A little about sets (Sw. mängder)

We can think of **sets** as ”bags” with ”objects” (or pointers to objects) in them. The ”objects” (which can also be sets) are called the **elements** of the set. Two sets are the same iff they contain the same elements.
$\{\pi, \sqrt{2}\}$ is the set with elements π and $\sqrt{2}$.
$\boldsymbol{\{x \mid Px\}}$ is the set of x with the property P, like $\{n \mid n \text{ heltal}, n^2 \equiv_5 2\}$.
The empty set $\varnothing = \{x \mid x \neq x\} = \{\}$, ”an empty bag”.
Note: $\{\varnothing\}$, with one element ($\varnothing$), is not the same set as $\varnothing$, with no elements.
The **universe** $\boldsymbol{\mathcal{U}}$, the set of all the ”objects” under consideration.

Standard notation for some sets of numbers: $\mathbb{Z}_+, \mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}, \mathbb{Z}_m$.

Important relations and operations on sets:

$a \in A$	a is an **element** of A ($a \notin A$: a is **not** an element of A)
$B \subseteq A$	B is a **subset** (Sw. delmängd) of A, i.e., $x \in B \Rightarrow x \in A$, for all x
$B \subset A$	B is a **proper** subset of A, i.e., $B \subseteq A$ and $B \neq A$
$\lvert A \rvert$	the number of elements in A, A:s **cardinality**
$A \cup B$	the **union** of A and B, $\{x \mid \text{at least one of } x \in A, x \in B\}$
$A \cap B$	the **intersection** (Sw. snittet) of A and B, $\{x \mid x \in A \text{ and } x \in B\}$
$A \setminus B$	the **difference** between A and B, $\{x \mid x \in A \text{ and } x \notin B\}$
$A \times B$	the **product set** (or Cartesian product) of A and B, $\{(a, b) \mid a \in A, b \in B\}$, where (a, b) is the **ordered pair** of a and b.
A^c	the **complement** of A, $\mathcal{U} \setminus A$
$\mathcal{P}(A)$	A:s **power set** (Sw. potensmängd), the set of subsets of A, $\{B \mid B \subseteq A\}$

Basic rules for $\cup, \cap, {}^c, \varnothing, \mathcal{U}$ (Boolean algebra) can be found below. They can be proved by studying **Venn diagrams** (useful, but hard to draw here) or reasoning from the definitions of the concepts.

Basic rules for operations on sets

$A \cap B = B \cap A$	$A \cup B = B \cup A$	commutativity
$(A \cap B) \cap C = A \cap (B \cap C)$	$(A \cup B) \cup C = A \cup (B \cup C)$	associativity
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	distributivity
$(A \cap B)^c = A^c \cup B^c$	$(A \cup B)^c = A^c \cap B^c$	De Morgan
$A \cap A = A$	$A \cup A = A$	idempotence
$A \cap (A \cup B) = A$	$A \cup (A \cap B) = A$	absorption
$(A^c)^c = A$		involution
$A \setminus B = A \cap B^c$		$\setminus$ expressed
$A \cap A^c = \varnothing$	$A \cup A^c = \mathcal{U}$	complementarity
$A \cap \varnothing = \varnothing$	$A \cup \mathcal{U} = \mathcal{U}$	
$A \cap \mathcal{U} = A$	$A \cup \varnothing = A$	

Relations

That $\mathcal{R}$ is a **binary relation** on the set X means that (for all $a, b \in X$) the statement $a\mathcal{R}b$ is either **true or false**.

The relation $\mathcal{R}$ can be given as

- A **subset** of $X^2 = X \times X$, $\{(a,b) \in X^2 \mid a\mathcal{R}b\}$,
  formally: $\mathcal{R} \subseteq X \times X$ or $\mathcal{R} \in \mathcal{P}(X \times X)$
- A **graph** with dots corresponding to the elements of X and
  an arrow from a to b iff $a\mathcal{R}b$
- A **matrix** with rows and columns corresponding to the elements of X,
  1 in position ab if $a\mathcal{R}b$, 0 if $a\not\mathcal{R}b$ (not $a\mathcal{R}b$)

Important **properties** for binary relations:

$\mathcal{R}$ is	**reflexive** iff	$x\mathcal{R}x$ for all $x \in X$
$\mathcal{R}$ is	irreflexive iff	$x\not\mathcal{R}x$ for all $x \in X$
$\mathcal{R}$ is	**symmetric** iff	$x\mathcal{R}y \;\Rightarrow\; y\mathcal{R}x$ for all $x, y \in X$
$\mathcal{R}$ is	**antisymmetric** iff	$x\mathcal{R}y$ and $y\mathcal{R}x \;\Rightarrow\; x = y$ for all $x, y \in X$
$\mathcal{R}$ is	**transitive** iff	$x\mathcal{R}y$ and $y\mathcal{R}z \;\Rightarrow\; x\mathcal{R}z$ for all $x, y, z \in X$

An **equivalence relation** on a set X is a binary relation $\mathcal{R}$ which is
reflexive, symmetric and transitive.

An equivalence relation partitions X into **equivalence classes** of elements standing in the relation to each other: $\mathcal{C}_x = [x] = \{y \in X \mid y\mathcal{R}x\}$

Conversely, for each partition of X, there is an equivalence relation with its parts as equivalence classes.

A non-strict **partial order** is a binary relation $\preceq$ on a set X which is
reflexive, antisymmetric and transitive.

The corresponding strict partial order $\prec$ is given by $x \prec y \Leftrightarrow x \preceq y$ and $x \neq y$. It is
irreflexive and transitive.

If $\prec$ is a partial order on the set X and $a \in A \subseteq X$, a is
a **minimal** element of A iff there is no $x \in A$ with $x \prec a$
a **least** element of A iff $a \preceq x$ for all $x \in A$,
correspondingly for **maximal** and **greatest** element.

If $x, y, z \in X$ and $z \preceq x$, $z \preceq y$, z is called a **lower bound** of x, y,
correspondingly for **upper bound**.

So, if $\preceq$ stands for $|$, divisibility, lower bound means common divisor. For arbitrary $\preceq$ there isn't necessarily for all $x, y \in X$ a **greatest** lower bound (like the gcd).

A partial order $\preceq$ is a **total order** (a linear order) iff for all $x, y \in X$:

$$x \preceq y \text{ or } y \preceq x.$$

(or, equivalently, exactly one of: $x \prec y$, $x = y$, $y \prec x$.)
For total orders, a minimal element is the same as a least element.

A **well-ordering** of a set X is a total order $\prec$, such that every $Y \subseteq X$, $Y \neq \varnothing$ has a **least** element (but not necessarily a greatest element).