FORMELBLAD 5B1928 LOGIK, VT 2004

INFERENCE RULES FOR SENTENTIAL LOGIC

Rule of Assumptions:

j (j) p Assumption

where p may be any formula.

Rule of &E:

$a_1,\ldots,a_n$ (j) $p \,\&\, q$
⋮
$a_1,\ldots,a_n$ (k) p (or q) j &E

Rule of &I:

$a_1,\ldots,a_n$ (j) p
⋮
$b_1,\ldots,b_u$ (k) q
⋮
$a_1,\ldots,a_n,b_1,\ldots,b_u$ (m) $p \,\&\, q$ j,k &I

Rule of →E:

$a_1,\ldots,a_n$ (j) $p \to q$
⋮
$b_1,\ldots,b_u$ (k) p
⋮
$a_1,\ldots,a_n,b_1,\ldots,b_u$ (m) q j,k →E

Rule of →I:

j (j) p Assumption
⋮
$a_1,\ldots,a_n$ (k) q
⋮
$\{a_1,\ldots,a_n\}$/j (m) $p \to q$ j,k →I

Rule of ~E:

$a_1,\ldots,a_n$ (j) $\sim q$
⋮
$b_1,\ldots,b_u$ (k) q
⋮
$a_1,\ldots,a_n,b_1,\ldots,b_u$ (m) $\curlywedge$ j,k ~E

Rule of ~I:

j (j) p Assumption
⋮
$a_1,\ldots,a_n$ (k) $\curlywedge$
⋮
$\{a_1,\ldots,a_n\}$/j (m) $\sim p$ j,k ~I

Rule of DN:

$a_1,\ldots,a_n$ (j) $\sim\sim p$
⋮
$a_1,\ldots,a_n$ (k) p j DN

Rule of ∨E:

$a_1,\ldots,a_n$ (g) $p \vee q$
⋮
h (h) p Assumption
⋮
$b_1,\ldots,b_u$ (i) r
⋮
j (j) q Assumption
⋮
$c_1,\ldots,c_w$ (k) r
⋮
X (m) r g,h,i,j,k ∨E

where $X = \{a_1,\ldots,a_n\} \cup \{b_1,\ldots,b_u\}/h \cup \{c_1,\ldots,c_w\}/j$.

Rule of ∨I:

$a_1,\ldots,a_n$ (j) p
⋮
$a_1,\ldots,a_n$ (k) $p \vee q$ j ∨I
or
$a_1,\ldots,a_n$ (k) $q \vee p$ j ∨I

Rule of Df:

If ⌜$(p \to q) \,\&\, (q \to p)$⌝ occurs as the entire formula at a line j, then at line k we may write ⌜$p \leftrightarrow q$⌝, labeling the line 'j, Df' and writing on its left the numbers from the left of j. Conversely, if ⌜$p \leftrightarrow q$⌝ occurs as the entire formula at a line j, then at line k we may write ⌜$(p \to q) \,\&\, (q \to p)$⌝, labeling the line 'j, Df' and writing on its left the numbers from the left of j.

Rule of EFQ:

$a_1,\ldots,a_n$ (j) $\curlywedge$
⋮
$a_1,\ldots,a_n$ (k) p j EFQ

Som alternativ till *Df* är det tillåtet att använda

Rule of ↔E:

$a_1,\ldots,a_m$ (j) $p \leftrightarrow q$
(or $a_1,\ldots,a_m$ (j) $q \leftrightarrow p$)
⋮
$b_1,\ldots,b_n$ (k) p
⋮
$a_1,\ldots,a_m,b_1,\ldots,b_n$ (l) q j,k ↔E

Rule of ↔I:

g (g) p Assumption
⋮
$a_1,\ldots,a_m$ (h) q
⋮
j (j) q Assumption
⋮
$b_1,\ldots,b_n$ (k) p
⋮
X (l) $p \leftrightarrow q$ g,h,j,k ↔I

where $X = \{a_1,\ldots,a_m\}/g \cup \{b_1,\ldots,b_n\}/j$

INFERENCE RULES FOR FIRST-ORDER LOGIC

Rule of ∀E:

$a_1,\ldots,a_n$ (j) $(\forall v)\phi v$
⋮
$a_1,\ldots,a_n$ (k) ϕt j ∀E

where ϕt is obtained from ϕv by replacing *every* occurrence of v in ϕv with t.

Rule of ∀I:

$a_1,\ldots,a_n$ (j) ϕt
⋮
$a_1,\ldots,a_n$ (k) $(\forall v)\phi v$ j ∀I

where t is not in any of the formulae on lines $a_1,\ldots,a_n$ and ϕv is obtained from ϕt by replacing *every* occurrence of t in ϕt with v, v a variable not already in ϕt.

Rule of ∃E:

$a_1,\ldots,a_n$ (i) $(\exists v)\phi v$
⋮
j (j) ϕt Assumption
⋮
$b_1,\ldots,b_u$ (k) ψ
⋮
X (m) ψ i,j,k ∃E

where $X = \{a_1,\ldots,a_n\} \cup \{b_1,\ldots,b_u\}/j$ and t is not in (i) ⌜$(\exists v)\phi v$⌝, (ii) ψ, or (iii) any of the formulae at lines $b_1,\ldots,b_u$ other than j.

Rule of ∃I:

$a_1,\ldots,a_n$ (j) ϕt
⋮
$a_1,\ldots,a_n$ (k) $(\exists v)\phi v$ j ∃I

where ϕv is obtained from ϕt by replacing *at least one* occurrence of t in ϕt with v, v a variable not already in ϕt.

Rule of =E:

$a_1,\ldots,a_n$ (j) $t_1 = t_2$
⋮
$b_1,\ldots,b_u$ (k) ϕt_1
⋮
$a_1,\ldots,a_n,b_1,\ldots,b_u$ (m) ϕt_2 j,k =E

where ϕt_2 is obtained from ϕt_1 by replacing *at least one* occurrence of t_1 in ϕt_1 with t_2.

Rule of =I:

(j) $t = t$ =I

where t is any individual constant.

Hela denna sida är SI-regler

Rule of Sequent Introduction: Suppose the sequent $r_1,...,r_n \vdash_{NK} s$ is a substitution-instance of the sequent $p_1,...,p_n \vdash_{NK} q$, that we have already proved the sequent $p_1,...,p_n \vdash_{NK} q$, and that the formulae $r_1,...,r_n$ occur at lines $j_1,...,j_n$ in a proof. Then we may infer s at line k, labeling the line '$j_1,...j_n$ SI (Identifier)' and writing on the left all the numbers which occur on the left of lines $j_1,...,j_n$. As a special case, when $n = 0$ and $\vdash_{NK} s$ is a substitution-instance of some theorem $\vdash_{NK} q$ which we have already proved, we may introduce a new line k into a proof with the formula s at it and no numbers on the left, labeling the line 'TI (Identifier)'.

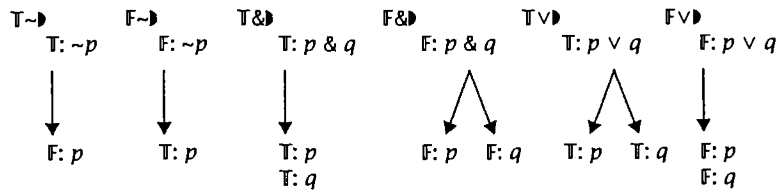
(a)	$A \vee B, \sim A \vdash_{NK} B$; or: $A \vee B, \sim B \vdash_{NK} A$	(DS)
(b)	$A \rightarrow B, \sim B \vdash_{NK} \sim A$	(MT)
(c)	$A \vdash_{NK} B \rightarrow A$	(PMI)
(d)	$\sim A \vdash_{NK} A \rightarrow B$	(PMI)
(e)	$A \vdash_{NK} \sim\sim A$	(DN+)
(f)	$\sim(A \,\&\, B) \dashv\vdash_{NK} \sim A \vee \sim B$	(DeM)
(g)	$\sim(A \vee B) \dashv\vdash_{NK} \sim A \,\&\, \sim B$	(DeM)
(h)	$\sim(\sim A \vee \sim B) \dashv\vdash_{NK} A \,\&\, B$	(DeM)
(i)	$\sim(\sim A \,\&\, \sim B) \dashv\vdash_{NK} A \vee B$	(DeM)
(j)	$A \rightarrow B \dashv\vdash_{NK} \sim A \vee B$	(Imp)
(k)	$\sim(A \rightarrow B) \dashv\vdash_{NK} A \,\&\, \sim B$	(Neg-Imp)
(l)	$A * B \vdash_{NK} B * A$	(Com)
(m)	$A \,\&\, (B \vee C) \dashv\vdash_{NK} (A \,\&\, B) \vee (A \,\&\, C)$	(Dist)
(n)	$A \vee (B \,\&\, C) \dashv\vdash_{NK} (A \vee B) \,\&\, (A \vee C)$	(Dist)
(p)	$\vdash_{NK} A \vee \sim A$	(LEM)
(q)	$A * B \dashv\vdash_{NK} \sim\sim A * \sim\sim B$; or: $\sim\sim A * B$; or: $A * \sim\sim B$	(SDN)
(r)	$\sim(A * B) \dashv\vdash_{NK} \sim(\sim\sim A * \sim\sim B)$; or: $\sim(\sim\sim A * B)$; or: $\sim(A * \sim\sim B)$	(SDN)

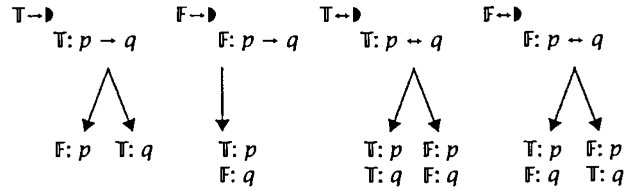
'$p \dashv\vdash_{NK} q$' står här för '$p \vdash_{NK} q$ och $q \vdash_{NK} p$'.
I (l) står '$*$' för '&', '$\vee$' eller '$\leftrightarrow$'.
I (q) och (r) står '$*$' för '&', '$\vee$', '$\rightarrow$' eller '$\leftrightarrow$'.

Extension (1) to the Rule of Sequent Introduction: If the formula at a line j in a proof has any of the forms ⌜$\sim(\forall v)\phi v$⌝, ⌜$(\exists v)\sim\phi v$⌝, ⌜$\sim(\exists v)\phi v$⌝ or ⌜$(\forall v)\sim\phi v$⌝, then at line k we may infer the provably equivalent formula of the form ⌜$(\exists v)\sim\phi v$⌝, ⌜$\sim(\forall v)\phi v$⌝, ⌜$(\forall v)\sim\phi v$⌝ or ⌜$\sim(\exists v)\phi v$⌝ respectively, labeling the line 'j SI (QS)' and writing on the left the same numbers as occur on the left of line j.

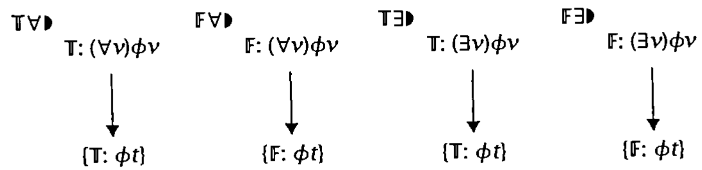
Extension (2) to the Rule of Sequent Introduction: For any closed sentence ϕv, if ϕv has been inferred at a line j in a proof and $\phi v'$ is a single-variable alphabetic variant of ϕv, then at line k we may write $\phi v'$, labeling the line 'j SI (AV)' and carrying down on its left the same numbers as are on the left of line j.

Tablåregler i satslogik





Tablåregler i predikatlogik



Peanos axiom

Språk (tolkningen i standardmodellen inom [ ]):
en 0-ställig funktionssymbol (dvs en individkonstant) 0 [talet 0]
en 1-ställig funktionssymbol S [nästa tal]
två 2-ställiga funktionssymboler $+$ och $*$ [addition och multiplikation]
(vi skriver t.ex. $x + y$ och $x * y$ i stället för $+(x, y)$ och $*(x, y)$.)

Axiom:

P1	$\forall x \forall y\, (S(x) = S(y) \rightarrow x = y)$	olika tal har olika efterföljare
P2	$\forall x\, S(x) \neq 0$	0 är inte efterföljare
P3	$\forall x\, x + 0 = x$	rekursiv definition av +, bas
P4	$\forall x \forall y\, x + S(y) = S(x + y)$	rekursiv definition av +, steg
P5	$\forall x\, x * 0 = 0$	rekursiv definition av $*$, bas
P6	$\forall x \forall y\, x * S(y) = (x * y) + x$	rekursiv definition av $*$, steg
P7	$\forall z_1 ... \forall z_n ((\phi 0 \,\&\, \forall x\, (\phi x \rightarrow \phi S(x))) \rightarrow \forall x\, \phi x)$	induktionsaxiom

I P7 är ϕx en godtycklig formel med alla fria variabler bland $x, z_1, ..., z_n$. P7 är alltså egentligen ett oändligt antal axiom, ett s.k. **axiomschema**.