$$\mathbb{Z}_{mn} \cong \mathbb{Z}_m \times \mathbb{Z}_n \textbf{ om } \mathbf{sgd}(\boldsymbol{m},\boldsymbol{n}) = \mathbf{1}, \text{ ty}$$

$$\boldsymbol{x} \equiv_{mn} \boldsymbol{y} \quad \Leftrightarrow \quad \begin{cases} \boldsymbol{x} \equiv_m \boldsymbol{y} \\ \boldsymbol{x} \equiv_n \boldsymbol{y} \end{cases}$$

ger

$$\boldsymbol{f} : \mathbb{Z}_{mn} \to \mathbb{Z}_m \times \mathbb{Z}_n,$$

$$\boldsymbol{f}([\boldsymbol{x}]_{mn}) = ([\boldsymbol{x}]_m, [\boldsymbol{x}]_n).$$

$\boldsymbol{f}$ är **1-1** och (således) **på** (dvs en bijektion).

Dessutom gäller

$$\boldsymbol{f}(\boldsymbol{a}+\boldsymbol{b}) = \boldsymbol{f}(\boldsymbol{a}) + \boldsymbol{f}(\boldsymbol{b}),\ \boldsymbol{f}(\boldsymbol{a}\cdot\boldsymbol{b}) = \boldsymbol{f}(\boldsymbol{a})\cdot\boldsymbol{f}(\boldsymbol{b}),$$

(komponentvisa operationer i HL) så $\boldsymbol{f}$ **är en isomorfi.**

Exempel ($m = 3,\ n = 5$):

$$\begin{array}{lll}
\mathbf{0 \mapsto (0,0)} & \mathbf{5 \mapsto (2,0)} & \mathbf{10 \mapsto (1,0)} \\
\mathbf{1 \mapsto (1,1)} & \mathbf{6 \mapsto (0,1)} & \mathbf{11 \mapsto (2,1)} \\
\mathbf{2 \mapsto (2,2)} & \mathbf{7 \mapsto (1,2)} & \mathbf{12 \mapsto (0,2)} \\
\mathbf{3 \mapsto (0,3)} & \mathbf{8 \mapsto (2,3)} & \mathbf{13 \mapsto (1,3)} \\
\mathbf{4 \mapsto (1,4)} & \mathbf{9 \mapsto (0,4)} & \mathbf{14 \mapsto (2,4)}
\end{array}$$

$f(8) = (2,3),\ f(11) = (2,1)$ och

$(2,3) + (2,1) = (1,4) = f(4)$ i $\mathbb{Z}_3 \times \mathbb{Z}_5$, **så** $8 + 11 = 4$ i $\mathbb{Z}_{15}$

$(2,3) \cdot (2,1) = (1,3) = f(13)$ i $\mathbb{Z}_3 \times \mathbb{Z}_5$, **så** $8 \cdot 11 = 13$ i $\mathbb{Z}_{15}$

$(2,3)^{-1} = (2^{-1}, 3^{-1}) = (2,2) = f(2)$ i $\mathbb{Z}_3 \times \mathbb{Z}_5$, **så** $8^{-1} = 2$ i $\mathbb{Z}_{15}$

$$\begin{array}{ccc}
\mathbb{Z}_{\mathbf{15}} & \xrightarrow{\ \boldsymbol{f}\ } & \mathbb{Z}_{\mathbf{3}} \times \mathbb{Z}_{\mathbf{5}} \\
\circ \downarrow & & \downarrow \quad \downarrow \circ \\
\mathbb{Z}_{\mathbf{15}} & \xleftarrow[\ \boldsymbol{f}^{-1}\ ]{} & \mathbb{Z}_{\mathbf{3}} \times \mathbb{Z}_{\mathbf{5}}
\end{array}$$

$\circ = +\,,\ \cdot$ eller $^{-1}$

$f(10) = (1,0),\ f(6) = (0,1)$ så $f(a \cdot 10 + b \cdot 6) = ([a]_3, [b]_5)$ och

$$\boldsymbol{f}^{-1}((\boldsymbol{a},\boldsymbol{b})) = [\boldsymbol{a}\boldsymbol{y}_1 + \boldsymbol{b}\boldsymbol{y}_2]_{\mathbf{15}}\,, \qquad y_1 = 10,\ y_2 = 6.$$