A Sensor Fusion Algorithm for Mobile Node Localization ⋆

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Abstract: Accurate distributed estimation of the position of network nodes is essential for many applications, including localization, geographic routing, and vehicular networks. When nodes are mobile and their mobility pattern is unknown, there are not yet adequate techniques to achieve high accuracy and low estimation errors. In this paper, a new distributed estimator of the position of mobile nodes is proposed. No model of the mobility is assumed. The estimator combines heterogeneous information coming from pre-existing ranging, speed, and angular measurements, which is jointly fused by an optimization problem where the squared mean and variance of the localization error is minimized. Challenges of this optimization are the characterization of the moments of the noises that affect the measurements. The estimator is distributed in that it requires only local processing and communication among the nodes of the network. Numerical results show that the proposed estimator outperforms traditional approaches based on the extended Kalman filter.

Keywords: Sensor fusion, positioning systems, distributed models, networks, optimization problems.

1. INTRODUCTION

In recent years, localization and positioning of nodes of wireless networks has been the object of considerable research efforts, Patwari et al. (2005). Relevant applications for this research field are vehicular traffic monitoring, asset tracking, process monitoring, and control of autonomous agents. Accurate position information is crucial for emergency personnel and first responders, see Rantakokko et al. (2010). While being relatively low-cost and easy to deploy, the commonly used Global Navigation Satellite Systems (GNSSs) provide us with an accuracy of about 3 to 10 m in outdoor scenarios. However, in indoor or electromagnetically challenged environments, GNSSs do not provide a sufficient coverage or accuracy. Therefore, in such scenarios, several alternative positioning technologies have been developed. An extensive survey of the indoor positioning field can be found in Liu et al. (2007).

Among the radio technologies useful to positioning, pulse-based Ultra-Wideband (UWB) has been largely studied, Sahinoglu et al. (2008). Its main advantage is the fine time resolution, which allows for centimeter-order accuracy distance measurement and for resilience to the multipath propagation effect. It is particularly suitable for the implementation of the round-trip-time (RTT) distance measurement method, which consists in the measurement of the two-way propagation time of a pulse travelling between two transceivers. The adoption of the RTT method allows to relax the synchronization requirements compared to other methods, such as time-of-arrival and time-difference-of-arrival, at the expense of calibration of the transceiver latencies. Furthermore, it can provide more accurate results with respect to the received-signal-strength method, and it requires a simpler architecture compared to the angle-of-arrival technique, Gezici et al. (2005).

To improve the performance of a positioning system in terms of reliability of the estimates (integrity), accuracy, and availability, it is appealing to integrate information obtained from a number of sensors by means of fusion techniques, Speranzon et al. (2008); Gustafsson (2010). These techniques involve the processing of different information sources, such as GNSSs, inertial navigation systems, odometry, and local radio technologies. An extensive survey of the most common information sources and sensor fusion approaches, in the context of automotive positioning, is provided in Skog and Händel (2009). In the field of information fusion for UWB indoor positioning, the commonly employed techniques are based on the extended Kalman filter (EKF) or other non-linear filtering approaches, Kailath et al. (2000). These include Monte Carlo methods such as the particle filter, which is used in Gonzalez, J. et al. (2007); Jourdan et al. (2005). In Hol et al. (2009), a 6 degrees-of-freedom tracking system is presented, which performs sensor fusion on a UWB distance-measuring device and an inertial sensor consisting of triaxial accelerometers and gyroscopes. The authors employ a tightly coupled approach, where the individual measurements obtained from the sensors are used directly in the EKF, without any previous preprocessing. In De Angelis et al. (2009b), an UWB/INS sensor fusion technique based on the complementary filter approach is presented, where the error states are estimated by an EKF using the UWB

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ranging measurements. Subsequently, the estimated errors are fed back to correct for the inertial navigation system biases, which would otherwise grow unbounded. Therefore, this system allows to exploit the complementary properties of the two individual subsystems.

In this paper, a new sensor fusion technique is investigated. It is assumed that a node wishing to estimate its position has already available ranging, speed and orientation (noisy) estimates. The information obtained from an UWB ranging measurement system, a speed sensor, and an absolute orientation sensor is then jointly processed. In particular, a loosely coupled approach is used, in which the UWB range measurements are preprocessed to obtain an initial position estimate. Subsequently, the information is combined with the speed and absolute orientation measurements, to provide a refined position estimate. The distinctive feature of the new method we propose in this paper is that it does not require any strong assumption on the dynamical model of the movement of the mobile unit. Therefore it is of a general nature and can be applied to any motion scenario.

The remainder of the paper is organized as follows. In Section 2 the problem of localization by sensor fusion is formulated. Thereafter, in Sections 3 and 4, the proposed technique is derived by solving this problem. To evaluate the proposed technique, simulation results are presented in Section 5. Finally, Section 6 provides conclusions.

2. PROBLEM FORMULATION

Consider a system in which a mobile UWB device (here denoted as master) needs to estimate its position. To do so, the master measures its distance with respect to n devices (here denoted as slaves), with $n \geq 4$. We do not assume any a-priori information on the mobility of the master, which could move by following a linear as well as a random trajectory. An example of such a system is the in-house developed experimental platform that has been described and characterized in De Angelis et al. (2009b,a). Then, we assume that the master is equipped with sensors that measure its speed and absolute orientation, such as those modelled in Mourikis and Roumeliotis (2006). The diagram in Fig. 1 shows a representation of the system model. The goal is to combine optimally, in a sense that will be clear later, information from the ranging measurements and from the on-board sensors to obtain an accurate estimate of the position of the master. The architecture of the sensor fusion system is shown in Fig. 2. In the following, we characterize this picture in detail.

An estimate of the master's position $\mathbf{z}(k+1) = [x(k+1) \quad y(k+1)]^T \in \mathbb{R}^2$ at time $k+1$ is denoted by

$$\hat{\mathbf{z}}\left(k+1 \mid k+1\right) = [\hat{x}\left(k+1 \mid k+1\right) \quad \hat{y}\left(k+1 \mid k+1\right)]^T ,$$

and is derived as follows:

$$\hat{x}\left(k+1 \mid k+1\right) = (1-\beta_{x,k})\hat{x}_r\left(k+1\right) + \beta_{x,k}\hat{x}_v\left(k+1\right) , \quad (1)$$

$$\hat{y}\left(k+1 \mid k+1\right) = (1-\beta_{y,k})\hat{y}_r\left(k+1\right) + \beta_{y,k}\hat{y}_v\left(k+1\right) , \quad (2)$$

where $\hat{\mathbf{z}}_r(k+1) = [\hat{x}_r(k+1) \quad \hat{y}_r(k+1)]^T$ is the position estimate based only on the ranging measurements, and $\hat{\mathbf{z}}_v(k+1) = [\hat{x}_v(k+1) \quad \hat{y}_v(k+1)]^T$ is the position estimate based on the dead reckoning block, which gives the position estimate at the previous time step and the on-board speed and orientation sensors. The terms $\beta_{x,k}$ and $\beta_{y,k}$ are the sensor-fusion design parameters that need to be optimally chosen, as we show later.

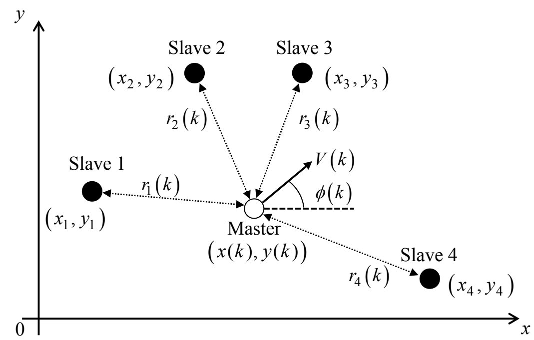

Fig. 1. Model of the localization system. The slave nodes are placed at fixed and known positions. At time k, the unknown-position master node measures its distance r_i with respect to each slave by measuring the round-trip-time of UWB pulses. Speed V and orientation ϕ of the master are measured using on-board sensors.

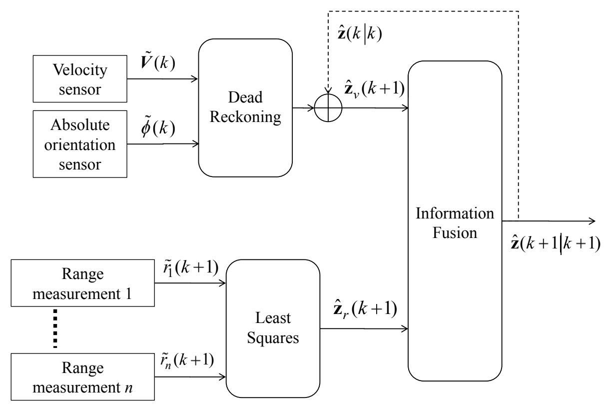

Fig. 2. Sensor fusion system architecture used at a master node for estimating its position at time $k+1$ given available information. The dead reckoning block gives speed and absolute orientation information, whereas the least square block gives ranging information as computed with respect to slave nodes. The information provided by these two blocks is then fused to provide an accurate estimation $\hat{\mathbf{z}}(k+1|k+1)$.

The dead reckoning block provides us with the estimates having the following expressions:

$$\hat{x}_v(k+1) = \hat{x}\left(k \mid k\right) + \tilde{V}(k)T\cos(\tilde{\phi}(k)) \quad (3)$$

$$\hat{y}_v(k+1) = \hat{y}\left(k \mid k\right) + \tilde{V}(k)T\sin(\tilde{\phi}(k)) , \quad (4)$$

where $\tilde{V}(k)$ is the measurement of the speed, $\tilde{\phi}(k)$ the measurement of the orientation and T is the sampling time interval. These measurements are expressed as

$$\tilde{V}(k) = V\left(k\right) + w_V\left(k\right)$$

$$\tilde{\phi}(k) = \phi\left(k\right) + w_\phi\left(k\right)$$

where the noise terms w_V and w_ϕ are zero-mean Gaussian random variables with known variances σ_V^2 and σ_ϕ^2, respectively.

The estimates provided by the dead reckoning block are biased. This is due to that the orientation measurement appears as the argument of a cosinus. It follows that the estimators of Eqs. (3) and (4) are biased and so are the estimates (1) and (2).

It is possible to show that the estimation problem is separable on the x and y axes, in the sense that the estimates on the y axis does not affect the x axis, and vice versa. Therefore in the following we will provide derivations only for the x component, because the derivations for the y component are similar to those for the x component.

We can model our estimator (1) as

$$\hat{x}\left(k+1 \mid k+1\right) \triangleq x(k+1) + w_x(k+1)\,, \tag{5}$$

where $w_x(k+1)$ is the error in the position estimate. This error has a non zero average, namely the estimator is biased, since the dead reckoning block gives a biased estimate. From (5) it follows that the error is

$$w_x(k+1) = \hat{x}\left(k+1 \mid k+1\right) - x(k+1)\,, \tag{6}$$

We denote the bias by $\mathbb{E}\left\{w_x(k+1)\right\}$, where $\mathbb{E}$ denotes statistical expectation.

The estimator proposed in this paper is based on the minimization of the correlation of the estimation error by choosing $\beta_{x,k}$ at each time instant, namely we consider the following problem:

$$\min_{\beta_{x,k}} \quad \mathbb{E}\left\{w_x^2(k+1)\right\} \tag{7}$$

$$\text{s.t.} \quad \beta_{x,k} \in \mathscr{B}\,, \tag{8}$$

where the cost function is minimized on the set $\mathscr{B}$, of which we give a precise definition later on. The minimization of this term is motivated by that we would like to reduce as much as possible both the average of the estimation error, namely the bias, and the variance of the estimation error. A sum of the squared mean and of the variance of the error is given by the cost function (7) we are minimizing. The constraint is motivated by that the estimator is biased and $\beta_{x,k}$ must ensure that the bias is stable and does not accumulate as time progresses, as we show by Lemma 3.4 in Subsection 3.3. The challenge for such an optimization is the analytical characterization of the cost function (7), which we study in the following section. Then, in Section 4 we solve the optimization problem.

3. CHARACTERIZATION OF THE ESTIMATION ERROR

In this section, we characterize the first and second order moments of the estimation error. From (6), after simple algebra, we derive the following expression for the position error at time instant k+1:

$$\begin{aligned} w_x(k+1) &= \left(1-\beta_{x,k}\right) w_r(k+1) + \beta_{x,k} w_x(k) \\ &+ \beta_{x,k} T\tilde{V}(k)\cos(\tilde{\phi}(k)) - \beta_{x,k} TV(k)\cos(\phi(k))\,. \end{aligned} \tag{9}$$

This error depends on the error affecting the ranging measurements, $w_r(k+1)$, and the error affecting the dead reckoning measurements. In the following subsections, we characterize the average and correlation of these errors.

3.1 Ranging estimate

Here we consider the problem of finding the first and second order moments of $\hat{x}_r(k+1)$. Recall that this is the estimate of the master's position, obtained by processing only the range measurements with respect to the slaves. We have that

$$\hat{x}_r(k+1) = x(k+1) + w_r(k+1),$$

where $w_r(k+1)$ is the error in the ranging estimate at time instant $k+1$. In the following, for the sake of generality, we focus on time instant k, and we recall some standard results on the expressions of this ranging estimate. We need these well known results to derive the expressions of the moments.

Since the estimate of the master's position is obtained by processing the range measurements with respect to n slaves, with $n \geq 4$, we have an over-determined system of quadratic constraints given by the Euclidean distances between each slave and the master:

$$\begin{array}{l} \left(x_1 - x\right)^2 + \left(y_1 - y\right)^2 = d_1^2 \\ \vdots \\ \left(x_n - x\right)^2 + \left(y_n - y\right)^2 = d_n^2 \end{array}, \tag{10}$$

where x_i and y_i, $i = 1, \ldots, n$, are known position of the slave devices, and x, y is the position of the master device, which is estimated by the ranging block. We rewrite the system of equations (10) as a system of linear equations, as shown in, e.g., Karl and Willig (2005), and we obtain an estimate of the position using the linear least squares approach. In particular, we define the matrix

$$\mathbf{H} = \begin{bmatrix} 2\left(x_1 - x_n\right) & 2\left(y_1 - y_n\right) \\ \vdots & \vdots \\ 2\left(x_{n-1} - x_n\right) & 2\left(y_{n-1} - y_n\right) \end{bmatrix}.$$

From this matrix, we can extract the estimates of the x and y component, which we collect in the vector $\hat{\mathbf{z}}_r(k) = [\hat{x}_r(k) \quad \hat{y}_r(k)]^T \in \mathbb{R}^2$. Notice that in the following derivations, we use only the first element of this vector, which is associated to the x component of the estimate and which we denote by $\hat{x}_r(k)$.

The ranging block is based on the least squares estimator applied to (10), which gives

$$\hat{\mathbf{z}}_r(k) = \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{r}'(k)\,, \tag{11}$$

where

$$\mathbf{r}'(k) = \tilde{r}_n^2(k) \cdot \mathbf{1} - \begin{bmatrix} \tilde{r}_1^2(k) \\ \vdots \\ \tilde{r}_{n-1}^2(k) \end{bmatrix} + \mathbf{a}\,, \tag{12}$$

where $\mathbf{a}$ is the constant vector given by the known coordinates of the slaves:

$$\mathbf{a} = \begin{bmatrix} x_1^2 - x_n^2 + y_1^2 - y_n^2 \\ \vdots \\ x_{n-1}^2 - x_n^2 + y_{n-1}^2 - y_n^2 \end{bmatrix},$$

and $\tilde{r}_i$ is the measured value of the distance between the master and the i-th slave affected by additive zero-mean Gaussian noise w_{r_i}:

$$\tilde{r}_i = r_i + w_{r_i}\,. \tag{13}$$

The variance of w_{r_i} depends on the range, according to the following exponential model that has been derived from real-world UWB measurement data in De Angelis et al. (2009b) and Nilsson et al. (2009):

$$\sigma^2_{w_{r_i}} = \sigma_0^2 \exp(\kappa_\sigma r_i)\,.$$

Lemma 3.1. The first component of the following vector gives the expectation of $w_r(k)$, namely $\mathbb{E}\left\{w_r(k)\right\}$:

$$\mathbb{E}\left\{\mathbf{w}_r(k)\right\}=(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\left(\mathbf{1}\sigma^2_{w_{r_n}}-\left[\sigma^2_{w_{r_1}},\dots,\sigma^2_{w_{r_{n-1}}}\right]^T\right).$$

Proof. By taking the mean of Eq. (11), we obtain:

$$\begin{aligned}\mathbb{E}\left\{\hat{\mathbf{z}}_r(k)\right\} &= \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\,\mathbb{E}\left\{\mathbf{r}'(k)\right\}\\ &= \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\Big(\mathbf{1}\,\mathbb{E}\left\{\tilde{r}_n^2(k)\right\}-\mathbb{E}\left[\tilde{r}_1^2(k),\dots,\tilde{r}^2_{n-1}(k)\right]^T+\mathbf{a}\Big)\\ &= \mathbf{z}(k)+\left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\left(\mathbf{1}\sigma^2_{w_{r_n}}-\left[\sigma^2_{w_{r_1}},\dots,\sigma^2_{w_{r_{n-1}}}\right]^T\right)\\ &= \mathbf{z}(k)+\mathbb{E}\left\{\mathbf{w}_r(k)\right\}\,,\end{aligned}$$

whereby the lemma follows. □

In the following, we derive $\mathbb{E}\left\{w_r^2(k)\right\}$. For the sake of simple notation, for the rest of this subsection we omit the k index from all expressions. We start by taking the square of Eq. (13):

$$\tilde{r}_i^2 = r_i^2 + w^2_{r_i} + 2r_i w_{r_i}\,, \tag{14}$$

then, using this expression in (12) and substituting in (11), we have the following expression for the ranging-based estimate:

$$\hat{\mathbf{z}}_r = \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\left\{\begin{bmatrix} r_n^2 - r_1^2\\ \vdots \\ r_n^2 - r^2_{n-1}\end{bmatrix} + \mathbf{B}_k + \mathbf{a}\right\}, \tag{15}$$

where

$$\mathbf{B}_k = \begin{bmatrix} w^2_{r_n} + 2r_n w_{r_n} - w^2_{r_1} - 2r_1 w_{r_1}\\ \vdots \\ w^2_{r_n} + 2r_n w_{r_n} - w^2_{r_{n-1}} - 2r_{n-1}w_{r_{n-1}}\end{bmatrix}. \tag{16}$$

Therefore the error can be expressed as

$$\mathbf{w}_r = \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{B}_k\,. \tag{17}$$

The second-order moment of the error is given by

$$\mathbb{E}\left\{\mathbf{w}_r^2\right\} = \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{C}\mathbf{H}\left(\mathbf{H}^T\mathbf{H}\right)^{-1}, \tag{18}$$

where $\mathbf{C} = \mathbb{E}\left\{\mathbf{B}_k\mathbf{B}_k^T\right\}$. In this matrix, the diagonal elements are

$$\mathbf{C}_{ll} = 3\sigma^4_{w_{r_n}} + 4r_n^2\sigma^2_{w_{r_n}} + 3\sigma^4_{w_{r_l}} + 4r_l^2\sigma^2_{w_{r_l}} - 2\sigma^2_{w_{r_n}}\sigma^2_{w_{r_l}}\,, \tag{19}$$

whereas the off-diagonal elements are

$$\mathbf{C}_{lj} = 3\sigma^4_{w_{r_n}} - \sigma^2_{w_{r_n}}\sigma^2_{w_{r_j}} + 4r_n^2\sigma^2_{w_{r_n}} - \sigma^2_{w_{r_l}}\sigma^2_{w_{r_n}} + \sigma^2_{w_{r_l}}\sigma^2_{w_{r_j}}\,. \tag{20}$$

3.2 Dead Reckoning estimate

In this subsection we give the first and second order moments of the dead reckoning contribution. The following simple lemma holds:

Lemma 3.2. Let $\tilde{\phi}(k)$ be Gaussian, and $\tilde{V}(k)$ and $\tilde{\phi}(k)$ statistically independent, then

$$\mathbb{E}\left\{\tilde{V}(k)\cos(\tilde{\phi}(k))\right\} = V(k)\cos(\phi(k))e^{-\frac{\sigma^2_\phi}{2}}\,. \tag{21}$$

Proof. Since $\tilde{V}(k)$ and $\tilde{\phi}(k)$ are independent, we have that

$$\begin{aligned}\mathbb{E}\left\{\tilde{V}(k)\cos(\tilde{\phi}(k))\right\} &= \mathbb{E}\left\{\tilde{V}(k)\right\}\mathbb{E}\left\{\cos(\tilde{\phi}(k))\right\}\\ &= V(k)\,\mathbb{E}\left\{\cos(\phi(k)+w_\phi(k))\right\}\\ &= V(k)\,\mathbb{E}\left\{\cos(\phi(k))\cos(w_\phi(k)) - \sin(\phi(k))\sin(w_\phi(k))\right\}\\ &= V(k)\left(\cos(\phi(k))\,\mathbb{E}\left\{\cos(w_\phi(k))\right\}\right.\\ &\quad\left. - \sin(\phi(k))\,\mathbb{E}\left\{\sin(w_\phi(k))\right\}\right)\,. \end{aligned} \tag{22}$$

By using the cosine series, we can write that

$$\begin{aligned}\mathbb{E}\left\{\cos(w_\phi(k))\right\} &=\\ &= \mathbb{E}\left\{1 - \frac{w^2_\phi(k)}{2} + \frac{w^4_\phi(k)}{4!} + \dots + (-1)^n\frac{w^{2n}_\phi(k)}{(2n)!}\right\}\\ &= 1 - \frac{\sigma^2_\phi}{2} + \frac{\sigma^4_\phi}{2^2\cdot 2!} + \dots + (-1)^n\frac{\sigma^{2n}_\phi}{2^n\cdot n!} = e^{-\frac{\sigma^2_\phi}{2}}\,,\end{aligned} \tag{23}$$

where we have used that $\tilde{\phi}(k)$ is Gaussian.

Similarly, by using the sine series we have that

$$\mathbb{E}\left\{\sin(w_\phi(k))\right\} = \mathbb{E}\left\{w_\phi + \dots + (-1)^n\frac{w^{2n+1}_\phi(k)}{(2n+1)!}\right\} = 0\,. \tag{24}$$

The lemma follows by substituting Eqs. (23) and (24) in Eq. (22). □

Furthermore, the following lemma holds:

Lemma 3.3. Let $\tilde{\phi}(k)$ be Gaussian, and $\tilde{V}(k)$ and $\tilde{\phi}(k)$ statistically independent, then

$$\mathbb{E}\left\{\tilde{V}^2(k)\cos^2(\tilde{\phi}(k))\right\} = \sigma_V^2\left(\frac{1}{2}+\frac{1}{2}\cos\left(2\phi(k)\right)e^{-2\sigma^2_\phi}\right).$$

Proof. Since $\tilde{V}(k)$ and $\tilde{\phi}(k)$ are independent, we have that

$$\mathbb{E}\left\{\tilde{V}^2(k)\cos^2(\tilde{\phi}(k))\right\} = \mathbb{E}\left\{\tilde{V}^2(k)\right\}\mathbb{E}\left\{\cos^2(\tilde{\phi}(k))\right\}. \tag{25}$$

For the second factor of the right-end side of this expression we have that

$$\begin{aligned}\mathbb{E}\left\{\cos^2(\tilde{\phi}(k))\right\} &= \mathbb{E}\left\{\frac{1}{2}+\frac{1}{2}\cos(2\tilde{\phi}(k))\right\}\\ &= \frac{1}{2}+\frac{1}{2}\mathbb{E}\left\{\cos\left(2\left(\phi(k)+w_\phi(k)\right)\right)\right\}\\ &= \frac{1}{2}+\frac{1}{2}\mathbb{E}\left\{\cos\left(2\phi(k)\right)\cos\left(2w_\phi(k)\right)\right.\\ &\quad\left. - \sin\left(2\phi(k)\right)\sin\left(2w_\phi(k)\right)\right\}\\ &= \frac{1}{2}+\frac{1}{2}\cos\left(2\phi(k)\right)\,\mathbb{E}\left\{\cos\left(2w_\phi(k)\right)\right\}\,.\end{aligned} \tag{26}$$

Using the cosine series, we can write:

$$\begin{aligned}\mathbb{E}\left\{\cos\left(2w_\phi(k)\right)\right\} &=\\ &= \mathbb{E}\left\{1 - 2^2\frac{w^2_\phi(k)}{2!} + 2^4\frac{3w^4_\phi(k)}{4!} + \dots + (-1)^n\frac{(2w_\phi)^{2n}}{(2n)!}\right\}\\ &= 1 - 2\frac{\sigma^2_\phi}{1!} + 2^2\frac{\sigma^4_\phi}{2!} + \dots + (-1)^n\frac{2^n\sigma^{2n}_\phi}{n!} = e^{-2\sigma^2_\phi}\,.\end{aligned} \tag{27}$$

By substituting (27) in (26), we obtain:

$$\mathbb{E}\left\{\cos^2(\tilde{\phi}(k))\right\} = \frac{1}{2} + \frac{1}{2}\cos\left(2\phi(k)\right) e^{-2\sigma_\phi^2}\,. \quad (28)$$

The lemma follows by substituting (28) in (25). □

In the following subsection, we use these results to characterize the estimation bias.

3.3 Estimation Bias

We are now in the position to derive the first and second order moment of the estimation error.

The following lemma holds:

Lemma 3.4. The bias in the position estimation is given by

$$\begin{aligned}\mathbb{E}\left\{w_x(k+1)\right\} &= \left(1-\beta_{x,k}\right)\mathbb{E}\left\{w_r(k+1)\right\} \\ &+\beta_{x,k}\,\mathbb{E}\left\{w_x(k)\right\}+\beta_{x,k}TV(k)\cos(\phi(k))\left(e^{-\frac{\sigma_\phi^2}{2}}-1\right) \quad (29)\end{aligned}$$

Proof. The lemma follows, after simple derivations, by taking the expectation of (1). □

Note that Eq. (29) can be sometimes positive and some other times negative, because $\cos\left(\phi(k)\right)$ can be both positive and negative and $e^{-\frac{\sigma_\phi^2}{2}}-1$ is negative. This is useful because it means that the average estimation error will not keep accumulating.

From the preceding lemma, we see that the average of the estimation error at time $k+1$ depends on the bias of time k. To avoid an increasing of the bias, we need to impose a condition on $\beta_{x,k}$ so that the absolute value of the average of the estimation error must be contractive. This can be easily achieved when $|\beta_{x,k}| \in [0,1)$, and thus $\mathscr{B} = (-1,1)$.

We can now give one of the core contributions of the paper:

Proposition 1. Consider Eq. (18). Then, the second order moment of the estimation error is

$$\mathbb{E}\left\{w_x^2(k+1)\right\} = \beta_{x,k}^2 a_k + 2\beta_{x,k} b_k + \mathbb{E}\left\{w_r^2(k+1)\right\}$$

where

$$\begin{aligned}a_k =&\ \mathbb{E}\left\{w_r^2(k+1)\right\} + \mathbb{E}\left\{w_x^2(k)\right\} \\ &+T^2\left(\mathbb{E}\left\{\tilde{V}^2(k)\cos^2(\tilde{\phi}(k))\right\}+V^2(k)\cos^2(\phi(k))\left(1-2e^{-\frac{\sigma_\phi^2}{2}}\right)\right) \\ &-2\left(\mathbb{E}\left\{w_x(k)\right\}\mathbb{E}\left\{w_r(k+1)\right\}-TV(k)\cos(\phi(k))\right. \\ &\left.\cdot\left(e^{-\frac{\sigma_\phi^2}{2}}-1\right)\left(\mathbb{E}\left\{w_x(k)\right\}-\mathbb{E}\left\{w_r(k+1)\right\}\right)\right)\,, \quad (30)\end{aligned}$$

and

$$\begin{aligned}b_k =& -\mathbb{E}\left\{w_r^2(k+1)\right\} + \mathbb{E}\left\{w_x(k)\right\}\mathbb{E}\left\{w_r(k+1)\right\} \\ &+\mathbb{E}\left\{w_r(k+1)\right\}TV(k)\cos(\phi(k))\left(e^{-\frac{\sigma_\phi^2}{2}}-1\right)\,. \quad (31)\end{aligned}$$

Proof. the proposition follows by substituting the expression of the position estimation error given by Eq. (9) in Eq. (7), and using the results of Lemmata 3.1, 3.2, 3.3 and 3.4. □

4. POSITIONING ALGORITHM

In the previous section, we characterized the first and second order moments of the estimation error. We are now in the position of solving the optimization problem (7).

Proposition 2. Consider optimization problem (7). Then, the optimal solution is

$$\beta_{x,k}^* = \max\left(-1, \min\left(-\frac{b_k}{a_k}, 1\right)\right)\,,$$

where a_k and b_k are given by Eq. (30) and Eq. (31), respectively.

Proof. First, note that the cost function of problem (7) is convex. This follows by considering that this function is always positive for any choice of $\beta_{x,k} \in \mathbb{R}$, which is possible only if $a_k > 0$. Since the cost function is a parabola, it follows that is convex. Therefore, the optimal solution is given by computing the derivative of the cost function, and observing that the optimal solution must lie in the interval $\mathscr{B} = [-1,1]$. □

Note that in the expressions for b_k and a_k the true speed $V(k)$ and orientation $\phi(k)$ terms are present. Since these are not available, we use approximations for these terms:

$$\begin{aligned}V_{\text{approx}}(k) \simeq& \frac{1}{T}\left(\left(\hat{x}\left(k \mid k\right) - \hat{x}\left(k-1 \mid k-1\right)\right)^2\right. \\ &\left.+\left(\hat{y}\left(k \mid k\right) - \hat{y}\left(k-1 \mid k-1\right)\right)^2\right)^{\frac{1}{2}}\,,\end{aligned}$$

$$\phi_{\text{approx}}(k) \simeq \arctan\left(\frac{\hat{y}\left(k \mid k\right) - \hat{y}\left(k-1 \mid k-1\right)}{\hat{x}\left(k \mid k\right) - \hat{x}\left(k-1 \mid k-1\right)}\right)\,.$$

Also, note that the second-order moment of the ranging error, given by (18)-(20), depends on the true ranges r_i. Therefore, for the calculation of the $\mathbb{E}\left\{w_r^2(k+1)\right\}$ term in b_k and a_k, we use the following approximations for the ranges:

$$r_{\text{approx}_i(k+1)} = \sqrt{(x_i-\hat{x}(k))^2+(y_i-\hat{y}(k))^2} \quad \forall i\,.$$

In the following section, we show that these approximations give good performance.

5. SIMULATION RESULTS

We carried out numerical simulations to validate the performance of the proposed sensor fusion technique. The parameter values used in the simulations are the following:

- Sampling time interval $T = 0.1$ s.
- Speed measurement noise standard deviation $\sigma_v = 0.05$ m/s, to model the worst-case performance of an odometry sensor, see e.g. Mourikis and Roumeliotis (2006).
- Orientation measurement noise standard deviation $\sigma_\phi = \pi/8$ rad, to model the worst-case performance of a magnetometer subject to disturbances due to the structure of the environment, see, e.g., Georgiev and Allen (2004), Abbott and Powell (1999).
- Ranging noise model parameter $\sigma_0 = 0.25$, see De Angelis et al. (2009b) and Nilsson et al. (2009).
- Ranging noise model parameter $\kappa_\sigma = 0.25$, see De Angelis et al. (2009b) and Nilsson et al. (2009).

The absolute value of β^*, provided by the algorithm, has been clipped to 0.99 to ensure that the average of the estimation error is contractive, as discussed in Section 3.3.

We performed the simulations in two scenarios:

A. Linear trajectory with constant speed of 0.1 m/s, see Fig. 3.
B. Piece-wise-linear trajectory with constant speed of 0.1 m/s in each linear segment, see Fig. 6.

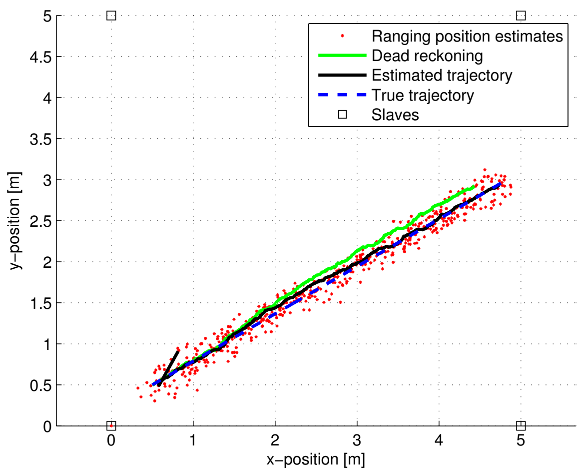

Fig. 3. Simulations in the linear trajectory case, scenario A. The ranging position estimates (points) exhibit a relatively high variance. The dead reckoning (green line) has a lower variance, but its estimation error accumulates over time since is biased. Our sensor fusion method (solid black line) allows to exploit the complementary properties of the two information sources and to accurately estimate the true trajectory.

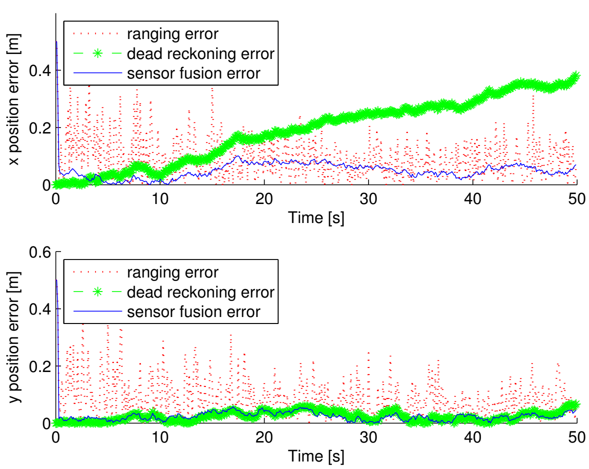

Fig. 4. Absolute errors in the linear trajectory case, scenario A, for the x and y components as obtained by our sensor fusion method.

The results of Fig. 3 and 4 show that the proposed algorithm can estimate the position of a master node moving with constant speed along a straight line with errors of less than 10 cm. Furthermore, the fusion method overcomes the limitation of dead reckoning, that is the

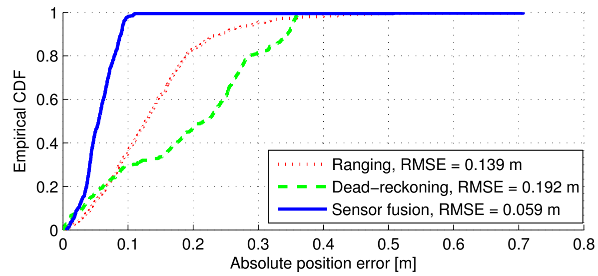

Fig. 5. Empirical CDF of the absolute positioning error in 2D for scenario A. Using the proposed method (blue line), about 90% of the position estimate has an error smaller than 9 cm.

slowly accumulating bias, while also reducing the relatively higher variance of the ranging measurements. The result is a smooth and accurate estimate of the trajectory of our proposed method. This behavior is evaluated in the empirical cumulative distribution function (CDF) plot shown in Fig. 5. Finally, Figs. 6, 7 and 8 show the results for simulation scenario B.

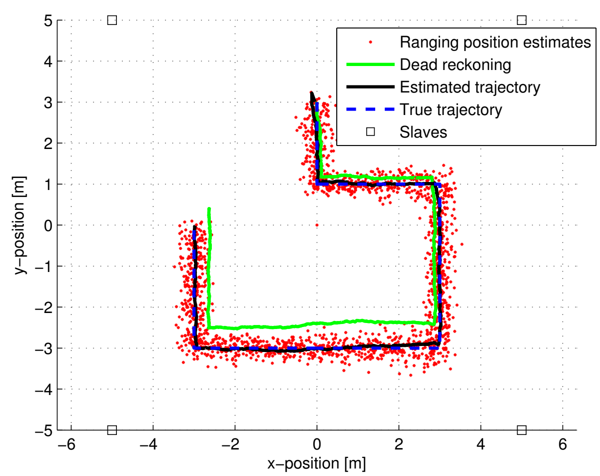

Fig. 6. Simulations in the piece-wise-linear trajectory case, scenario B. Similarly to the linear case, the unaided dead reckoning estimate diverges, while the proposed technique closely tracks the true trajectory.

5.1 Comparison with other approaches

We compared the performance of our technique to a number of EKF-based solutions. For these solutions we need to assume mobility models. We used the following standard mobility models for the system, which are taken from Gustafsson (2010):

I Constant velocity, where the master node is assumed to follow a linear trajectory.
II Random walk acceleration, where the master node is assumed to follow a random non linear trajectory, with a Gaussian process noise in the acceleration of zero average and standard deviation of 1 m/s^2.

A summary of this evaluation is presented in Table 1. In scenario A, our method has worse performance than the EKF. Conversely, in scenario B, the proposed method can

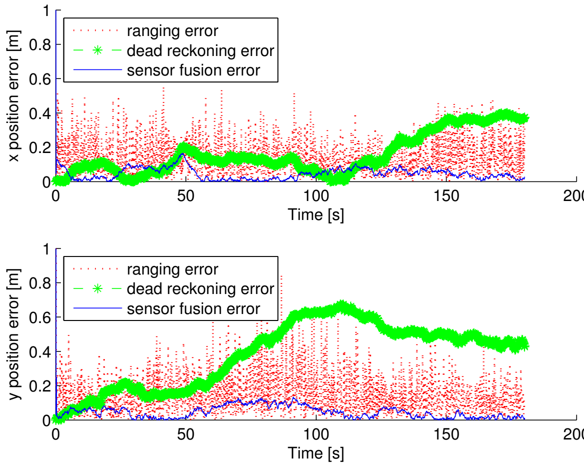

Fig. 7. Absolute errors in the piece-wise-linear trajectory case, scenario B, for the x and y components.

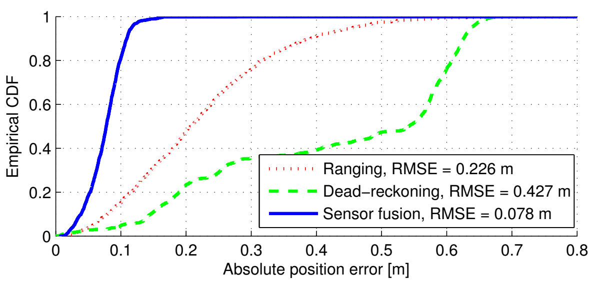

Fig. 8. Empirical CDF of the absolute positioning error in 2D for scenario B. About 90% of the position estimate has an error smaller than 12 cm.

Table 1. RMSE [m] of the estimators.

Scenario	*Proposed method*	*EKF model I*	*EKF model II*
A	0.059	0.037	0.029
B	0.078	3.685	0.139

achieve better performance than the EKF. In this scenario, the localization problem is highly non-linear, therefore the models in the EKF do not provide good approximation for the system.

6. CONCLUSION

In this paper, we presented a new method to sensor fusion for localization. We showed that by optimally combining heterogeneous measurements of the position of a mobile node, it is possible to achieve better accuracy than methods based on extended Kalman filtering when the trajectories are not linear. Future studies include the study of the fundamental performance limitations of our proposed scheme.

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