A generalized utility maximization problem with outage constraints in CDMA networks

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Abstract: The problem of maximizing a utility function while limiting the outage probability below an appropriate threshold is investigated. A coded-division multi access wireless network under mixed Nakagami-lognormal fading is considered. Solving such a utility maximization problem is difficult because the problem is non-convex and non-geometric with mixed integer and real decision variables and no explicit functions of the constraints are available. In this paper, two methods to the solution of the utility maximization problem are proposed. By the first method, a simple explicit outage approximation is used and the constraint that rates are integers is relaxed yielding a standard convex programming optimization that can be solved quickly but at the price of a reduced accuracy. The second method uses a more accurate outage approximation, which allows one solving the utility maximization problem by the Lagrange duality for non-convex problems and contraction mapping theory. Numerical results show that the first method performs well for average values of the outage requirements, whereas the second one is always more accurate, but is also more computationally expensive.

Keywords: Radio Power Control, CDMA, Outage, Non-Convex Optimization.

1. INTRODUCTION

In this paper, we propose a method to maximize the utility for code-division multi access (CDMA) wireless systems by allocating rates and powers. We focus on delay-limited services, such as video transmission, where the QoS is expressed by outage probability constraints [Caire, G. and Taricco, G. and Biglieri, E., 1998]. Typical measures of utility are the sum of transmit rates, the throughput, or the negative sum of radio powers (i.e., the utility maximization is the minimization of the powers) [Meshkati et al., 2005, 2007]. In CDMA, the utility can be maximized by adapting the transmit rates and the radio powers to the wireless channel conditions and traffic load such that multi-access interference caused by co-channel transmitters is kept within acceptable levels. Therefore, the utility maximization can be cast as a constrained optimization, where the objective function is the utility and the constraints are the outage probability.

The paper is organized as follows: In Section 2, we summarize the existing work from the literature and highlight the original contribution. We present the system model in Section 3 and the mathematical formulation of the utility maximization problem in Section 4. In Section 5 we solve the utility maximization problem. Numerical results are reported in Section 6 to demonstrate the properties of our approach to the optimization. Section 7 concludes the paper.

2. BACKGROUND AND ORIGINAL CONTRIBUTION

The utility maximization problem is complex, because of the relations among objective function, constraints, radio powers, and transmit rates. In many wireless propagation scenarios, it is impossible to derive a closed form expression for the outage constraints, and one has to resort to approximations of adequate accuracy and computational complexity. Moreover, in CDMA systems, transmit rates are integers whereas powers are real, which makes the optimization problem a mixed integer-real one. As a result of this complexity, several approaches have been proposed in the literature, which differ for the approximations adopted for the outage probability, and for the method employed to solve the optimization problem, as we survey in the following.

In the simpler cases of utility functions given by the radio powers minimization or rates maximization, there have been many attempts to solve utility maximization problems with outage constraints. Minimizing the powers under outage constraints was investigated in [Heikkinen, T., 2001] for lognormal fading channels. The same channel was considered in [Kandukuri, S. and Boyd, S., 2002], where the authors resorted to a Gaussian approximation for the computation of the outage probability. This approach was further developed in [Hsiung, K.-L. and Kim, S.-J. and Boyd, S., 2005], where the channel included Rayleigh fading distributions. Both [Kandukuri, S. and Boyd, S., 2002] and [Hsiung, K.-L. and Kim, S.-J. and Boyd, S., 2005], showed that power control under outage constraints can be cast as a geometric program. In [Papandriopoulos, J. and Evans, J. and Dey, S., April 2006], the constraints were relaxed by an upper bound provided by the Jensen’s inequality. In [Fischione et al., 2009], a framework to solve the rate maximization problem with outage constraints in CDMA systems was introduced. The channel was modelled by a general Nakagami-lognormal distribution. A survey of the utility maximization problem can be found in [Meshkati et al., 2005, 2007], where...
Our original contribution is as follows: First, we pose a novel
utility-maximization problem with outage constraints. The solution of such a problem is challenging because no exact expressions are known for the constraints and the problem is a non-convex and non-geometric optimization problem with mixed real-integer decision variables. To the best of our knowledge, no paper in the literature poses such a utility maximization problem when considering the outage probability. Second, using two approximations of the outage probability we have proposed in [Fischione et al., 2009] and [D’Angelo et al., 2008], we solve the utility-maximization problem by two methods. By the first approach, we relax the constraint on rates to be real values so that there are no integer variables in the optimization, which becomes a convex mathematical programming problem. The second approach is the solution of the utility maximization problem by means of the duality theory for non-convex optimization problems and integer programming.

Despite our deep scanning of the literature, no papers can be found that deal with a utility maximization in a general fading set-up. Specifically, in the literature most closely related to our problem, as [Heikkinen, T., 2001, Kandukuri, S. and Boyd, S., 2002, Hsiung, K.-L. and Kim, S.-J. and Boyd, S., 2005, Papandriopoulos, J. and Evans, J. and Dey, S., April 2006, Kim, D. I. and Le, L. and Hossain, E., 2008, Papandriopoulos et al., 2008], the authors either assume special cases of the fading, or they resort to bounds for the outage constraints. In contrast to our previous work [Fischione et al., 2009], here we consider sigmoidal cost functions, which are much more general and challenging to tackle than the sum-rate function we considered previously.

3. SYSTEM MODEL

We consider a system scenario where $K$ mobile transmitters communicate towards a receiver. Each transmitter $i = 1, \ldots, K$ is associated with a traffic source type (voice, video, data, etc.). Let the channel gain of signals of transmitter $i$ be $h_i = l_i z_i \Omega_i$, where $l_i$ is the path loss, $z_i$ is the fast fading, and $\Omega_i$ is the shadowing [Stüber, G. L., 1996, pag. 91]. The fast fading $z_i$ is modelled as the square of a Rayleigh-distributed random variable. For any $i \neq j$, $z_i$ and $z_j$ are considered statistically independent. The shadow fading is $\Omega_i = \exp(\xi_i)$, where $\xi_i$ is a Gaussian random variable having zero mean and standard deviation $\sigma_{\xi_i}$. Let the binary random variable $\nu_i$ be the activity status (on/off) of the source. The probability mass function is such that $P(\nu_i = 1) = \alpha_i$ and $P(\nu_i = 0) = 1 - \alpha_i$, where $\alpha_i$ is the activity factor of source $i$. Let $h = [h_1, \ldots, h_K]^T$ and $\nu = [\nu_1, \ldots, \nu_K]^T$. Independence is assumed between any pair of variables of the vectors $h$ and $\nu$. We model the rate of each transmitter as $R_i = R_{0i} n_i$, where $R_{0i} = 1/T_i$ is the basic rate (with basic bit time $T_{0i}$), and $n_i$ is an integer denoting the assigned rate. Such a rate is a power of two due to the spreading code structure [Stüber, G. L., 1996]. Consequently, the spreading factors are expressed as $G_i = G_{0i}/n_i$, where $G_{0i} = T_{0i}/T_i$ corresponds to the basic rate (when $n_i = 1$) and $T_i$ is the chip time. Let the radio power be $P_i = p_i n_i$, where $p_i$ is the power at the basic rate. Let $p = [p_1, \ldots, p_K]^T$ and $n = [n_1, n_2, \ldots, n_K]^T$. Notice that $p$ is a vector of positive real elements, whereas $n$ is a vector of integer elements because transmit rates must be integer in CDMA.

The Signal to Interference plus Noise Ratio (SINR) is [Goldsmith, 2005]

$$\text{SINR}_i(n, p) = \frac{p_i h_i \exp(\xi_i)}{\sum_{j \neq i}^K p_j h_j \nu_j}.$$  \hspace{1cm} (1)

In the next section, the SINR is used to formulate the utility maximization problem we are interested in this paper.

4. UTILITY MAXIMIZATION WITH OUTAGE CONSTRAINTS

The objective function of the utility maximization problem is a function of the transmit rates and the transmit powers, and the constraints are bounds on the outage probabilities, transmit powers and rates:

$$\max_{p, n} f(n, p)$$ \hspace{1cm} (2a)

s.t. \hspace{0.5cm} \Pr[\text{SINR}_i(n, p) < \gamma_i] \leq P_{\text{out}}, \forall i = 1, \ldots, K \hspace{1cm} (2b)
$$
$$\nu^T(n \circ E[h]) \leq P_T,$$ \hspace{1cm} (2c)

$$1 \leq n_i \leq G_{0i}, \hspace{0.25cm} n_i \in \mathbb{N}, \forall i = 1, \ldots, K \hspace{1cm} (2d)$$

The decision variables are the powers $p$ and rates $n$. $P_{\text{out}}$ is the maximum allowed outage probability of the SINR for transmitter $i$ with respect to the threshold $\gamma_i$. The constraint (2c) is due to that the receiver antenna can accept only a maximum amount of power without distorting the signal. Note that $\circ$ denotes that Hadamard product. For obvious physical reasons, powers cannot be smaller than $p_{\min}$ or larger than $p_{\max}$. The rate constraint (2e) is motivated by that the spreading factor is $G_i = G_{0i}/n_i \geq 1$. The set of rates, $\mathbb{N}$, must be a power of two. The utility function of the problem is $f(n, p) : \mathbb{R}^K \times \mathbb{R}^K \rightarrow \mathbb{R}$. In this paper, we are interested to two classes of utility functions that include many cases of practical interests: sigmoidal and monotonic cost functions [Meshkati et al., 2005, 2007]. We give details in the following subsections.

4.1 Sigmoidal Cost functions

Let (2a) be a sigmoidal function. Such a function has an S shape in the decision variables $n$ and $p$. A typical example includes the minimization of the throughput, or net number of information bits that are transmitted without error per unit time [Meshkati et al., 2005, 2007]:

$$f(n, p) = \sum_{i=1}^K n_i (1 - \text{BER}(\text{SINR}_i(n, p))),$$ \hspace{1cm} (3)

where BER($\text{SINR}_i(n, p)$) is the bit error rate. A sigmoidal function is a quasi-convex function in $n$ and $p$. This property is useful to compute the optimal solution to the optimization problem (2), as we see later.
4.2 Monotonic Cost functions

Suppose that the cost function (2a) is a monotonic function of the decision variables \( n, p \). As for the case of sigmoidal cost functions, a monotonic function is quasi-convex function in \( n, p \).

A typical example is

\[
f(n, p) = \beta^T n - \vartheta^T p, \tag{4}
\]

where \( \beta \in \mathbb{R}^K \) and \( \vartheta \in \mathbb{R}^K \) are two constant vectors, with \( \beta \succeq 0 \) and \( \vartheta \succeq 0 \), where \( \succeq \) denotes the component-wise inequality. For example, if \( \beta = 0 \) and \( \vartheta \neq 0 \), then we have a problem of minimization of the radio powers. Conversely, if \( \vartheta = 0 \) and \( \beta \neq 0 \), then (2) is the problem of maximizing the transmit rates. The special case when \( \beta = 1 \) and \( \vartheta = 0 \) was investigated in [Fischione et al., 2009]. Another example of monotonic cost function is

\[
f(n, p) = \sum_{i=1}^{K} \ln n_i. \tag{5}
\]

It can be shown that this gives a fair rate allocation [Kim, D. I. and Le, L. and Hossain, E., 2008].

For both the sigmoidal cost function and monotonic cost function, it is challenging to solve problem (2): the outage constraints are complicated functions of rates and power, for which no closed form expression is available for Nakagami-lognormal fading, and the rates are integers. Moreover, the problem is in general non-convex, which makes difficult to compute the optimal solution by computationally affordable algorithms. We resort to two different approximations of the outage constraint that we have proposed in [D’Angelo et al., 2008] and [Fischione et al., 2009], which allow us to reformulate and solve the optimization problem as described in the following section.

5. SOLVING THE UTILITY MAXIMIZATION PROBLEM

The outage constraints can be modelled by two approximations, which were developed in [Fischione et al., 2009] and [D’Angelo et al., 2008]. The first approximation is a linear one in the decision variables and is denoted by \( I_{i}^{(1)}(n, p) \). It has the drawback of a lower accuracy. The second approximation is much more accurate but is a non-linear one in the decision variables. It is denoted by \( I_{i}^{(2)}(n, p) \). See [Fischione et al., 2009] and [D’Angelo et al., 2008] for the expressions of \( I_{i}^{(1)}(n, p) \) and \( I_{i}^{(2)}(n, p) \). By using the \( I_{i}^{(2)}(n, p) \), we can approximate the optimization problem (2) as

\[
\begin{align*}
\max_{n, p} & \quad f(n, p) \tag{6a} \\
s.t. \quad & -p_i + I_{i}^{(2)}(n, p) \leq 0, \quad \forall i = 1, \ldots, K, \tag{6b} \\
& p_t^{T}(n \circ E[h]) - P_T \leq 0, \tag{6c} \\
& p_{i0} \leq p_i \leq p_{\max}, \quad \forall i = 1, \ldots, K, \\
& 1 \leq n_i \leq G_{i0}, \quad n_i \in N, \quad \forall i = 1, \ldots, K. \tag{6d}
\end{align*}
\]

This optimization problem is a mixed integer-real one, since the transmit rates are integer numbers and the radio powers are real positive variables. We assume that the problem is feasible, which means that the constraints are satisfied at the minimum rate and power. This is equivalent to say that transmitters are admitted into the system after an admission control policy (see, e.g., [Evans and Everitt, Jan. 1999]).

To solve problem (6), we propose two methods:

1. A solution is achieved by a convex relaxation where the integrality constraint of the rates is relaxed. We will show in the numerical results section that this method performs well only in some cases.

2. We retain the transmit rates integer. Then, we use two steps. First, the rate vector \( n \) is fixed and a novel algorithm based on Lagrange duality for non-convex non-geometric problems is developed to find the optimal radio power vector \( p^* (n) \) that solves (2) as a function of the fixed rate vector. Second, the optimal solution is given by searching the rate \( n \in N \) that maximizes the utility function \( f(n, p^* (n)) \) computed by the powers given in the previous step. We will show that this method is the most accurate.

We address the details in the following subsections.

5.1 Convex Relaxation

Here we describe the first method. If we relax the integrality constraint on the rates, and consider the simple approximation of the outage probability \( I_{i}^{(2)}(n, p) \) in [D’Angelo et al., 2008, Eq. (7)], we can cast the rate maximization problem as a convex optimization problem. Rewriting problem (2) according to the canonic form [Boyd, S. and Vandenberghe, L., 2004], we obtain

\[
\begin{align*}
\min_{n, p} & \quad f(n, p) \tag{7} \\
s.t. \quad & \sum_{j=1}^{K} b_j p_j n_j - a_i p_i - c \leq 0 \quad \forall i = 1, \ldots, K, \\
& p_{i0} \leq p_i \leq p_{\max}, \quad \forall i = 1, \ldots, K, \\
& 1 \leq n_i \leq G_{i0}, \quad n_i \in N, \quad \forall i = 1, \ldots, K.
\end{align*}
\]

where the outage constraint has been rewritten after simple algebra, and where

\[
a_j = G_{i0} l_i \phi_i, \tag{8} \\
b_j = l_j \mu_2 \alpha_j \exp \left( \frac{1}{2} \sigma^2 \right), \tag{9} \\
c = -\frac{N_0}{2T}, \tag{10}
\]

with \( \phi_i = \exp(\sigma \xi_i + \mu \xi_i)/\gamma_i \). All the constraints of problem (7) are convex. The cost function is quasi-convex since so is a sigmoidal or monotonic function. A feasible solution is given by the minimum radio powers and minimum transmit rates. Therefore, strong duality holds and a global optimal solution exists, which can be obtained quickly by interior point methods [Boyd, S. and Vandenberghe, L., 2004].

5.2 Optimal Powers

In this section, we describe the second method to solve problem (6). We suppose that the transmit rate vector \( n \) is fixed, and we solve the following optimization problem:

\[
\begin{align*}
\max_{p} & \quad f(n, p) \tag{11a} \\
s.t. \quad & -p_i + I_{i}^{(2)}(n, p) \leq 0, \quad \forall i = 1, \ldots, K \tag{11b} \\
& p_t^{T}(n \circ E[h]) - P_T \leq 0, \tag{11c} \\
& p_{i0} \leq p_i \leq p_{\max}, \quad \forall i = 1, \ldots, K, \\
& 1 \leq n_i \leq G_{i0}, \quad n_i \in N, \quad \forall i = 1, \ldots, K.
\end{align*}
\]
where we recall that \( I_i^{(2)}(n, p) \) is based on the more accurate approximation given in [Fischione et al., 2009, Eq. (2)]. We remark that in this problem the decision variable is \( p \), while \( n \) is fixed. This problem is a non-convex optimization problem in the variable \( p \), since \( I_i^{(2)}(n, p) \) is non-convex. Nevertheless, we can solve it efficiently, as we show in the following fundamental result:

**Theorem 1.** Let problem (11) be feasible. Then,

- if the cost function is sigmoidal, the optimal solution \( p^*(n) \) is given by the solution to the following system of \( 2K + 1 \) equations in the variables \( p \) and \( \lambda = (\lambda_1, \ldots, \lambda_K, \lambda_{K+1}) \):
  \[
  \nabla f(n, p^*) - \sum_{i=1}^{K} \lambda_i (1 - \nabla I_i^{(2)}(n, p^*)) + \lambda_{K+1} \nabla p^T(n \circ E[h]) = 0
  \]
  \[
  \lambda_{K+1} \nabla p^T(n \circ E[h]) = 0
  \]
  where \( \nabla \) is the gradient operator with respect to the variable \( p \). The solution to this system of equation can be achieved with a complexity of \( O(K^3) \).
- if the cost function is monotonic, the optimal solution \( p^* \) is given by the solution to the system of equations given by the constraints (11b):
  \[
  p_i^* = I_i^{(2)}(n, p^*), \quad i = 1, \ldots, K. \tag{12}
  \]

**Proof 1.** A proof is provided in [C. Fischione, M. D’Angelo, M. Butussi, 2010].

The previous theorem tells us that for the sigmoidal functions, given that the Lagrange duality theory applies, the optimal solution can be computed with the efficient interior point methods [Boyd, S. and Vandenberghe, L., 2004]. Therefore, the computational complexity to the solution of (11) is the same as the one needed to solve a convex optimization problem. Moreover, from Theorem 1, it follows that if the cost function is monotonic, then the computational complexity necessary to solve problem (11) is even smaller than the case of sigmoidal cost function, since the optimal solution of problem (11) is given by the solution to the set of equations (12). An approximate solution to this system of equations can be solved in closed form if we use the approximation \( I_i^{(1)}(n, p) \) given in [D’Angelo et al., 2008, Eq. (7)]. We have the following corollary:

**Corollary 1.** Consider the system of equations \( p_i = I_i^{(1)}(n, p) \), \( i = 1, \ldots, K \), in the variable \( p \), where \( n \) is fixed. Then, the solution to this system of equations is

\[
  p_i = \frac{c - \sum_{j=1 \atop j \neq i}^{K} b_{ij} n_j}{a_{ii} + \sum_{j=1}^{K} b_{ij} n_j - a_{ii}}, \tag{13}
\]

where \( a_{ij}, b_{ij}, \) and \( c \) are given in (8), (9), and (10), respectively.

**Proof 2.** A proof is provided in [C. Fischione, M. D’Angelo, M. Butussi, 2010].

On the other hand, if we use the approximation \( I_i^{(2)}(n, p) \), the system of equations (12) can be solved easily by an iterative algorithm that is initialized by the solution (13) provided by the previous Corollary. In fact, it is possible to show that \( I_i^{(2)}(n, p) \) is contractive in \( p \) [C. Fischione, M. D’Angelo, M. Butussi, 2010]. Therefore, the solution to the system (12) can be obtained easily by asynchronous contraction mappings

Algorithm I. Optimal Solution to the Utility Maximization Problem

1. Initialize the rate vector;
2. Solve problem (11) by Theorem 1;
3. Reduce the set of rates \( N \) by using Proposition 1;
4. for any rate vector in the remaining set of rates do
5. Solve problem (11) by Theorem 1;
6. Reduce the set of rates by Proposition 1;
7. end for;

(see [Bertsekas, D. P. and Tsitsiklis, J. N., 1997, Pag. 431]):

\[
  p_i(t) = I_i^{(2)}(n, p(t-1)) \gamma_i \quad \forall i = 1, \ldots, K. \tag{14}
\]

The initialization \( p(0) \) of this iterative algorithm is given by Corollary 1. By such a choice, Monte Carlo simulations show that the iterative algorithm takes less than 5 iterations to converge and give the solution to (12).

### 5.3 Optimal Rates and Powers

In the previous section, we have shown how to solve efficiently problem (11), given that \( n \) was fixed. For every \( n \in N \), one could solve problem (11) and then check the pair \( n, p(n) \) that maximizes the utility. Clearly, solving (6) for each \( n \in N \) may be prohibitive. It is possible to reduce the set \( N \) over which making the checking of the optimal solution by exploiting the following proposition:

**Proposition**

**Proposition 1.** Assume that problem (6) is infeasible at \( n \). If \( n \geq \bar{n} \), then problem (6) is infeasible at \( n \).

**Proof 3.** A proof is provided in [C. Fischione, M. D’Angelo, M. Butussi, 2010].

Given a rate \( \bar{n} \in N \) that makes problem (6) infeasible, then Proposition 1 allows us to remove from the set of possible rates all the rates \( n \geq \bar{n} \).

In Algorithm 1, we summarize how to yield the solution to the utility maximization problem (6). This algorithm is the second method proposed in this paper.

### 6. NUMERICAL RESULTS

In this section we solve the optimization problem by the Convex Relaxation approach and by Algorithm 1, and we compare the results.

The simulation parameters are taken from the 3GPP specifications [3GPP TS 25.214 V6.1.0, 2004]: the chip time is \( T_c = 2.6 \times 10^{-7} \) s, and the maximum spreading factor is \( G_{\alpha 0} = 256 \). We assume a thermal noise \( N_0 = -170 \) dBm/Hz, and we set \( P_T = 1.5239 \times 10^{-12} \), such that \( P_T/T_c/N_0 = 16 \) dB. We assume a homogeneous propagation environment for the path loss \( l_i \), which is set to 90 dB for each transmitter. The system is considered in outage when the SINR is below \( \gamma_i = 3.1 \). All transmitters have the same activity factor \( \alpha_i = 0.5 \).

The plots in Figs. 1 – 3 and Figs. 4 – 6 show the optimal values for the utility maximization problems with the sigmoidal (throughput maximization) and monotonic (rate sum) cost function for a scenario with 4, 8 and 12 transmitters. We assumed \( \sigma_{\xi_i} = 0.5 \). Other values of \( \sigma_{\xi_i} \) provide the same trend in the numerical results, as those discussed below. Each plot reports the solution of problem (6) as obtained by the proposed methods, namely:
Fig. 1. Optimal cost function for a scenario with $K = 4$ transmitters and $\sigma_{\xi_i} = 0.5$, for $i = 1, \ldots, K$. The sigmoidal cost function has been considered.

Fig. 2. Optimal cost function for a scenario with $K = 8$ transmitters and $\sigma_{\xi_i} = 0.5$, for $i = 1, \ldots, K$. The sigmoidal cost function has been considered.

Fig. 3. Optimal cost function for a scenario with $K = 12$ transmitters and $\sigma_{\xi_i} = 0.5$, for $i = 1, \ldots, K$. The sigmoidal cost function has been considered.

Fig. 4. Optimal cost function for a scenario with $K = 4$ transmitters and $\sigma_{\xi_i} = 0.5$, for $i = 1, \ldots, K$. The monotonic cost function has been considered.

Fig. 5. Optimal cost function for a scenario with $K = 8$ transmitters and $\sigma_{\xi_i} = 0.5$, for $i = 1, \ldots, K$. The monotonic cost function has been considered.

- Convex Relaxation: optimum of problem (7) (first method);
- Algorithm 1: optimum of problem (6) as obtained by Algorithm 1 (second method).

In Figs. 1 – 6, the curves obtained by Algorithm 1 are the benchmark, since they are obtained by the approximation $I^{(2)}_i(n, p)$. Recall that Convex Relaxation is obtained by approximation $I^{(1)}_i(n, p)$. In the figures, a zero rate means that the rate maximization problem is not feasible with respect to the given outage constraint $P_{out}$.

Comparing the two result sets, we observe that the solution achieved by Convex Relaxation are close to the ones obtained by the mixed integer-real approach for intermediate ranges of $P_{out}$. Notice that in all the scenarios there is always a cross-point between the curves for certain values of the outage requirements. For outage requirements lower than the crossing point, Convex Relaxation provides an over-allocation of the transmit rates. This is due to the less accurate approximation of the SINR, and may result in a violation of the outage constraints. For higher values of the outage probability require-
Fig. 6. Optimal cost function for a scenario with $K = 12$ transmitters and $\sigma_{u_i} = 0.5$, for $i = 1, \ldots, K$. The monotonic cost function has been considered.

ment, Convex Relaxation underestimates the achievable rates, particularly for very high outage requirements. Obviously, an under allocation of the rates as a consequence of the approximation guarantees the satisfaction of the actual outage constraints.

7. CONCLUSIONS

We proposed an approach to the solution of a generalized utility maximization problem with outage constraints. The generalization consisted in that we dealt with both sigmoidal cost functions and monotonic functions in Nakagami-lognormal wireless channels. To solve the problem, we exploited a simple approximation of the outage probability and a more complex approximation. Then, we proposed two methods of different complexity. The first method is the less complex and is based on a convex relaxation. The second is more complex and uses an accurate approximation of the constraints and is based on the duality theory for non-convex problems and combinatorial optimization.

Since the solution methods investigated in this paper require a computational load of different complexity, networks with limited processing capabilities can use the least complex method but at the potential expense of a reduction of the quality of the solution. Numerical results showed that the solution obtained by the convex relaxation performs well for low values of the outage requirements, where the actual probability is underestimated with a maximum error that is less than 2%. We showed that the second method performs well in any situation, but at the cost of a higher computational complexity.

REFERENCES


