Power and Rate Control Outage Based in CDMA Wireless Networks under MAI and Heterogeneous Traffic Sources

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Abstract—We characterize the maximum throughput achievable for the up-link of a power-controlled WCDMA wireless system with variable spreading factor. Our system model includes multi access interference caused by users with heterogeneous data sources, and quality of service expressed in terms of outage probability. Inner loop and outer loop power control mechanisms are also explicitly taken into account. We express the system throughput as the sum of the users transmission rates, and propose a mixed integer optimization program where the objective function is the sum of the rates under outage probability constraints. Then, we solve the optimization problem proposing an efficient optimal approach based on two steps: firstly, a modified problem provides feasible solutions, and then the optimal solution is obtained with branch and bound criteria. Numerical results confirm the validity of our approach, and show how the throughput depends on the power control fluctuation, activity of the sources, and quality of service.

Index Terms: CDMA, Rate and Power control, Outage, Branch and Bound Criteria, Combinatorial Optimization.

I. INTRODUCTION

In many wireless networks, code-division multiple-access (CDMA) is the access interface developed to enable a multiplicity of services, as voice, video, and wireless access to the Internet. Multi-media services require, among others, suitable throughput performance. For instance, the maximum transmission rate allowed at the physical layer provides a bound to the performance of the Transmission Control Protocol (TCP) when it is implemented on wireless systems.

Rate maximization in CDMA systems has been thoroughly investigated during last years (see [1]-[2]). Here, we limit the discussion to some contributions related to the up-link with outage constraints. The problem of radio resource management under outage constraints dates back at least to the work presented in [3] (see also [4]). In [5] the authors have proposed a maximization of the system capacity, intended as number of users, under outage requirements. In [6] and [7], a novel Geometric Programming approach to solve a relaxed version of the problem of minimization the users powers has been proposed. In this context, an interesting line of research can be found in [8], where the problem of joint power minimization and multi-user detection has been explored. In that paper, the authors have presented a general method to solve iteratively the power allocation problem, under mixed slow and fast fading. Furthermore, by relaxing the outage constraints with an upper bound provided by the Jensen’s inequality on the statistical average, the outage specifications are mapped onto the average SINR.

In this paper, we propose a theoretical framework to maximize the up-link throughput that can be achieved in WCDMA wireless system by power allocation. We express Quality of Service (QoS) requirements in terms of the outage probability for any user, that means we refer to delay-limited scenarios [9]. The approach we propose is original, since the system model includes variable spreading factors under a detailed model of multi-access interference with heterogeneous data sources. We investigate a mixed integer optimization problem which is different from the existing relevant contributions we have mentioned above. In particular, the constraints on the outage are expressed by resorting to an accurate log-normal approximation (see [5] and [10]), which allows to solve the program even for very low outage probabilities (less then 1%).

The interesting works [6], [7], and [8] are instead focused on mapping the outage requirement onto the average SINR, and they do not include a detailed model of the multi access interference (neglecting the source behavior). We propose an optimal method to solve the mixed integer optimization problem, which is based on branch and bound criteria. For the best of our knowledge, this is an unexplored study when considering outage specifications. Furthermore, despite we apply our method to a wireless system with inner loop and outer loop power control, we believe that the approach is general and can be used also for scenario with shadow fading and mixed fading scenarios.

The remainder of the paper is organized as follows. In Section II the system model is presented, and in Section III the optimization problem for the rate maximization is formulated. In Section IV, the constraints on the outage probability are addressed. In section V we propose a two steps optimal approach for the problem solution. Numerical results are reported and commented in Section VI. Finally, Section VII concludes the paper.

II. SYSTEM DESCRIPTION

We consider a scenario with $K$ mobile users connected to a MS, where each user is associated to a traffic source type (voice, video, data, ...), and uses the same chip time $T_c$. In order to meet signal intensity (or SINR) requirements at
the BS receiver input for each user signal in the presence of both slow and fast channel variations, open-loop and closed-loop power control algorithms are adopted. The outer loop power control is responsible for setting the target Signal-to-(Interference + Noise) Ratio (SINR) at the receiver input. The inner loop power control aims at compensating channel variations induced by fast fading phenomenon and, hence, tries to adjust the powers of each MS in order to meet a target level for the SINR. Operations of such an algorithm are envisaged according to [5] and [11].

The behavior of the above mentioned power control mechanisms is included in our model of the physical layer for the up-link of a single-cell asynchronous BPSK DS/CDMA system. Following the same approach outlined in [5], it can be shown that the SINR at the output of a coherent correlation receiver matched to signal related to the generic user i, has the following expression:

\[
SINR_i(\xi(t), \nu(t)) = \sqrt{\frac{P_{0i} \xi_i t_0}{\sum_{j \neq i} \frac{N_0 T_{0i}}{4n_i} + \Psi_i(\xi(t), \nu(t))}}^{1/2},
\]

where the numerator includes the energy associated to the useful signal, and the denominator includes the multi-user interference, which is expressed as follows:

\[
\Psi_i(\xi(t), \nu(t)) = \sum_{j=1}^{K} \frac{P_{0j} T_{0j}^2}{6G_{0j} n_i} e^{\xi_i(t)\nu_j(t)}. \tag{2}
\]

In (1), we have modelled the rate of each user as \( R_i = R_{0i} n_i = n_i / T_{0i} \), where \( R_{0i} = 1 / T_{0i} \) is the basic rate of the user i, and \( n_i \) is an integer number denoting the rate to assign. Note that such rates are power of 2 due to the structure of the spreading codes [12]. The spreading factors are expressed as \( G_i = \frac{2^m}{n_i} \), where \( G_{0i} \) is the spreading factor corresponding to the basic user’s rate \( R_{0i} \), i.e. \( G_{0i} = \frac{T_{0i}}{T_{0i}} \). The power settings of each user (the power of each user as seen at the receiver) is expressed as

\[
P_i = P_{0i} n_i, \quad i = 0, ..., K,
\]

where \( P_{0i} \) is the power at the basic rate \( R_{0i} \) (i.e., when \( n_i = 1 \)). Such a power model is obviously motivated by the fact that users requiring larger rates need larger transmit powers. Such an assumption has been included also in [13], and has the further advantage of keeping the bit energy to noise ratio constant with respect to the variations of \( n_i \). The system bandwidth is indicated with \( W \), and \( N_0/2 \) denotes the two-sided power spectral density of thermal noise. The meaning of other variables and parameters in the (1) is as follows:

- \( \xi(t) = [\xi_1(t), ..., \xi_K(t)]^T \), where \( \xi_i(t) \) denotes the residual (inner loop) power control error for the user signal i, and is represented (in log units) by a zero mean Gaussian process with standard deviation \( \sigma_{\xi_i} \) (in Neper units);
- \( \nu(t) = [\nu_1(t), ..., \nu_K(t)]^T \), where \( \nu_i(t) \) is a binary random process indicating the activity status (On/Off) of the source at time t; its first order probability mass function is such that \( P[\nu_i = 1] = \alpha_i \) and \( P[\nu_i = 0] = 1 - \alpha_i \), where \( \alpha_i \) is said to be the activity factor of sources i.

Independence is assumed between any pair of processes of the vectors \( \xi(t) \) and \( \nu(t) \). Finally, note that (2) is a stochastic process with respect to the vectors \( \xi(t) \) and \( \nu(t) \). The power coefficients, for notational convenience, are grouped in the vector \( P = [P_{10}, ..., P_{K0}]^T \), whereas the rate coefficients are grouped in the vector \( n = [n_1, n_2, ..., n_K]^T \).

As a consequence of fluctuations of the instantaneous power of user signal and MAI, the outage probability is introduced as a performance measure and the QoS requirement for the generic user i, and is expressed as

\[
Pr[SINR_i(\xi, \nu) < \gamma_i] \leq \bar{P}_{out}^i,
\]

\( \bar{P}_{out}^i \) denoting the outage requirement.

### III. PROBLEM FORMULATION

We express the problem of rate maximization as

\[
\max_{P} \sum_{i=1}^{K} n_i
\]

s.t. \( Pr[SINR_i(\xi, \nu) < \gamma_i] \leq \bar{P}_{out}^i \)

\[
\sum_{i=1}^{K} P_i \leq P_T
\]

\( 0 < P_{0i} \)

\( 1 \leq n_i \leq G_{0i} \), \( n_i \in \mathbb{N} \), \( i = 1, ..., K \)

In (4), the constraint on the total power \( P_T \) is motivated by the fact that the receiver input of the BS can receive only a maximum amount of power [5]. The minimum constraints on the rates are motivated by the fact that the rates are multiple of the basic ones \( R_{0i} \), while the maximum values are due to the fact that, after the rate variation, the spreading factor \( G_{0i} \) is divided by \( n_i \).

The solution of the program (4) is a difficult task: firstly, the outage constraints are complicated functions of the rates and power; secondly, the rates are powers of 2, and the constraint on \( P_T \) is difficult to manage. We approach these issues in the next sections.

### IV. OUTAGE PROBABILITY CONSTRAINTS

The solution of the problem (4) requires knowledge of the outage probability, which, in turn, requires availability of the SINR statistics. Due to the presence of the MAI term, the SINR statistics are unknown, hence they must be approximated. We adopt the well known extended Wilkinson Moment matching method ([5] and [10]) to approximate the SINR. The complete expression of the MAI (2) can be included into (1), and the SINR is thus expressed as follows:

\[
SINR_i(\xi(t), \nu(t)) = L_i(\xi(t), \nu(t))^{-\frac{1}{2}},
\]

where

\[
L_i(\xi(t), \nu(t)) = \frac{N_0}{2P_{0i}G_{0i}T_c} e^{-\xi_i(t)} + \sum_{j=1}^{K} \frac{P_{0j} n_j}{3G_{0j}P_{0i}} e^{\xi_i(t) - \xi_j(t)} \nu_j(t). \tag{5}
\]

Looking at (5), the SINR is a combination of Log-normal processes weighted by constants and binary on-off processes,
hence we can rely upon the extension of the Wilkinson Moment matching method. Specifically, we make the following approximation:

\[ L_i(\xi(t), \nu(t)) \approx e^{Z_i(t)} \]  \hspace{1cm} (6)

where \( Z_i(t) \) is a Gaussian random process having average \( m_{Z_i} \) and standard deviation \( \sigma_{Z_i} \), which are derived as function of the first and second moments of the SINR:

\[
m_{Z_i} = \ln \left( \frac{M_{m1}^2}{\sqrt{M_{m2}}} \right) \quad \text{and} \quad \sigma_{Z_i}^2 = \ln \left( \frac{M_{m2}}{M_{m1}} \right),
\]

where:

\[
M_{m1} \triangleq E_{\xi(t),\nu(t)} \left\{ L_i(\xi(t), \nu(t)) \right\},
\]

\[
M_{m2} \triangleq E_{\xi(t),\nu(t)} \left\{ L_i^2(\xi(t), \nu(t)) \right\},
\]

where we have denoted with \( E_{\xi(t),\nu(t)} \{ \cdot \} \) the expectation w.r.t. the distribution of \( \xi(t) \) and \( \nu(t) \). Due to lack of space in this paper, we provide the expressions of (7) and (8) in the Appendix A of the technical report [14].

With the Log-normal approximation (6), the SINR in dB is a Gaussian process, hence it is possible the derivation of the outage probability as follows:

\[
Pr[SINR_i(\xi, \nu) < \gamma_i] = Q \left( \frac{-2 \ln \gamma_i - m_{Z_i}}{\sigma_{Z_i}} \right),
\]

where \( Q(x) = \frac{1}{\sqrt{2 \pi}} \int_x^{\infty} e^{-t^2/2} dt \) is the complementary standard Gaussian distribution.

It is straightforward to show that, using (9), the constraints on the outage probability can be rewritten as

\[
\frac{-2 \ln \gamma_i - m_{Z_i}}{\sigma_{Z_i}} \geq q_i,
\]

where \( q_i = Q^{-1}(\tilde{P}_{\text{out}}) \), with \( Q^{-1}(\cdot) \) having denoted the inverse of \( Q(\cdot) \). Now, let us define the Interfering function \( I_i(n_{-i}, P_{-i}) \) as

\[
I_i(n_{-i}, P_{-i}) = \sqrt{G_i(n_{-i}, P_{-i})} \left( -2 \ln \gamma_i - m_{Z_i} \right) e^{-2 \ln \gamma_i - m_{Z_i}} \left( H_i(n_{-i}, P_{-i}) \right)^2,
\]

where

\[
G_i(n_{-i}, P_{-i}) = M_{m1}P_{i0},
\]

\[
H_i(n_{-i}, P_{-i}) = M_{m2}P_{i0}^2,
\]

and

\[
P_{-i} = [P_{i0}, P_{i1}, \ldots, P_{(i-1)0}P_{(i+1)0}, \ldots, K0]^T,
\]

\[
n_{-i} = [n_{11}, n_{12}, \ldots, n_{i-1, i}, n_{i+1, i}, \ldots, n_{K}]^T.
\]

Note that \( H_i(n_{-i}, P_{-i}) \) and \( G_i(n_{-i}, P_{-i}) \) are not function of \( P_{i0} \). This can be easily proved by looking at the expressions of (7) and (8). The definition (11) allows to rearrange the constraints on the outage probability (10) as

\[
\frac{P_{i0}}{I_i(n_{-i}, P_{-i})} \geq \gamma_i^2 \quad \forall i = 1, \ldots, K.
\]

V. Optimal Solution

The function (14) allows to rewrite the optimization problem (4) as

\[
\max P \sum_{i=1}^{K} n_i
\]

\[
s.t. \quad \frac{P_{i0}}{I_i(n_{-i}, P_{-i})} \geq \gamma_i^2 \quad \forall i = 1, \ldots, K \quad (15)
\]

\[
\sum_{i=1}^{K} n_i P_{i0} \leq P_T
\]

\[
P_{i0} > 0 \quad \forall i = 1, \ldots, K
\]

\[
1 \leq n_i \leq G_i, \quad n_i \in \mathbb{N} \quad \forall i = 1, \ldots, K
\]

In order to find a solution of the program (15), let us consider the following definition:

**Definition 1 (Feasible Rates Vector):** A rates vector, \( \mathbf{n} \), is said feasible if and only if there exists a powers vector \( \mathbf{P} \) such that the couple \( (\mathbf{n}, \mathbf{P}) \) verifies all the constraints of problem (15).

A. Modified problem

Let us restrict our attention to the following program, which we define modified problem:

\[
\min P \sum_{i=1}^{K} n_i
\]

\[
s.t. \quad \frac{P_{i0}}{I_i(n_{-i}, P_{-i})} \geq \gamma_i^2 \quad \forall i = 1, \ldots, K \quad (16)
\]

\[
P_{i0} > 0 \quad \forall i = 1, \ldots, K
\]

\[
1 \leq n_i \leq G_i, \quad n_i \in \mathbb{N} \quad \forall i = 1, \ldots, K
\]

The pair of vectors \( \mathbf{P} \) and \( \mathbf{n} \) that solve (16) obviously provides a feasible rate vector if the cost function is less than \( P_T \). Hence, it follows that \( \mathbf{n} \) is feasible. Therefore, (16) takes relevance and we can explore the feasibility of a rate vector \( \mathbf{n} \) just by solving the problem (16).

To study the problem (16), we resort to the well known theory of standard interference function [15]. Specifically, it is possible to prove the following proposition:

**Proposition 1:** for any given rate vector \( \mathbf{n} \) the function defined as \( T(n, P) = [I_1(n_{-1}, P_{-1}), \ldots, I_N(n_{-1}, P_{-1}), \ldots, N(n_{-1}, P_{-1})]^T \) is a standard interference function.

**Proof:** The proof of this proposition follows directly from the Remark 1 and Theorem 1 in the Appendix B of the technical report [14].

Using the interference function theory, we can say that for any given rate vector \( \mathbf{n} \) the problem (16) has a unique solution, \( \mathbf{P} \), that can be found using distributed and asynchronous algorithms. Specifically, it is interesting to see that \( \mathbf{P} \) is the solution of (16) if and only if the \( N \) following equations are verified:

\[
\frac{P_{i0}}{I_i(n_{-i}, P_{-i})} = \gamma_i^2 \quad \forall i = 1, \ldots, K.
\]
Previous equality comes from Theorem 1 in Appendix B of the technical report [14]. The theorem implies that for any given vector \( \mathbf{n} \), then \( \mathbf{P} \) is solution of (16) if and only if it verifies the \( K \) constraints (16) with the equality. In such a case, the solution of the optimization problem (16) is unique and it can be solved using a large family of distributed algorithms (see e.g. [16] and [15]). Moreover, when there exists a solution of (16), in [16] it is shown that such solution can be computed using a low computational cost distributed algorithm using sequences of asynchronous transmitted powers. Specifically, the solution of (16) can be found by the following algorithm:

1) \( t = 0 \), initialize the vector \( \mathbf{n}(0) \) with the basic rates, and the vector \( \mathbf{P}(0) \) with the basic powers;
2) \( t = t + 1 \)
3) \( \forall i = 1 \ldots K \) \( P_{i0}(t) = I_i(n_{-i}(t - 1), P_{-i}(t - 1))\gamma_i^2 \)
4) go to 2;

Since \( \mathbf{n} \) assumes discrete and finite values, the rate vector, \( \mathbf{n}^* \), that solves the problem (15) could be found using an exhaustive search among all feasible rate vectors. Nonetheless, such a technique has large computational costs to be implemented. In the next section we will present some useful rate vector properties, which, employed along with a branch and bound criteria, allow to find the optimal rate vector with reduced computational cost.

B. Branch and bound Criteria

The rates vector \( \mathbf{n} \) belongs to a discrete set, which we denote by \( \mathcal{N} \), hence integer computation techniques are a good choice to solve problem (15).

Recalling the consideration done about problem (16), we can say that given \( \mathbf{n} \), it is feasible if and only if (16) has a solution \( \mathbf{P} \), and a cost function that verifies: \( \mathbf{n}^T \cdot \mathbf{P} \leq P_f \).

To test all possible elements of \( \mathcal{N} \), as well as to verify their feasibility and compare their objective functions, has a prohibitive computational cost increasing exponentially with \( \mathcal{N} \)'s cardinality. Nevertheless, using local knowledge about specific values of \( \mathbf{n} \), the set \( \mathcal{N} \) can be reduced. In the following we propose two criteria for the \( \mathcal{N} \)’s reduction. The first one is given using the well known cutting planes approach [16] whereas the second one is carried out by considerations on the program (15). Specifically, the criteria can be summarized in the following propositions (their proof is easily based on the \( T \)’s properties and is omitted).

Proposition 2: If \( \mathbf{n} \) is feasible, the couple \((\mathbf{n}, \mathbf{P})\) can be the optimal solution only if \( \|\mathbf{n}\|_1 \geq \|\mathbf{n}\|_1 \).

Using the Proposition 2 it follows that, if we find a feasible rates vector \( \mathbf{n} \) the research of optimal solution can be restricted to the set:

\[
\mathcal{N}_\Sigma(\mathbf{n}) = \mathcal{N} \setminus \{ \mathbf{n} \text{ such that } \|\mathbf{n}\|_1 \leq \|\mathbf{n}\|_1 \} . \quad (17)
\]

Proposition 3: If \( \mathbf{n} \) is infeasible, then any \( \mathbf{n} \) such that \( \|\mathbf{n}\|_1 \geq \|\mathbf{n}\|_1 \) is infeasible too.

Knowing that \( \mathbf{n} \) is infeasible the set \( \mathcal{N} \) can be reduced coherently with the following rule:

\[
\mathcal{N}_I(\mathbf{n}) = \mathcal{N} \setminus \{ \mathbf{n} \text{ such that } \|\mathbf{n}\|_1 \geq \|\mathbf{n}\|_1 \} . \quad (18)
\]

Note that \( \mathcal{N}_\Sigma \cap \mathcal{N}_I \subseteq \mathcal{N} \).

Starting from Proposition 2 and 3, the search of optimal rates vector is based on a neural network architecture, where computation and information distribution is optimized. The neural network can be modelled using an oriented graph, which allows to reduce the computational cost of the solution. Specifically, to properly define the oriented graph we have to introduce an equivalent representation of the \( \mathbf{n} \) vector. Since the elements of \( \mathbf{n} \) are in a finite and numerable set, there is a biunivocal correspondence between the \( \mathcal{N} \) set and the set, denoted with \( \mathcal{R}_g \), which is composed by vector having dimension equal to \( N \), and elements assuming integer values between 1 the number of possible rates of each user. Namely,

\[
\mathcal{R}_g = \{ \mathbf{r} = [r_1, \ldots, r_N]^T \mid r_i \in \mathbb{N} \text{ and } 1 \leq r_i \leq g \} .
\]

where \( g_{i0} = 2^g \), and in \( g \) we have omitted the dependence on \( i \) for the sake of notational simplicity. The relation between a given rate \( \hat{n}_i \) and its correspondent \( \hat{r}_i \) is given by the following equation:

\[
\hat{n}_i = G_{i0}2^{-\hat{r}_i - 1} . \quad (19)
\]

Note that rewriting \( I(\mathbf{n}, \mathbf{P}) \) as a function of \( \mathbf{r} \), \( I(\mathbf{r}, \mathbf{P}) \), the interference function properties are not modified for any given \( \mathbf{r} \). Indeed, using the relation (19), the validity of the Propositions 2 and 3 can be easily proved also with the \( \mathbf{r} \) vector.

Using the \( \mathbf{r} \) vector, we can define the following oriented graph:

Definition 2 (Rate Graph): Given a set \( \mathcal{R}_g \) the associated Rate Graph is defined as an oriented graph having the number of nodes equal to \( \mathcal{R}_g \)’s cardinality. Each graph’s node has associated a correspondent \( \mathbf{r} \) vector. The graph is then defined by the couple \( (\mathcal{E}, \mathcal{R}_g) \), where \( \mathcal{R}_g \) the set of nodes and \( \mathcal{E} \) is the set of oriented edges, \( \mathcal{E} \subset \mathcal{R}_g \times \mathcal{R}_g \). The couple \((\mathbf{r}, \mathbf{r}')\) is in \( \mathcal{E} \) if and only if \( ||\mathbf{r} - \mathbf{r}'|| = 1 \) and \( ||\mathbf{r}'|| < ||\mathbf{r}|| \).

The Rate Graph is an efficient structure to implement branch and bound distributed techniques. Indeed, an efficient algorithm can be implemented so that each graph’s node is able to evaluate its own feasibility solving the problem (16) for its associated rate vector. The \( \mathbf{r} \) node has to compute its own feasibility function:

\[
f(\mathbf{r}) = \begin{cases} 1 & \text{if } \mathbf{r} \text{ is feasible} \\ 0 & \text{otherwise} \end{cases} .
\]

Each node sends the result of the feasibility function to its adjacent nodes, and, by exploiting the Propositions 2 and 3, the information exchange reduces the candidate rate set \( \mathcal{N} \). Moreover, using Proposition 3, the information exchange can be used to simplify the computation of some nodes.

In order to formalize the algorithm, we introduce the definition of forward and backward nodes.

Definition 3 (Forward and Backward Nodes): Given a Rate Graph and a node \( \mathbf{r} \), the set of the Forward Nodes is defined as:

\[
\mathcal{F}(\mathbf{r}) = \{ \mathbf{r} \in \mathcal{R}_g | (\mathbf{r}, \mathbf{r}') \in \mathcal{E} \} ,
\]
while the set of the Backward Nodes is defined as:
\[ B(r) = \{ r \in R_g | (r, r) \in \mathcal{E} \}. \]

Adopting previous definition, the algorithm based on branch and bound criteria can be summarized as follows:
1) \( k = 0 \); any \( A(k) \subset R_g \); \( S(k) = R_g \setminus A(k) \); \( D = \{\emptyset\} \).
2) Compute the feasibility of any \( r \in A(k) \).
3) For any \( r \in A(k) \) such that \( f(r) = 0 \) then:
   \[ D = D \cup r \cup \mathcal{F}(r) \]
4) For any \( r \in A(k) \) such that \( f(r) = 1 \) then:
   \[ D = D \cup r \cup B(r) \]
5) \( S = R_g \setminus D \); \( A(k+1) = \{\emptyset\} \)
6) For any \( r \in A(k) \) such that \( f(r) = 0 \) then:
   \[ A(k+1) = A(k+1) \cup [S \cap B(r)] \]
7) For any \( r \in A(k) \) such that \( f(r) = 1 \) then:
   \[ A(k+1) = A(k+1) \cup [S \cap \mathcal{F}(r)] \]
8) if \( A(k+1) \neq \{\emptyset\} \) then \( k = k + 1 \) and go to step (2) otherwise the search of the optimum rate vector is finished.

The algorithm can interpreted in the following way. The graph nodes are split in three sets:
- **A**: Active Node Set. It is the set of the nodes that are computing their own feasibility.
- **S**: Stand By Set. It is the set of the nodes that are waiting to compute their own feasibility.
- **D**: Deleted Node Set. It is the set of the nodes that are deleted from the graph. A node can be deleted from the graph by itself, if it has already computed its feasibility, or by the other nodes coherently to proposition (2) and (3).

Assume that an active node \( r \) computes its \( f(r) \) function, and that it is feasible. That means that we have found a feasible solution, and that any node in \( B(r) \) has an objective function smaller than \( r \). Since such nodes cannot improve the solution, they can be deleted by the graph. To improve the solution it is necessary to activate the set \( \mathcal{F}(r) \).

Now, let us assume that an active node \( r \) computes its \( f(r) \) function, and that it is infeasible. That means that we have to find a feasible solution, such solution can be found only in the set \( B(r) \), and then any node in \( B(r) \) is activated. Using proposition (3) we are sure that any node in \( \mathcal{F}(r) \) is infeasible, and therefore it can be deleted from the graph.

Note that a node can be activated when it is in the set \( S \), or, equivalently, if it has not been delete yet.

In the next section, we will apply the branch and bound algorithm to find the optimal solution of the problem (4).

VI. NUMERICAL RESULTS

In this section we present numerical solutions of the problem (4) for six scenarios for a system with \( K = 4 \) users. The system parameters are assumed to be compatible with the 3GPP specifications [17]. We have used a chip time \( T_c = 2.610^{-7} \) s, and a bandwidth of \( W = 5 \) MHz. The power spectral density of the thermal noise has been set to \( P_{\text{th}} = T_c/10 \). The maximum spreading factors are assumed to be \( G_{\text{max}} = 256 \). The value of the SINR threshold has been set to \( \gamma_t = 3.1 \). Furthermore we assumed that \( P_T G_{\text{max}} T_c / N_0 = 22 \) dB, thus getting \( P_T = 0.0391 \). We have considered five scenarios, denoted A, B, C, D, and E, with uniform standard deviation of the power fluctuation \( \xi_i \) and the activity of the sources \( \alpha_i \), and a fifth scenario, denoted M, with mixed values. In Tab. 1, the values adopted for \( \sigma_{\xi_i} \) and \( \alpha_i \) are reported.

In Fig. 1 the convergence of the powers for the modified program (16) for the case M is reported as obtained with the iterative algorithm. The outage probability \( P_{\text{out}} \) has been set to 0.01. As it can be observed, the convergence is very fast, and employs less than 5 iterations. For different settings of the parameters, the behavior of the convergence remains the basically the same. Hence, these results confirm the validity of Remark 1 and Theorem 1.

![Fig. 1. Convergence of the power for the dual program (16) for the case M. The powers are reported in log units.](image-url)

In Figs. 2 and 3, the optimal solution of the program (15) obtained with our optimal algorithm are reported for each scenario. Each dot of the curves is referred to a value of the outage probability requirement (which is reported on the x axis, and it is assumed to be the same for each user). In the y axis, the optimal objective function of (15) is reported. The algorithm has been initialized in the worst case possible, with the rates of the users set to 1. We observed that, in the range of parameters adopted in the scenarios, the maximum achievable rate per user is 22. Furthermore, as the outage requirements
becomes less stringent (i.e. higher values of the outages are allowed) the objective function obviously increases. It is also interesting to observe that better performance is obtained when the users have low activity factors (\(\alpha_i = 0.2\)). In fact, in such a case, the MAI seen from the users is reduced, thus enabling higher transmit rates. Also, observe that as the standard deviations of the power control errors increase, the total rate decreases. Once again, the reason can be found in the fact that larger fluctuations of the power control error increases the MAI.

probability constraints can be tracked down to the standard interference function theory. This result enabled to derive the optimal solution of the problem in two steps: first a modified program has been investigated to provide feasible solutions with a simple but efficient iterative procedure, and then an algorithm, based on branch and bound criteria, has been described. Numerical results in several scenarios of practical interest have been derived. The fast solution of the modified problem confirms the validity of our approach based on the interference function.

Ongoing work is focused on the extension of our approach to a multi-cell scenario, and with mixed Rayleigh/Nakagami Log normal fading. For system model with limited computational resources (as wireless sensor or ad hoc networks), the investigation of simpler solution is a challenging task. Initial results are being obtained employing a Geometric Programming approach to a relaxed version of the throughput maximization problem.

**REFERENCES**


[17] 3GPP TS 25.214 V6.1.0. 3rd generation partnership project; technical specification group radio access network; physical layer procedures (fd).