the integer values $n = 1, \dots, N$. The single data stream is transmitted for each user, and we denote the set of all data streams in the system by $\mathcal{L}$ and label them with the integer values $l = 1, \dots, L$. The transmitter node (i.e., the BS) of the lth data stream is denoted by $tran(l)$, and the receiver node of the lth data stream is denoted by $rec(l)$. We have $\mathcal{L} = \cup_{n \in \mathcal{N}} \mathcal{O}(n)$, where $\mathcal{O}(n)$ denotes the set of data streams transmitted by BS n.

[1]Note that the efficiency of a branch and bound algorithm greatly depends on the specific bounding method.

The antenna signal vector transmitted by the nth BS is given by

$$\mathbf{x}_n = \sum_{l\in\mathcal{O}(n)} d_l \mathbf{m}_l \tag{1}$$

where $d_l \in \mathbb{C}$ and $\mathbf{m}_l \in \mathbb{C}^T$ represent the information symbol and the transmit beamformer associated to lth data stream, respectively. We assume that data stream for different users are independent, i.e., $\mathrm{E}\{d_l d_j^*\} = 0$ for all $l, j \in \mathcal{L}$, where $l \neq j$; and also assume d_l is normalized to $\mathrm{E}|d_l|^2 = 1$.

The signal received at $rec(l)$ can be expressed as

$$y_l = d_l \mathbf{h}_{ll}^{\mathrm{H}} \mathbf{m}_l + \sum_{j\in\mathcal{L}, j\neq l} d_j \mathbf{h}_{jl}^{\mathrm{H}} \mathbf{m}_j + z_l \tag{2}$$

where $\mathbf{h}_{jl}^{\mathrm{H}} \in \mathbb{C}^{1\times T}$ is the channel matrix between $tran(j)$ and $rec(l)$, and z_l is circular symmetric complex Gaussian noise with variance σ_l^2. The received signal-to-interference-plus-noise ratio (SINR) of lth data stream is given by

$$\gamma_l = \frac{|\mathbf{h}_{ll}^{\mathrm{H}} \mathbf{m}_l|^2}{\sigma_l^2 + \sum_{j\in\mathcal{L}, j\neq l} |\mathbf{h}_{jl}^{\mathrm{H}} \mathbf{m}_j|^2}. \tag{3}$$

Let β_l be an arbitrary nonnegative weight associated with data stream l, $l \in \mathcal{L}$. We consider the case where all receivers are using *single-user detection* (i.e., a receiver decodes its intended signal by treating all other interfering signals as noise). Assuming that the power allocation is subject to a maximum power constraint $\sum_{l\in\mathcal{O}(n)} \|\mathbf{m}_l\|_2^2 \leq p_n^{\max}$ for each BS $n \in \mathcal{N}$, the problem of WSRMax can be expressed as

$$\begin{aligned} \text{maximize} \quad & \sum_{l\in\mathcal{L}} \beta_l \log(1+\gamma_l) \\ \text{subject to} \quad & \gamma_l = \frac{|\mathbf{h}_{ll}^{\mathrm{H}} \mathbf{m}_l|^2}{\sigma_l^2 + \sum_{j\in\mathcal{L}, j\neq l} p_j |\mathbf{h}_{jl}^{\mathrm{H}} \mathbf{m}_j|^2}, \quad l \in \mathcal{L} \\ & \sum_{l\in\mathcal{O}(n)} \|\mathbf{m}_l\|_2^2 \leq p_n^{\max}, \quad n \in \mathcal{N} \end{aligned} \tag{4}$$

where the optimization variables are γ_l and $\mathbf{m}_l$ for all $l \in \mathcal{L}$.

By relaxing the SINR equalities constraints and changing the sign of the objective function, problem (4) can be expressed equivalently as

$$\begin{aligned} \text{minimize} \quad & \sum_{l\in\mathcal{L}} -\beta_l \log(1+\gamma_l) \\ \text{subject to} \quad & \gamma_l \leq \frac{|\mathbf{h}_{ll}^{\mathrm{H}} \mathbf{m}_l|^2}{\sigma_l^2 + \sum_{j\in\mathcal{L}, j\neq l} |\mathbf{h}_{jl}^{\mathrm{H}} \mathbf{m}_j|^2}, \quad l \in \mathcal{L} \\ & \sum_{l\in\mathcal{O}(n)} \|\mathbf{m}_l\|_2^2 \leq p_n^{\max}, \quad n \in \mathcal{N}. \end{aligned} \tag{5}$$

The equivalence between problems (4) and (5) follows from the monotonically increasing property of the $\log(\cdot)$ function which ensures that the SINR inequality constraints of problem (5) are tight (i.e., they holds with equality at the optimal solution).

III. Branch and Bound Algorithm

We start by equivalently reformulating problem (5) as minimization of a nonconvex function over an L-dimensional rectangle. Then, we apply BB techniques [20] to minimize the nonconvex function over the L-dimensional rectangle.

Let us first denote the objective function of problem (5) as $f_0(\boldsymbol{\gamma}) = \sum_{l\in\mathcal{L}} -\beta_l \log(1+\gamma_l)$ and the feasible set for variables $\boldsymbol{\gamma} = [\gamma_1, \ldots, \gamma_L]^T$ (or the achievable SINR values) by $\mathcal{G}$, i.e.,

$$\mathcal{G} = \left\{ \boldsymbol{\gamma} \,\middle|\, \begin{array}{ll} \gamma_l \leq \frac{|\mathbf{h}_{ll}^{\mathrm{H}} \mathbf{m}_l|^2}{\sigma_l^2 + \sum_{j\in\mathcal{L}, j\neq l} |\mathbf{h}_{jl}^{\mathrm{H}} \mathbf{m}_j|^2}, & l \in \mathcal{L} \\ \sum_{l\in\mathcal{O}(n)} \|\mathbf{m}_l\|_2^2 \leq p_n^{\max}, & n \in \mathcal{N} \end{array} \right\}. \tag{6}$$

For clarity, let us define a new function $\tilde{f} : \mathbb{R}_+^L \to \mathbb{R}$ as

$$\tilde{f}(\boldsymbol{\gamma}) = \begin{cases} f_0(\boldsymbol{\gamma}) & \text{if } \boldsymbol{\gamma} \in \mathcal{G} \\ 0 & \text{otherwise,} \end{cases} \tag{7}$$

and note that for any $\mathcal{S} \subseteq \mathbb{R}_+^L$ such that $\mathcal{G} \subseteq \mathcal{S}$, we have

$$\inf_{\boldsymbol{\gamma}\in\mathcal{S}} \tilde{f}(\boldsymbol{\gamma}) = \inf_{\boldsymbol{\gamma}\in\mathcal{G}} f_0(\boldsymbol{\gamma}) = p^\star \tag{8}$$

where $p^\star$ is the optimal value of problem (5). Note that the first equality follows from the fact that for any $\boldsymbol{\gamma} \in \mathbb{R}_+^L$ we have $f_0(\boldsymbol{\gamma}) \leq 0$. It is also worth noting that the function $\tilde{f}$ is nonconvex over $\mathcal{S}$ and f_0 is a global lower bound on $\tilde{f}$, i.e., $f_0(\boldsymbol{\gamma}) \leq \tilde{f}(\boldsymbol{\gamma})$ for all $\boldsymbol{\gamma} \in \mathcal{S}$.

Let us define the L-dimensional rectangle $\mathcal{Q}_{\text{init}}$ as

$$\mathcal{Q}_{\text{init}} = \left\{ \boldsymbol{\gamma} \,\middle|\, 0 \leq \gamma_l \leq \frac{\|\mathbf{h}_{ll}\|_2^2}{\sigma_l^2} p_{tran(l)}^{\max}, l \in \mathcal{L} \right\}. \tag{9}$$

It is easy to check that $\mathcal{G} \subseteq \mathcal{Q}_{\text{init}}$.[2] Therefore, from (8), it follows that $\inf_{\boldsymbol{\gamma}\in\mathcal{Q}_{\text{init}}} \tilde{f}(\boldsymbol{\gamma}) = p^\star$. Thus, we have reformulated problem (5) equivalently as a minimization of the nonconvex function $\tilde{f}$ over the rectangle $\mathcal{Q}_{\text{init}}$. To maintain a cohesive presentation, in the sequel, we review briefly the BB method introduced in [16] to minimize $\tilde{f}$ over $\mathcal{Q}_{\text{init}}$ for the single-input single-output (SISO) case.

For any L-dimension rectangle $\mathcal{Q} = \{\boldsymbol{\gamma} | \gamma_{l,\min} \leq \gamma_l \leq \gamma_{l,\max}, l \in \mathcal{L}\}$ such that $\mathcal{Q} \subseteq \mathcal{Q}_{\text{init}}$, let us define a function $\phi_{\min}(\mathcal{Q})$ as

$$\phi_{\min}(\mathcal{Q}) = \inf_{\boldsymbol{\gamma}\in\mathcal{Q}} \tilde{f}(\boldsymbol{\gamma}). \tag{10}$$

By using (8) and (10), it can be easily verified that $\phi_{\min}(\mathcal{Q}_{\text{init}}) = \inf_{\boldsymbol{\gamma}\in\mathcal{Q}_{\text{init}}} \tilde{f}(\boldsymbol{\gamma}) = p^\star$.

The key idea of the BB algorithm is to generate a sequence of asymptotically tight upper and lower bounds for $\phi_{\min}(\mathcal{Q}_{\text{init}})$. At each iteration k, the lower bound L_k and the upper bound U_k are updated by partitioning $\mathcal{Q}_{\text{init}}$ into smaller rectangles. To ensure the convergence, the bounds should become tight as the number of rectangles in the partition of $\mathcal{Q}_{\text{init}}$ grows. To do this, the BB uses two functions $\phi_{\text{ub}}(\mathcal{Q})$ and $\phi_{\text{lb}}(\mathcal{Q})$, defined for any rectangle $\mathcal{Q} \subseteq \mathcal{Q}_{\text{init}}$ s.t. following conditions are satisfied [20], [21].

C1) The functions $\phi_{\text{lb}}(\mathcal{Q})$ and $\phi_{\text{ub}}(\mathcal{Q})$ compute a lower bound and an upper bound, respectively on $\phi_{\min}(\mathcal{Q})$, i.e.,

$$\phi_{\text{lb}}(\mathcal{Q}) \leq \phi_{\min}(\mathcal{Q}) \leq \phi_{\text{ub}}(\mathcal{Q}).$$

C2) As the maximum half length of the sides of $\mathcal{Q}$ (i.e., $\text{size}(\mathcal{Q}) = \frac{1}{2}\max_{l\in\mathcal{L}}\{\gamma_{l,\max} - \gamma_{l,\min}\}$) goes to zero, the difference between the upper and lower bounds uniformly converges to zero, i.e.,

$$\begin{aligned} &\forall \epsilon > 0 \; \exists \delta > 0 \text{ s.t. } \forall \mathcal{Q} \subseteq \mathcal{Q}_{\text{init}}, \\ &\text{size}(\mathcal{Q}) \leq \delta \Rightarrow \phi_{\text{ub}}(\mathcal{Q}) - \phi_{\text{lb}}(\mathcal{Q}) \leq \epsilon. \end{aligned} \tag{11}$$

[2]It follows from Cauchy–Schwartz inequality (i.e., $|\mathbf{h}_{ll}^{\mathrm{H}} \mathbf{m}_l|^2 \leq \|\mathbf{h}_{ll}\|_2^2 p_{tran(l)}^{\max}$ for all $\|\mathbf{m}_l\|_2^2 \leq p_{tran(l)}^{\max}$) after neglecting the interference terms in the denominator of SINR constraints in (6).

Finding accurate and easy to compute upper and lower bound functions $\phi_{\rm ub}(\mathcal{Q})$ and $\phi_{\rm lb}(\mathcal{Q})$ is one of the most difficult part in deriving a BB algorithm. For clarity, we first summarize the generic BB algorithm and the bounding functions are defined in Section IV.

Algorithm 1: Branch and Bound Algorithm

1) Initialization: given tolerance $\epsilon>0$. Set $k=1$, $\mathcal{B}_1=\{\mathcal{Q}_{\rm init}\}$, $U_1=\phi_{\rm ub}(\mathcal{Q}_{\rm init})$, and $L_1=\phi_{\rm lb}(\mathcal{Q}_{\rm init})$.
2) Stopping criterion: if $U_k-L_k>\epsilon$ go to Step 3, otherwise STOP.
3) Branching:
   a) Pick $\mathcal{Q}\in\mathcal{B}_k$ for which $\phi_{\rm lb}(\mathcal{Q})=L_k$ and set $\mathcal{Q}_k=\mathcal{Q}$.
   b) Split $\mathcal{Q}_k$ along one of its longest edge into $\mathcal{Q}_I$ and $\mathcal{Q}_{II}$.
   c) Let $\mathcal{B}_{k+1}=(\mathcal{B}_k\backslash\{\mathcal{Q}_k\})\cup\{\mathcal{Q}_I,\mathcal{Q}_{II}\}$.
4) Bounding:
   a) Set $U_{k+1}=\min_{\mathcal{Q}\in\mathcal{B}_{k+1}}\{\phi_{\rm ub}(\mathcal{Q})\}$.
   b) Set $L_{k+1}=\min_{\mathcal{Q}\in\mathcal{B}_{k+1}}\{\phi_{\rm lb}(\mathcal{Q})\}$.
5) Set $k=k+1$ and go to step 2.

The convergence of the above algorithm is established by the following theorem.

Theorem 1: If for any $\mathcal{Q}\subseteq\mathcal{Q}_{\rm init}$ with $\mathcal{Q}=\{\boldsymbol{\gamma}\,|\gamma_{l,\min}\le\gamma_l\le\gamma_{l,\max},\ l\in\mathcal{L}\}$, the functions $\phi_{\rm ub}(\mathcal{Q})$ and $\phi_{\rm lb}(\mathcal{Q})$ satisfy the conditions C1) and C2), then Algorithm 1 converges in a finite number of iterations to a value arbitrarily close to $p^\star$, i.e., $\forall\epsilon>0,\ \exists K>0$ s.t $U_K-p^\star\le\epsilon$.

Proof: The proof is similar to the one provided in [20], [21] and it is provided here for the sake of completeness. First note that, there are k rectangles in the set $\mathcal{B}_k$. Let ${\rm vol}(\mathcal{Q}_{\rm init})$ denote the volume of rectangle $\mathcal{Q}_{\rm init}$. Thus, we have

$$\min_{\mathcal{Q}\in\mathcal{B}_k}\ {\rm vol}(\mathcal{Q})\le\frac{{\rm vol}(\mathcal{Q}_{\rm init})}{k}\,. \tag{12}$$

Therefore, as k increases at least one rectangle in the partition become small. Then it is required to show that, the smaller ${\rm vol}(\mathcal{Q})$ the smaller ${\rm size}(\mathcal{Q})$. To do this, we first define the *condition number* of rectangle $\mathcal{Q}=\{\boldsymbol{\gamma}\,|\gamma_{l,\min}\le\gamma_l\le\gamma_{l,\max},\ l\in\mathcal{L}\}$ as

$${\rm cond}(\mathcal{Q})=\frac{\max_l(\gamma_{l,\max}-\gamma_{l,\min})}{\min_l(\gamma_{l,\max}-\gamma_{l,\min})}\,. \tag{13}$$

Note that the branching rule we use (see Algorithm 1, Step 3), always ensures that for any k and any rectangle $\mathcal{Q}\in\mathcal{B}_k$ [21, Lemma 1]

$${\rm cond}(\mathcal{Q})\le\max\{{\rm cond}(\mathcal{Q}_{\rm init}),2\}\,. \tag{14}$$

Moreover, we have

$$\begin{aligned}{\rm vol}(\mathcal{Q})&=\prod_{l=1}^{L}(\gamma_{l,\max}-\gamma_{l,\min}) &(15)\\ &\ge\max_l(\gamma_{l,\max}-\gamma_{l,\min})\left(\min_l(\gamma_{l,\max}-\gamma_{l,\min})\right)^{L-1} &(16)\\ &=\frac{(2\,{\rm size}(\mathcal{Q}))^L}{({\rm cond}(\mathcal{Q}))^{L-1}} &(17)\\ &\ge\left(\frac{2\,{\rm size}(\mathcal{Q})}{{\rm cond}(\mathcal{Q})}\right)^L &(18)\end{aligned}$$

where the last inequality follows by noting that ${\rm cond}(\mathcal{Q})\ge 1$. Thus, from (18), we have

$${\rm size}(\mathcal{Q})\le\frac{1}{2}{\rm cond}(\mathcal{Q}){\rm vol}(\mathcal{Q})^{\frac{1}{L}}\,. \tag{19}$$

By using (12), (14), and (19), we get

$$\min_{\mathcal{Q}\in\mathcal{B}_k}\ {\rm size}(\mathcal{Q})\le\frac{1}{2}\max\{{\rm cond}(\mathcal{Q}_{\rm init}),2\}\frac{{\rm vol}(\mathcal{Q}_{\rm init})}{k}\,. \tag{20}$$

We are now ready to show that there exist a positive integer K such that for any $\epsilon>0$, $U_K-p^\star\le\epsilon$. To see this, we select K as the maximum number of iterations such that

$$\frac{1}{2}\max\{{\rm cond}(\mathcal{Q}_{\rm init}),2\}\frac{{\rm vol}(\mathcal{Q}_{\rm init})}{K}\le\delta\,. \tag{21}$$

Thus, from (20), for some $\tilde{\mathcal{Q}}\in\mathcal{B}_K$, ${\rm size}(\tilde{\mathcal{Q}})\le\delta$ and from C2) [see (11)], we have $\phi_{\rm ub}(\tilde{\mathcal{Q}})-\phi_{\rm lb}(\tilde{\mathcal{Q}})\le\epsilon$. However, note that $U_K\le\phi_{\rm ub}(\tilde{\mathcal{Q}})$ (since $U_K=\min_{\mathcal{Q}\in\mathcal{B}_K}\{\phi_{\rm ub}(\mathcal{Q})\}$) and $p^\star\ge\phi_{\rm lb}(\tilde{\mathcal{Q}})$. Thus, $U_K-p^\star\le\epsilon$, and the result follows. ∎

IV. Upper and Lower Bound Functions

In this section, we derive the bounding functions $\phi_{\rm ub}(\mathcal{Q})$ and $\phi_{\rm lb}(\mathcal{Q})$ for Algorithm 1 by exploiting the monotonic nonincreasing property of f_0. First, basic bounding functions are established, and then a method to improve the basic lower bounding function is proposed.

A. Basic Upper and Lower Bounds

The basic bounding functions $\phi_{\rm lb}$ and $\phi_{\rm ub}$ proposed in [16] for the case of SISO networks can be formally expressed as

$$\phi_{\rm lb}^{\rm Basic}(\mathcal{Q})=\begin{cases}f_0(\boldsymbol{\gamma}_{\max}) & \boldsymbol{\gamma}_{\min}\in\mathcal{G}\\ 0 & \text{otherwise}\end{cases} \tag{22}$$

and

$$\phi_{\rm ub}^{\rm Basic}(\mathcal{Q})=\tilde{f}(\boldsymbol{\gamma}_{\min})=\begin{cases}f_0(\boldsymbol{\gamma}_{\min}) & \boldsymbol{\gamma}_{\min}\in\mathcal{G}\\ 0 & \text{otherwise}\end{cases} \tag{23}$$

where

$$\boldsymbol{\gamma}_{\max}=\Big[\gamma_{1,\max},\ldots,\gamma_{L,\max}\Big]^T,\boldsymbol{\gamma}_{\min}=\Big[\gamma_{1,\min},\ldots,\gamma_{L,\min}\Big]^T,$$

and $\mathcal{G}$ is defined in (6). These general expressions hold true for the case of MISO system as well. Furthermore, it is easy to show that functions $\phi_{\rm lb}^{\rm Basic}(\mathcal{Q})$ and $\phi_{\rm ub}^{\rm Basic}(\mathcal{Q})$ satisfy conditions C1) and C2); the proof is similar to the one provided in [16, Lemmas 1 and 2]. However, checking the condition $\boldsymbol{\gamma}_{\min}\in\mathcal{G}$, which is central to computing $\phi_{\rm lb}^{\rm Basic}$ and $\phi_{\rm ub}^{\rm Basic}$, is much more difficult in the case of multiple transmit antenna. Thus, a computationally efficient method based on SOCP is presented in the sequel.

Let $\{\gamma_l\}_{l\in\mathcal{L}}$ be a specified set of SINR values. Testing if these values are achievable (i.e., testing if $\{\gamma_l\}_{l\in\mathcal{L}}\in\mathcal{G}$) is equivalent to solving the following feasibility problem [22, Sect. 4.1.1]:

$$\begin{array}{ll}\text{find} & \mathbf{m}_1,\ldots,\mathbf{m}_L\\ \text{subject to} & \dfrac{|\mathbf{h}_{ll}^{\rm H}\mathbf{m}_l|^2}{\sigma_l^2+\sum\limits_{j\in\mathcal{L},j\ne l}|\mathbf{h}_{jl}^{\rm H}\mathbf{m}_j|^2}\ge\gamma_l,\quad l\in\mathcal{L}\\ & \sum\limits_{l\in\mathcal{O}(n)}\|\mathbf{m}_l\|_2^2\le p_n^{\max},\quad n\in\mathcal{N},\end{array} \tag{24}$$

with variables $\mathbf{m}_l$, $l\in\mathcal{L}$. Feasibility problem (24) determines whether the SINR constraints are achievable, and if so, returns a set of feasible transmit beamformers $\{\mathbf{m}_l^\star\}_{l\in\mathcal{L}}$ that satisfies them.

Problem (24) is not convex as such, but following the approach of [19, Sec. IV-B], it can be reformulated as a standard SOCP and solved efficiently via interior points methods [22], [23]. Let

$\mathbf{M}_n = [\mathbf{m}_l]_{l \in \mathcal{O}(n)}$. Then, problem (24) can be cast as the following SOCP feasibility problem:

$$\begin{array}{ll} \text{find} & \mathbf{m}_1, \ldots, \mathbf{m}_L \\ \text{subject to} & \begin{bmatrix} \sqrt{\left(1+\frac{1}{\gamma_l}\right)} \mathbf{m}_l^{\mathrm{H}} \mathbf{h}_{ll} \\ \left[\mathbf{m}_1^{\mathrm{H}} \mathbf{h}_{1l}, \ldots, \mathbf{m}_L^{\mathrm{H}} \mathbf{h}_{Ll}\right]^T \\ \sigma_l \end{bmatrix} \succeq_{\mathrm{SOC}} 0, \quad l \in \mathcal{L} \\ & \begin{bmatrix} \sqrt{p_n^{\max}} \\ \operatorname{vec}(\mathbf{M}_n) \end{bmatrix} \succeq_{\mathrm{SOC}} 0, \quad n \in \mathcal{N} \end{array} \tag{25}$$

where the optimization variable is $\{\mathbf{m}_l\}_{l \in \mathcal{L}}$; and we use notation $\succeq_{\mathrm{SOC}}$ to denote the generalized inequalities with respect to the second order cone [19], [22], i.e., for any $x \in \mathbb{R}$ and $\mathbf{y} \in \mathbb{C}^T$, $[x, \mathbf{y}^T]^T \succeq_{\mathrm{SOC}} 0$ is equivalent to $x \geq \|\mathbf{y}\|_2$.

B. Improved Lower Bound

Tighter bounds[3] are very important as they can increase substantially the convergence speed of the BB algorithm. By exploiting the monotonically nonincreasing property of f_0, an improved lower bound is proposed in this subsection.

Note that, in the case of $\boldsymbol{\gamma}_{\min} \notin \mathcal{G}$ [i.e., $\mathcal{Q} \cap \mathcal{G} = \emptyset$, see Fig. 1(a)], $\tilde{f}(\boldsymbol{\gamma}) = 0$ for any $\boldsymbol{\gamma} \in \mathcal{Q}$. Thus, both the basic lower bound (22) and the basic upper bound (23) are trivially zero and no further improvement is possible since they are tight. Consequently, tighter bounds can be found only in the case $\boldsymbol{\gamma}_{\min} \in \mathcal{G}$ [i.e., $\mathcal{Q} \cap \mathcal{G} \neq \emptyset$, see Fig. 1(b)]. Thus, we consider only this case in the sequel.

Roughly speaking, a tighter lower bound can be obtained as follows. We first construct the smallest rectangle $\bar{\mathcal{Q}}^\star \subseteq \mathcal{Q}$ which encloses the intersection $\mathcal{Q} \cap \mathcal{G}$ (see Fig. 1(b)). Let us denote this rectangle as $\bar{\mathcal{Q}}^\star = \{\boldsymbol{\gamma} \,|\, \gamma_{l,\min} \leq \gamma_l \leq \bar{\gamma}_l^\star,\ l \in \mathcal{L}\}$. The improved lower bound is given by $f_0(\bar{\gamma}_1^\star, \ldots, \bar{\gamma}_L^\star)$.

Recall that $\mathcal{Q} = \{\boldsymbol{\gamma} \,|\, \gamma_{l,\min} \leq \gamma_l \leq \gamma_{l,\max},\ l \in \mathcal{L}\}$. For any $\mathcal{Q} \subseteq \mathcal{Q}_{\mathrm{init}}$, the improved lower bound can be formally expressed as

$$\phi_{\mathrm{lb}}^{\mathrm{Imp}}(\mathcal{Q}) = \begin{cases} f_0(\bar{\boldsymbol{\gamma}}^\star) & \text{if } \boldsymbol{\gamma}_{\min} \in \mathcal{G} \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

where $\bar{\boldsymbol{\gamma}}^\star = [\bar{\gamma}_1^\star, \ldots, \bar{\gamma}_L^\star]^T$ is the maximum corner of the rectangle $\bar{\mathcal{Q}}^\star$, and $\bar{\gamma}_i^\star$ can be found by using bisection search on each edge of the rectangle $\mathcal{Q}$ as discussed below.

Let us define a corner point along $\mathbf{e}_i$ edge of the rectangle $\mathcal{Q}$ as $\mathbf{a}_i = \boldsymbol{\gamma}_{\min} + (\gamma_{i,\max} - \gamma_{i,\min})\mathbf{e}_i$. If a corner point $\mathbf{a}_i$ lies inside $\mathcal{G}$, i.e., $\mathbf{a}_i \in \mathcal{G}$ [see $\mathbf{a}_1$ in Fig. 1(c)], then $\bar{\gamma}_i^\star = \gamma_{i,\max}$. Otherwise (i.e., $\mathbf{a}_i \notin \mathcal{G}$), a bisection search over the line segment between the points $\boldsymbol{\gamma}_{\min}$ and $\mathbf{a}_i$ can be used to find $\bar{\gamma}_i^\star$. The bisection search used to find $\bar{\gamma}_i^\star$ (when $\mathbf{a}_i \notin \mathcal{G}$) is summarized below.

Algorithm 2: Bisection Search for Finding $\bar{\gamma}_i^\star$

1) Initialization: $\mathbf{l} = \boldsymbol{\gamma}_{\min}$ and $\mathbf{u} = \mathbf{a}_i$, and tolerance $\epsilon_b > 0$.
2) If $\|\mathbf{u} - \mathbf{l}\|_2 < \epsilon_b$ return $\bar{\gamma}_i^\star = [\mathbf{u}]_i$ and STOP.
3) Set $\mathbf{t} = (\mathbf{l} + \mathbf{u})/2$.
4) If $\mathbf{t} \in \mathcal{G}$ set $\mathbf{l} = \mathbf{t}$. Otherwise, set $\mathbf{u} = \mathbf{t}$. Go to step 2.

Note that the SOCP feasibility problem formulation (25) is used for checking if $\mathbf{t} \in \mathcal{G}$ at step 4 of the bisection search.

V. Numerical Example

In this section, we first evaluate the impact of the proposed bounds (Section IV) on the convergence of Algorithm 1. Next, we use the proposed Algorithm 1 to evaluate the performance loss of several suboptimal algorithms.

[3]We say a bound is tighter in the following sense: $\phi_{\mathrm{lb}}(\mathcal{Q})$ is a tighter lower bound if for any $\mathcal{Q} \subseteq \mathcal{Q}_{\mathrm{init}}$, we have $\phi_{\min}(\mathcal{Q}) \geq \phi_{\mathrm{lb}}(\mathcal{Q}) \geq \phi_{\mathrm{lb}}^{\mathrm{Basic}}(\mathcal{Q})$.

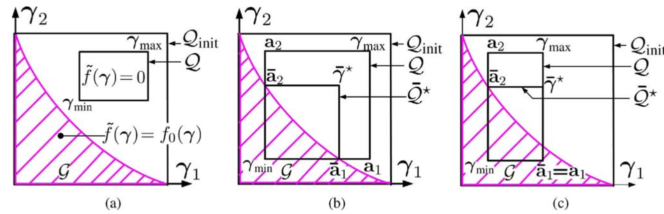

Fig. 1. Illustration of $\mathcal{G}$, $\mathcal{Q}_{\mathrm{init}}$, $\mathcal{Q}$, and $\bar{\mathcal{Q}}^\star$ in a 2–dimensional space.

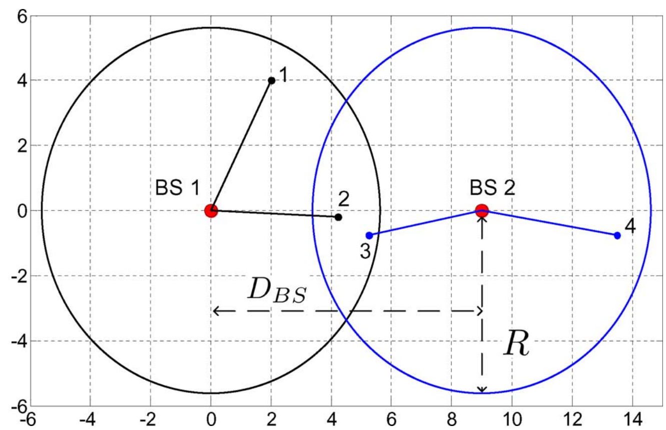

Fig. 2. MISO downlink wireless network with $N = 2, T = 2$, and $L = 4$.

We consider a multicell wireless downlink system as shown in Fig. 2. There are $N = 2$ base stations, each with $T = 2$ transmit antennas. The distance between the BSs is denoted by D_{BS}. We assume circular cells, where the radius of each one is denoted by R. For simplicity, we assume two users per cell. The locations of users associated with BSs are arbitrarily chosen as shown in Fig. 2.

We assume an exponential path loss model, where the channel matrix between BSs and users is modeled as

$$\mathbf{h}_{jl} = \left(\frac{d_{jl}}{d_0}\right)^{-\frac{\eta}{2}} \mathbf{c}_{jl}$$

where d_{jl} is the distance from the transmitter of data stream j (i.e., BS $tran(j)$) to the receiver of data stream l (i.e., user $rec(l)$), d_0 is the *far field reference distance* [24], η is the path loss exponent, and $\mathbf{c}_{jl} \in \mathbb{C}^T$ is arbitrarily chosen from the distribution $\mathcal{CN}(0, \mathbf{I})$ (i.e., frequency-flat fading channel with uncorrelated antennas). Here, we refer an arbitrarily generated set of fading coefficients $\mathcal{C} = \{\mathbf{c}_{jl} | j, l \in \mathcal{L}\}$ as a *single fading realization*.

We set $p_n^{\max} = p_0^{\max}$ for all $n \in \mathcal{N}$, and $\sigma_l = \sigma$ for all $l \in \mathcal{L}$. We define the signal-to-noise ratio (SNR) operating point at a distance r as

$$\mathrm{SNR}(r) = \left(\frac{r}{d_0}\right)^{-\eta} \frac{p_0^{\max}}{\sigma^2}. \tag{27}$$

In the following simulations, we set $d_0 = 1$, $\eta = 4$, $\sigma^2 = 1$, and the cell radius R is fixed throughout the simulations such that $\mathrm{SNR}(R) = 10$ dB for $\frac{p_0^{\max}}{\sigma^2} = 40$ dB. Furthermore, we let $\frac{D_{\mathrm{BS}}}{R} = 1.6$.

Fig. 3 shows the evolution of upper and lower bounds for the optimal value of problem (5) for a single fading realization, and $\beta_l = 0.25$ for all $l \in \mathcal{L}$. Specifically, in Fig. 3, we used the basic upper bound ($\mathrm{UB_{Basic}}$) in conjunction to both the basic lower bound ($\mathrm{LB_{Basic}}$) and the improved lower bound ($\mathrm{LB_{Imp}}$). Results show that both lower/upper bound pairs ($\mathrm{LB_{Imp}}, \mathrm{UB_{Basic}}$) and ($\mathrm{LB_{Basic}}, \mathrm{UB_{Basic}}$) become

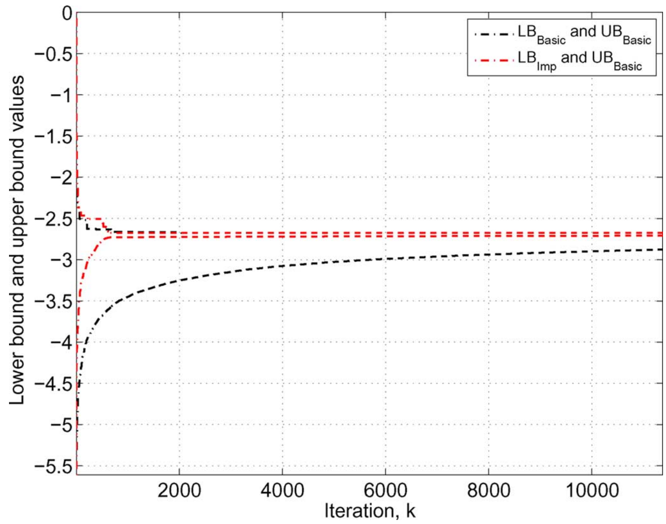

Fig. 3. Upper and lower bound evolution.

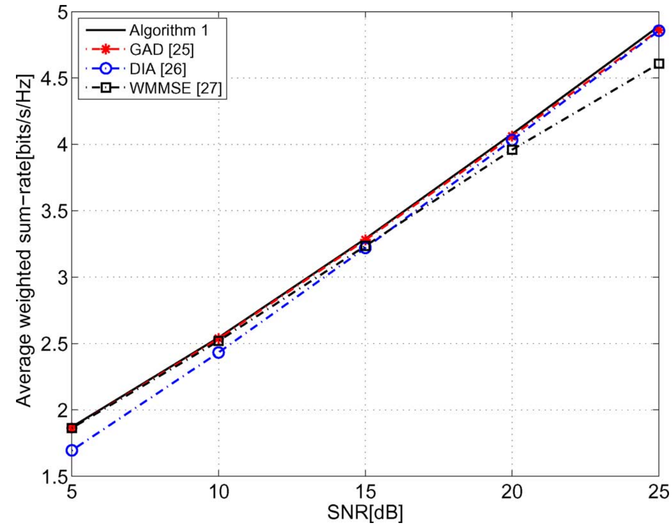

Fig. 5. Average weighted sum-rate on SNR.

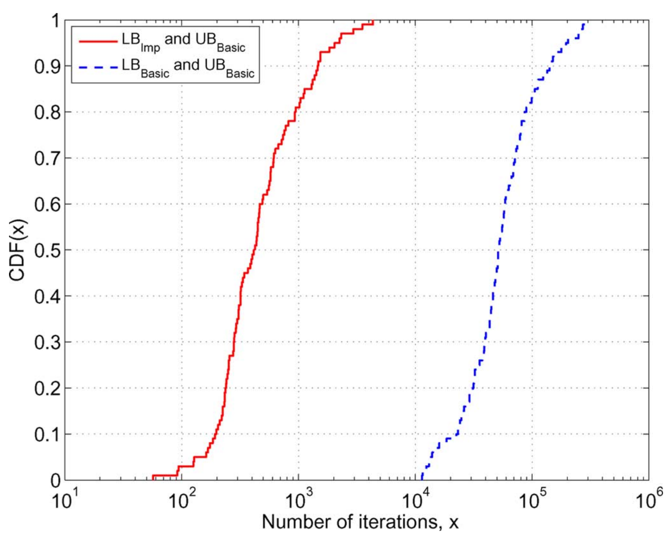

Fig. 4. Empirical CDF plot of total number of iterations.

tighter as the number of iterations grows. However, the convergence speed of Algorithm 1 is substantially increased by the improved lower bound as compared to the basic one. For example, when $\epsilon = 0.2$, the basic bound requires more than 10^4 iterations to converge, where as the improved lower bound, with the bisection search tolerance $\epsilon_b = 0.1$, achieves the same level of accuracy in only 525 iterations.

In order to provide a statistical description for the speed of convergence, we run Algorithm 1 for 100 fading realizations. For each one we store the number of iterations required to find the optimal value of problem (5) within an accuracy of $\epsilon = 0.1$ with both lower/upper bound pairs $(\mathrm{LB}_{\mathrm{Imp}}, \mathrm{UB}_{\mathrm{Basic}})$ and $(\mathrm{LB}_{\mathrm{Basic}}, \mathrm{UB}_{\mathrm{Basic}})$, respectively.

Fig. 4 shows the empirical cumulative distribution function (CDF) plots of total number of iterations required to terminate Algorithm 1. Results show that the improved lower bound increases significantly (about 100 times) the convergence speed of Algorithm 1. For example, when the improved lower bound is used the algorithm finishes in less than 1500 iterations for more than 90% of the simulated cases, but with the basic lower bound the algorithm needs about 1.5×10^5 iterations to find the optimal solution with the same probability.

In the sequel, we use the proposed Algorithm 1 to evaluate numerically the performance loss of the following suboptimal algorithms: 1) generalized asynchronous distributed (GAD) algorithm [25, Sec. II], 2) distributed interference alignment (DIA) algorithm [26], and 3) weighted sum mean-square error minimization (WMMSE) algorithm [27]. The algorithms GAD and DIA can handle only interference channels (IC), and therefore we limit our simulations to a two-user MISO IC. Specifically, in the following simulation we consider one user per BS in Fig. 2, i.e., only user 2 of BS 1 and user 3 of BS 2 are considered.

Fig. 5 shows the weighted sum-rate of the considered algorithms for different SNR values.[4] Each curve is averaged over 500 fading realizations. Transmit beamforming vectors with full transmit power are used for initializing the beamformers of suboptimal algorithms. For Algorithm 1, the accuracy ϵ is set to 0.01. Results show that, the performance of GAD algorithm is very close to the optimal value irrespective of SNR values for the considered system setup. The DIA algorithm has a noticeable performance loss at low SNR values, however, it approaches to the optimal value at high SNR values. In contrast, the performance of WMMSE algorithm is close to the optimal value at low SNR values and exhibits a noticeable performance loss at high SNR values.

VI. Conclusion

We have considered the problem of weighted sum-rate maximization (WSRMax) in multicell downlink multi-input single-output (MISO) systems. In fact, this problem is NP-hard. A solution method, based on the branch and bound technique has been proposed for solving the nonconvex WSRMax problem globally with an optimality certificate. Efficient bounding methods based on conic optimization are proposed. The convergence speed of the proposed algorithm can be substantially increased by improving the lower bound. Performance benchmarks for various network design problems can be obtained by back-substituting the proposed algorithm into any network design method which relies on WSRMax. Moreover, the method proposed here is not restricted to WSRMax. It can also handle *any* system performance metric that can be expressed as a Lipschitz continuous and increasing function of signal-to-interference-plus-noise ratio.

References

[1] L. Tassiulas and A. Ephremides, "Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks," *IEEE Trans. Autom. Control*, vol. 37, no. 12, pp. 1936–1949, Dec. 1992.

[4]For fixed radius R in Fig. 2, different SNRs (i.e., different $\mathrm{SNR}(R)$) are obtained by changing $\frac{p_0^{\max}}{\sigma^2}$ in (27).

[2] M. J. Neely, *Stochastic Network Optimization with Application to Communication and Queueing Systems*. San Rafael, CA: Morgan & Claypool, 2010, vol. 7, Synthesis Lectures on Communication Networks.

[3] M. J. Neely, E. Modiano, and C. E. Rohrs, "Power allocation and routing in multibeam satellites with time-varying channels," *IEEE/ACM Trans. Netw.*, vol. 11, no. 1, pp. 138–152, Feb. 2003.

[4] X. Lin and N. B. Shroff, "Joint rate control and scheduling in multihop wireless networks," Purdue Univ., Richmond, IN, Tech. Rep., 2004 [Online]. Available: http://cobweb.ecn.purdue.edu/~linx/papers.html

[5] M. J. Neely, E. Modiano, and C. E. Rohrs, "Dynamic power allocation and routing for time varying wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 1, pp. 89–103, Jan. 2005.

[6] A. L. Stolyar, "Maximizing queueing network utility subject to stability: Greedy primal-dual algorithm," *Queueing Syst.*, vol. 50, no. 4, pp. 401–457, Aug. 2005.

[7] A. Eryilmaz, R. Srikant, and J. R. Perkins, "Stable scheduling policies for fading wireless channels," *IEEE/ACM Trans. Netw.*, vol. 13, no. 2, pp. 411–424, Apr. 2005.

[8] X. Wu and R. Srikant, "Regulated maximal matching: A distributed scheduling algorithm for multi-hop wireless networks with node-exclusive spectrum sharing," in *Proc. IEEE Conf. Decision Control/Eur. Control Conf.*, Seville, Spain, Dec. 12–15, 2005, pp. 5342–5347.

[9] Z. Luo and S. Zhang, "Dynamic spectrum management: Complexity and duality," *IEEE J. Sel. Areas Commun.*, vol. 2, no. 1, pp. 57–73, Feb. 2008.

[10] R. Horst, P. Pardolos, and N. Thoai, *Introdiction to Global Optimization*, second ed. Boston, MA: Kluwer, 2000, vol. 48.

[11] C. Audet, P. Hansen, and G. Savard, *Essays and Surveys in Global Optimization*. New York: Springer Science + Business Media, Inc., 2005.

[12] Y. Xu, T. Le-Ngoc, and S. Panigrahi, "Global concave minimization for optimal spectrum balancing in multi-user DSL networks," *IEEE Trans. Signal Process.*, vol. 56, no. 7, pp. 2875–2885, Jul. 2008.

[13] H. Al-Shatri and T. Weber, "Optimizing power allocation in interference channels using D.C. programming," in *Proc. Workshop Resource Alloc. Wireless Netw.*, Avignon, France, Jun. 4, 2010, pp. 367–373.

[14] P. Tsiaflakis, J. Vangorp, M. Moonen, and J. Verlinden, "A low complexity optimal spectrum balancing algorithm for digital subscriber lines," *Elsevier Signal Process.*, vol. 87, no. 7, pp. 1735–1753, Jul. 2007.

[15] L. Qian, Y. J. Zhang, and J. Huang, "MAPEL: Achieving global optimality for a non-convex wireless power control problem," *IEEE Trans. Wireless Commun.*, vol. 8, no. 3, pp. 1553–1563, Mar. 2009.

[16] P. C. Weeraddana, M. Codreanu, M. Latva-aho, and A. Ephremides, "Weighted sum-rate maximization for a set of interfering links via branch and bound," presented at the Annu. Asilomar Conf. Signals, Syst., Comput., Pacific Grove, CA, Nov. 7–10, 2010.

[17] S. Shuying, M. Schubert, and H. Boche, "Rate optimization for multiuser MIMO systems with linear processing," *IEEE Trans. Signal Process.*, vol. 56, no. 8, pp. 4020–4030, Aug. 2008.

[18] S. Shuying, M. Schubert, and H. Boche, "Per-antenna power constrained rate optimization for multiuser MIMO systems," in *Proc. ITG Workshop Smart Antennas*, Feb. 2008, pp. 270–277.

[19] A. Wiesel, Y. C. Eldar, and S. Shamai, "Linear precoding via conic optimization for fixed MIMO receivers," *IEEE Trans. Signal Process.*, vol. 54, no. 1, pp. 161–176, Jan. 2006.

[20] S. Boyd, Branch-and-Bound Methods, 2007 [Online]. Available: http://www.stanford.edu/class/ee364b/lectures/bb_slides.pdf

[21] V. Balakrishnan, S. Boyd, and S. Balemi, "Branch and bound algorithm for computing the minimum stability degree of parameter-dependent linear systems," *Int. J. Robust Nonlinear Control*, vol. 1, no. 4, pp. 295–317, Oct. 1991.

[22] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.

[23] M. Grant and S. Boyd, Cvx: Matlab Software for Disciplined Convex Programming, 2011 [Online]. Available: http://www.stanford.edu/~boyd/cvx/

[24] A. Kumar, D. Manjunath, and J. Kuri, *Wireless Networking*. Burlington, MA: Elsevier, 2008.

[25] C. Shi, R. A. Berry, and M. L. Honig, "Distributed interference pricing with MISO channels," in *Proc. Annu. Allerton Conf. Commun., Control, Comput.*, 2008, pp. 539–546.

[26] K. Gomadam, V. R. Cadambe, and S. A. Jafar, "Approaching the capacity of wireless networks through distributed interference alignment," in *Proc. IEEE Global Telecommun. Conf.*, Dec. 2008, pp. 1–6.

[27] Q. Shi, M. Razaviyayn, Z. Luo, and C. He, "An iteratively weighted MMSE approach to distributed sum-utility maximization for a MIMO interfering broadcast channel," *IEEE Trans. Signal Process.*, vol. 59, no. 9, pp. 4331–4340, Sep. 2011.