Standard and Quasi-Standard Stochastic Power Control Algorithms

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Outline

- Power control in wireless networks
- Standard power control
- Standard stochastic power control
- Quasi-standard stochastic power control
- Convergence of stochastic power control algorithms
- Example
Power Control in Wireless Networks

- Uplink scenario:

![Diagram of wireless network with hexagonal cells and signal reception]

- Quality of service requirement:  
  \[ \text{SINR}_i = \frac{h_{ii}p_i}{\sum_{j \neq i} h_{ij}p_j + \sigma^2} \geq \gamma_i \]

- Optimization problem:

\[
\min_{\sum_{j=1}^{K} p_j} \quad \text{s.t. } \forall i \quad \text{SINR}_i \geq \gamma_i
\]
Central/distributed power control:
- **Central**: high complexity, global CSI
- **Distributed**: low complexity, local CSI, adaptive to channel variations

Deterministic/stochastic power control:
- **Deterministic**: perfect knowledge of interference power
- **Stochastic**: noisy observations of interference power
SINR requirements can be represented as $p \succeq l(p)$

$l(p)$ is a standard function if $\forall p \succeq 0$
- **Positivity**: $l(p) \succ 0$
- **Monotonicity**: if $p \succeq p'$, then $l(p) \succeq l(p')$
- **Scalability**: $\eta l(p) \succ l(\eta p)$, $\forall \eta > 1$

Standard power control algorithm

$$p(n + 1) = (1 - \alpha)p(n) + \alpha l(p(n)), \quad \forall 0 < \alpha \leq 1$$

Converges to $p^*$, $p^* = l(p^*)$ if the problem is feasible.

$p^*$ is the feasible power vector with smallest elements.
Standard Stochastic Power Control

- $\tilde{I}(p, \theta)$ is a random estimate of $I(p)$
- Standard stochastic interference function satisfies $\forall p \geq 0$:
  - **Mean condition**:
    $$E[\tilde{I}(p, \theta)|p] = I(p)$$
  - $I(p)$ is a deterministic standard interference function.
  - **Lipschitz condition**:
    $$\exists K_1 > 0, \forall p_1, p_2, \|I(p_1) - I(p_2)\|^2 \leq K_1\|p_1 - p_2\|^2$$
  - **Growing condition** (constraint on estimation noise):
    $$\exists K_2 > 0, E[\|\tilde{I}(p, \theta) - I(p)\|^2|p] \leq K_2(1 + \|p\|^2)$$

- Standard stochastic power control algorithm:
  $$p(n+1) = (1 - \alpha(n))p(n) + \alpha(n)\tilde{I}(p(n), \theta(n))$$
Quasi-standard Stochastic Power Control

- Quasi-standard stochastic interference function satisfies:
  - **Mean condition:**
    \[
    E[\tilde{I}(p, \theta) | p] = I(p) + g(p)
    \]
    
    $I(p)$ is a deterministic standard interference function, and $g(p)$ is a bias term.
  - **Bias condition:**
    \[
    \exists K_3 > 0, 0 \leq \beta(n) \leq 1; \quad \|g(p)\| \leq K_3 \beta(n)(1 + \|p(n)\|)
    \]
  - **Lipschitz condition:**
    \[
    \exists K_1 > 0, \forall p_1, p_2; \quad \|I(p_1) - I(p_2)\|^2 \leq K_1 \|p_1 - p_2\|^2
    \]
  - **Growing condition:**
    \[
    \exists K_2 > 0; \quad E[\|\tilde{I}(p, \theta) - I(p) - g(p)\|^2 | p] \leq K_2 (1 + \|p\|^2)
    \]

- Standard stochastic power control algorithm:
  \[
  p(n + 1) = (1 - \alpha(n))p(n) + \alpha(n)\tilde{I}(p(n))
  \]
Deterministic and Stochastic Standard Power Control

- **Deterministic** standard power control:

  \[ p(n+1) = p(n) - \alpha(n)(p(n) - l(p(n))) \]

- **Stochastic** standard power control:

  \[ p(n+1) = p(n) - \alpha(n)(p(n) - l(p(n))) + \alpha(n)\left(\tilde{l}(p(n)) - l(p(n))\right) \]

  estimation noise
If $\sum_{n=0}^{\infty} \alpha(n) = \infty$, $\sum_{n=0}^{\infty} \alpha(n)^2 < \infty$, then the **standard** stochastic power control algorithm converges to $p^*$, given by $p^* = I(p^*)$, with probability one.

If $\sum_{n=0}^{\infty} \alpha(n) = \infty$, $\sum_{n=0}^{\infty} \alpha(n)^2 < \infty$, $\sum_{n=0}^{\infty} \alpha(n) \beta(n) < \infty$ then the **quasi-standard** stochastic power control algorithm converges to $p^*$, given by $p^* = I(p^*)$, with probability one.
Convergence in Probability

If $\exists \alpha^* \geq 0$, $N > 0$, such that $\alpha(n) \leq \alpha^*$, $\beta(n) \leq \sqrt{\alpha^*}$, $\forall n \geq N$ then $\forall \varepsilon > 0$, $\exists K_8(\varepsilon) > 0$, such that in the standard and quasi-standard stochastic power control algorithm:

$$\limsup_{n \to \infty} Pr(\|p(n) - p^*\| \geq \varepsilon) \leq K_8\alpha^*$$
Example

- Uplink of CDMA system with K users and M base stations
- Chip match filter output: \( z_i = \sum_{j=1}^{K} \sqrt{p_j h_{ij}} b_j s_j + v_i \)
- Output of linear filter: \( y_i = \sum_{j=1}^{K} \sqrt{p_j h_{ij}} b_j (c_i^T s_j) + c_i^T v_i \)
- Fixed base station assignment with match filter receiver:
  - \( c_i = s_i \)
  - \( l_i(p) = \frac{\gamma_i}{h_{ii}} (\sum_{j \neq i} p_j h_{ij} (s_i^T s_j)^2 + \sigma^2) \)
  - Received signal power: \( E[y_i^2] = \sum_j p_j h_{ij} (s_i^T s_j)^2 + \sigma^2 \)
  - \( l_i(p) = \frac{\gamma_i}{h_{ii}} E[y_i^2] - \gamma_i p_i \)
  - The stochastic estimate of \( l_i(p) \) is: \( \tilde{l}_i = \frac{\gamma_i}{h_{ii}} y_i^2 - \gamma_i p_i \)
  - \( \tilde{l} \) is a **standard** stochastic interference function
Thank you!