Outage Performance of Power Controlled DS-CDMA Wireless Systems with Heterogeneous Traffic Sources

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Abstract

This paper proposes a performance analysis of a Direct Sequence-Code Division Multiple Access (DS-CDMA) wireless system with heterogeneous traffic in terms of second order outage statistics. Imperfections in closed-loop power control are modelled in their first order distribution and autocorrelation function. System capacity and optimal power allocation has been previously derived [13] in the presence of requirements expressed only in terms of signal-to-(noise+interference) ratio and outage probability of every user in the system. Therefore, in this paper the effectiveness of power allocation is evaluated also in terms of average outage rate and average outage duration for the generic user link of each traffic class. This allows to gain insights on the effects of power allocation and feedback control on channel burstiness for each class of users, so that forward error correction and retransmission strategies can be properly tuned.

Keywords
Multimedia, Wireless systems, CDMA, Outage.

1. Introduction

DS/CDMA is a basic access technique for the radio interface of third generation wireless systems, e.g. W-CDMA and CDMA2000. These systems will be required to support heterogeneous traffic, with a variety of source rates and quality of service requirements. The achievement of large capacities and adequate performance in this context is a challenging task, and requires a proper allocation of system resources. Moreover, as the environment is time-varying, dynamic allocation strategies are needed. Power allocation is a relevant issue in a DS/CDMA system, where the power of a certain user signal determines the actual bandwidth resource occupied by that user. Since different types of traffic have different bit rates and Quality of Service (QoS) requirements, it is intuitive that larger powers should be allocated to those services having larger bit rates and/or more stringent QoS requirements. Power allocation strategies have been identified in recent literature as solutions to various optimization problems. In [1], minimization of the sum of transmitted powers for a given configuration of path gains and QoS requirements was achieved in a single-cell environment by solving a linear optimization problem. Similar findings for the power allocation strategy were reported in [7], where accurate modelling of intercell interference and soft handover operations have been taken into account. An approach to throughput maximization was presented in [11] for a dual-class system, with a class having constant bit rate and stringent delay requirements, and the other class being delay-tolerant. Traffic design issues and admission control policies for multiservice CDMA networks have been also addressed (e.g. [9],[3]). Other contributions (e.g. [10]) have pointed out the effects of on/off activity or variable bit-rate of multimedia sources.

In a recent paper [13] we proposed a detailed analysis of the capacity of a multimedia DS/CDMA wireless system in the presence of residual (closed-loop) power control errors. In fact, practical power control algorithms may suffer the limited capacity of the power control (sub-) channel and loop delay, so that residual fluctuations exist. Moreover, in the presence of heterogeneous traffic sources, power control performance may be different for different services. In particular, burstiness of the source plays a significant role and for data sources transmitting short messages closed-loop power control might be unfeasible, as noted in [2]. The scope of the analysis in [13] was twofold: 1) to investigate the effects of power control imperfections on the capacity of a DS/CDMA system optimized for ideal power control; 2) to incorporate the statistics of power control errors in a modified optimal power allocation policy. QoS requirements were expressed in terms of the outage probability for any class of sources, that means we considered a delay-limited case [5],[6], and the capacity was referred to as radio capacity. From numerical results, it was observed that residual power control errors may determine a noticeable capacity decrease. The capacity loss becomes larger if the error standard deviation of one class is quite larger than those of other classes and/or the difference among QoS requirements of different classes becomes larger. Moreover, a proper (optimal) power allocation policy was found, and it reduces the above-mentioned capacity loss. The optimal target power levels can be easily found in terms of relative power levels in the presence of a reasonable approximation of the multiple access interference term.

In the present paper we propose an extension of our previous analysis [13] to consider second order outage statistics as more meaningful performance measures. In fact, as stated e.g. in [12], knowledge of second order outage statistics of a mobile radio channel can be effectively used to design error recovery algorithms and protocols for both delay-constrained and delay-tolerant transmissions. Closed-loop power control modifies the memory properties of the fading channel, however the controlled channel may still exhibit a bursty behavior, as noted in [8], where an attempt to derive the autocovariance function of the controlled channel was proposed. In [13] and [4] a characterization of closed-loop power control algorithms has been also proposed. In particular, numerical results were presented
in order to support the assumptions on the standard deviation of residual power control fluctuations that were made in the power allocation problem. In the present paper we use also estimates of the autocovariance of residual closed-loop power control errors, obtained from simulation of fixed step power control algorithms, to complete the channel model in the power allocation problem. By assuming a Gaussian model (in dB) for power control errors, the analysis can be carried out by resorting to an extended log-normal approximation [14] for the Signal-to-(Noise + Interference) ratio, where the activity status of each source is modelled as a random process. Performance results for the generic user of each traffic class are then expressed in terms of average outage rate and average outage duration, other than in terms of outage probability.

The paper is organized as follows. In Section 2 the system model is described and the power allocation strategy based on first order outage analysis [13] is briefly presented. In Section 3 the analysis of second order outage statistics is addressed. In Section 4 a collection of numerical results is presented and discussed. Finally, conclusions and future perspectives are given in Section 5.

2. Power Allocation Strategy

2.1. System Model

According to [13], we consider the return link of a single-cell asynchronous BPSK DS/CDMA system, with $S$ classes of mobile users. There are $K_i$ active users of the class $i$ (with $i = 0, ..., S - 1$). The generic class $i$ is characterized by its own bit rate $R_i$ (or the bit interval $T_i = \frac{1}{R_i}$) and signal-to-(noise + interference) ratio requirement $\gamma_i$. The same fixed bandwidth $W$, and thus the same chip interval $T_c$, is allocated to every user. Therefore, each class of users has a processing gain $G_i$ which is expressed as $G_i = \frac{W}{T_i}$. Following the same approach outlined in [13] and references therein, it can be shown that the signal-to-(noise + interference) ratio at the output of a coherent correlation receiver matched to the user $0$ of Class $m$, after averaging with respect to carrier phases, time delays, data and signature symbols, has the following expression:

$$SNR_{m0}(\xi(t), \nu(t)) = \frac{\left(\frac{\sigma_{\rho}^2}{4} + \text{var}_{m0}(t)\right)^{1/2} T_m}{N_0 T_m/4 + \text{var}_{m0}(t)}$$

with

$$\text{var}_{m0}(t) = \sum_{k=1}^{K_{m1}} \nu_{mk}(t) P_{mk} e^{\xi_{mk}(t)} T_m^2 6G_{mk}$$

$$+ \sum_{j=0}^{S-1} \sum_{k=0}^{K_{j1}} \nu_{jk}(t) P_{jk} e^{\xi_{jk}(t)} T_m^2 6G_{jk}.$$  

The meaning of various variables and parameters in the above expressions are as follows:

- $N_0/2$ denotes the two-sided power spectral density of thermal noise at the receiver input;
- $\text{var}_{m0}(t)$ denotes the variance of multiple access interference (MAI);
- $P_i$ denotes the target power level at the receiver input for the generic user signal of Class $i$;
- $\nu_{ih}(t)$ is a binary random process indicating the activity status (On/Off) of the source at time $t$; its first order probability mass function is such that $P[\nu_{ih} = 1] = \alpha_i$ and $P[\nu_{ih} = 0] = 1 - \alpha_i$, where $\alpha_i$ is said to be the activity factor of sources of the class $i$;
- $\xi_{ih}(t)$ denotes the residual power control error for the user signal $k$ of class $i$ and is represented (in log units) by a zero mean Gaussian process with standard deviation $\sigma_{\xi_i}$ (in dB);
- the vectors $\xi(t)$ and $\nu(t)$ are defined as $\xi(t) = (\xi_0(t), ..., \xi_{K_{01}-1}(t), ..., \xi_{S-1}(t), ..., \xi_{K_{S1}-1}(t))$ and $\nu(t) = (\nu_0(t), ..., \nu_{K_{01}-1}(t), ..., \nu_{S-1}(t), ..., \nu_{K_{S1}-1}(t))$, and independence is assumed between any pair of processes.

2.2. Power Allocation

As in [13], the outage probability is introduced as a performance measure in power allocation and the QoS requirement for the generic user $k$ of class $i$ is expressed as

$$Pr[SNR_{ik}(\xi, \nu) < \gamma_i] \leq P_{out}^i$$

$P_{out}^i$ denoting the outage requirement and $\gamma_i$ denoting a threshold above which satisfactory performance is guaranteed by the coding/modulation format. The radio capacity of the single-cell system is evaluated in terms of the maximum number of users of a generic class that can be accepted for a given configuration of users belonging to each other class. It is also assumed that Class $0$ has the lowest source rate ($R_0$) and $P$ is a power level such that $E_b = \frac{P}{R_0}$, where $E_b$ is determined by assigning $E_b/N_0$. The generic target power level $P_i$ is expressed as $P_i = \beta_i P$, with $\{\beta_i\}_{i=0,...,S-1}$ being a set of coefficients such that $\sum_{i=0}^{S-1} \beta_i = S$. Finally, for easier notation, it is assumed $x_i = \beta_i / \beta_0$ for $i = 1, ..., S - 1$. An approximate solution to the optimal power allocation problem, that corresponds to replace $\text{var}_{m0}(t)$ with its average, can be expressed in closed form in terms of $x_i$ ($i = 1, ..., S - 1$) as:

$$x_i = \frac{3G_0 \phi_0 + \alpha_i \epsilon_{\xi_i}^2 / 2}{3G_i \phi_i + \alpha_i \epsilon_{\xi_i}^2 / 2},$$

where $\sigma_{\xi_i} = \ln \frac{1}{\phi_i} \sigma_{\xi_i}$ and $\phi_i = \frac{1}{\epsilon_{\xi_i}} e^{\epsilon_{\xi_i}} Q(1 - P_{out}^i)$, $Q^{-1}$ denoting the inverse function of the complementary standard Gaussian distribution, i.e. $Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} dt$. (4) evidences the effects of main system parameters on power allocation and may drive to a quite accurate capacity estimation in some situations. In any case, it can be used to feed exhaustive optimization methods that are required when complete first order statistics of $\text{var}_{m0}(t)$ is retained [13].

3. Second Order Outage Statistics

In this section we propose an approach for deriving second order outage statistics, which is based on approximating with a log-normal process a linear combination of independent log-normal processes (that model residual power fluctuations), whose weights are other random processes (that model the activity status of sources). A generalized Moment Matching approach is developed, that moves from the one presented in [13] and imposes matching between the autocorrelation of the weighted sum and the autocorrelation of the approximating log-normal process. At this point, in the logarithmic scale the level crossing analysis of Gaussian processes can be applied to obtain the average outage rate. The average outage duration can
be finally obtained as the ratio between the outage probability and the average outage rate.

As a preliminar to the analysis, the processes $\xi_k(t)$ and $\nu_k(t)$ are characterized in terms of their autocorrelation function.

- $\xi_k(t)$ is assumed to be a Gaussian stationary random process with autocorrelation (covariance) function denoted as $C_{\xi_k}(\tau)$. The proper model and related parameters can be chosen by proper fitting of trends estimated by simulation (some trends were reported in [13]). As our analysis is carried out with reference to classes of users, it will be assumed in the following that $C_{\xi_k}(\tau)$ is dependent only on the Class $i$ and independent on the user $k$. This does not limit the significance of our approach. However, as closed loop power control performance is strictly dependent on Doppler spread and each terminal may have its own speed, each user signal should be considered with its own characteristics. For the same reason, the stationarity assumption should only be retained as a local assumption, since the user speed may vary and power control error statistics may vary as a consequence.

- To derive the autocorrelation function of $\nu_k(t)$, we consider the sequence of pairs of activity interval-subsequent inactivity interval can be modelled according to renewal processes. Moreover, it is assumed that the generic activity interval and the subsequent inactivity interval are independent exponential RVs, with average $1/\mu_{i,\text{on}}$ and $1/\mu_{i,\text{off}}$, respectively. It trivially results that $\alpha_i = \frac{1}{\mu_{i,\text{on}}} + \frac{1}{\mu_{i,\text{off}}}$. The autocorrelation function of $\nu_k(t)$ can be derived and it results

$$ r_{\nu_k}(\tau) = \frac{\mu_{i,\text{off}} \mu_{i,\text{on}} e^{-\left[\frac{1}{\mu_{i,\text{on}}} + \mu_{i,\text{off}}\right] |\tau|}}{\mu_{i,\text{off}} + \mu_{i,\text{on}}} + \frac{\mu_{i,\text{on}} e^{-\left[\frac{1}{\mu_{i,\text{on}}} + \mu_{i,\text{off}}\right] |\tau|}}{\mu_{i,\text{off}} + \mu_{i,\text{on}}} $$ (5)

It is important to remark that the exponential behavior of the above autocorrelation has a singularity in its second order derivative at the origin. This leads to an infinite level crossing rate for the SNR process and it can actually be explained in terms of the ideal transitions between On and Off states. This would not allow to carry out a level crossing analysis. However, we can consider that, at the transmitter side, the actual transitions between states are not ideal (for example, the effects of ramp-up and ramp-down of power amplifiers should be considered). From the point of view of our model, this can be accounted for by replacing $\nu_k(t)$ with its low-pass filtered version. In particular, a first order filter is assumed, whose impulse response is $h(t) = \alpha e^{-\beta t} u(t)$. In this way the average of the process is not modified. Moreover, in numerical computations, the decay constant $1/\beta$ is chosen small enough with respect to the decay constant in (5), in order not to affect the asymptotic behavior of the autocorrelation. Finally, the expression of the autocorrelation of the filtered process is obtained as:

$$ R_{\nu_k}(\tau) = r(\tau) \otimes h(\tau) \otimes h(-\tau) $$ (6)

$$ = d \left( \frac{c}{d} + \frac{ab}{b^2 - d^2} e^{-d|\tau|} - \frac{ad}{b^2 - d^2} e^{-b|\tau|} \right) $$

where: $a = \frac{\mu_{i,\text{off}} \mu_{i,\text{on}}}{(\mu_{i,\text{off}} + \mu_{i,\text{on}})^2}$, $b = \mu_{i,\text{off}} + \mu_{i,\text{on}}$, and $c = \frac{\mu_{i,\text{off}}^2}{(\mu_{i,\text{off}} + \mu_{i,\text{on}})^3}$.

Let (1), with consideration of (2), be rewritten as follows:

$$ SNR_m(\xi(t), \nu(t)) = D_m e^{-\xi_m(t)} $$

$$ = \left\{ \begin{array}{l}
D_m e^{-\xi_m(t)} \\
+ \sum_{k=1}^{K_m-1} A_m e^{\xi_{mk}(t)} - \xi_m(t) \nu_{mk}(t) \\
+ \sum_{j=0}^{S-1} \sum_{j\neq m} B_{mj} e^{\xi_{jk}(t)} - \xi_m(t) \nu_{jk}(t) \end{array} \right\} $$ (7)

where $A_m = \frac{1}{3d_m}$, $B_{mj} = \frac{p_j}{2\beta_{m} \alpha_m}$, and $D_m = \frac{\beta_{m} \alpha_m}{2\beta_{m} x_m}$. Moreover, let $L_m(t)$ be defined as $SNR_m(\xi(t), \nu(t)) = L_m^{-1/2}(t)$ (the dependence on $\xi, \nu$ is not evidenced for easier notation). $L_m(t)$ is a randomly weighted sum of correlated log-normal r.v.s. To compute its second order statistics, we can resort to a method based on the assumption that $L_m(t)$ can be approximated with another log-normal random process, i.e. $L_m(t) \approx e^{Z_m(t)}$, where $Z_m(t)$ is a Gaussian random process. The Wilkinson’s approach can be extended to second order statistics, so that the mean and the autocovariance function of $Z_m(t)$ can be obtained by imposing matching of the first moment and of the autocovariance of $L_m(t)$ and $e^{Z_m(t)}$.

Matching of the first moment can be carried out as in [13], thus leading to:

$$ E\{e^{Z_m(t)}\} = e^{m x_m + \frac{1}{2} \sigma_{Z_m}^2} = E\{L_m(t)\} \triangleq M_{m1} $$ (8)

and the following expression holds for $M_{m1}$:

$$ M_{m1} = D_m e^{\frac{1}{2} \sigma_{L_m}^2} + A_m \alpha_m (K_m - 1) e^{\xi_m} + \sum_{j=0}^{S-1} B_{mj} \alpha_j K_j e^{\frac{\sigma_{Z_m}^2 + \sigma_{j}^2}{2}} $$ (9)

Matching of the autocorrelation yields:

$$ E\{e^{Z_m(t)}Z_m(t+\tau)\} = e^{2m x_m + \frac{1}{2} \sigma_{Z_m}^2 + C_{Z_m}(\tau)} = E\{L_m(t)L_m(t+\tau)\} \triangleq M_2(\tau) $$ (10)

where $C_{Z_m}(\tau)$ denotes the autocovariance function of $Z_m(t)$. By solving for $C_{Z_m}(\tau)$ eq. (10), and considering eq. (9) one obtains:

$$ C_{Z_m}(\tau) = \ln M_2(\tau) - 2m x_m - \sigma_{Z_m}^2 $$

$$ = \ln M_{m1}(\tau) - 2m M_{m1} $$

$$ = \ln \frac{M_{m1}(\tau)}{M_{m1}} $$ (11)

Moreover, the following expression is obtained for $M_2(\tau)$ after manipulations:
Numerical results are shown in this section, as obtained for a two-class system. The two classes correspond to Class 0 and Class 2 considered in [13]. Thus, it is assumed $W = 5$ MHz, $E_b/N_0 = 14$ dB, $R_0 = R_2 = 8$ kbit/s, $P_{out}^0 = 10^{-2}$ and $P_{out}^2 = 10^{-5}$. The power parameter is $\mu_0_{on} = 1.5 s^{-1}$, $\mu_0_{off} = 1 s^{-1}$ for Class 1 (voice source) and $\mu_2_{on} = 1.25 s^{-1}$, $\mu_2_{off} = 1 s^{-1}$ for Class 2 (generic data source). The parameter of the filter used to smooth on-off transitions of sources has been set to $d = 15 s^{-1}$ (values of $d$ larger than 0.01 $s^{-1}$ do not affect significantly the obtained results). The following autocovariance model has been considered for fitting the estimated autocovariance function of power control errors:

$$C_{\xi k}(r) = \frac{\sigma_{\xi k}^2}{\tau_{\xi k}} \exp\left(-\frac{r}{\tau_{\xi k}}\right)^2$$

where $\tau_{\xi k}$ represents the autocovariance delay constant. The values of $\sigma_{\xi k}$ and $\tau_{\xi k}$ that will be considered have been observed to fall in ranges of practical interest, according to simulation results of closed loop power control.

In Fig. 1 the outage occurrence rate is plotted as obtained for the generic user of Class 0 when the standard deviation of power control error for one of the two classes varies. In this case it is assumed that the autocorrelation decay constants are set to $0.45 m/s$, that corresponds to 4 bit intervals at the bit rate of both source classes. Moreover, it is assumed that 40 terminals of each class are in the system, that represents a traffic load lower than the capacity bound derived in [13]. The power allocation policy has been derived according to (4) with $\sigma_{e1} = \sigma_{e2} = 1 dB$. It can be seen from the figure that the outage rate increases remarkably with $\sigma_{e1}$, while it increases very slowly with $\sigma_{e2}$. It is thus confirmed that, in the presence of a relatively large number of terminals, interfering signal statistics mainly contributes to the average of the $SNR$. Moreover, if power allocation coefficients are adapted to $\sigma_{e1}$, a performance degradation is observed for Class 0 when $\sigma_{e1}$ is above 1 dB with respect to the case where power allocation is not adapted. This can be understood, as an adaptive power allocation would try to optimize resource allocation, by protecting Class 2 against larger fluctuations of Class 0.

The average outage duration for the generic user of Class 0 is plotted in Fig. 2 versus $\sigma_{e1}$ and $\sigma_{e2}$. Considerations similar to those related to Fig. 1 still apply. However, it can be seen that relative variations are less remarkable than those observed for the outage rate.

In Fig. 3 the outage occurrence rate is plotted as obtained for the generic user of Class 0 versus the autocorrelation decay constant $\tau_{\xi k}$. It is still assumed $\sigma_{e1} = \sigma_{e2} = 1 dB$ and power allocation has been applied accordingly. It can be seen that the outage rate decreases as the decay constant increases. Relative variations of are of the order of 1 decade. Finally, it can be observed from Fig. 4 that the average outage duration increases almost linearly with $\tau_{\xi k}$ and relative variations are more remarkable than those observed when varying $\sigma_{e1}$.

4. Numerical Results

5. Conclusions

An approach to performance analysis of a DS-CDMA wireless systems with multiple classes of sources has been presented, with emphasis on second order outage statistics. The approach is based on an extension of Wilkinson’s method to derive second order statistics of the power sum of Gaussian processes.
Figure 1: The average outage occurrence rate for the generic terminal of Class 0 versus the standard deviation of power control errors of the users of Class 0 and Class 2. When one parameter is let to vary, that other one is kept fixed to 1 dB.

Figure 2: The average outage duration for the generic terminal of Class 0 versus the standard deviation of power control errors of the users of Class 0 and Class 2. When one parameter is let to vary, that other one is kept fixed to 1 dB.

Figure 3: The average outage occurrence rate for the generic terminal of Class 0 versus the autocovariance decay constant of Class 0.

Figure 4: The average outage duration for the generic terminal of Class 0 versus the autocovariance decay constant of Class 0.

with random weights. Numerical results have shown that, for a fixed load in the system, the outage occurrence rate seen from a generic user is very sensitive to the standard deviation of power control error of that user. Moreover, the average outage duration is more sensitive to the autocorrelation decay distance of power control fluctuations. Ongoing work is focused on application of the proposed analysis to improve power allocation strategies, so that second order outage constraints can be accounted for. In perspective, an interesting investigation could concern power allocation and control strategies along with error recovery mechanisms in a multimedia environment in order to let a recovery scheme (typically operating at various layers for delay-tolerant services) of each user to operate with the 'desired' channel.

6. References


